Bayesian Analysis of Simple Step-stress Model under Weibull Lifetimes

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Censoring
Censoring

- Quite useful technique in reliability life testing.
- Possible termination of experiment before failing all the experimental units.
- Lower cost in terms of money and time than full experiment.
- Survival experimental units can be used for further experiments.
1. Censoring

Type-I Censoring

- $n$: Number of items put on the test.
- $\tau$: Pre-fixed time.
- $\tau^* = \tau$: Experiment termination time.
## Type-I Censoring

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1. Censoring

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Diagram: 

- $n$ is the number of items tested.
- $\tau^*$ indicates the experiment termination time.
- $0$ to $t_{1:n}$ represents the duration of the test.
1. Censoring

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Diagram:

- $n$:
- $t_1:n$
- $t_2:n$
- $\tau^*$
- $\tau$
1. Censoring

Type-I Censoring

- \( n \): Number of items put on the test.
- \( \tau \): Pre-fixed time.
- \( \tau^* = \tau \): Experiment termination time.

![Diagram showing Type-I Censoring with \( n \) items tested up to time \( \tau^* \).]
1. Censoring

Type-I Censoring

- $n$: Number of items put on the test.
- $\tau$: Pre-fixed time.
- $\tau^* = \tau$: Experiment termination time.

The diagram illustrates the time axis with $n$ items tested, with $t_1:n$, $t_2:n$, ..., $t_N:n$ indicating the times of failure or censoring. The experiment terminates at $\tau^* = \tau$.

- Number of failures is a random variable.
Type-I Censoring

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Number of failures is a random variable.

- **Advantage**: Pre-fixed experiment termination time.
- **Disadvantage**: Very few failures, even no failure, before time \( \tau \).
1. Censoring

**Type-II Censoring**

- $n$: Number of items put on the test.
- $r \leq n$: Pre-fixed integer.
- $\tau^* = t_{r:n}$: Experiment termination time.
1. Censoring

Type-II Censoring

- $n$: Number of items put on the test.
- $r (\leq n)$: Pre-fixed integer.
- $\tau^* = t_{r:n}$: Experiment termination time.

Diagram:

```
 n
  |
  v
 0 | t_{1:n}
```
Type-II Censoring

- $n$: Number of items put on the test.
- $r (\leq n)$: Pre-fixed integer.
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Diagram:

- $n$:
- $0$ to $t_{1:n}$ to $t_{2:n}$
1. Censoring

Type-II Censoring

- $n$: Number of items put on the test.
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![Diagram of censored data points over time]

0 $\rightarrow$ $t_{1:n}$ $\rightarrow$ $t_{2:n}$ $\rightarrow$ $\ldots$ $\rightarrow$ $t_{r:n}$
1. Censoring

Type-II Censoring

- \( n \): Number of items put on the test.
- \( r (\leq n) \): Pre-fixed integer.
- \( \tau^* = t_{r:n} \): Experiment termination time.

```
0  t_{1:n}  t_{2:n}  \cdots  t_{r:n}
```

[Diagram of Type-II Censoring]
1. Censoring

Type-II Censoring

- $n$: Number of items put on the test.
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**Type-II Censoring**

- $n$: Number of items put on the test.
- $r (\leq n)$: Pre-fixed integer.
- $\tau^* = t_{r:n}$: Experiment termination time.

Duration of experiment is a random variable.

Advantage: Pre-fixed number of failures.

Disadvantage: Long experimental duration.
Other Censoring Schemes

- Hybrid Censoring Schemes: Hybridization of Type-I and Type-II censoring.
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- Progressive Censoring Schemes: Allow to remove items from the test before completion of the experiment.
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Other Censoring Schemes

- Hybrid Censoring Schemes: Hybridization of Type-I and Type-II censoring.
- Progressive Censoring Schemes: Allow to remove items from the test before completion of the experiment.
- Progressive Hybrid Censoring Schemes: Mixture of hybrid and progressive censoring schemes.
- All the censoring schemes suffer from the disadvantage of either Type-I or Type-II censoring scheme.
Step-stress Life Tests
Accelerated Life Tests

- Useful experimental technique to obtain data on the lifetime distribution of highly reliable products.
- Put a sample of products on the test in some extreme environmental conditions to get early failures.
- Need to extrapolate to estimate the lifetime distribution under the normal condition.
Step-stress Life Tests

- A particular type of accelerated life test.
- Allows the experimenter to change the stress levels during the life-testing experiments.
- $n$: Number of items put on the test.
- $s_1, s_2$: Stress levels (Simple SSLT).
- $\tau$: Stress changing time (Pre-fixed).
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- \( \tau \): Stress changing time (Pre-fixed).

