Studies on accelerating universe with bulk viscous Zel'dovich fluid

Thesis submitted to
Cochin University of Science and Technology
in partial fulfillment of the requirements
for the award of the degree of

DOCTOR OF PHILOSOPHY

by

Rajagopalan Nair K
Department of Physics
Cochin University of Science and Technology
Kochi - 682022

May 2019

Studies on accelerating universe with bulk viscous Zel'dovich fluid
PhD thesis in the field of Cosmology

Author
Rajagopalan Nair K
Department of Physics
Cochin University of Science and Technology
Kochi-682022
rajagopal_physics_61@cusat.ac.in
rajagopal_61@yahoo.co.in

Research Supervisor
Prof. Titus K Mathew
Department of Physics
Cochin University of Science and Technology
Kochi-682022
titus@cusat.ac.in

Dedicated to my family and friends.

CERTIFICATE

Certified that the work presented in this thesis entitled "Studies on accelerating universe with bulk viscous Zel'dovich fluid" is a bonafide research work done by Mr. Rajagopalan Nair K, under my guidance in the Department of Physics, Cochin University of Science and Technology, Kochi-682022, India and has not been included in any other thesis submitted previously for the award of any degree. All the relevant corrections and modifications suggested by the audience during the pre-submission seminar and recommendations by the doctoral committee have been incorporated in this thesis.

Kochi - 682022
May, 2019

Prof. Titus K. Mathew
(Supervising Guide)

DECLARATION

I hereby declare that the work presented in this thesis entitled "Studies on accelerating universe with bulk viscous Zel'dovich fluid" is based on the original research work done by me under the guidance of Prof. Titus K. Mathew, Department of Physics, Cochin University of Science and Technology, Kochi- 682022, India, and has not been included in any other thesis submitted previously for the award of any degree.

Kochi - 682022 — Rajagopalan Nair K.

May, 2019

PREFACE

Cosmological observations provide an increasingly precise picture of our universe. The first giant step towards this was by Edwin Hubble who obtained distances to various galaxies external to Milkyway and their velocities using Cepheid variable star as the standard candle. This ultimately led to the realization that the universe is expanding. Besides this, it led to the understanding of the constituents of the universe. The baryon density was found to be roughly 5% of the total content of the universe. Further a multitude of observations such as galaxy rotation curves, cosmic microwave background radiation, baryon acoustic oscillations etc, indicated that 23% of the total density of the cosmic component is in the form of weakly interacting matter called dark matter.

Yet another spectacular discovery followed in the year 1998 as two teams - the Supernova Cosmology Project led by Saul Perlmutter and the High-Z Supernova Search Team lead by Brian P. Schmidt and Adam G. Riess based on their observations on Type I a Supernovae, independently reported that the expansion of the universe is accelerating. This seems to be indicating further that about 72% of the total density is in the form of yet another exotic component called dark energy.

Even though the exotic components like dark matter and dark energy are adequate for the explanation of the known evolution of the universe so far, the cosmological data doesn't rule out the existence of other exotic cosmic components. For example, it was observed by S. Dutta et al that the presence of dark radiation in the cosmos, responsible for interaction between proposed particles of dark matter, will not go against any prevailing data. Another exotic component not violating the data on the universe is the Zeldovich fluid proposed by Ya B. Zeldovich, which is the stiffest possible fluid reaching optimum pressure, having the speed of sound in it equaling that of light. Its equation of state is $\frac{p_z}{\rho_z} = 1$, where ρ_z and p_z are density and pressure of the stiff fluid respectively. There had been several theoretical models, such as by R. Stiele et al, P. Horava, E. Kiritsis et al, T.P. Sotiriou et al, C. Bogdanos et al, A. Ali et al, S. Dutta et al, J.D. Barrow, and L. Fernandez-Jambrina et al speculated the presence of Zeldovich fluid in the early universe. The equation of state fo the Zeldovich, through cosmological conservation implies that, the density of this fluid behaves as, $\rho_z \propto a^{-6}$. Due to this ever faster decrease in the density of Zeldovich fluid, if at all it has any effect on the evolution of the universe it must in the early stage. The process of primordial nucleosynthesis, an important process which took place in the early stage of the universe has been invoked by S. Dutta and R.J. Scherrer to constrain the density of the Zeldovich fluid as $\rho_z/\rho_\gamma < 30$, at a fiducial temperature of 10 MeV, where ρ_γ is the density of radiation in the early stage of the universe. As $\rho_z \propto a^{-6}$, the role of Zeldovich fluid is insignificant in the late universe. However, an extended model of Zeldovich fluid by incorporating a bulk viscosity can alter the density evolution of the universe, which can have appreciable effect in the late universe. Our thesis is aiming at a detailed analysis of this aspect in the context of the late acceleration of the universe.

