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Dynamics of coupled Josephson junctions under the influence of applied fields

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Abstract

We investigate the effect of the phase difference of applied fields on the dynamics of mutually coupled Josephson junctions. A phase difference between the applied fields desynchronizes the system. It is found that though the amplitudes of the output voltage values are uncorrelated, a phase correlation is found to exist for small values of applied phase difference. The dynamics of the system is found to change from chaotic to periodic for certain values of phase difference. We report that by keeping the value of phase difference as π, the system continues to be in periodic motion for a wide range of values of system parameters. This result may find applications in devices like voltage standards, detectors, SQUIDS, etc., where chaos is least desired.

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Keywords: Phase effect; Synchronization; Control of chaos

1. Introduction

The study of the dynamics of Josephson junction (JJ) is of great interest from theoretical as well as experimental points of view. The interaction of Josephson junctions with external fields have played important roles in the development of physics and chaotic dynamics of Josephson junctions [1–4]. The existence of chaos in rf-biased Josephson junction has been verified through theory, numerical simulation and experiments [5]. The rf-biased junction finds applications as voltage standards, detectors, etc., where chaotic behavior is least desired [6]. Control of chaos continues to be an active area of research [7] because of the many undesirable effects chaos brings in mechanical systems and other devices. It was shown earlier that it would be possible to control chaos both theoretically and experimentally using different methods such as giving a feed back [8], application of a weak periodic force [9], etc. The problem of controlling spatio-temporal chaotic pattern induced by an applied rf signal in a Josephson junction has earlier been discussed [10]. By controlling chaos in rf-biased Josephson junctions it was shown that even in the presence of thermal noise, they could be used as voltage standards [11]. Suppression of temporal and spatio-temporal chaos allows complex systems to be operated in highly nonlinear regimes. This is required in many physical systems. By applying a small time-dependent modulation to a parameter of the system, a chaotic system can be stabilized. However in practical applications this method requires that the characteristic times of the system is not too short compared with the times of the feed back. In the case of JJ oscillators the characteristic times of the dynamics response are of the order of few picoseconds which is too short for any electronic feedback control system.

Since it was shown that chaotic systems could be synchronized by linking them to a common signal [12], many works have been done in this direction because of its application in secure communication [13]. Synchronization in Josephson junctions has been an interesting area of research [14–16]. The role of phase difference of the applied sinusoidal fields on Duffing

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oscillators has been studied earlier [17,18]. In the present work, we consider the effect of phase difference of the applied rf-fields on a mutually coupled Josephson junctions. We discuss the coupled JJ in Section 2 and arrive at a dimensionless first order equation of motion. In Section 3 the parameter range in which the system may be synchronized is found and the effect of phase difference of the applied rf-fields on synchronization is found out. The dynamics of the Josephson junction after applying the phase difference and the effect of other parameters on the junction after fixing the phase difference between the applied fields to a value at which system is in periodic motion are also discussed in this section. In Section 4 the results are discussed.

2. Coupled Josephson junction

Josephson junction can be represented by a resistively and capacitively shunted junction (RCSJ) model and the dynamics of the system can be explored by writing the equation of motion [19]. The equation of a single Josephson junction for this model can be written by solving Kirchoff's law as

$$\frac{\hbar C}{2e}\frac{d^2\phi}{dt^2}+\frac{\hbar}{2eR}\frac{d\phi}{dt}+i_c\sin\phi=i_{dc}+i_0\cos(\omega t), \tag{1}$$

where ϕ is the phase difference of the wave function across the junction, $i_0\cos(\omega t)$ is the driving rf-field and i_{dc} is the dc bias. The junction is characterized by a critical current i_c, capacitance C and normal resistance R. The coupled JJ considered here consists of a pair of such junctions wired in parallel with a linking resistor R_s [15]. Schematic representation of the system is given in Fig. 1 and the dynamical equations can be written as

$$\frac{\hbar C_1}{2e}\frac{d^2\phi_1}{dt'^2}+\frac{\hbar}{2eR_1}\frac{d\phi_1}{dt'}+i_{c1}\sin\phi_1 = i'_{dc}+i'_0\cos(\omega t')-i_s, \tag{2}$$

