MATHEMATICAL MODELLING OF THE DYNAMICS OF GRANULAR MATERIALS IN A ROTATING CYLINDER

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SCIENCE

by

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under the supervision of

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Declaration

I, Hema V., do hereby declare that the matter embodied in this thesis entitled "**Mathematical Modelling of the Dynamics of Granular Materials in a Rotating Cylinder**" is the result of the investigations carried out by me in the Computational Modelling and Simulation Unit of the Regional Research Laboratory (CSIR), Thiruvananthapuram, under the guidance of **Dr. S. Savithri**, and that the same has not been submitted elsewhere for any other degree.

In keeping with the general practice of reporting scientific observations, due acknowledgement has been made wherever the work described is based on the findings of other investigators.

Hema

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CERTIFICATE

This is to certify that the work outlined the thesis entitled "**Mathematical Modelling of the Dynamics of Granular Materials in a Rotating Cylinder**" is an authentic record of the research work carried out by **V. Hema** under my supervision in partial fulfillment of the requirements for the degree of **Doctor of Philosophy** of the **Cochin University of Science and Technology** and further that no part thereof has been submitted elsewhere for any other degree.

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Abstract

A fundamental understanding of mixing and blending of granular materials in horizontal rotating cylinders will be beneficial to a wide range of industries: pharmaceuticals, metallurgy, ceramics, composites, polymers, food processing and agriculture. Yet, in relation to its industrial prevalence, the understanding of granular mixing lags considerably when compared to that of liquid mixing.

Granular bed motion in the transverse cross section of a horizontal rotating cylinder is fundamental in determining advective heat transfer as well as axial flow of the bed material; but to date studies have been mainly empirical aimed at determining the macroscopic bed motion but devoid of detailed information of granular bed dynamics at a microscopic level.

The current study is therefore aimed at the development of a theoretical simulation tool based on Discrete Element Method (DEM) to interpret granular dynamics of solid bed in the cross section of the horizontal rotating cylinder at the microscopic level and subsequently apply this model to establish the transition behaviour, mixing and segregation.

The simulation of the granular motion developed in this work is based on solving Newton's equation of motion for each particle in the granular bed subjected to the collisional forces, external forces and boundary forces. At every instant of time, the forces are tracked and the positions, velocities and accelerations of each particle is

The software code for this simulation is written in VISUAL FORTRAN 90. After checking the validity of the code with special tests, it is used to investigate the transition behaviour of granular solids motion in the cross section of a rotating cylinder for various rotational speeds and fill fraction.

This work is hence directed towards a theoretical investigation based on Discrete Element Method (DEM) of the motion of granular solids in the radial direction of the horizontal cylinder to elucidate the relationship between the operating parameters of the rotating cylinder geometry and physical properties of the granular solid.

The operating parameters of the rotating cylinder include the various rotational velocities of the cylinder and volumetric fill. The physical properties of the granular solids include particle sizes, densities, stiffness coefficients, and coefficient of friction. Further the work highlights the fundamental basis for the important phenomena of the system namely; (i) the different modes of solids motion observed in a transverse cross-section of the rotating cylinder for various rotational speeds, (ii) the radial mixing of the granular solid in terms of active layer depth (iii) rate coefficient of mixing as well as the transition behaviour in terms of the bed turnover time and rotational speed and (iv) the segregation mechanisms resulting from differences in the size and density of particles.

The transition behaviour involving its six different modes of motion of the granular solid bed is quantified in terms of Froude number and the results obtained are validated with experimental and theoretical results reported in the literature. The transition from slumping to rolling mode is quantified using the bed turnover time and a linear relationship is established between the bed turn over time and the inverse of the rotational speed of the cylinder as predicted by Davidson *et al.* [2000]. The effect of the rotational speed, fill fraction and coefficient of friction on the dynamic angle of repose are presented and discussed. The variation of active layer depth with respect to fill fraction and rotational speed have been investigated. The results obtained through simulation are compared with the experimental results reported by Van Puyvelde *et. al.* [2000] and Ding *et al.* [2002].

The theoretical model has been further extended, to study the mixing and segregation in the transverse direction for different particle sizes and their size ratios. The effect of fill fraction and rotational speed on the transverse mixing behaviour is presented in the form of a mixing index and mixing kinetics curve. The segregation pattern obtained by the simulation of the granular solid bed with respect to the rotational speed of the cylinder is presented both in graphical and numerical forms. The segregation behaviour of the granular solid bed with respect to particle size, density and volume fraction of particle size has been investigated. Several important macro parameters characterising segregation such as mixing index, percolation index and segregation index have been derived from the simulation tool based on first principles developed in this work.