The late acceleration of the universe was discovered around 1998 by two teams, one team led by Saul Perlmutter and the other by Adam Riess, by observing the Type Ia, supernovae. The acceleration was thought to be caused by an exotic component of the fluid called dark energy, whose nature is still unknown. The successful candidate for dark energy is the cosmological constant, which lead to the most successful model of the late accelerating universe, the standard ΛCDM model. Despite its highly laudable success, the model has two major flaws: (i) the huge discrepancy between the theoretical and observed value of the density of the Λ called Cosmological constant problem and (ii) cosmological coincidence problem, the observation that the order of magnitude of the present densities of the constituents cold dark matter and dark energy are the same. This paved way to a variety of dynamical dark energy models like Quintessence, K-essence, Tachyon field, Phantom ghost field, Dilatonic dark energy, Chaplygin gas model, Holographic dark energy model, etc. To explain the accelerating universe there had been attempts with alternative theories of gravity, such as $f(R)$ gravity, $f(T)$ gravity, Gauss-Bonnet theory, Lovelock gravity, Horava- Lifshitz gravity, scalar-tensor theories, braneworld model, etc. Despite this intense theoretical physics activities, the nature of dark energy still continues to be a mystery. An alternative way of understanding the late acceleration without invoking to any exotic dark energy form is by incorporating viscosity into the matter sector. There were attempts to explain the early inflation by using viscous fluids by Padmanabhan and others. In the context of late acceleration also the viscous models have been analyzed extensively by many. In the present thesis we study the late universe having bulk viscous Zeldovich fluid as the dominant component. We analyzed the background evolution and found that it is predicting a transition from a prior decelerated epoch to a late accelerated epoch by accounting for the cosmological parameters in a reasonable way. The model was constrained with the supernovae data thus extracted the values of the corresponding model parameter including the present value of the Hubble parameter. A statefinder analysis of the model by using the conventional parameter (r, s) has also been performed, which shows that our model is arguably different from the ΛCDM model. We further studied the dynamical system behavior of the model, thus obtained the nature of the asymptotic behavior. We also explore the thermodynamic evolution of the model in detail, which implies that the universe evolves as an ordinary macroscopic system with an upper bound to the growth of entropy. It was emerged from our analysis that, the model predicts under age for the current universe. To alleviate this problem, we extended the model by incorporating the decaying vacuum energy as an additional cosmic component. By this analysis we found the previous age problem can be solved to a great extend. The thesis consists of five chapters. Underneath, we give a brief account of the facts presented in different chapters of the thesis.

Chapter 1

This chapter gives an introduction to standard cosmology, with emphasize on the ΛCDM model. It begins with Einstein's field equations and ends by briefing with the disadvantages of the ΛCDM model, alongside its successful features of explaining the cosmic dynamics. This chapter also briefly discusses the discovery of accelerating universe, and motivates the bulk viscous model of the universe.

Chapter 2

This chapter describes our work on the background evolution of the late universe dominated with bulk viscous Zeldovich fluid. We describe the evolution of the normal Zeldovich (stiff) fluid. Following this we briefly discuss the origin of bulk viscosity and negative pressure which can arise from the adiabatic thermodynamical disturbances and due to the tendency of the overall cosmos towards restoration of equilibrium. We adopt the Eckart's approach in accounting the viscosity, using which we analytically obtain expressions for scale factor, acceleration, q factor and the state function ω and obtain their evolution profiles for various possible values of bulk viscosity coefficient $\bar{\zeta}$, thereby obtaining clues on their range of values compatible with the observed features of the evolution of the late universe. This includes seeing the criteria by which the age of the universe is defined. We also analyze the thermal evolution of the model and the validity of GSL (Generalized Second Law) has been checked. The use of r-s plot as a diagnostic tool is explained, as the evolution of q factor and state function ω are not satisfactory enough to contrast the given model with the standard one. It is discussed how the bulk viscous Zeldovich fluid compares with the ΛCDM model regarding the prediction of future of the universe in terms of its asymptotic limit. Overall, we arrive at the conclusion that the dimensionless bulk viscous coefficient $\bar{\zeta}$ must be $\bar{\zeta} < 6$ for predicting the conventional evolution of the universe. For $0 < \bar{\zeta} < 4$ the transition point of acceleration from a prior decelerated epoch is found to be occurred in future. For $\bar{\zeta} = 4$ the transition is at present. The transition would appear in the past for $4 < \bar{\zeta} < 6$. For $\bar{\zeta} \geq 6$ the cosmos is eternally accelerating, having no big bang. The analytically obtained age of the universe is ~ 13.7 GY(Giga Years) for $\bar{\zeta} \sim 5.7$.

