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INVARIANT CHARACTERIZATION OF NEURAL SYSTEMS

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It is shown that the invariant integral, viz., the Kolmogorov second entropy, is eminently suited to characterize EEG quantitatively. The estimation obtained for a "clinically normal" brain is compared with a previous result obtained from the EEG of a person under epileptic seizure.

Keywords: EEG, epilepsy

Human brains with 10^{10} neurons exhibit a wide variety of responses for a given stimulus, so much so that it is difficult to identify a cause-and-effect framework which will eventually lead to neural modelling. The various attempts till 1975 are given by MacGregor and Lewis (1977). It is increasingly becoming evident that mechanisms which play in different responses are different. A recent review on neural mechanisms in cognition has been given by Harth et al. (1987). A general model of evolution in a neural network has been given by Parikh and Pratap (1984). It is also realized that the processes that are capable of explaining higher functions of brain are highly nonlinear and non-Marcovian (Pratap, 1987) resulting in collective and synergic modes.

While extensive experimental studies have been conducted to understand the properties of individual neurons, it is only recently that attention has been directed to the investigation of collective modes by the study of electroencephelogram (EEG). In this paper, we propose to give an account of our study of EEG of a "clinically" normal brain and compare the results with that obtained from the study of a brain during an epileptic seizure (Varghese, et al., 1987). The significant results are: (a) the system acts as an attractor of dimension about 5 embedded in a subspace of dimension 10 to 15 in an infinite dimensional phase space. (b) The second Kolmogorov entropy is a sensitive parameter and can be used with profit to classify neural systems. This parameter is found to be higher for "normal" brains as compared to one subjected to seizure. (c) We have also observed that in some "clinically normal" neural networks when subjected to Fourier analysis, there exists only odd harmonics. This conspicuous absence of even harmonics is also observed in optical bistability in lasers. This aspect is not elaborated here, as this requires a more extensive study. As the system is a multifaceted one, it is still not known why in some cases we observed this and in others not. Probably this may be due to psychological conditions which differ from patient to patient depending on the environment, genetic factors etc. (d) The second Kolmogorov entropy as a global

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property is found to be a sensitive parameter and hence can be used as a diagnostic tool to study the neural systems, and since this is a dynamic parameter, this also indicates the existence of multiple time scales in the system resulting in synergism (Patro, 1982; Patro & Pratap, 1983).

Studies of nonlinear dynamical systems have evolved significant methods of analysis of time series. Nonlinear effects in dynamical systems can arise in two different ways: a system having a single fundamental characteristic frequency can have various higher harmonics in the system which can interact amongst themselves nonlinearly producing coherence. On the other hand, if there exists more than one characteristic frequency in the system, which are incommensurate, this would produce synergism (Schwinger et al., 1976). The only condition for this mechanism is that these frequencies should be close by. In the present study, we have shown the existence of multiple time scales from the study of dimensionality of the attractor.

ANALYSIS

In the study of time series, one recognizes two invariant quantities which are independent of the nature of interactions in the system: characteristic dimensionality of the attractor is a static invariant and Kolmogorov entropy is a dynamic invariant, and hence they depend on time scales in the system. In the present discussion, we shall follow the definitions given by Atmanspacher and Scheingraber (1986). We shall adopt the method developed by Abraham et al. (1987) which is specially suited for small data sets.

The EEG amplitudes, considered as a time series, can be measured at various times $t_1 \ldots t_N$ and this is given by

$$X(t) = \{X(t_1), X(t_2), \ldots X(t_N)\}. \tag{1}$$

We can now construct additional data sets by introducing a time delay τ which can be written as

$$\begin{array}{c} X(t_1) \ldots\ldots\ X(t_N), \\ X(t_1+\tau) \ldots\ldots\ X(t_N+\tau), \\ \ldots\ldots \\ X(t_1+\tau d) \ldots\ldots\ X(t_N+\tau d). \end{array} \tag{2}$$

where d is an integer. The above matrix which is rectangular, can be considered as a set of N vectors, each column being a vector of d dimension—or (2) gives a set of N vectors defined in a d dimensional space and this can be written as

$$X(t_i) = \{X(t_i) \ldots X(t_i+\tau d)\}. \tag{3}$$

Figure 1 is a Poincaré plot of the data set (1). We have plotted only 40 points as X_{n+1} against X_n. This gives the intersection of the Poincaré surface by the plane (X_{n+1}, X_n). One may observe that the trajectories do not show a tendency to retrace their paths thereby implying the existence of an attractor. One could plot X_{n+j}

against X_m, and it is known that the diagram becomes narrower for larger j—j being an integer. As one can easily see, this set of vectors is topologically equivalent to the data set (1).

