

**S.m.14. MADHUSOODAN, T.P.—Time dependent solutions for some queuing and inventory models—1989—  
Dr. A. Krishnamoorthy**

In this thesis we study the time dependent behaviour of some complex queuing and inventory models. In our analysis-renewal theory plays an important role. In each model, identifying the regeneration points and using matrix convolutions, we obtain the required transition probability densities.

The thesis is divided into seven chapters and the first chapter gives a brief introduction to the subject matter and some related topics. In the second chapter we consider a finite capacity bulk service queuing system. The arrival of customers is according to a homogeneous Poisson process and the service times are generally distributed independent random variables and the distribution is depending on the size of the batch being served (i.e.  $M/G^{(b)}/1$  System). Using renewal theoretic arguments, we derive the probability distribution of the busy period and the time dependent system size probabilities at arbitrary epochs. Also we derive the probability distribution for the virtual waiting time in the queue at any time  $t$ .

Next chapter deals with an infinite capacity  $M^x/G/1$  queuing system with vacations to the server. Under exhaustive service discipline, server takes vacations for a random duration having general distribution following multiple vacation rule. Here also we derive the time dependent system size probabilities at arbitrary epochs and the probability distribution of the virtual waiting time.

A finite capacity  $M/G^{(b)}/1$  queuing system with vacations to the server is analysed in chapter four. The general bulk service rule is modified to allow the arriving customers to enter for service. Arriving customers enter for partial service with probability  $p$  and wait for full service with probability  $1-p$ , till the service capacity is attained. Server goes for vacation whenever there are less than 'a' customers in the system and this is a multiple vacation. The time dependent system size probabilities and the probability distribution of virtual waiting time are derived explicitly.

In chapter five, a finite capacity  $M/G^{(b)}/1$  vacation system with Bernoulli schedules is analysed. After each service completion server starts a new service, if customers are present, with probability  $p$  and takes a vacation with probability  $1-p$ . If the system is empty at a service completion point or vacation completion point, server always takes vacation. At vacation completion points, if there is at least one customer server resumes service. Here also we derive explicit expressions for the time dependent system size probabilities and the probability distribution of the virtual waiting time.

In the next two chapters we analyse some single item continuous review (s,S) inventory systems with random lead times. In chapter six, we study an inventory system in which the quantity replenished is a random variable. The inter arrival time  $\alpha$ , unit demands and lead times are independent sequences of i.i.d. random variables following general distribution. Order is placed at level  $s$  for  $S-s$  units and the quantity replenished is a random variable that can assume values

$s+1, \dots, S-s$ . We derive explicit expressions for the probability mass function for the stock level at arbitrary epochs. An expression for the total cost over a period of time is obtained. Then we consider the special case of zero lead time and discuss the associated optimization problem in detail.

In the last chapter, we consider an  $(s, S)$  inventory system with random lead time depending on the size of the order. The time between successive demands and the quantities demanded at these points are independent sequences of i.i.d. random variables following general distribution. Whenever the inventory level falls to zero, the server goes for vacation for a random duration following a general distribution. Using renewal theoretic arguments, we derive explicit expressions for the inventory level probabilities at arbitrary epochs. Some directions for further work are given in the last section.