$$\frac{\hbar C_2}{2e}\frac{d^2\phi_2}{dt'^2}+\frac{\hbar}{2eR_2}\frac{d\phi_2}{dt'}+i_{c2}\sin\phi_2 = i'_{dc}+i'_0\cos(\omega t'+\theta)+i_s, \tag{3}$$

where i_s is the current flowing through the coupling resistor and is given as

$$i_s=\frac{\hbar}{2eR_s}\left[\frac{d\phi_1}{dt'}-\frac{d\phi_2}{dt'}\right]. \tag{4}$$

In order to express Eqs. (2) and (3) in dimensionless form the junction plasma frequencies ω_{J1} and ω_{J2} given by $\omega_{J1}=(2ei_{c1}/\hbar C_1)^{1/2}$ and $\omega_{J2}=(2ei_{c2}/\hbar C_2)^{1/2}$ are introduced. The normalized time scale is written as $t=\omega_{J1}t'$. The dimensionless damping parameter β is defined as

$$\beta=\frac{1}{R_1}\sqrt{\frac{\hbar}{2ei_{c1}C_1}}.$$

The dc bias current i'_{dc} and the rf amplitude i'_0 are normalized to the critical current i_{c1}. The actual frequency ω is rescaled to $\Omega=\omega/\omega_{J1}$ and the coupling factor is defined as $\alpha_s=(R_1/R_s)\beta$. For identical Josephson junctions, Eqs. (2) and (3) may be written as

$$\ddot{\phi}_1+\beta\dot{\phi}_1+\sin\phi_1=i_{dc}+i_0\cos(\Omega t)-\alpha_s[\dot{\phi}_1-\dot{\phi}_2],$$
$$\ddot{\phi}_2+\beta\dot{\phi}_2+\sin\phi_2=i_{dc}+i_0\cos(\Omega t+\theta)-\alpha_s[\dot{\phi}_2-\dot{\phi}_1]. \tag{5}$$

It can be seen that the coupling arises as a natural consequence of the exchange of current through the resistor R_s and it depends on the differential voltage $(\psi_1-\psi_2)$. For Josephson junction devices, phase derivatives are of central importance because they are proportional to junction voltages. In order to study the system numerically Eq. (5) is written in the first order differential form as

$$\dot{\phi}_1=\psi_1,$$
$$\dot{\psi}_1=-\beta\psi_1-\sin\phi_1+i_{dc}+i_0\cos(\Omega t)-\alpha_s[\psi_1-\psi_2],$$
$$\dot{\phi}_2=\psi_2,$$
$$\dot{\psi}_2=-\beta\psi_2-\sin\phi_2+i_{dc}+i_0\cos\big[(\Omega t)+\theta\big]-\alpha_s[\psi_2-\psi_1]. \tag{6}$$

Eq. (6) is studied using fourth order Runge–Kutta method and the maxima of normalized voltage values are plotted to study the dynamics. The values of the system parameters were fixed as $\beta=0.15$, $i_0=0.7$, $i_{dc}=0.3$ and $\Omega=0.6$. From Fig. 2 we observe that the maximum Lyapunov exponent is positive for all values of coupling strength and it remains at the same positive value. Hence the system exhibits chaotic behavior for all values of coupling strength in the parameter range we selected.

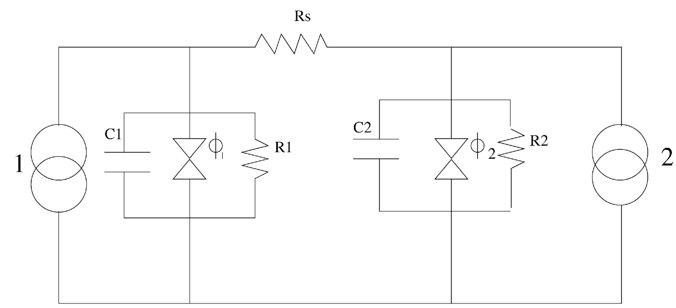

Fig. 1. Schematic representation of a coupled Josephson junction connected in parallel with a linking resistor R_s. 1 and 2 represent the applied fields.

