NONLINEAR ANALYSIS OF AN ELECTROENCEPHALOGRAM (EEG) DURING EPILEPTIC SEIZURE

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ABSTRACT

A mathematical analysis of an electroencephalogram of a human brain during an epileptic seizure shows that the K_2 entropy decreases as compared to a clinically normal brain while the dimension of the attractor does not show significant deviation.

INTRODUCTION

Recent progress in nonlinear dynamics has provided new methods for the investigation of multivariable complex systems for the analysis of experimental data obtained as a one-dimensional time series. Their chief merit is to discriminate between random and deterministic nature of dynamical systems by evaluating parameters such as dimensional entropy, Lyapunov exponents etc. These quantities also provide a means of following the system as it evolves from one kind of behaviour to another. In addition, the dimensions defined from the attractor provide estimates of the number of independent variables that may ultimately be needed to model the attractor.

Recently Abraham *et al*[1] developed a method of analysis for calculating dimensions of attractors from small data sets and this has been applied recently to analyse instabilities in multimode laser oscillation and hydrodynamic turbulence. We propose to use this method of analysis to the study of an electroencephalogram (EEG) of human brain. It is known that brain is one of the complex systems encountered in nature consisting of about 10^{10} neurons. The EEG taken from eight points on the scalp (agreed upon internationally) reflects the resultant of elemental sustained neuronal activity of a relatively low frequency (0.5–40 Hz). Rapp *et al*[2] observed chaotic behaviour of an isolated neuron of a mamalian (Squirrel monkey) central nervous system. The existence of chaos in a single neuron as well as a randomly connected network of 26 neurons was also shown by Kurten and Clark[3]. An EEG however will give a collective effect of a large number of nonlinearly interacting neurons and hence would manifest in an entirely new manner as compared to the dynamics of a single neuron. To understand this collective mode, we recently analysed an EEG of a normal brain as a time series and found that the dynamics has a deterministic component[4]. We established the existence of an attractor with dimensionality 4.5 and the Kolmogorov entropy was found to be 72. In the present paper, we extend this study to the EEG of a human brain during an epileptic seizure.

METHOD OF ANALYSIS

In the following section, we describe the salient features of the estimation of correlation dimension D_2 and second order Kolmogorov entropy K_2 from an observed time series. Time series is digitized with time interval τ and is written as

$$X_0(t)=\{X_0(t_1), X_0(t_2)....X_0(t_N)\}, \qquad (1)$$

so that $t_{i+1} - t_i = \tau$ and from this we construct d additional data sets

$$\{X_0(t_1 + d\tau), X_0(t_2 + d\tau)....X_0(t_N + d\tau)\}, \qquad (2)$$

where d = 1, 2, 3....we can now define N vectors $\vec{X}_i$ in a d dimensional space as

$$\vec{X}_i = \{X_0(t_i+\tau),\ldots\ldots X_0(t_i+d\tau)\}. \qquad (3)$$

The correlation is now defined as

$$C(r) = Lt_{N\to\infty}N^{-2}\sum_{i,j=1}^{N}\Theta(r|\vec{X}_i - \vec{K}_j|), \qquad (4)$$

where $\Theta(x)$ is the Heaviside function $C(r)$ counts the number of pairs of points with distance $|\vec{X}_i - \vec{X}_j|$ (which is the Euclidean norm) less than r. It is known that for small values of r, $C(r)\sim r^{\nu}$, where ν is the correlation dimension of the attractor.

Dimension of the attractor

Figure 1 gives the plot of log $C(r)$ against log(r) for various values of d and as one can see they all

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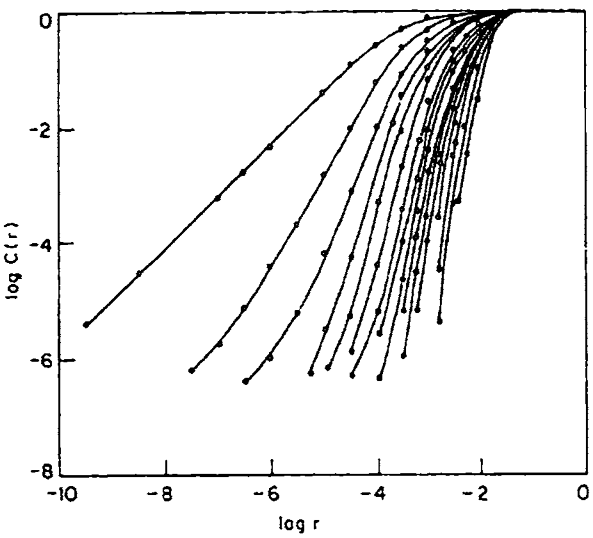

Figure 1. A plot of log $[C(r)]$ against log $[r]$ for various d. As d increases, the lower part of the curves become more and more parallel to each other. The curves saturate at the top.