\[ n \:
\begin{align*}
\tau & \quad \tau \\
&& \tau \\
st_1 & \quad \tau \\
&t_1:n \quad t_2:n \quad \cdots \quad t_N:n \quad t_{N+1:n} \quad \cdots \quad t_{n:n}
\end{align*}
\]
Step-stress Life Tests

- Generalization
  - \( n \): No of items placed on the test.
  - \( s_1, s_2, s_3, \ldots, s_{m+1} \): Stress levels.
  - \( \tau_1 < \tau_2 < \ldots < \tau_m \): Stress changing times (Pre-fixed).
Consider a simple SSLT, i.e., only two stress levels, $s_1$ and $s_2$, present.

$F_i(\cdot)$ : CDF of lifetime of an item under the stress level $s_i$, $i = 1, 2, \ldots, m + 1$.

$F(\cdot)$ : CDF of lifetime of an item under the step-stress pattern.

Model needed to relate $F(\cdot)$ to $F_i(\cdot)$, $i = 1, 2, \ldots, m + 1$.

Popular models
  - Cumulative exposure model.
  - Tampered failure rate model.
  - Khamis-Higgins model.
Cumulative Exposure Model

- First proposed by Seydyakin (1966)\(^4\) and later studied by Nelson (1980)\(^5\).
- \(F_i(\cdot)\) is the CDF of lifetime of an item under the stress level \(s_i, i = 1, 2, \ldots, m + 1\).
- \(F(\cdot)\) is the CDF of lifetime of an item under the step-stress pattern.

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The CEM assumptions are:

- The remaining life of an item depends only on the current cumulative fraction accumulated, regardless how the fraction accumulated.
- If the stress level is fixed, the survivors will fail according to the distribution function of that stress level but starting at previous accumulated fraction failed.
2. Step-stress Life Tests

Cumulative Exposure Model

Figure: Example of CEM

Here $F_1(\cdot)$ and $F_2(\cdot)$ are CDF of $Exp(14)$ and $Exp(1)$ respectively.
Cumulative Exposure Model

Under the assumptions of CEM, the CDF of the lifetime is given by

\[ F_{\text{CEM}}(t) = F_i(t - \tau_{i-1} + h_{i-1}) \quad \text{if} \quad \tau_{i-1} \leq t < \tau_i, \; i = 1, 2, \ldots, m + 1, \]

where \( \tau_0 = 0, \) \( \tau_{m+1} = \infty, \) \( h_0 = 0 \) and \( h_i, \; i = 1, 2, \ldots, m, \) is the solution of

\[ F_{i+1}(h_i) = F_i(\tau_i - \tau_{i-1} + h_{i-1}). \]
2. Step-stress Life Tests

Tampered Failure Rate Model

- Proposed by Bhattacharyya and Soejoeti (1989)\(^1\) for simple SSLT.
- Generalized by Madi (1993)\(^2\) for multiple step SSLT.

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Tampered Failure Rate Model

- Proposed by Bhattacharyya and Soejoeti (1989)\(^1\) for simple SSLT.
- Generalized by Madi (1993)\(^2\) for multiple step SSLT.
- Effect of switching the stress level is to multiply the failure rate of the first stress level by a positive constant.

\[
\lambda_{TFRM}(t) = \left( \prod_{j=0}^{i-1} \alpha_j \right) \lambda(t) \text{ if } \tau_{i-1} \leq t < \tau_i, \ i = 1, 2, \ldots, m+1.
\]


Khamis-Higgins Model

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Khamis-Higgins Model

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- Under KHM, the CDF is given by

\[
F_{\text{KHM}}(t) = 1 - e^{-\lambda_i(t^\beta - \tau_i^{\beta})} - \sum_{j=1}^{i-1} \lambda_j(\tau_j^\beta - \tau_{j-1}^\beta) \quad \text{if} \quad \tau_{i-1} \leq t < \tau_i.
\]


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  \]
- Xu and Tang (2003)\(^2\) showed that KHM is a particular case of TFRM.

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Advantages

- By increasing the stress level, reasonable number of failure can be obtained.
- Experimental time is reduced.
2. Step-stress Life Tests

Disadvantages

- Exact relationship between the stress level and lifetime of the product is needed.
- Model must take into account the effect of stress accumulated.
- Model becomes more complicated.
A Brief Literature Review
3. A Brief Literature Review

Literature Review

- Balakrishnan et al. (2007)\(^1\).
  - Type-II censoring.
  - Exponentially distributed failure times.
  - Cumulative exposure model.