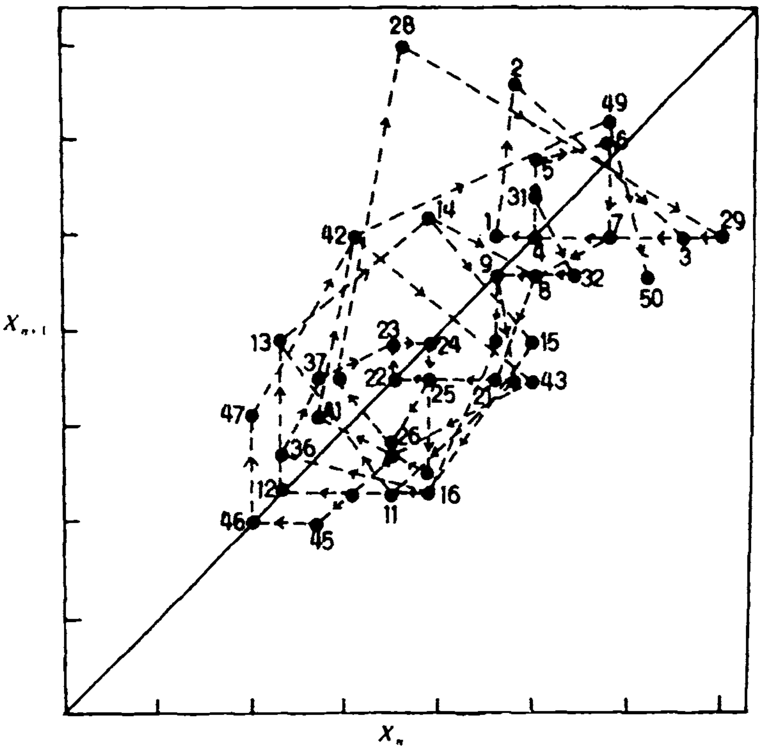

FIGURE 1 Poincaré plot of the data set used in the analysis. The plot is X_{n+1} against X_n. This is the intersection of the surface with the plane.

We can now define a correlation integral in its discrete form as

$$C_d = N^{-2} \sum_{i,j} \Theta(\varepsilon - |X_i - X_j|), \tag{4}$$

where N is the number of data points and Θ is the Heaviside function which is zero when the argument is negative and is unity if it is greater than zero. Hence (4) gives the number of pairs of points whose absolute distance is less than ε—a preassigned quantity. The correlation function (4) is plotted in Figure 2, for the various values of d, for a normal brain. The plots have been made up to $d = 20$, and the higher ones get more and more crowded. There seems to be a tendency for bunching up of the curves.

For sufficently large data sets, and for large embedding dimension d, it is shown (Grassberger & Procaccia, 1983) that

$$C_d(\varepsilon) \sim \varepsilon^{\nu}, \tag{5}$$

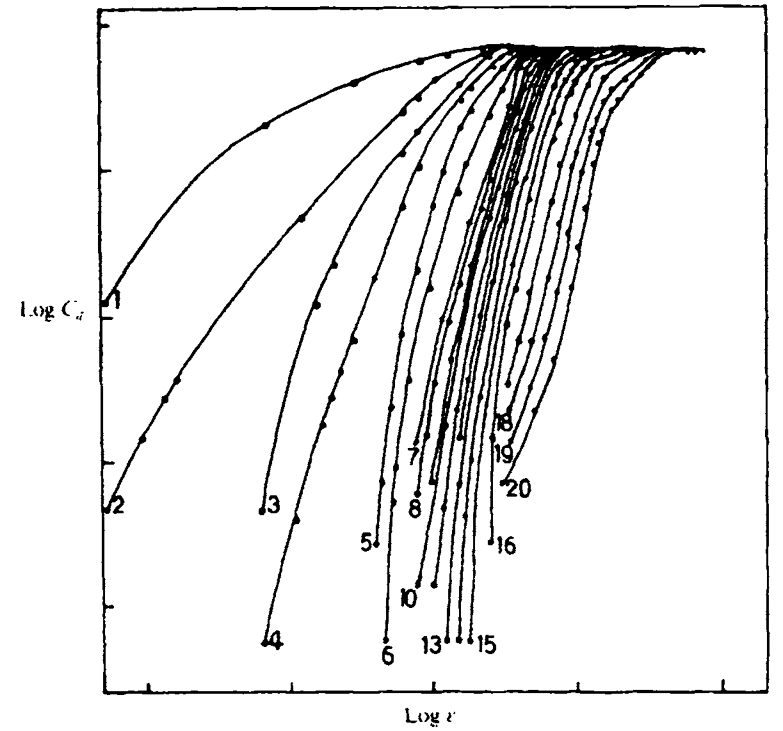

FIGURE 2 Correlation function plot. The higher dimensions are not plotted, they crowd together, and show the same slope.

