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Numerical study of reverse period doubling route from chaos to stability in a two-mode intracavity doubled Nd-YAG laser

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We have numerically studied the behavior of a two-mode Nd-YAG laser with an intracavity KTP crystal. It is found that when the parameter, which is a measure of the relative orientations of the KTP crystal with respect to the Nd-YAG crystal, is varied continuously, the output intensity fluctuations change from chaotic to stable behavior through a sequence of reverse period doubling bifurcations. The graph of the intensity in the *X*-polarized mode against that in the *Y*-polarized mode shows a complex pattern in the chaotic regime. The Lyapunov exponent is calculated for the chaotic and periodic regions. © *1999 American Institute of Physics.* [S1054-1500(99)01501-3]

Laser systems are good candidates for observing nonlinear effects. It has been experimentally observed that a Nd-YAG laser with an intracavity KTP crystal shows chaotic intensity fluctuations in the frequency doubled output. This behavior is explained considering the multimode operation of the laser with mode–mode coupling. Here we numerically study the experimentally interesting case of the Nd-YAG laser with two orthogonally polarized longitudinal modes. Our results show that the system undergoes a sequence of reverse period doubling route from chaos to stability.

I. INTRODUCTION

Transition to chaos through a sequence of successive period doubling bifurcations[1] in nonlinear dynamical systems has been studied extensively over the last few decades. One of the earliest experimental observations of this period doubling route to chaos was in a CO_2 laser in which cavity loss was modulated by an electro-optic modulator.[2] Period doubling and subsequent remerging of the branches leading to a reverse sequence of period doubling bifurcation in many interesting physical systems have also been reported.[3–6] In the present paper we investigate numerically a model which was introduced for explaining the intensity fluctuations in a diode pumped multimode Nd-YAG laser with an intracavity KTP crystal for frequency doubling. Large amplitude fluctuations in intensity of the frequency-doubled output have been observed when the laser was allowed to operate in several longitudinal modes.[7,8] Since the frequency-doubled output was in the green region this was known as ''the green problem.'' Baer suggested that the sum frequency generation in the KTP crystal could lead to a nonlinear coupling among various longitudinal modes which is essential for the observation of chaos in the system.[9] The steady state solutions of Baer's rate equations and their stability analysis have been done by Wu and Mandel[10,11] and by Oka and Kubota.[12] In a series of papers Roy *et al.* showed that the fluctuations in intensity are manifestations of deterministic chaos in the system.[13–18] The instability of laser system with nonlinear mode–mode coupling was predicted by Arecchi and Ricca.[19]

II. THE MODEL

In order to describe the multimode operations of the laser, Baer modeled the system[10] by a set of coupled nonlinear differential equations. He considered the frequency doubling in the KTP crystal by both the second harmonic generation and the sum frequency generation. The sum frequency generation was assumed to give the necessary coupling between the different modes. Later Bracikowski and Roy[15,16] observed that the intensity fluctuations could be eliminated by suitably changing the relative orientation between the fast axes of the Nd-YAG and the KTP crystals. They modified the earlier model by introducing a geometric factor, which is a measure of the relative orientation and obtained the following set of coupled differential equations for the intensity I_k and gain G_k associated with the kth longitudinal mode,

$$\tau_c \frac{dI_k}{dt} = \left(G_k - \alpha - g\epsilon I_k - 2\epsilon \sum_{j\neq k} \mu_{jk} I_j \right), \tag{1a}$$

$$\tau_f \frac{dG_k}{dt} = \gamma - \left(1 + I_k + \beta \sum_{j\neq k} I_j \right) G_k . \tag{1b}$$

Here $k=1,2,...,n$ is the mode number, τ_c is the cavity round trip time (0.2 ns), τ_f is the fluorescence lifetime of the Nd^{3+} ion (240 μs), α is the cavity loss parameter (0.01), γ is the small signal gain related to the pump rate (0.05), β is the cross saturation parameter (0.7), ϵ is a nonlinear coefficient describing the conversion efficiency of the crystal (5×10^{-6}) and g is a geometrical factor which depends on the phase delays of the YAG and KTP crystal as well as on the angle between their fast axes. The value[15] of g varies from 0 to 1. Each of the cavity modes can be polarized in either one of the two orthogonal directions (X or Y). For modes j having the same polarization as the kth mode $\mu_{jk}=g$ and for modes having orthogonal polarization $\mu_{jk}=(1-g)$.