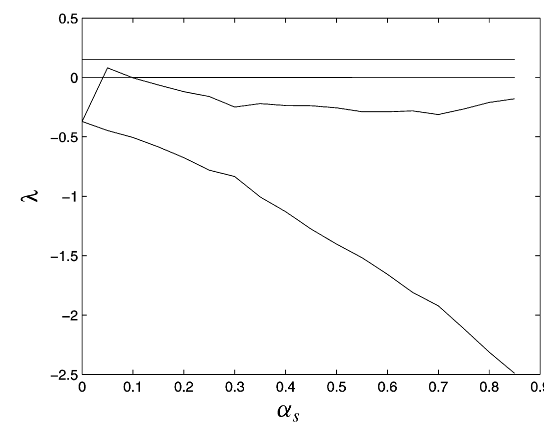

Fig. 2. Lyapunov exponent spectrum is plotted for different values of coupling strength α_s with $\theta=0$, $\beta=0.15$, $i_0=0.7$ and $\omega=0.6$. It can be seen that the system is in chaotic motion for all values of coupling strength.

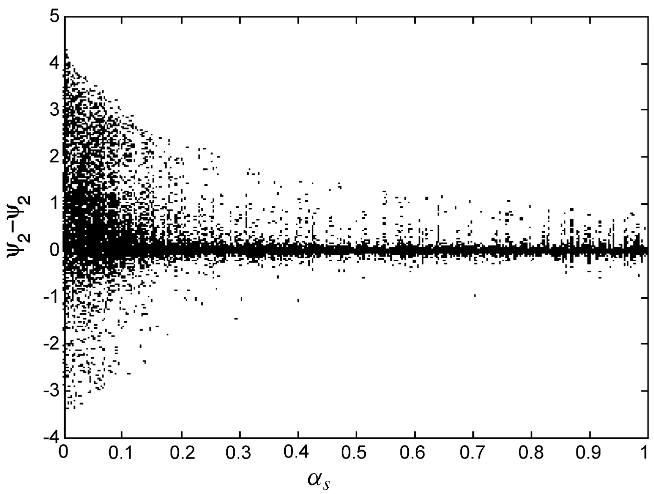

Fig. 3. Maxima of the difference in voltage against coupling strength α_s. $\theta = 0$, $\beta = 0.15$, $i_0 = 0.7$ and $\omega = 0.6$.

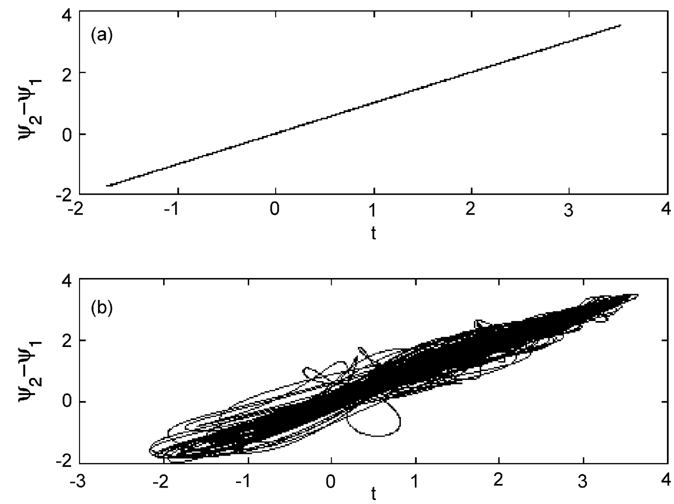

Fig. 4. (a) shows the system is synchronized for $\theta = 0$, $\beta = 0.15$, $i_0 = 0.7$, $i_{dc} = 0.3$, $\alpha_s = 0.45$ and $\omega = 0.6$. (b) shows the system is desynchronized for a phase difference of $\theta = 0.1\pi$.

However as the coupling strength is increased, the difference in voltage becomes smaller as can be seen from Fig. 3. Hence we can suitably select a value for the coupling strength such that the system is in the synchronization manifold.