Chapter 3

This chapter is devoted to the analysis of the asymptotic behavior of the model. First we have obtained the best fit for value of $\bar{\zeta}$ and the present value of Hubble parameter H_0 by contrasting the model with the standard Union 307 data of type Ia supernovae. This is done by a $\chi-$square analysis method. We found that Hubble parameter is $H_0 = 70.20 \pm 0.58$ km s^{-1} Mpc^{-1}, which is comparable to the observed values and that predicted by the standard ΛCDM model. The best fit value of bulk viscous coefficient is 5.25 ± 0.14. Using the best estimated parameters we obtain the redshift at which the universe entered the acceleration as $z \sim 0.6$, which is agreeing with the observations. The model predicts an asymptotic de Sitter phase, agreeing with ΛCDM predictions. However, the age predicted for the mentioned best fit $\bar{\zeta}$ and H_0 is 10 GY which is not reasonable enough, compared to the 13.7 GY obtained from Globular clusters observationally and from ΛCDM model theoretically. The dynamical system analysis shows that the model possess stable critical point corresponding to the end de Sitter epoch. However, when the radiation is included as an additional cosmic component there arise no stable critical point and hence the model does not accommodate the radiation dominated era.

Chapter 4

In this chapter, we discuss the extended Zeldovich fluid model by including decaying vacuum as an additional component. By this we hope to improve the age prediction of the model. The justification of the two component fluid is derived from scalar field models. The decaying vacuum of the form $\Lambda = \alpha H^2$, having α a constant borrowed from Bessada et al's research on star formation rate analysis. We analyzed the evolution of scale factor, state function and q factor for various values of $\bar{\zeta}$ and found there is big bang for $\bar{\zeta} < 1.72$. For values $\bar{\zeta} \geq 1.72$, besides having no big bang, the universe remains eternally accelerating. Therefore the age is not predicted for $\bar{\zeta} \geq 1.72$, The mentioned analysis indicates the asymptotic de Sitter phase too. Later, after having α fixed at 0.14(following the work of Bessada et al), we constrained the values of $\bar{\zeta}$ and H_0 with $\chi-$square analysis using the type Ia supernova, CMB(Cosmic Microwave Background) and BAO (Baryon Acoustic Oscillations) data. We obtained the best fit value for $\bar{\zeta}$ as about 1.445, besides obtaining the best fit H_0=70.03 km s^{-1} Mpc^{-1}, closer than the previous mono component fluid model to that predicted by ΛCDM model. The transition point of acceleration is also around the experimentally observed $z \sim 0$. We also did the dynamical system analysis which gave a stable critical point, again supporting the asymptotic de Sitter phase. The theoretically predicted age of the universe for the best fit values of H_0 and $\bar{\zeta}$ with $\alpha = 0.14$ had remarkably improved, having the value to be about 11.8GY.

Chapter 5

In this chapter we summarize the overall work and present our conclusions. We have also presented the future scope of the work.

ACKNOWLEDGMENTS

The research work and this thesis that followed became a possibility because of the painstaking, affectionate support and helps from my guide, family members and friends.

First and foremost, I express my sincere thanks to my supervisor, Prof. Titus K Mathew, for the patient guidance, encouragement and advises he had provided throughout my research period. I have been extremely lucky to have a supervisor who cared so much about my work, and who responded to my questions and queries so promptly. His most crucial help was in introducing me to his totally new approach in modeling the late universe, using bulk viscous Zel'dovich fluid. He also displayed his professionalism as a research guide by keeping me focused towards the right approach for resolving the riddles those are the reliable empirical data as regards the science of universe viewed macroscopically.

I gratefully acknowledge all the funding sources - Cochin University of Science and Technology for providing me UJRF and USRF.