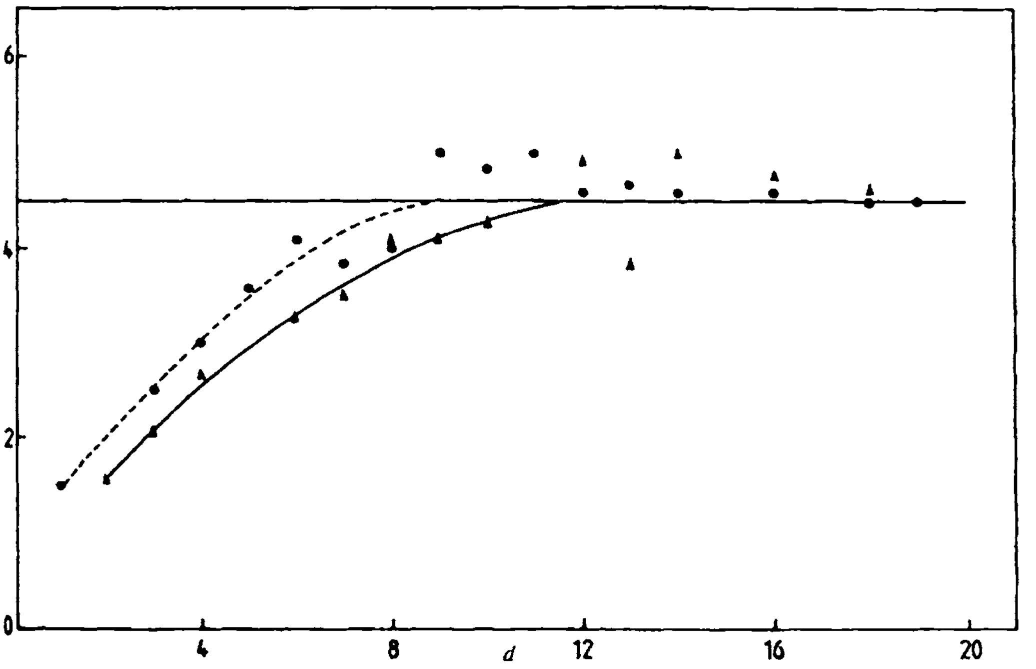

FIGURE 3 Slope versus dimension. The dash curve is for the normal brain and the solid curve is for the one under epileptic seizure. Both curves attain saturation around dimension 10.

and hence a plot of $\log[C_d(\varepsilon)]$ against $\log(\varepsilon)$ would give a slope ν. ν is called the correlation exponent or correlation dimension. Figure 3 gives the plot of slope against dimension d. We have given along with this the curve for the Grand Mal epileptic case (Varghese et al., 1987) for comparison. It may be observed that the asymptotic line gives the same correlation dimension value as 4.5. This could very well be expected as the characteristic dimension of the attractor is a static invariant and hence independent of time scales. In their analysis of EEG of a person with mental activity also Dvorak and Siska (1986) get the same result. However, they get different asymptotic limits depending on the choice of data set, which seems to be peculiar. In the study of Petit Mal epilepsy considered by Babloyantz and Dastexhe (1986), a limiting value of about 2 is obtained. It may well be realized that classification of various of epilepsy is qualitative and that causes for different kinds may indeed be different. Hence, various cases of different values could be indicative of different origins of these malfunctionings.