3. The effect of phase difference

In order to study the effect of phase difference of the applied sinusoidal driving fields on JJ system we write $S_\phi = \phi_1 - \phi_2$ and $S_\psi = \psi_1 - \psi_2$ and from Eq. (6) we get

$$\begin{aligned}\dot{S}_\phi &= S_\psi,\\ \dot{S}_\psi &= -\beta S_\psi - \sin\phi_1 + \sin\phi_2 - 2\alpha_s S_\psi\\ &\quad + 2i_0 \sin\left(\Omega t + \frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right).\end{aligned} \tag{7}$$

When $\theta = 0$ the term $2i_0\sin(\Omega t + \theta/2)\sin(\theta/2)$ vanishes. The values of α_s may be chosen such that the difference in voltage is negligible, i.e., $\psi_1 \approx \psi_2$. Now both $\dot{S}_\phi$ and $\dot{S}_\psi$ go to zero. From Fig. 3 the value of α_s is chosen as 0.45 which satisfies this condition. However even small values of applied phase differences desynchronizes the system. Fig. 4(a) shows that the system is synchronized and Fig. 4(b) shows that the system is desynchronized by an applied phase difference of $\theta = 0.1\pi$. The level of mismatch of chaotic synchronization can be given quantitatively by taking the similarity function $S(\tau)$ as a time averaged difference between the variables ψ_1 and ψ_2 taken with time shift τ [20]

$$S^2(\tau) = \frac{\langle[\psi_1(t+\tau) - \psi_2(t)]^2\rangle}{[\langle\psi_1^2(t)\rangle][\langle\psi_2^2(t)\rangle]^{1/2}}. \tag{8}$$

The value $S(\tau)$ plotted against τ for different values of phase difference θ is shown in Fig. 5. It is observed that for $\theta = 0$, the system is in complete synchronization. For a finite value of phase difference, a minimum of $S(\tau_0)$ appears which indicates the existence of a certain phase difference between the interacting systems. $S(\tau_0)$ is finite in these cases which means that in this regime the amplitudes are uncorrelated.

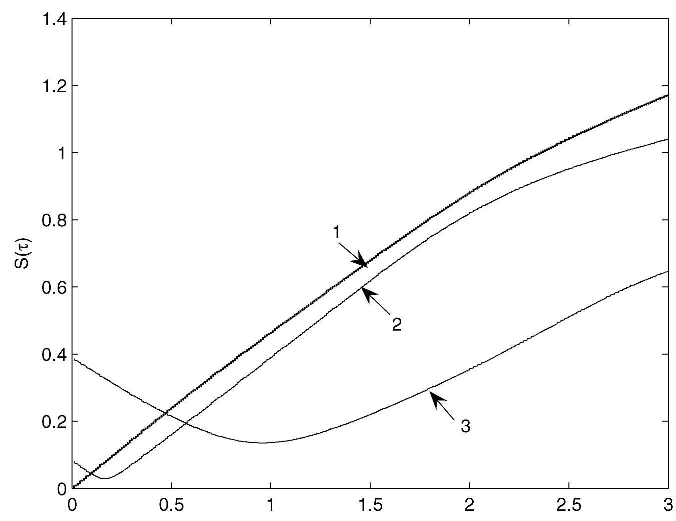

Fig. 5. Similarity function $S(\tau)$ versus τ for different values of phase difference θ. Curve 1 is with phase difference $\theta = 0$, curve 2 for $\theta = 0.1\pi$ and curve 3 for $\theta = 0.5\pi$.