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CHAPTER 1
INTRODUCTION

1.1 Granular materials: A fascinating field

Granular materials are large conglomerations of discrete microscopic particles of varying size, shape and material properties, which displace from one another and interact only at the contact points. Granular materials are obiquitous; a few examples include food products such as rice, corn and breakfast cereal flakes, building materials (sand, gravel and soil) and chemicals (coal, plastics and pharmaceuticals). They play an important role in several of our industries such as mining, agriculture, civil engineering and pharmaceutical engineering. They are also important in geological processes where landslides and erosion occur on a larger scale.

The behaviour of granular materials is determined by the material and geometric properties of the particles at the contact points. Depending on these conditions the granular materials exhibit properties reminiscent of the various states of matter; they may be deformed as solids, may have flowability similar to liquids or compressibility like that of gases. For example, a sand pile at rest with a slope lower than the angle of repose, behaves like a solid: the material remains at rest even though gravitational forces create macroscopic stresses on its surface. If the pile is tilted several degrees above the angle of repose, grains start to flow. However, this flow is clearly not that of an ordinary fluid because it only exists in a boundary layer at the pile's surface with no movement in the bulk.

A distinguishing feature of granular flows over other flows of solid-fluid mixtures is that in granular flows, the direct interaction of particles plays an important role in the flow mechanics. The dynamics of granular materials are highly dissipative and a significant fraction of the energy dissipation and momentum transfer in granular flows occur when particles are in contact with each other or with a boundary. Unlike the flows of solid-liquid mixtures, the temperature has no effect on grain motion since external forces such as gravity dominate the behaviour. Further the frictional interactions between the individual grains are highly nonlinear and, for static friction, even discontinuous. Also the particle size in a granular system is comparable to the length scale of flow variation; for example, the patterns such as waves resulting from the flow of granular materials occur on scales only 10-100 times that of the individual grain. Due to these unique properties exhibited by granular systems they are usually considered as an additional state of matter in its own right.

The unusual and unique character exhibited by granular material systems have led to a resurgence of interest within several scientific and engineering disciplines ranging from physics, soil mechanics and chemical engineering (Jaeger and Nagel [1992]; Behringer [1993, 1995]; Bideau and Hansen [1993]; Jaeger *et al.* [1994, 1996a, 1996b]; Mehta [1994]; Hayakawa *et al.* [1995]).

The science of granular media has a long history. Much of the engineering literature has been devoted to understanding how to deal with these materials. Notable contributions in the literature include Coulomb [1773], who proposed the ideas of static friction; Faraday [1831], who discovered the convective instability in a vibrated container filled with powder, and Reynolds [1885], who introduced the notion of dilatancy, which implies that a compacted granular material must expand in order for it to undergo shear.

Finally, there is another vitally important reason for the recent activity in this field. As mentioned above, many of our industries rely on transporting and storing granular materials. These include the pharmaceutical industry which relies on the processing of powders and pills, agriculture and the food processing industry where seeds, grains and foodstuffs are transported and manipulated, as well as all construction based industries. Similarly, manufacturing processes in the automotive industry rely on casting large metal parts in carefully packed beds of sand. Thus it is important to develop appropriate technology for handling and controlling granular materials effectively, and this still remains as an area not yet fully explored.

1.2 Why study granular materials?

A good understanding of the physics of granular materials is desirable in designing efficient processing and handling systems. The significance of this is apparent when one considers the following statistics:

- In chemical industry approximately one-half of the products and at least three-quarters of the raw materials are in granular form (Neddermann [1992]).
- Ennis *et al.* [1994] estimate that $61 billion in the chemical industry is linked to particle technology.
- Approximately 1.3% of the U.S. electrical power production goes toward grinding particles or ores (Ennis *et al.* [1994]).
- Landslides cause a minimum of $1.5 billion dollars of property damage and at least 25 fatalities in the United States annually.
- Each year over 1,000 silos, bins, and hoppers fail in North America (Knowlton *et al.* [1994]).
- In Mexico, 5 million tons of corn is handled each year, 30% of which is lost due to poor handling systems.

In the chemical industry alone, it is estimated that half the products and three-quarters of the raw materials are in the form of particulates (Neddermann [1992]) and also enormous costs are associated with handling these materials. For example, a