A VARIATIONAL CALCULATION OF TE AND TM CUTOFF WAVENUMBERS IN CIRCULAR ECCENTRIC GUIDES BY CONFORMAL MAPPING

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ABSTRACT: *The cutoff wavenumbers of higher order modes in circular eccentric guides are computed with the variational analysis combined with a conformal mapping. A conformal mapping is applied to the variational formulation, and the variational equation is solved by the finite-element method. Numerical results for TE and TM cutoff wavenumbers are presented for different distances between the centers and ratio of the radii. Comparisons with numerical results found in the literature validate the presented method.* © 2001 John Wiley & Sons, Inc. Microwave Opt Technol Lett 31: 381–384, 2001.

Key words: *circular eccentric guides; conformal mapping; variational analysis; finite-element method*

I. INTRODUCTION

Many investigations on the calculation of TE and TM cutoff wavenumbers in circular eccentric guides have been carried out. Analytical formulations were developed and used to calculate exact values of the cutoff wavenumbers employing different methodologies in finding the roots of the characteristic equations [1–4].

Kuttler [5] obtained the weighted Helmholtz equations using a conformal mapping. He found the lower bounds of the cutoff wavenumbers using the intermediate methods and the upper bounds of the cutoff wavenumbers using the Rayleigh–Ritz method. Zhang, Zhang, and Wang [1] found that most of Kuttler's upper bounds were close to the exact values. From the conceptual and computational point of view, the Rayleigh–Ritz method is found to be complex, even though it is a powerful technique. On the other hand, the variational formulation together with the finite-element method is found to be much simpler than the Rayleigh–Ritz method. But the mesh generation processes of waveguides having complex boundaries are very laborious.

In this paper, the conformal mapping used by Kuttler is applied to the eigenvalue problems of circular eccentric guides, and the variational form is developed and solved by the finite-element method. The cutoff wavenumbers of TE and TM modes in several circular eccentric guides are calculated by this procedure. The numerical results are compared with those by other authors [1, 5]. It is found that this approach is a very simple and powerful method to find the cutoff wavenumbers of higher order modes in waveguides whose mapping functions are known as analytical.

II. METHOD OF ANALYSIS

A. Conformal Mapping Applied to the Variational Formulation. The cutoff frequencies of the higher order modes in a uniform waveguide with an arbitrary cross section and homogeneous medium are derived from the eigenvalues k of the Helmholtz equation

$$(\nabla_t^2 + k^2)\phi = 0 \tag{1}$$

where $k^2 = \omega^2\mu_0\varepsilon_0 + k_z^2$, and $\phi = H_z$ or E_z for TE or TM modes, respectively. H_z and E_z are the longitudinal components of magnetic and electric fields. The governing variational equation of the wave equation (1) is well known as

$$\delta F = 0$$

$$F = \frac{1}{2}\iint_\Omega \left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 - k^2\phi^2\,dx\,dy. \tag{2}$$

The relationship between the original coordinate $w = (x, y)$ and the new coordinate $z = (x', y')$ is defined by an analytical complex function:

$$z = f(w). \tag{3}$$

By conformal mapping (3), the variational equation (2) becomes, in the new coordinate system,

$$\delta F = 0$$

$$F = \frac{1}{2}\iint_{\Omega'} \left(\frac{\partial\phi}{\partial x'}\right)^2 + \left(\frac{\partial\phi}{\partial y'}\right)^2 - k^2|J|\phi^2\,dx'\,dy' \tag{4}$$

where the Jacobian $|J|$ is related to the complex function by

$$|J| = \left|\frac{dw}{dz}\right|^2_{z=(x',y')}. \tag{5}$$

Consider a uniform waveguide with a circular outer conductor and an eccentric circular inner conductor as in Figure 1. The analytic function

$$w = R_1 \sinh x_1 \coth\frac{z}{2} \tag{6}$$

transforms the cross section in the w-plane (Fig. 1) to a rectangle in the z-plane (Fig. 2).

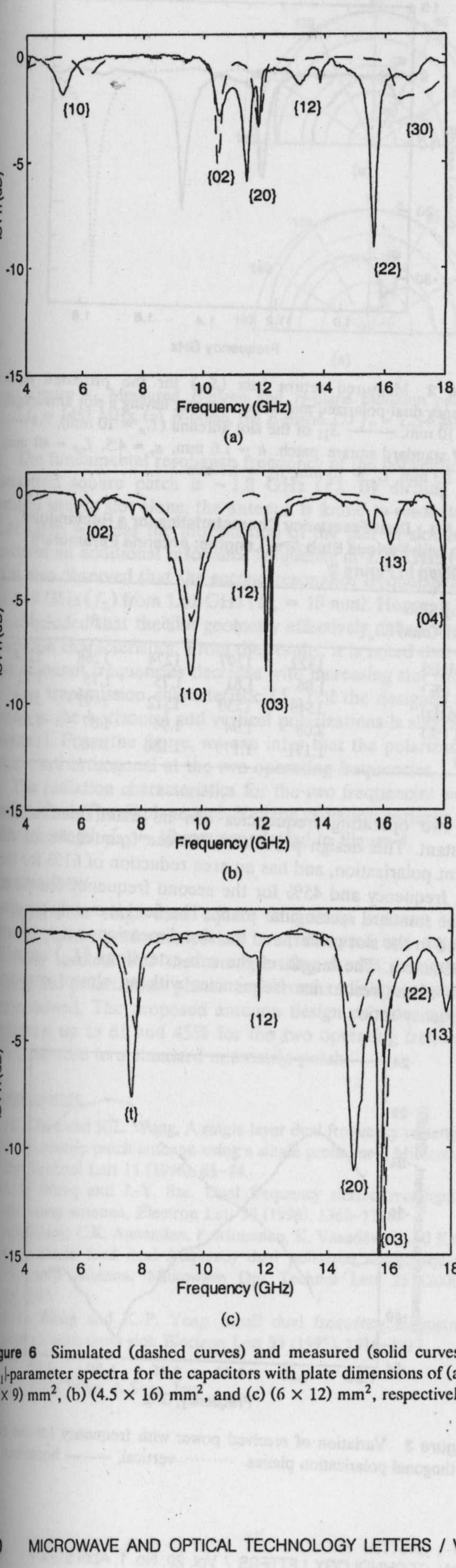

Figure 6 Simulated (dashed curves) and measured (solid curves) $|S_{11}|$-parameter spectra for the capacitors with plate dimensions of (a) $(8 \times 9)\ mm^2$, (b) $(4.5 \times 16)\ mm^2$, and (c) $(6 \times 12)\ mm^2$, respectively

the measurements of S_{11}-parameter spectra of planar capacitors, and good agreement was found.

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