The values of θ is varied from 0 to 2π and the maxima of the difference in voltage is plotted against the phase in Fig. 6. In the synchronization manifold, i.e., when $\phi_1 \approx \phi_2$ and $\psi_1 \approx \psi_2$ we can write $P_\phi = [\phi_1 + \phi_2]/2$ and $P_\psi = [\psi_1 + \psi_2]/2$ and from Eq. (6) we get

$$\begin{aligned}\dot{P}_\phi &= P_\psi,\\ \dot{P}_\psi &= -\beta P_\psi - \sin(P_\phi) + i_{dc} + i_0\cos\left(\frac{\theta}{2}\right)\cos\left(\Omega t + \frac{\theta}{2}\right).\end{aligned} \tag{9}$$

Eq. (9) is equivalent to Eq. (1) and we can see that i_0 is replaced by $i_0\cos(\theta/2)$ and the phase of the driving field leads by $(\theta/2)$ as a result of coupling. This may correspond to a parameter space where the system is periodic which explains the change in system dynamics. However the effect of applying a phase difference between the driving fields and that of changing the

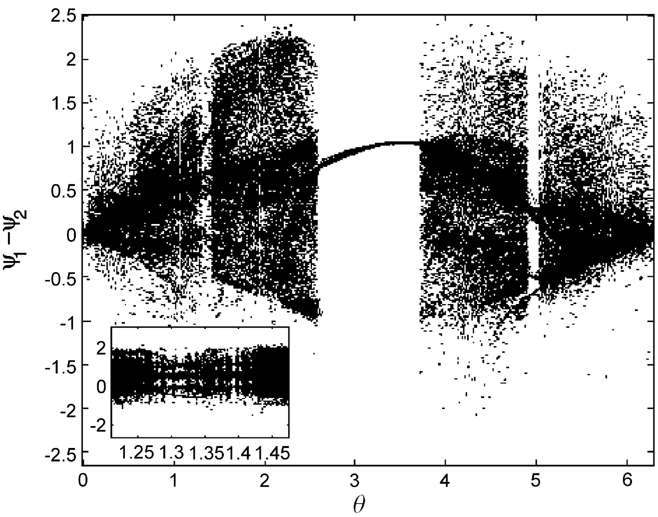

Fig. 6. Maxima of the difference in voltage is plotted against phase difference applied, $\theta = 0$–2π. $\alpha_s = 0.45$, $i_{dc} = 0.3$, $\beta = 0.15$, $i_0 = 0.7$ and $\omega = 0.6$.

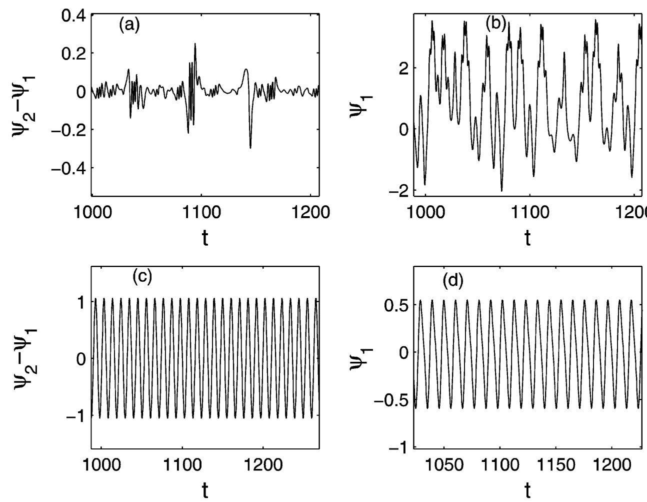

Fig. 8. (a) shows the differential voltage plotted against time and (b) is the voltage of one junction with $\theta = 0$. It is observed that the variation is chaotic and the maximum difference in voltage is 0.05. (c) and (d) show the corresponding voltages with an applied phase difference of π. Other parameter values are $\alpha_s = 0.45$, $\beta = 0.15$, $i_0 = 0.7$ and $\omega = 0.6$.

