

# RRNS-Convolutional encoded Concatenated Code for OFDM based Wireless Communication

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**Abstract-** The modern telecommunication industry demands higher capacity networks with high data rate. Orthogonal frequency division multiplexing (OFDM) is a promising technique for high data rate wireless communications at reasonable complexity in wireless channels. OFDM has been adopted for many types of wireless systems like wireless local area networks such as IEEE 802.11a, and digital audio/video broadcasting (DAB/DVB). The proposed research focuses on a concatenated coding scheme that improve the performance of OFDM based wireless communications. It uses a Redundant Residue Number System (RRNS) code as the outer code and a convolutional code as the inner code. The bit error rate (BER) performances of the proposed system under different channel conditions are investigated. These include the effect of additive white Gaussian noise (AWGN), multipath delay spread, peak power clipping and frame start synchronization error. The simulation results show that the proposed RRNS-Convolutional concatenated coding (RCCC) scheme provides significant improvement in the system performance by exploiting the inherent properties of RRNS.

## I. INTRODUCTION

The increasing demand for broadband communication systems with a greater range of services like video conferencing, internet services and digital multimedia applications has promoted the development of orthogonal frequency division multiplexing (OFDM) based systems. The OFDM is a digital multicarrier modulation method which distributes the data over a large number of closely spaced orthogonal carriers. The spectrum of each carrier has null at the centre frequency of each of the other carriers in the system. The available bandwidth is divided among the orthogonal carriers. Each carrier is then modulated by a low data rate stream with a conventional modulation scheme such as quadrature amplitude modulation (QAM) or quadrature phase shift keying (QPSK). OFDM provides high spectral efficiency by spacing the channels close together. This will not result in interference between the carriers, as they are orthogonal to each other. In a coded OFDM (COFDM) system, signals are coded before transmission for forward error correction (FEC). The efficiency in spectrum usage and robustness to multipath fading make COFDM as a popular scheme for wideband digital communication.

Several papers are available in literature that deals with different coding schemes to improve the performance of multicarrier wireless communication systems. A multi-code direct sequence code division multiple access (DS-CDMA) system based on redundant residue number system (RRNS) as

inner code and Reed-Solomon (RS) code as outer code is presented in [1]. The performance of a DS-CDMA system over bursty communication channels and multipath environment is concerned in [2], using a concatenated coding with convolutional code as outer code and RRNS as inner code. The design rules and general analytical upper bounds for parallel concatenated, serial concatenated, hybrid concatenated and self concatenated codes over AWGN and Rayleigh fading channels are presented in [3]. The suitability of OFDM as a modulation technique for wireless communication system is investigated in [4] in which a comparison with CDMA system is provided. A concatenated code for IEEE 802.11a system is proposed in [5], where a block Hamming code joins with a convolutional code, to achieve better system performance under fixed power and bit error rate (BER) requirements.

One of the major drawbacks of OFDM is that it generates signals with large amplitude variations resulting in high peak to average power ratio (PAPR). These large peaks increase the amount of intermodulation distortion resulting in an increase in the error rate. The system performance can be improved by minimizing the PAPR which allows a higher average power to be transmitted for a fixed peak power. A lot of research has been done that reduces the PAPR for OFDM based systems [6-8]. A method to enhance the bandwidth efficiency of a multicarrier CDMA system by using RNS representation for information symbols combined with PSK/QAM modulation and orthogonal spreading is presented in [9]. The performance evaluation of RNS based parallel communication scheme using orthogonal signaling with ratio static test over AWGN channel and multipath fading channel is demonstrated by Yang and Hanzo [10, 11].

This paper proposes a concatenated coding scheme consisting of RRNS as outer code and convolutional code as inner code for OFDM based wireless communication system. The proposed coding scheme combines the error detection and correction properties of RRNS with convolutional codes. The simulation results show that the RRNS-Convolutional concatenated coding (RCCC) scheme offers significant improvement in BER performance under different channel conditions. The performance of the system is analyzed for AWGN channel and multipath fading channel. The effect of frame start synchronization error and peak power clipping for PAPR reduction for the proposed coding scheme are also analyzed. The rest of the paper is organized as follows: Section

II describes the principle of error correction with RRNS coding. Section III deals with the system model and explains the structure of transmitter and receiver. Section IV gives the simulation results, and the BER performance under different operating conditions is analyzed. Finally, Section V concludes the paper.