I express my heart-felt gratitude to Prof. M. Junaid Bushiri, Head of the Department of Physics and all the former Heads of this Department for providing the necessary facilities for my research work. I thank all the members of the faculty, library and office staff of the department for the help they rendered during the entire period of my research study. Special thanks to Dr. Ramesh Babu T., Dr. M. Raveendran, Dr. Rhine Kumar, Dr. Charles Jose and Dr. M. Sabir for their help and always being available with their comments and suggestions for the improvement of the present work. I would also like to acknowledge all the teachers I learnt from since my childhood, I would not have been here without their guidance, blessings and support.

I also like to thank Inter University Center for Astronomy and Astrophysics (IUCAA), Pune for the phenomenal support they had extended in using the computational facility for many of our computations. We have also used the excellent library in IUCAA for reference purposes.

I would like to thank all my fellow researchers, Jerin, Paxy, Manosh, Athul, Sarath, Dinto, Thaskeena, Athira Sasidharan, Krishna, etc for their moral support and motivation. I also like to thank all my seniors, specially Lini, for all the support and guidance. I would also like to thank all other research scholars, M. Phil. and M. Sc. students of the Department for the moments I had with them.

The rare support at the cost of sacrifices from my Mrs and my daughter, daring to relieve me off my share of burdens in the family life by compounding their schedule, with the former being a busy scientist and the latter undergoing a tough course in architecture for B-arch, needs to be mentioned. I also express my gratitude to my two brothers, their family, my mother and father (late) for their valuable helps and moral support. Besides, there had been valuable encouragements from other members of my extended family. Besides, I had great friends who were concerned and were rectifying my going through criticism and guiding with valuable advises to carry through. I also had fruitful, though with a fuzzy logic like connections of helps from interactions with the generous faculty members of physics Dept. University of Pune, BARC Mumbai, IISc Bangalore, NDA Pune, scientists in physical chemistry section of NCL Pune and project section of CDAC Pune and CWPRS Pune.

Above all, I express my gratefulness to the will of Providence keeping humanity hungry for pure knowledge that the appropriate environments for the research outputs such as this is a possibility. I also acknowledge great spiritual leaders world all over, contributing in inspiring love towards work in me proverbially like the dews emanating in the night unnoticed and yet crucial for sustaining life.

Rajagopalan Nair K.

LIST OF PUBLICATIONS

Papers published in refereed international journals and those presented in seminars and conferences are listed below.

Refereed Journals.

1. K. Rajagopalan Nair and Titus K. Mathew, "Bulk viscous Zeldovich fluid model and its asymptotic behavior," *Eur. Phys. J. C.* **76**, 519 (2016).

2. K. Rajagopalan Nair and Titus K. Mathew, "A model of the late universe with viscous Zeldovich fluid and decaying vacuum," *Astrophys. Space Sci.* **368**, 183 (2018).

Conferences.

1. K. Rajagopalan Nair, "A bulk viscous Zeldovich fluid model of accelerating universe" in Regional Astronomers Meeting on Astronomy Research Opportunties and Challenges -IV by IUCAA Pune, from 1^{st} December to 2^{nd} December 2017, held in WMO Arts & Science College, Wayanad, Kerala.

2. K. Rajagopalan Nair, "The acceleration of the universe as its viscosity", in National Science Day Celebrations 2018 'theory thoughts' by Kerala State Council for, Science, Technology and Environment from 27^{th} February to 28^{th} February 2018, held in St. Paul's College, Kalamassery, Kerala.

3 K. Rajagopalan Nair, "Modeling the universe as a bulkviscous Zel'dovich fluid in decaying dark energy background," in Regional Astronomers Meeting on Astronomy Research Opportunties and Challenges by IUCAA Pune, from 8^{th} February to 9^{th} February 2019, held in CUSAT, Kochi-22, Kerala.