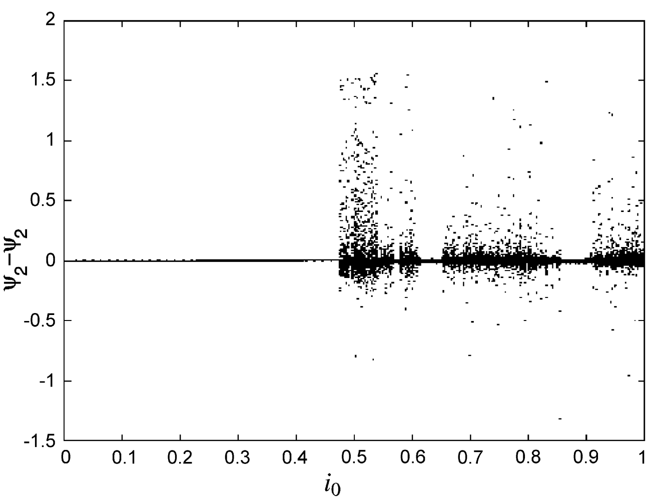

Fig. 7. Maxima of the difference in voltage is plotted against amplitude of driving field, $\theta = 0$. $\alpha_s = 0.45$, $i_{dc} = 0.3$, $\beta = 0.15$ and $\omega = 0.6$.

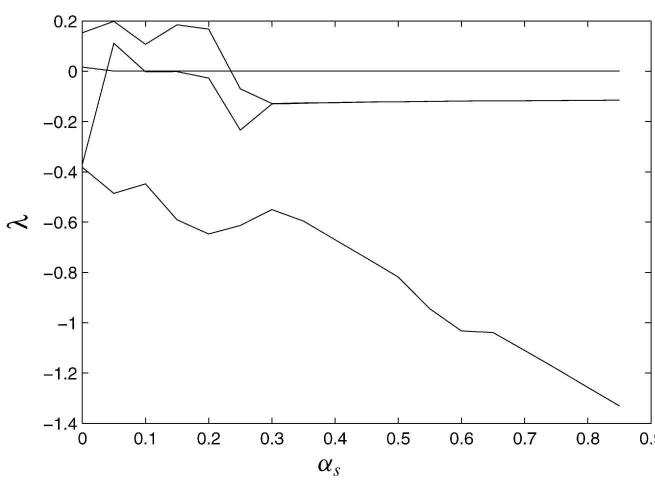

Fig. 9. Lyapunov exponent spectrum plotted against coupling strength α_s for $\theta = \pi$, $\beta = 0.15$, $i_0 = 0.7$ and $\omega = 0.6$.

amplitude of the driving fields on the dynamics of the system is different as can be seen from Figs. 7 and 6.

From the inset of Fig. 6 it can be seen that the system exhibits periodic window in the region where $\theta = 0.34\pi$ to 0.4π. However in this region even a slight change in system parameter values would bring the system back to chaotic regime. For a phase difference of $\theta = 0.95\pi$ to 1.5π the system exhibits periodic motion. The difference in voltage and voltage of a single junction for $\theta = \pi$ plotted against time are shown in Figs. 8(c) and 8(d). Figs. 8(a) and 8(b) show difference in voltage and voltage of a single junction against time for an applied phase difference of $\theta = 0$.

From the Lyapunov exponent spectrum in Fig. 9 with $\theta = \pi$ it can be seen that the system is in periodic motion for most of the coupling values. The system turns from hyperchaos (two positive Lyapunov exponent) to chaos and then to a limit cycle (two negative exponents) on increasing the coupling strength. Fixing the phase difference between the driving fields as π, the change in the response of the system to other parameter variations are then studied.

Fixing the value of α as 0.45 and the amplitude of the driving rf-field is changed from 0 to 1. Without an applied phase difference the system exhibits chaotic motion from a value of 0.43 onwards with some periodic windows in between (Fig. 10). However, on the application of a phase difference the system stays in periodic state for a wide range of amplitude values which were chaotic earlier. Fig. 11 shows the response of the system when the amplitude of the driving field is changed from 0 to 1 with an applied phase difference of π.