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\[
\hat{f}_{\theta_1}(t) = \sum_{j=1}^{r-1} \sum_{k=0}^{j} c_{jk} f_G(t - \tau_{ik}; j, \frac{j}{\theta_1}).
\]

- \(c_{jk}\) involves \((-1)^k, \binom{n}{j}, \binom{j}{k}\), and \(e^{-\frac{\tau}{\theta_1}(n-j+k)}\).
- \(f_G(\cdot)\) is the PDF of Gamma distribution.

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- Point and interval estimation are considered.
- Exact distributions of model parameters are obtained.
- These exact distributions are used to construct confidence intervals.

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Balakrishnan et al. (2009).  
- Type-I and Type-II censored data.  
- Exponentially distributed failure times.  
- Cumulative exposure model.  
- Order restriction among means of lifetimes.
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Literature Review

Balakrishnan et al. (2009)\(^1\).

- Type-I and Type-II censored data.
- Exponentially distributed failure times.
- Cumulative exposure model.
- Order restriction among means of lifetimes.
- MLE does not exist in explicit form.
- Further analysis depends on asymptotic results.

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Kateri and Balakrishnan (2008).\(^1\)

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Model Description and Prior Assumptions
Model Description

- $n$: Number of item put on the test.
- $s_1, s_2$: Stress levels.
- $\tau_1$: Stress changing time (Pre-fixed).
- Type-I censored data.
- $\tau_2 (> \tau_1)$: Censoring time (Pre-fixed).
Model Description

- $n$: Number of item put on the test.
- $s_1, s_2$: Stress levels.
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- $\tau_2 (> \tau_1)$: Censoring time (Pre-fixed).

Life time at stress level $s_i$, $i = 1, 2$, has a Weibull($\beta, \lambda_i$) distribution, i.e., its CDF is given by

$$F_i(t) = \begin{cases} 1 - e^{-\lambda_i t^\beta} & \text{if } t > 0 \\ 0 & \text{otherwise.} \end{cases}$$
Model Description

Under CEM, the CDF is given by

\[ F_{CEM}(t) = \begin{cases} 
0 & \text{if } t < 0 \\
1 - e^{-\lambda_1 t^\beta} & \text{if } 0 \leq t < \tau_1 \\
1 - e^{-\lambda_2 \left( t - \tau_1 + \frac{\lambda_1}{\lambda_2} \tau_1 \right)^\beta} & \text{if } t \geq \tau_1. 
\end{cases} \]
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\]

- Under KHM, the CDF is given by

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F_{\text{KHM}}(t) = \begin{cases} 
0 & \text{if } t < 0 \\
1 - e^{-\lambda_1 t^\beta} & \text{if } 0 \leq t < \tau_1 \\
1 - e^{-\lambda_2 (t^\beta - \tau_1^\beta) - \lambda_1 \tau_1^\beta} & \text{if } \tau_1 < t < \infty.
\end{cases}
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Model Description

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F_{\text{CEM}}(t) = \begin{cases} 
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- Under KHM, the CDF is given by

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1 - e^{-\lambda_2 (t^\beta - \tau_1^\beta) - \lambda_1 \tau_1^\beta} & \text{if } \tau_1 < t < \infty.
\end{cases}
\]

- KHM is mathematically tractable than CEM.
- It is difficult to distinguish between CEM and KHM.
Prior Assumptions I

- $\lambda_1 \sim \text{Gamma}(a_1, b_1)$.
- $\lambda_2 \sim \text{Gamma}(a_2, b_2)$.
- $\beta \sim \text{Gamma}(a_3, b_3)$.
- $\lambda_1, \lambda_2, \text{and } \beta$ are independently distributed.
Main aim of SSLT is to get rapid failure by imposing extreme environmental condition.

Plausible to assume that the mean life time at stress level $s_2$ is smaller than that at stress level $s_1$.