## II. ERROR CORRECTION WITH RRNS

The residue number system (RNS) is primarily used for high speed digital signal processing due to the modular carry free arithmetic operations. The nonweighted and nonpositional nature of residues offer fault tolerant properties to RNS. The RNS is defined by a set of relatively prime integers ( $m_1, m_2, \dots, m_v$ ) which are called the nonredundant moduli. Error detection and correction properties are introduced by inserting few redundant moduli. Thus redundant residue number system (RRNS) is defined by the moduli set ( $m_1, m_2, \dots, m_v, m_{v+1}, \dots, m_u$ ). The redundant moduli should be relatively prime to the nonredundant moduli and should satisfy the condition ( $m_{v+1}, \dots, m_u$ )  $>$   $\max(m_1, m_2, \dots, m_v)$ . The total dynamic range of RRNS

is given by  $M_T = \prod_{i=1}^u m_i$ . This total range  $[0, M_T]$  is divided

into two adjacent intervals in terms of the ranges defined by the nonredundant and redundant moduli. The interval  $[0, M]$  is

called the *legitimate range*, where  $M = \prod_{i=1}^v m_i$  and the

interval  $[M, M_T]$  is called the *illegitimate range*. In order for RRNS to have self checking, error detection and error correction properties, the information or data has to be constrained within the legitimate range. It has been shown that RRNS with  $(u - v)$  redundant moduli can detect  $(u - v)$  errors and can correct up to  $\lfloor (u - v) / 2 \rfloor$  errors, where  $\lfloor \cdot \rfloor$  denotes the integer part [12].

An error correction scheme with RRNS [13, 14] is given in Fig. 1. Here, the binary number  $Z'$  is generated from the received nonredundant residue digits ( $z_1, z_2, \dots, z_v$ ) using a reverse converter based on Chinese Remainder Theorem (CRT). An auxiliary set of residues  $|Z'|_{m_{v+1}}, |Z'|_{m_{v+2}}, \dots$

$|Z'|_{m_u}$  corresponding to the redundant channels are generated

from the output  $Z'$  by forward conversion. The error syndrome for each redundant channel is calculated as in (1), by comparing the received redundant residue digit with the auxiliary residue generated.

$$S_{m_i} = z_{m_i} - |Z'|_{m_i} \text{ for } i = v+1, \dots, u. \quad (1)$$

If all the error syndromes are zeros, then all the received residue digits are correct and hence there is no error present. If any one of the redundant residue channel is in error, the corresponding error syndrome is nonzero and the other syndromes are zeros. In such cases the output calculated using the nonredundant residues is correct. If there is an error in the nonredundant channel, all the syndromes are nonzeros. In this

case a correction has to be applied to the output  $Z'$ . There is a unique error corresponding to each combination of the syndromes. So error correction can be done with the help of a look up table (LUT). The LUT is addressed by the syndrome values, and the size of the LUT required to store the correction factor is determined using (2). The correction factor (CF) from the LUT is added to the output  $Z'$  from reverse converter to produce the correct output  $Z$ .

$$N = \left( 2 \sum_{i=1}^v m_i - 1 \right) + \left( \sum_{i=v+1}^u m_i \right) - 1 \quad (2)$$