The combined effect of phase difference and the applied dc bias on the system is also studied. For this all other parameter values were fixed and i_{dc} value is changed from 0 to 0.4. Here also we observe that the system continues to be in periodic motion for a large range of i_{dc} values. The comparison can be obtained from Figs. 12 and 13. Thus we show that the system

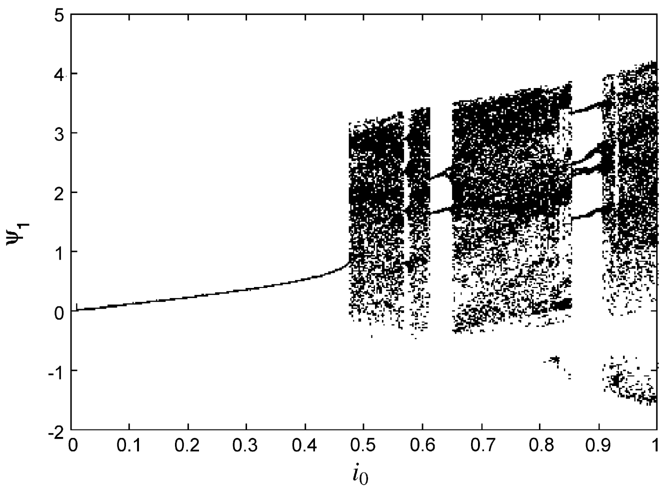

Fig. 10. Maxima of the normalized voltage against amplitude of applied field i_0 with $\theta = 0$. The other parameter values are $\alpha_s = 0.45$, $i_{\mathrm{dc}} = 0.3$, $\beta = 0.15$ and $\omega = 0.6$.

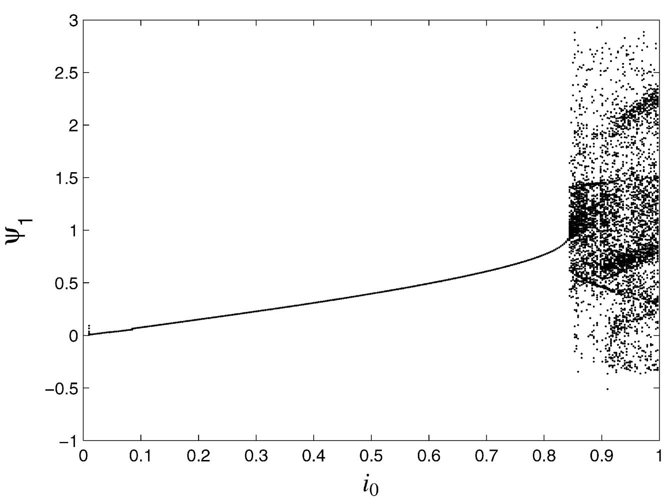

Fig. 11. Maxima of the normalized voltage is plotted against the amplitude of applied field. It can be seen that the system is in periodic motion for a wide range of amplitude values. $\theta = \pi$, $\alpha_s = 0.45$, $i_{\mathrm{dc}} = 0.3$, $\beta = 0.15$ and $\omega = 0.6$.

exhibits periodic motion for a wide range of parameter values for an applied phase difference between the driving fields. This may be of great practical importance in Josephson junction devices like voltage standards, SQUIDS, detectors, etc.

An important point to be noted here is that the parameter values at which we apply phase difference is to be chosen carefully. If the values we choose is in a region where the difference in voltage ($\psi_1 - \psi_2$) is large, then by just applying a phase difference we may not be able to control chaos. Fig. 14 shows the voltage across a junction plotted against θ with coupling strength $\alpha_s = 0.25$. It is seen that a periodic motion cannot be observed in this case.

4. Conclusion

In this work we have found that by applying a phase difference between the driving fields to coupled Josephson junctions we can control chaos if the system is in the synchronization

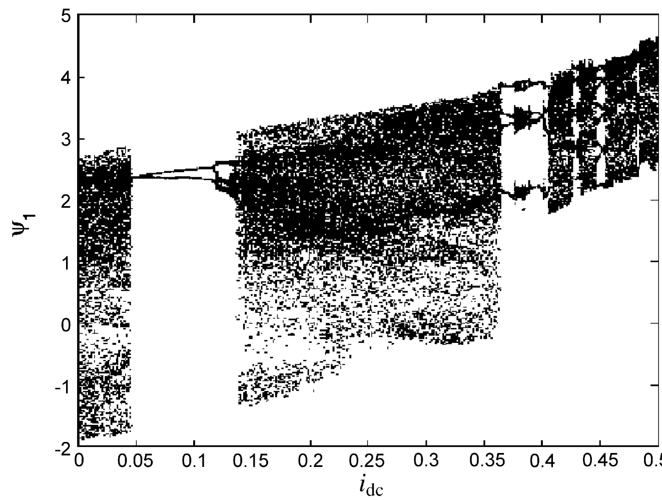

Fig. 12. Maxima of the normalized voltage against the i_{dc}. $\theta = 0$, $\alpha_s = 0.45$, $\beta = 0.15$, $i_0 = 0.7$ and $\omega = 0.6$.