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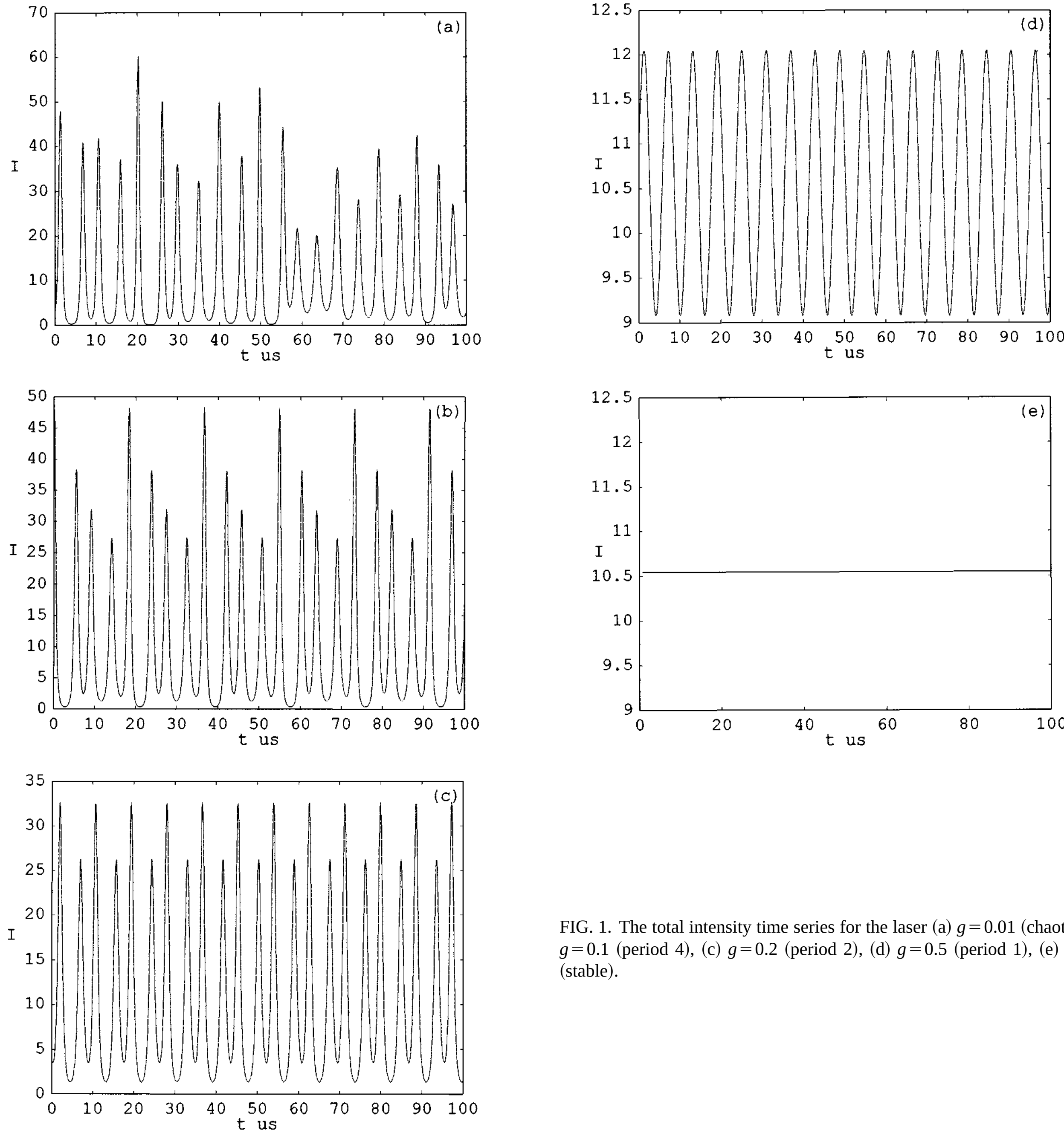


FIG. 1. The total intensity time series for the laser (a) $g=0.01$ (chaotic), (b) $g=0.1$ (period 4), (c) $g=0.2$ (period 2), (d) $g=0.5$ (period 1), (e) $g=0.6$ (stable).