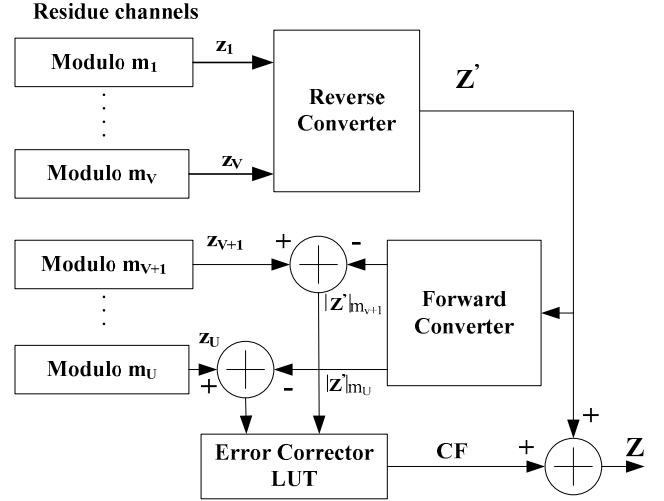


Figure 1. Principle of error correction with RRNS

## III. SYSTEM DESCRIPTION

This section illustrates the signal flow through a typical wireless digital communication system that includes concatenated forward error control coding, interleaving and deinterleaving, orthogonal digital modulation and channel impairments. Concatenated coding is a good way to create long powerful codes with large coding gain and reduced decoding complexity by combining relatively simple channel codes [15]. The proposed RRNS-Convolutional concatenated coding corrects the errors using the outer RRNS decoder that are not corrected by the inner decoder. Thus better BER performance is achieved by exploiting the properties of RRNS.

### A. Transmitter Model

The functional block diagram shown in Fig. 2 illustrates the transmitter section of the proposed communication system. The N-bit information symbols are first converted to RRNS representation by a forward converter. The moduli set of RRNS is selected such a way that it offers redundancy and sufficient dynamic range for unique and unambiguous information representation. An information symbol  $X$ , is represented in RRNS as  $(x_1, x_2, \dots, x_v, x_{v+1}, \dots, x_u)$  with respect to a moduli set  $(m_1, m_2, \dots, m_v, m_{v+1}, \dots, m_u)$ , where  $x_i = X \text{ modulo } m_i$  for  $i = 1$  to  $u$ . This is the outer RRNS coding. In the proposed system, 8-bit information symbols are considered and

RRNS moduli set is selected as (5, 7, 8, 9, 11) where (5, 7, 8) forms the nonredundant moduli and (9, 11) forms the redundant moduli. This allows detection of errors in two residue digits and can correct single residue digit errors.

The residue digits are interleaved and then applied to a convolutional encoder for inner encoding. The proposed system uses industry standard  $\frac{1}{2}$  rate convolutional encoder with constraint length of 7, as shown in Fig. 3. The generator polynomials are:  $g_1 = 133_8$  and  $g_2 = 171_8$ . The interleaved and rearranged data bits are mapped into signal constellation points according to the type of modulation used. Differential QPSK is used as the modulation scheme for this work. Serial-to-parallel conversion is done for the modulated data. An Inverse Fast Fourier Transform (IFFT) is taken for implementing OFDM. The IFFT transforms the subcarriers from the frequency domain into the corresponding time domain. The OFDM signal is represented as in (3).

$$S(n) = \frac{A}{M} \sum_{i=0}^{M-1} x_i(n) \exp(2\pi f_i n) \text{ for } 0 \leq n, i < M \quad (3)$$

where A is the scaling factor, M is the total number of subcarriers,  $x_i(n)$  is the  $n^{\text{th}}$  bit of the  $i^{\text{th}}$  data stream and  $f_i = f_c + \frac{i}{T}$ , for  $i = 0, 1, \dots, M - 1$ . 'T' is the symbol duration for the information sequence, and  $f_c$  is the centre frequency of the subcarriers. A guard band interval is inserted to avoid intersymbol interferences (ISI) and intercarrier interferences (ICI) caused by the multipath fading. Finally, the signal is transmitted after radio frequency up-conversion.

### B. Receiver Model

The channel attenuates the transmitted signals, delays it in time, and corrupts them by addition of Gaussian noise. The effects of multi-path delay spread are accounted by using a lowpass finite impulse response (FIR) filter model. The length of the filter represents the maximum delay spread, and the magnitude of filter coefficients represents the reflected signal amplitudes. The received signal can be represented as in (4), assuming  $P$  resolvable frequency selective paths for the multipath channel,

$$r(t) = \sum_{p=0}^{P-1} \alpha_p(t) s(t - \tau_p) + N(t) \quad (4)$$

Here,  $N(t)$  represents a stationary zero-mean Gaussian random process with single sided power spectral density of  $N_0$ ,

and  $\alpha_p$  and  $\tau_p$  are the complex valued channel gain and time delay of the  $p^{\text{th}}$  path respectively.