Contents

List of Tables

List of Figures

1

Standard Cosmology

Cosmology is the science dealing with the universe at large scale and its evolution. The understanding of the cosmos in the large scale requires telescopes good enough to see astronomical objects at distances more than hundreds of megaparsecs. Having no such facilities available till the end of nineteenth century, there were not much clues about galaxies external to Milky Way and LSS(Large Scale Structures). The beginning of 20^{th} century initiated explosive revolutions in cosmology in terms of new theories and observations. With his general theory of relativity, Einstein revolutionized the concept of gravity, which led to the development of expanding universe models. The idea of expanding universe turned out to be a reality due to the land mark discovery by Edwin Hubble that the galaxies are receding from each other. He observed the cosmos with a large telescope, the first of its kind in optical astronomy having a diameter 2.5 m, paving way to discoveries that Milky way is not the center of the universe and there are other galaxies similar and different in shapes and sizes. Yet again, it was a chain of spectacular discoveries in theory and experiment those followed - big bang theory which logically and empirically chased out the majority supported steady state theory, discovery of CMBR (Cosmic Microwave Background Radiation), discovery of acceleration of universe and recently the detection of gravitational waves in 2016.

Gravity is the major controlling factor of the universe at large scale. The concept of gravity evolved from astronomical observations since prehistoric period. Based on Kepler's laws of planetary orbits, Isaac Newton arrived at the first widely successful law of gravity. It states that two point masses, say m_1 and m_2 separated by a distance $|\vec{r}_{12}|$ will exert on each other the forces given by

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r_{12}^3}\vec{r}_{12}, \quad \text{and} \quad \vec{F}_{21} = -G\frac{m_1 m_2}{r_{12}^3}\vec{r}_{21}; \tag{1.1}$$

Newton stated the law as universal. However, the idea of instantaneous action at a distance between point masses was later discarded because,
(i) No communication in terms of momentum/energy exchange can be faster than light, as prescribed by special relativity theory,
(ii) The statement that gravity acts instantaneously, violates the relativity of simultaneity and therefore the equations are not relativistically covariant,
(iii) The law partially failed in predicting the shift in the perihelion of planet Mercury[1] and
(iv)The law was also ignorant about the effect of gravity on light.

After around 250 years, the idea of gravity had undergone a paradigm shift due to Albert Einstein. Unlike Newton's idea that gravity is an action at a distance, Einstein revolutionized the concept, that gravity is the curvature of the space-time due to presence of matter/energy. So gravity lost its status as a force, instead it has become the geometrical feature of space-time. The triumphs were remarkable. The new law explained the discrepancy on the perihelion shift of Mercury as observed by LeVerrier[1]. But more thrilling is the prediction of the bending of light, an effect which was unknown to Newtonian gravity. This prediction was observationally verified by Eddington in 1919 during a total solar eclipse. An important achievement of Einstein's gravity is its viability in applying to explain the origin and evolution of the whole universe. However, Einstein missed the chance of predicting the expanding universe due to his strong prejudice for a static universe. Friedmann and Lemaitre, contemporaries of Einstein, were able to predict the expanding universe using Einstein's theory of gravity independently. Even though the idea of expanding universe was in frozen state for a short period of time, it was resurrected and had become widely accepted due to the seminal observational discovery by Edwin Hubble in 1929. This led to the Big-bang theory of the universe, which addresses the issues of the origin and evolution of the universe. The milestones in the wide acceptance of the big-bang model are the prediction and discovery of microwave background radiation, prediction and verification of the abundance of light elements etc.

The observational capacities since 1930s, led to the measurements of rotational velocities in the outskirts of many galaxies, which finally paved way to the speculation of new cosmic component called dark matter. It is an invisible form of matter interacting with surrounding matter-energy only gravitationally. Later it was confirmed that galaxies have more dark matter than the luminous matter[2–11]. In the days to follow, the gravitational lens effects due to dark matter also were observed[7, 12, 13].

The year 1998 was that of yet another surprise, the discovery of late acceleration of the universe which won the 2011 Nobel prize. Having only normal matter and radiation, where matter includes both luminous and dark matter as the cosmic components, it was expected that the velocity of expansion must decrease. On attempting to observe this retardation in the expansion of the universe, two teams, supernova cosmology project team and High redshift supernovae search team, as a matter of total surprise, observed that in fact the expansion is accelerating and this acceleration had begun in the recent past of the universe[16–21]. This has opened up a new area in cosmology, aiming to explain the fundamental reason for this observed acceleration. On giving a general overview of the current cosmology in this direction we start with the Einstein's theory of gravity.

1.1 Gravity and geometry of space-time

> *"The experimentally known matter independence of the acceleration of fall is therefore a powerful argument for the fact that relativity postulate has to be extended to coordinate systems, which, relative to each other, are in non uniform motion"* - Albert Einstein, Nature[issue dedicated to Relativity], Feb 17, 1921 .