III. NUMERICAL RESULTS

We have numerically studied the system for the case in which the laser is operating in two modes with orthogonal states of polarization (X and Y). Equations (1) were numerically integrated using the Runge–Kutta fourth order method for different values of g (the parameter which depends on the relative orientation between the Nd-YAG and KTP crystal) and the results are shown in Figs. 1–4. When g is very small the numerically obtained total infrared intensity showed chaotic fluctuation. As the value of g is increased the fluctuations become periodic and finally become stable through a sequence of reverse period doubling bifurcations. In Fig. 1 we have plotted the variations in the total intensity against time for different values of g. Figure 1(a) shows the chaotic intensity variations corresponding to $g=0.01$. Figure 1(b) shows periodic variations with period 4 for $g=0.1$ and Figs. 1(c) and 1(d) show periodic variations with period 2 and period 1, respectively, corresponding to $g=0.2$ and $g=0.5$. As g is increased still further, the variations in intensity gradually decreases and finally become stable. Figure 1(e) shows the stable behavior of the laser intensity corresponding to $g=0.6$. It is found to be stable for higher values of g. It was analytically showed that when $g=1$ there was no sum frequency generation and hence there was no nonlinear coupling between the different modes so that the output intensity was stable.[18]

Figure 2 shows the phase portraits for the laser. Here the total infrared intensity is plotted against total gain for different values of g. The phase portrait shows a complex behavior in the chaotic region as shown in Fig. 2(a) for $g=0.01$. As