The functional block diagram of the receiver section is shown in Fig. 4. The received signal is down-converted from radio frequency and synchronized with the symbol interval. The guard band, which is inserted for eliminating the ISI and ICI effects, is removed. The symbol constellations corresponding to the original transmitted spectrum are recovered by passing the signal through FFT. The resulting data are deinterleaved and channel decoded. The convolutional encoding applied to the data is decoded by Viterbi decoding. The output data is deinterleaved and given to RRNS decoder. The binary symbol is generated from the nonredundant residue digits using a reverse converter based on CRT. The error corrector LUT addressed by the error syndromes gives out a correction factor. This is added to the output of the reverse converter to get the corrected binary symbol. The proposed model uses two redundant moduli and three nonredundant moduli. So the size of the LUT becomes  $[ 2 * \{ (5-1) + (7-1) + (8-1) \} + \{ 9 + 11 \} - 1 ] = 53$  address locations. Thus RRNS decoding corrects single residue digit errors that are not corrected by Viterbi decoding.

The subcarriers in OFDM can add constructively and destructively. This creates the potential for a large variation in the signal power resulting in a large PAPR. The PAPR is defined as:

$$PAPR = \frac{\max_{t \in [0, T]} |S(t)|^2}{\mathcal{E}\{|S(t)|^2\}} \quad (5)$$

where  $|S(t)|^2$  is the instantaneous power of the transmitted signal, T is the symbol duration and  $\mathcal{E}\{\}$  indicates expectation. The large dynamic range of OFDM systems presents a particular challenge for the power amplifier (PA) design. The PA is required to operate in the linear region to minimize the amount of distortion and to reduce the amount of out-of-band energy generated by the transmitter. This means that OFDM needs to keep its average power low in order to accommodate the signal power peaks. It corresponds to lower output power for the majority of the signal in order to accommodate the infrequent peaks. However, lowering the average power affects the efficiency and range. Peak power clipping is a solution to reduce PAPR. The amount of peak clipping can be increased with a proper coding scheme without affecting the BER performance.