$\lambda_1 < \lambda_2$. 
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Reparameterize $\lambda_1 = \alpha \lambda_2$ with $0 < \alpha < 1$. 
Prior Assumptions II

- Main aim of SSLT is to get rapid failure by imposing extreme environmental condition.
- Plausible to assume that the mean life time at stress level $s_2$ is smaller than that at stress level $s_1$.
- $\lambda_1 < \lambda_2$.
- Reparameterize $\lambda_1 = \alpha \lambda_2$ with $0 < \alpha < 1$.
- $\lambda_2 \sim \text{Gamma}(a_2, b_2)$.
- $\beta \sim \text{Gamma}(a_3, b_3)$.
- $\alpha \sim \text{Beta}(a_4, b_4)$.
- $\alpha$, $\beta$, and $\lambda_2$ are independently distributed.
Motivation

- Weibull distribution is quite flexible and fits a large range of lifetime data.
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- Weibull distribution is quite flexible and fits a large range of lifetime data.
- MLEs of the model parameters do not have explicit form and all inferences rely on asymptotic distributions.
Posterior Analysis
For $\beta > 0$, $\lambda_1 > 0$, and $\lambda_2 > 0$

\[
l_1(\beta, \lambda_1, \lambda_2 \mid \text{Data}) \propto \beta^{n^* + a_3 - 1} \lambda_1^{n_1^* + a_1 - 1} \lambda_2^{n_2^* + a_2 - 1} \times e^{-(b_3 - c_1)\beta - \lambda_1 A_1(\beta) - \lambda_2 A_2(\beta)},
\]

\[
n^* = n_1^* + n_2^*, \quad c_1 = \sum_{j=1}^{n^*} \ln t_{j:n},
\]

\[
A_1(\beta) = b_1 + \sum_{j=1}^{n_1^*} t_{j:n}^\beta + (n - n_1^*)\tau_1^\beta,
\]

\[
A_2(\beta) = b_2 + \sum_{j=n_1^* + 1}^{n^*} (t_{j:n}^\beta - \tau_1^\beta) + (n - n^*)(\tau_2^\beta - \tau_1^\beta).
\]
For $\beta > 0$, $\lambda_1 > 0$, and $\lambda_2 > 0$

\[ l_1(\beta, \lambda_1, \lambda_2 \mid \text{Data}) \propto \beta^{n^* + a_3 - 1} \lambda_1^{n_1^* + a_1 - 1} \lambda_2^{n_2^* + a_2 - 1} \]

\[ \times e^{-(b_3 - c_1)\beta - \lambda_1 A_1(\beta) - \lambda_2 A_2(\beta)}, \]

\[ n^* = n_1^* + n_2^*, \quad c_1 = \sum_{j=1}^{n^*} \ln t_{j:n}, \]

\[ A_1(\beta) = b_1 + \sum_{j=1}^{n_1^*} t_{j:n}^{\beta} + (n - n_1^*)\tau_1^{\beta}, \]

\[ A_2(\beta) = b_2 + \sum_{j=n_1^*+1}^{n^*} (t_{j:n}^{\beta} - \tau_1^{\beta}) + (n - n^*)(\tau_2^{\beta} - \tau_1^{\beta}). \]

$l_1(\beta, \lambda_1, \lambda_2 \mid \text{Data})$ is integrable if proper priors are assumed on all the unknown parameters.
For $0 < \alpha < 1$, $\beta > 0$, and $\lambda_2 > 0$

$$l_2(\alpha, \beta, \lambda_2 \mid \text{Data}) \propto \alpha^{n_1^* + a_4 - 1}(1 - \alpha)^{b_4 - 1}\beta^{n_1^* + a_3 - 1}\lambda_2^{n_2^* + a_2 - 1} \times e^{-(b_3 - c_1)\beta - \lambda_2(\alpha D_1(\beta) + D_2(\beta) + b_2)},$$

$$n^* = n_1^* + n_2^*, \quad c_1 = \sum_{j=1}^{n^*} \ln t_{j:n},$$

$$D_1(\beta) = \sum_{j=1}^{n_1^*} t_j^{\beta:n} + (n - n_1^*)\tau_1^\beta,$$

$$D_2(\beta) = \sum_{j=n_1^*+1}^{n^*} (t_j^{\beta:n} - \tau_1^\beta) + (n - n^*)(\tau_2^\beta - \tau_1^\beta).$$
For $0 < \alpha < 1$, $\beta > 0$, and $\lambda_2 > 0$

$$l_2(\alpha, \beta, \lambda_2 \mid \text{Data}) \propto \alpha^{n^*_1 + a_4 - 1}(1 - \alpha)^{b_4 - 1} \beta^{n^*_2 + a_3 - 1} \lambda_2^{n^* + a_2 - 1}$$

$$\times e^{-(b_3 - c_1)\beta - \lambda_2(\alpha D_1(\beta) + D_2(\beta) + b_2)},$$

$$n^* = n^*_1 + n^*_2, \ c_1 = \sum_{j=1}^{n^*} \ln t_{j:n},$$

$$D_1(\beta) = \sum_{j=1}^{n^*_1} t_{j:n}^\beta + (n - n^*_1) \tau_1^\beta,$$

$$D_2(\beta) = \sum_{j=n^*_1 + 1}^{n^*} (t_{j:n}^\beta - \tau_1^\beta) + (n - n^*)(\tau_2^\beta - \tau_1^\beta).$$