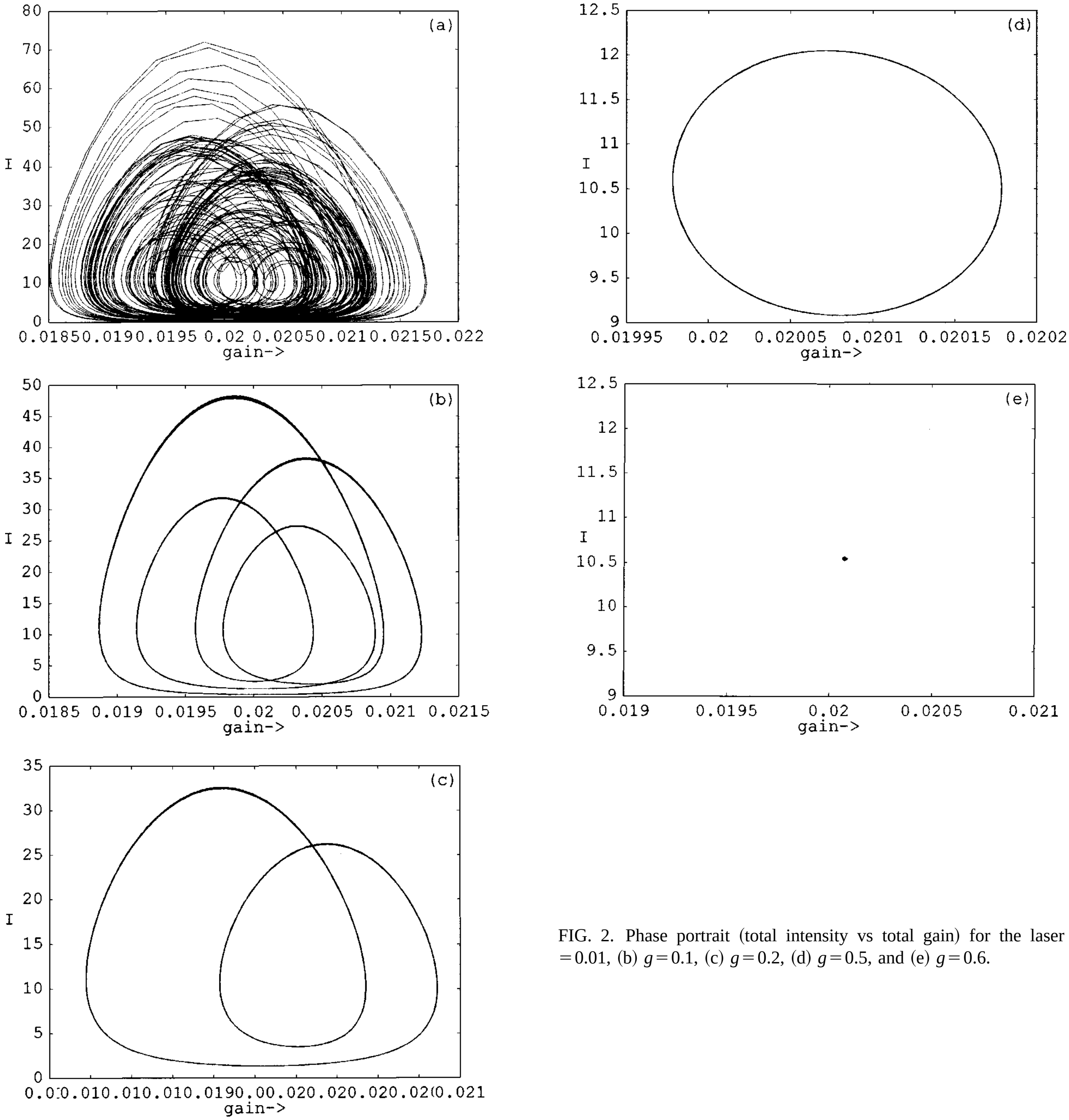


FIG. 2. Phase portrait (total intensity vs total gain) for the laser (a) $g = 0.01$, (b) $g = 0.1$, (c) $g = 0.2$, (d) $g = 0.5$, and (e) $g = 0.6$.

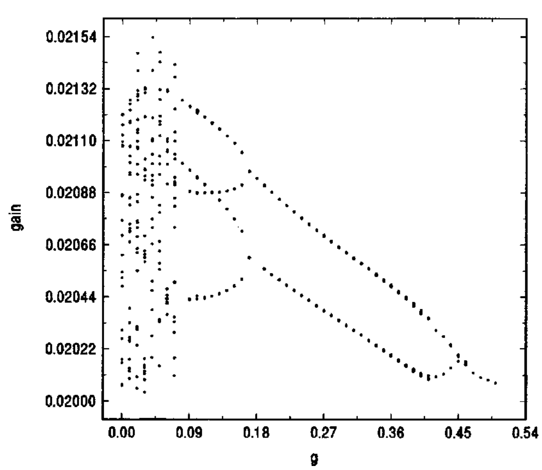
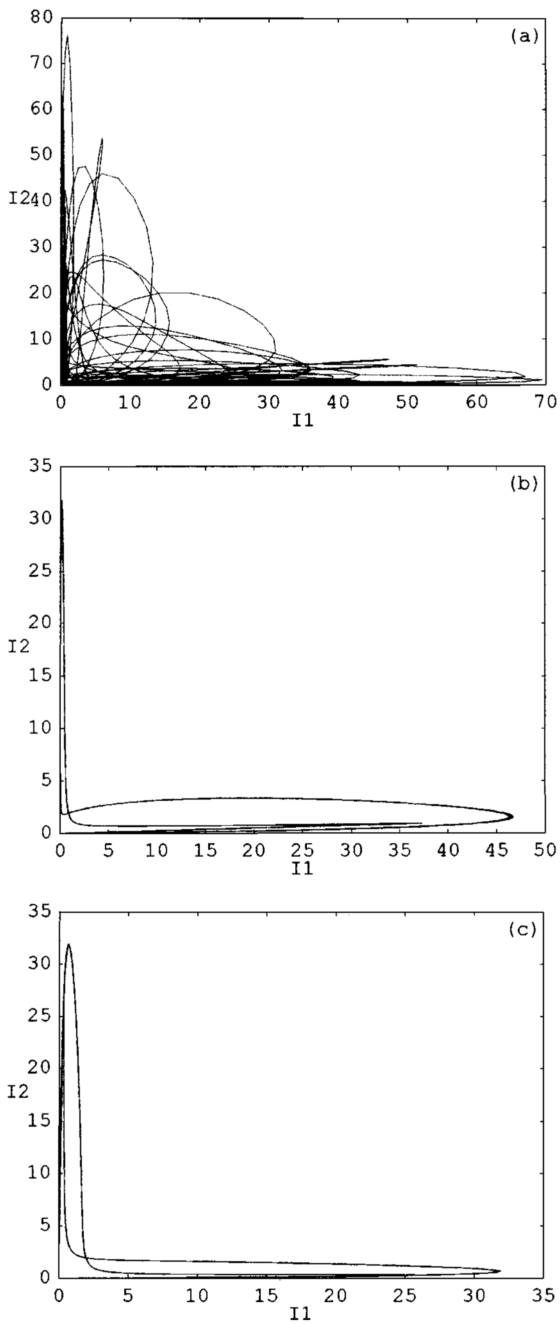


FIG. 3. Bifurcation diagram for the laser.

the value of g is increased it changes to four loops for $g = 0.1$ [Fig. 2(b)], corresponding to the presence of period four oscillation of intensity and to double and single loops for $g = 0.2$ and 0.5 corresponding to, respectively, two and one periodic oscillations of intensity [Figs. 2(c) and 2(d)]. At $g = 0.6$ it becomes a single point showing stable operation. The reverse period doubling sequence is also evi-

TABLE I. Lyapunov characteristic exponent for the laser.