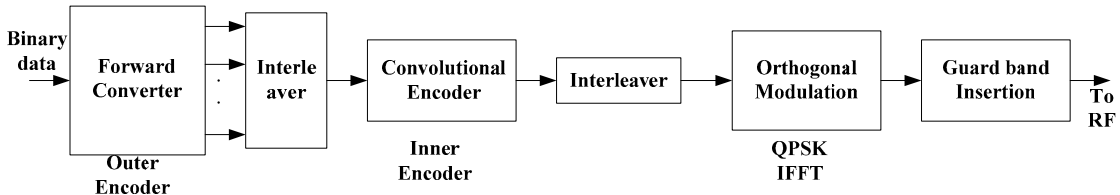


Figure 2. Block diagram of transmitter section

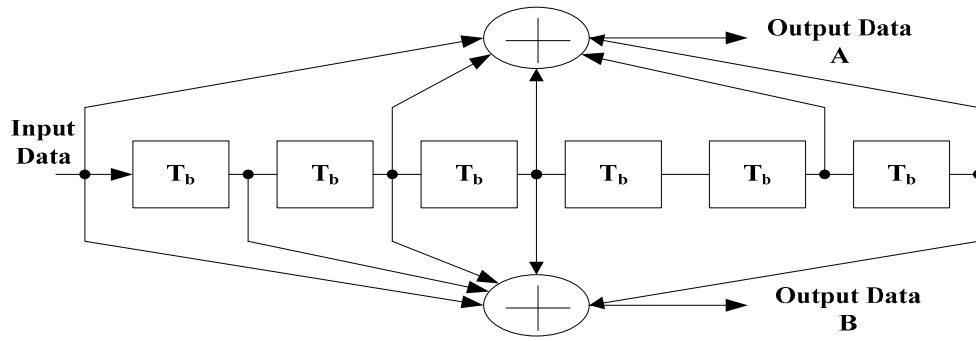


Figure 3. Convolutional encoder

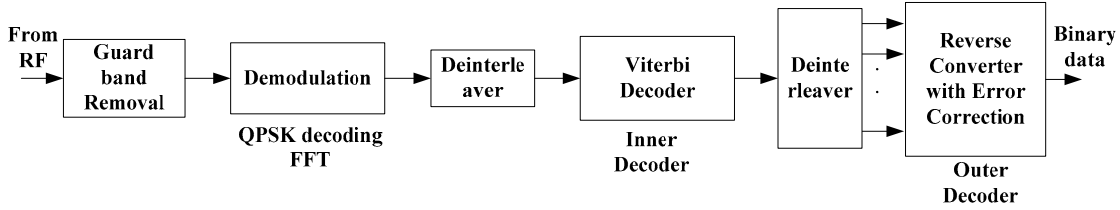


Figure 4. Block diagram of receiver section

#### IV. SIMULATION RESULTS AND PERFORMANCE ANALYSIS

The BER performance of the proposed system using RCCC scheme is evaluated under different operating conditions. This coding scheme offers significant improvement in BER performance for the OFDM system. The communication model is implemented in MATLAB<sup>®</sup>. The OFDM system is simulated with 800 subcarriers using differential QPSK modulation scheme and with the FFT and IFFT sizes of 2048 points. The simulations are carried out to evaluate the system performance under additive white Gaussian noise and multipath fading effects. The PAPR reduction by peak clipping under different clip compression ratios is found out for this coding scheme. The BER performance with cyclic prefixed guard band for different frame start synchronization errors is also analyzed.

##### A. Additive White Gaussian Noise Tolerance

The channel adds zero-mean Gaussian noise to the transmitted signal and the BER performances of uncoded, convolutional coded, and concatenated coded system are obtained. The simulations are carried out by varying the signal-to-noise ratio (SNR), and the BER values are plotted against the channel SNR for different cases as shown in Fig. 5. The simulation results show that the proposed RCCC scheme offers a coding gain of about 4 dB at BER of  $10^{-2}$ . This system can tolerate SNR > 8 – 10 dB with QPSK modulation and RRNS-Convolutional concatenated coding scheme.

##### B. Multipath Delay Spread Immunity

One of the important properties of OFDM is its robustness to multipath delay spread. This is achieved by distributing the digitally encoded symbols over several orthogonal subcarriers in order to reduce the symbol rates. In a frequency-selective multipath fading channel, the base pulses of the original OFDM signal and the delayed version of the signal are no

longer orthogonal. This leads to severe ISI as the orthogonality of the signals is lost. To address this problem, a guard interval is inserted between OFDM symbols.

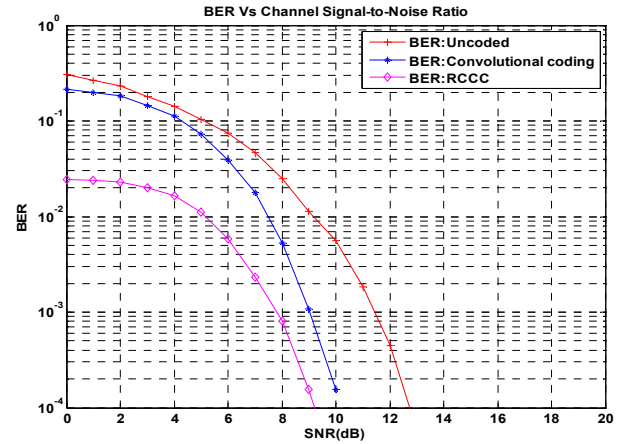


Figure 5. BER versus SNR for uncoded, convolutional coded and RCCC OFDM system

The proposed OFDM system uses cyclic prefix as guard band where the last 256 samples are copied and inserted in front of the symbol. Now a multipath reflection that stays within the guard interval will not cause interference problems. For a channel bandwidth of 1.25 MHz, 256 samples as the guard period correspond to a reflected signal with an additional path length of 61.4 km. The simulation is carried out for a multipath signal containing single reflected signal which is 3 dB weaker than the direct signal. It is sufficient to take a 3 dB weaker signal as the signals weaker than this do not cause measurable errors. The multipath modeling is done by using a lowpass FIR filter function. The length of the filter corresponds to the delay in terms of number of samples and filter coefficients correspond to the reflected signal amplitudes. The BER performance for different delay spreads is obtained for

the three types of OFDM systems as shown in Fig. 6. The tolerable multipath delay spread corresponds to the time of cyclic prefix of the guard period. The results show that the proposed coding scheme offers additional multipath delay spread immunity for the OFDM system. When the delay spread is longer than the guard period, the BER increases rapidly due to the increased ISI. But the RCCC scheme causes the BER to increase at a lesser rate compared to the other two schemes.