The second invariant quantity is the Kolmogorov entropy K_2, which is defined as

$$K_2 = \underset{\substack{\varepsilon \to 0 \\ d \to \infty}}{Lt} \tau^{-1} \operatorname{Log}[C_d(\varepsilon)/C_{d+1}(\varepsilon)]. \tag{6}$$

This is a dynamical invariant and is sensitive to time scales within the system. $K_2 = 0$ implies a completely ordered system, while $K_2 = \infty$ indicates total randomness. A finite K_2 is a sufficient condition for deterministic chaos. In Figure 4 we give the plot of K_2 for a normal brain and for comparison we give the same

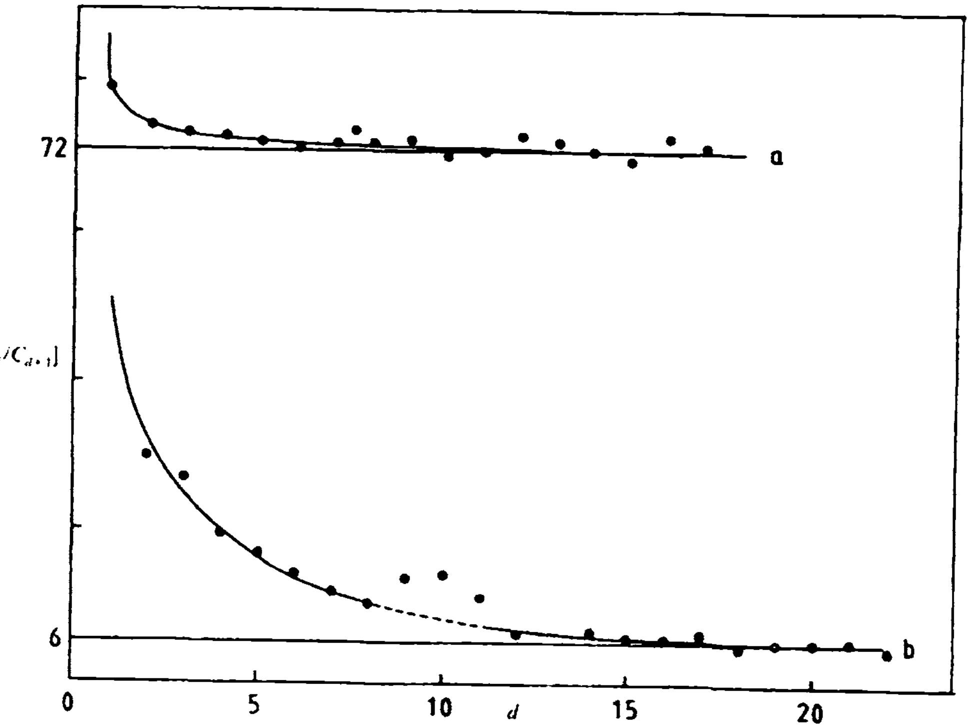

FIGURE 4 The Kolmogorov second entropy given as a function of dimension. Asymptotically, K_2 attains different values thereby showing the variability. (a) Normal brain, (b) epileptic case.

for the Grand Mal epilepsy (Varghese et al., 1987). The present value of K_2 is about 72, while for the epileptic case it is only 6. It becomes obvious that K_2 depends critically on the state of the system. The dependence of time scales and hence K_2 on the dynamics of the system requires a deeper understanding of the mechanism in the system. Hence, these results should be considered only as preliminary. But the result gives sufficient indication to show that this should be the parameter which could be considered as one for classifying the various neural systems.

CONCLUSIONS

The most significant result in this paper is the identification of the Kolmogorov second entropy as a parameter to characterize a neural system. It has recently been shown (Caputo & Atten, 1987) that of all the Kolmogorov entropies, K_2 is the most significant one. Furthermore, K_2 depends on time scales within the system and that the variations of K_2 with the normal and epileptic cases indicate that different time scales play significant roles in this malfunction of the neural system. Again, K_2 in the case of epilepsy, with less scatter and lower in value, signifies that the system is more ordered (in the information sense) than the normal case. In the normal case, K_2 exhibits a small oscillatory behavior in the asymptotic limit, which probably could be traced to spontaneous firings of the neurons. If this is so, then these firings get suppressed in the epileptic case and this needs further investigation.

The characteristic dimension of the attractor has been found to be more or less stationary in our case, while in cases reported elsewhere, they have found significant variations. In the present case we found an attractor of higher dimension 5, while in others, the attractor has been traced to lower dimension 2. This indicates that the present classification of EEGs is more heuristic and that a more rigorous classification is needed. This work is in progress and we shall be reporting this elsewhere. We have, however, shown that Kolmogorov second entropy could be used as a diagnostic tool in the study of EEG. It is also increasingly felt that the Kernal function G given in the general evolution equation of Parkh and Pratap (1984) would take different forms for different processes. Hence G should be considered as a class of functions in explaining the higher processes of brain, rather than a unique function to explain all the different pattern formations in the neural system. This obviously is true, since various time scales in different combinations are responsible for the various functions of the brain, and this is detailed in Pratap (1987).

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