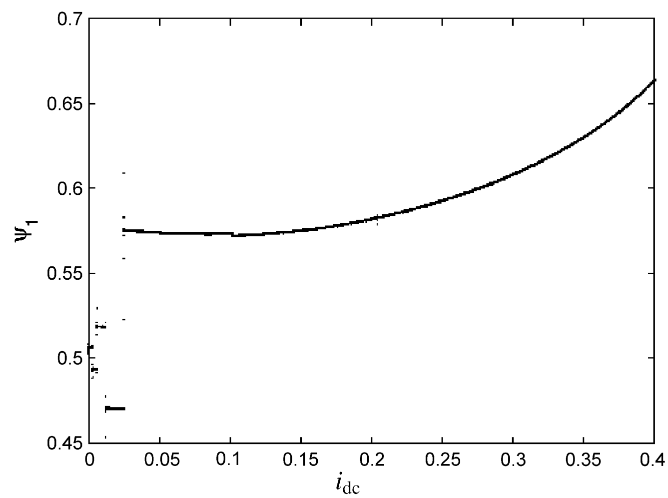

Fig. 13. Maxima of the normalized voltage against the i_{dc}. $\theta = \pi$, $\alpha_s = 0.45$, $\beta = 0.15$, $i_0 = 0.7$ and $\omega = 0.6$.

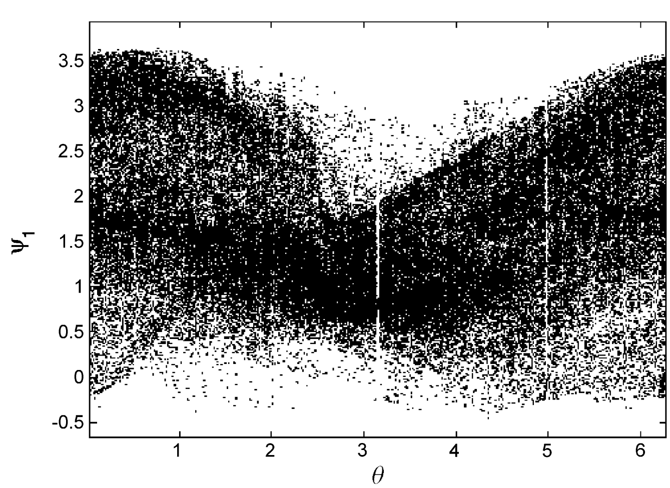

Fig. 14. Maxima of the normalized voltage against the θ with $\alpha_s = 0.25$. $i_{\mathrm{dc}} = 0.3$, $\beta = 0.15$, $i_0 = 0.7$ and $\omega = 0.6$.

manifold. However if the difference in voltages between the two junction is not negligible chaos cannot be controlled by just applying a phase difference between the driving fields. Though the application of a phase difference between the applied fields desynchronizes the system, a phase correlation has been found to exist for small values of applied phase differences. The difference between changing the amplitude of the driving fields and applying a phase difference between the fields has been discussed. For a phase difference of $\theta = 0.95\pi$ to 1.5π the dynamics of the system has been found to change from chaotic to periodic. Then fixing the phase difference as π and varying other parameters such as dc bias, amplitude of applied field and coupling strength the change in the response of the system has been studied. It has been found that even for large variation of these parameters, the system continues to be in periodic motion. So this may be of great practical importance as phase difference can be easily applied to the rf-field in an experimental set up. Thus it offers an easier way to control chaos and thus will provide an enhanced capability to design superconducting circuits in such a way as to maximize the advantages of nonlinearity while minimizing the possibility of instabilities.

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