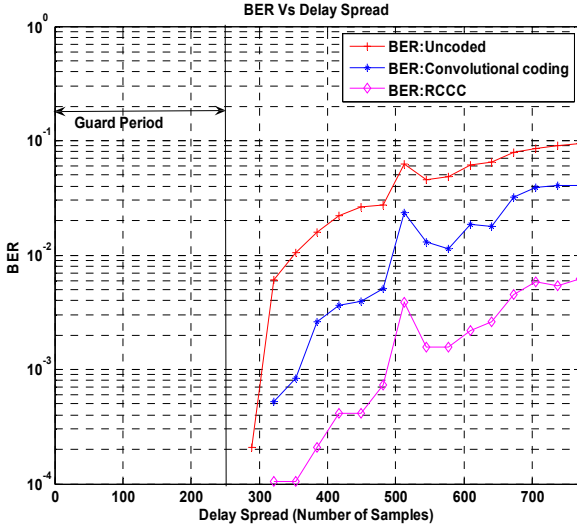


Figure 6. BER versus multipath delay spread for uncoded, convolutional coded and RCCC OFDM system

### C. Effect of Frame Synchronization Errors

The cyclic prefix guard band insertion provides tolerance to frame start time error as well. The BER performance of the system for different timing errors specified in terms of number of samples is shown in Fig. 7. The results show that the starting synchronization errors up to the guard band period are tolerable. This is due to the fact that the orthogonality is maintained during the guard period. Also the proposed coding scheme keeps the BER of the system less than that for the uncoded and convolutional coded systems. If multipath delay spread is taken into consideration, this will reduce the effective stable time of the guard period. Hence multipath delay spread leads to reduced timing error tolerance. But the RCCC scheme offers better timing error tolerance in presence of multipath signals for a particular BER.

### D. Peak Power Clipping for PAPR Reduction

The signal peak at the transmitter is clipped to reduce the PAPR value without much increase in the BER. As the clipping level is increased the PAPR reduces, but the BER is increased. The BER performance for different clip compression ratios in dB is shown in Fig. 8, where the clip compression ratio (CR) is defined as the ratio of the peak power of the signal before clipping to the peak power of the clipped signal. The proposed RCCC coding scheme allows the signal to clip heavily without significant increase in BER. The results show that the proposed system can operate at a BER of

$10^{-3}$  with a clip compression ratio of 15 dB. Fig. 9 shows the BER performance of the proposed system for different clip compression ratios with varying amount of channel noise. For high value of CR, more signal amplitude is clipped resulting in high BER. Hence as CR is increased, the required SNR to achieve the same BER performance is increased. This is due to the increased probability of existence of OFDM signal amplitudes higher than the clipping level. The PAPR values for different clip compression ratios are shown in Table I. The performance of the system for CR = 2 dB is very close to the no clipping performance. There is a trade off between BER performance and PAPR reduction.

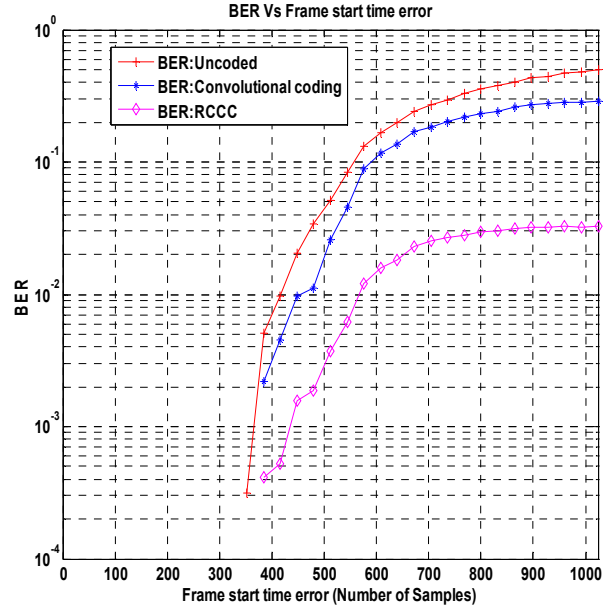


Figure 7. BER versus frame start time error for uncoded, convolutional coded and RCCC OFDM system