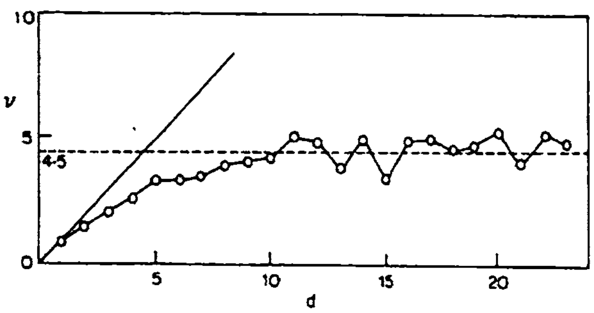

Figure 2. Slope of the linear part of the curves in figure 1 is plotted against the dimension d. The curve saturates as d increases. The saturation value of the slope is called D_2, the second order dimension of the attractor.

saturate. The slopes ν of the linear part of the curves are plotted against respective dimension d and is given in figure 2. For a completely random dynamics, this plot will be a straight line with slope unity. However, in the case of an EEG data, the curve saturates for large values of the dimension d and the saturation value of the slope D_2 (4.5 in the present case), is called the second order dimension of the attractor. Moreover, this saturation behaviour proves the existence of deterministic component of the EEG. The noninteger value of the attractor dimension D_2 establishes the existence of determi-

Second order Kolmogorov entropy

Another quantity of great significanc second order Kolmogorov entropy which is in terms of the correlation function as

$$K_2 = \mathop{\mathrm{Lt}}_{r\to 0, d\to\infty} \frac{1}{\tau} \log\,[C_d(r)/C_{d+1}(r)],$$

where τ corresponds to temporal reso measurements. $K_2 = 0$ implies an ordered while $K_2 = \infty$ corresponds to totally stocha tion. Thus $K_2 > 0$ is a sufficient condition f ministic chaos. Experimentally K_2 is obtaine limiting value in the plot of

$$\frac{1}{\tau} \log\,[C_d\,(r)/C_{d+1}(r)]$$

against d. Figure 3 gives such a plot and over to a value 6 asymptotically for large va

CONCLUSIONS

The results presented were obtained by the EEG of a person during an epilep which is classified as a Grand Mal and the the frontal right side. It was found that the dimension is not sensitive for epileptic sei

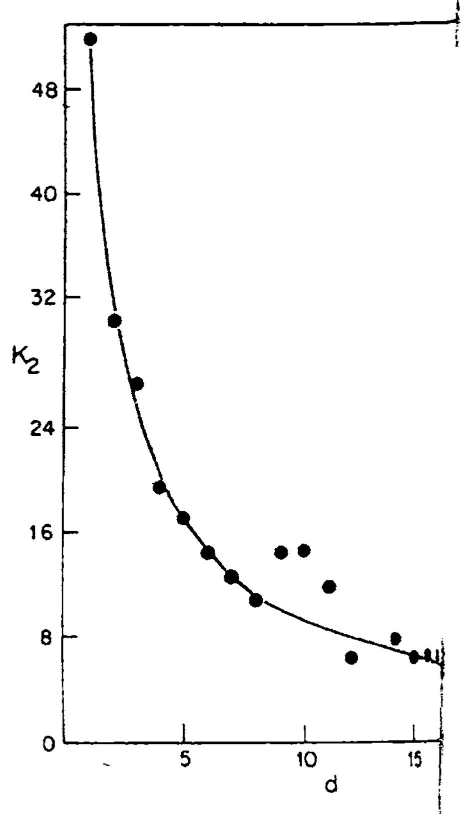

Figure 3. The second order Kolmogor given as a function of d. This again s

K_2 is reduced drastically from 72 in the case of normal brain to 6 in the case of epileptic one. The present results are at variance with those obtained by Babloyantz *et al*[5] in which they have analysed a Petit Mal case and obtained a dimension of 2.05 ± 0.09. However, they have not calculated the value of K_2 and have gone only up to $d = 7$. Dvorak and Siska[6] obtained the dimension to fluctuate between 3.8 and 5.4 as they have considered the trace from Oxypetal with two different values of the number of data points N (1000 and 12000). It is now realized that our results agree with the above with lesser data points as claimed by Abraham *et al*[1]. The simultaneous records from the 8 channels of EEG of the same patient would enable one to obtain more informations about the dynamics. Furthermore, K_2 seems to be a more reliable parameter to look into, since it is critically dependent on time scales. This work is in progress.

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1. Abraham, N. B., *et al.*, *Phys. Lett.*, 1986, **A114**, 217.
2. Rapp, P. E., Zimmerman, I. D., Albano, A. M., De Guzman, G. C. and Greenbaum, N. N., *Phys. Lett.*, 1985, **A110**, 335.
3. Kurten, K. E. and Clark, J. W., *Phys. Lett.*, 1986, **A114**, 413.
4. Lalaja Varghese, Nampoori, V. P. N. and Pratap, R., *J. Neurosci.*, 1987 (Communicated).
5. Babloyantz, A. and Dexlexhe, A., *Proc. Natl. Acad. Sci. (USA)*, 1986, **83**, 3513.
6. Dvorak, I. and Siska, J., *Phys. Lett.*, 1986, **A118**, 63.

ANNOUNCEMENTS

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Nominations for the award are invited from environmental organizations, government agencies, trade associations, companies, international organizations, academic and scientific societies and from individuals.

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