$l_2(\alpha, \beta, \lambda_2 \mid \text{Data})$ is integrable if proper priors are assumed on all the unknown parameters.
Bayes Estimate and Credible Interval

- Squared error loss function.
Bayes Estimate and Credible Interval

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- \[ g_B(\beta, \lambda_1, \lambda_2) = \int \int \int g(\beta, \lambda_1, \lambda_2) h(\beta, \lambda_1, \lambda_2) d\lambda_2 d\lambda_1 d\alpha. \]
Bayes Estimate and Credible Interval

- Squared error loss function.
- \( g_B(\beta, \lambda_1, \lambda_2) = \int \int \int g(\beta, \lambda_1, \lambda_2) h_1(\beta, \lambda_1, \lambda_2) d\lambda_2 d\lambda_1 d\alpha. \)
- Bayes estimate of \( g(\beta, \lambda_1, \lambda_2) \) cannot be obtained explicitly in general.
Bayes Estimate and Credible Interval

- Squared error loss function.
- \( g_B(\beta, \lambda_1, \lambda_2) = \int \int \int g(\beta, \lambda_1, \lambda_2) h_1(\beta, \lambda_1, \lambda_2) d\lambda_2 d\lambda_1 d\alpha \).
- Bayes estimate of \( g(\beta, \lambda_1, \lambda_2) \) cannot be obtained explicitly in general.
- An algorithm based on importance sampling is proposed to compute \( \hat{g}_B(\beta, \lambda_1, \lambda_2) \) and to construct CRI for \( g(\beta, \lambda_1, \lambda_2) \) in both the cases.
Bayes Estimate and Credible Interval

\[
l_1(\beta, \lambda_1, \lambda_2 \mid \text{Data}) = l_3(\lambda_1, \mid \beta, \text{Data}) \times l_4(\lambda_2, \mid \beta, \text{Data}) \times l_5(\beta \mid \text{Data}),
\]

where

\[
l_3(\lambda_1, \mid \beta, \text{Data}) = \frac{\{A_1(\beta)\}^{n_1^*+a_1}}{\Gamma(n_1^* + a_1)} \lambda_1^{n_1^*+a_1-1} e^{-\lambda_1 A_1(\beta)} \text{ if } \lambda_1 > 0,
\]

\[
l_4(\lambda_2, \mid \beta, \text{Data}) = \frac{\{A_2(\beta)\}^{n_2^*+a_2}}{\Gamma(n_2^* + a_2)} \lambda_2^{n_2^*+a_2-1} e^{-\lambda_2 A_2(\beta)} \text{ if } \lambda_2 > 0,
\]

\[
l_5(\beta \mid \text{Data}) = c_2 \frac{\beta^{n_2^*+a_3-1} e^{-(b_3-c_1)\beta}}{\{A_1(\beta)\}^{n_1^*+a_1} \{A_2(\beta)\}^{n_2^*+a_2}} \text{ if } \beta > 0.
\]
Data Analysis
6. Data Analysis

Illustrative Example

- An artificial data is generated from KHM with $n = 40$, $\beta = 2$, $\lambda_1 = 1/1.2 \approx 0.833$, $\lambda_2 = 1/4.5 \approx 2.222$, and $\tau_1 = 0.6$. 
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- Prior I: $\hat{\beta} = 2.35$, $\hat{\lambda}_1 = 0.93$, $\hat{\lambda}_2 = 2.61$.
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- Prior I: 95% symmetric CRI for $\beta$ is (1.12, 4.04).
- Prior II: 95% symmetric CRI for $\beta$ is (0.45, 2.44).
Conclusion
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- Extensive simulation has been done to judge the performance of the proposed procedures.
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- MSEs of BE of all unknown parameters are smaller in case of Prior II than those in case of Prior I.
- Other loss functions and other censoring schemes can be handled in a very similar fashion.
Future Works
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- Optimality of SSLT under Bayesian framework.
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- Optimality of SSLT under Bayesian framework.
- Prior elicitation is becoming a popular topic among Bayesian. It will be a challenging task to find a subjective prior for step-stress life testing models.
Thank You