Value of g	Lyapunov exponent
0.01	$+2.5475\times10^{4}\ s^{-1}$
0.1	$-2.6323\times10^{3}\ s^{-1}$
0.2	$-2.0263\times10^{3}\ s^{-1}$
0.5	$-2.5475\times10^{-3}\ s^{-1}$



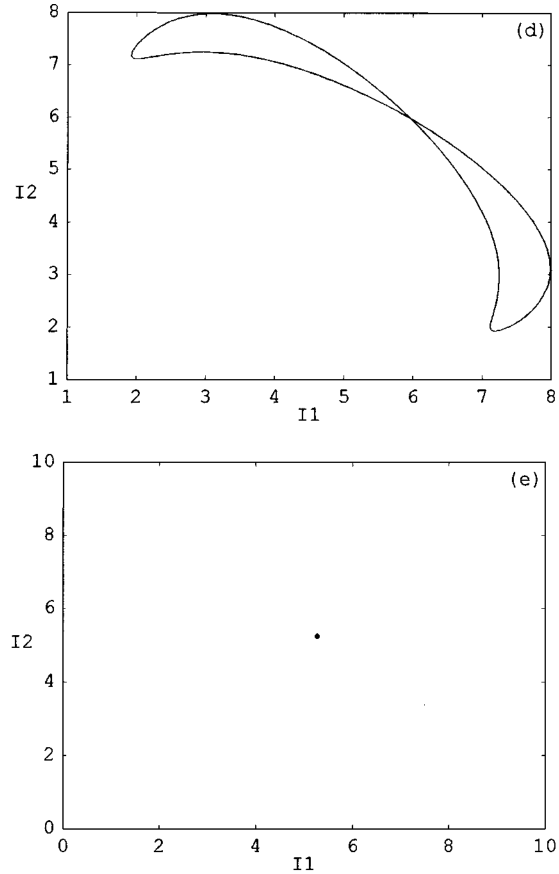


FIG. 4. Plot of intensity in the *X*-polarized direction against that in the *Y*-polarized direction (a) $g=0.01$, (b) $g=0.1$, (c) $g=0.2$, (d) $g=0.5$, and (e) $g=0.6$.

dent from the bifurcation diagram for the laser shown in Fig. 3.

In Fig. 4 the intensity in the *x*-polarized mode is plotted against that in the *y*-polarized mode for the different values of *g*. It forms a complex pattern corresponding to $g=0.01$ [Fig. 4(a)] which shows that the exchange of energy between the two modes takes place in a chaotic manner, while it is a closed loop corresponding to $g=0.1$, 0.2 and 0.5 showing a periodic energy exchange between the modes [Figs. 4(b)–4(d)]. Again, in the stable region it is a single point showing that there is no exchange of energy between the modes [Fig. 4(e)].

In order to characterize the chaotic and periodic regions we have calculated the maximum Lyapunov characteristic exponent (LCE) of the numerically obtained time series of the total infrared intensity. The Wolfs *et al.*[20] algorithm is used for the calculation of the LCE. The LCE for different values of *g* are tabulated in Table I. For $g=0.01$ the LCE is positive, showing the intensity variations are chaotic while it is negative for $g=0.1$, $g=0.2$, and $g=0.5$ corresponding to periodic behavior.

IV. CONCLUSION

Our numerical studies show that the Nd-YAG laser with intracavity frequency doubling crystal operating in a two-mode regime will exhibit a reverse sequence of period doubling bifurcation from chaotic to stable behavior when the value of *g* is changed from 0 to 1. The range of *g* values for which the chaotic behavior can be observed is very small in the two mode case compared to the case with more than two modes.[21] It may be the reason why only periodic behavior has been observed experimentally when the laser is operating in two longitudinal modes.[16]

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