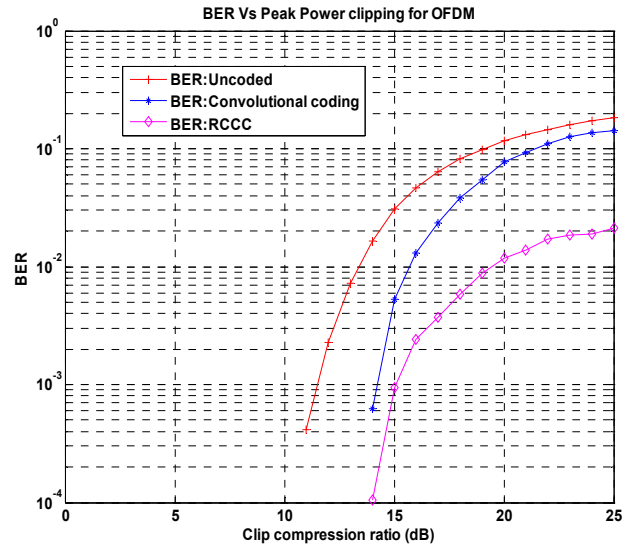


Figure 8. BER versus peak power clipping for uncoded, convolutional coded and RCCC OFDM system

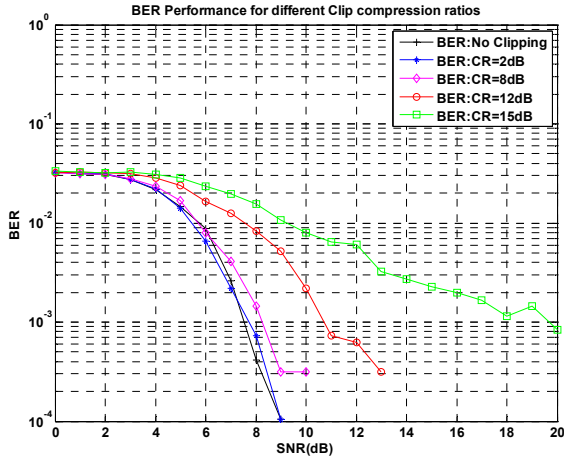


Figure 9. BER versus channel noise of the RCCC OFDM system for different peak power clipping levels

TABLE I  
PAPR FOR DIFFERENT PEAK POWER CLIPPING

Clipping Ratio CR (dB)	Peak to Average Power Ratio, PAPR (dB)	
	Maximum	Average
No clipping (0 dB)	9.1417	5.5961
2 dB	9.126	5.5905
5 dB	8.7477	5.4783
8 dB	8.3789	4.8693
10 dB	8.2788	4.1188
12 dB	7.8339	3.3014

## V. CONCLUSION

The research proposes RRNS-convolutional concatenated coding (RCCC) scheme for an OFDM based wireless communication system. The concatenated code uses RRNS code as the outer code and convolutional code as the inner code. At the receiver, errors that are not corrected by Viterbi decoding will get corrected by the RRNS decoding due to the redundancy introduced. The performance of this system is analyzed for different channel conditions. The simulation results show that RCCC scheme offers improved BER performance in presence of additive white Gaussian noise and multipath delay spread. The guard band insertion with cyclic prefixing provides tolerance to multipath delay spread and frame start synchronization errors. This coding scheme makes the OFDM system more robust against multipath effects and timing errors. Also, the signal can be heavily clipped to reduce the PAPR without significant increase in BER for the RCCC

OFDM system. The performance analysis shows that the proposed RCCC scheme is suitable for OFDM as it improves the tolerance of system to channel noise, multipath effects, timing errors and peak power clipping.

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