Though Einstein's attempt to find a covariant expression for gravity in the special relativistic limit was far from fulfilling the objective, his efforts led to discoveries of expression for gravitational red shifts with sufficiently small path intervals of light in a gravitational field where principle of equivalence holds[28]. The results indicated that the expression was of first approximation kind to a more general treatment. As per Albert Einstein's acknowledgment, as regards his final hitting the mark with non Euclidean, dynamic geometry of space-time depending on the presence of matter-energy, his inspiration dates back to his days as a student attending the lectures on Gaussian surfaces by a top mathematician Geiser. Next he commemorates his contemporary mathematicians Ricci and Levi-Civita. Yet again he acknowledges the help from his class mate and colleague at teaching post, Grossmann, as of to the optimum. Einstein in his venture, came to realize Minkowsky space-time is a special case to the (now known as a pseudo manifold in) space-time with a metric of the form,

$$dS^2 = g_{ij} dx_i dx_j, \tag{1.2}$$

Einstein summation convention being followed for the elementary components along the four vector space-time axes x_i and x_j; dS being the line element and dx_i its component along i^{th} coordinate axis. At once Grossmann supplied with the valuable guidance that what Einstein required was Riemann space[28]. Finally a thankful Einstein obtained the field equations for gravity as a space-time curvature, as

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \tag{1.3}$$

where $T_{\mu\nu}$ is the energy momentum tensor of matter and energy and $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$, the Einstein tensor representing the curvature of the space-time. The second rank tensor $R_{\mu\nu}$ is the Ricci tensor and R the Ricci scalar obtained by the contraction of the Ricci tensor expressed by the equation $R = g^{\mu\nu}R_{\mu\nu}$. The Ricci tensor is defined in terms of the Christoffel symbol as,

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} - \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\alpha\nu} \tag{1.4}$$

where $\Gamma^{\lambda}_{\mu\nu}$ is the Christoffel symbol defined as,

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}g^{\lambda\alpha}(g_{\alpha\nu,\mu} + g_{\alpha\mu,\nu} - g_{\mu\nu,\alpha}). \tag{1.5}$$

Einstein theory thus eliminated the status of gravity as an interaction between masses, instead it replaces it as the manifestation of curvature of the space-time due to matter. Thus in a sense it is geometrical theory of space-time dynamics.

The theoretical task is to find suitable solutions to the Einstein equation and is so hard as it is of non-linear kind. However a few solutions were derived corresponding to respective standard conditions. The prominent among them is the Schwarzchild metric, the solution corresponding to a spherically symmetric matter distribution[24–27, 29]. This solution finds its immediate application in resolving the half a century old problem regarding the perihelion shift of planet Mercury exactly[30]. In fact this is considered to be the first triumph of the Einstein's equation of gravity. The same solution was applied in calculating the bending of light when it enters the premises of an astronomical body [32]. The corresponding prediction of the theory was put in to observational test by Eddington and others during a total solar eclipse who indeed verified the light from a star very close to Sun in the field of view undergoes bending exactly as per the theoretical prediction.

The most profound application of Einstein's gravity is in understanding the origin and evolution of the universe. As it is envisaged by the Einstein's equation, if one knows the average density distribution of matter in the whole universe, it is possible to predict the space-time geometry of the entire universe in a macroscopic scale. The first attempt in this direction was initiated by Einstein himself. By knowing that his own equation is leading to a dynamic universe, which he first hesitated to believe, he introduced what is known as the cosmological constant to make the solution corresponding to a static universe. In the mean time, Alexander Friedmann and George Lemaitre came forward with expanding universe solutions using Einstein's gravity equation. The expanding universe model was firmly established with the historic discovery of Edwin Hubble that the distant galaxies were receding from us. The farther the galaxies are, the faster they recede, in accordance with the standard law, what we now call as the Hubble's law.

1.2 Expanding universe and standard ΛCDM model

Edwin Hubble in the late 1920s observed the distant Cepheid variable stars and confirmed the existence of galaxies other than ours, the Milkyway. He measured both the redshift and the distances of these variable stars. The distance measurements were done using Heavit's theoretical relation between the period and absolute brightness of the Cepheid variables. He observed around 29 galaxies and came to the conclusion that, at large scale, the galaxies were moving away from us [33]. He found a linear relation between velocity and distance, such that the velocity V, of the galaxies are found depending on the distance d as,

$$V = Hd \tag{1.6}$$

where H was then known as the Hubble constant.

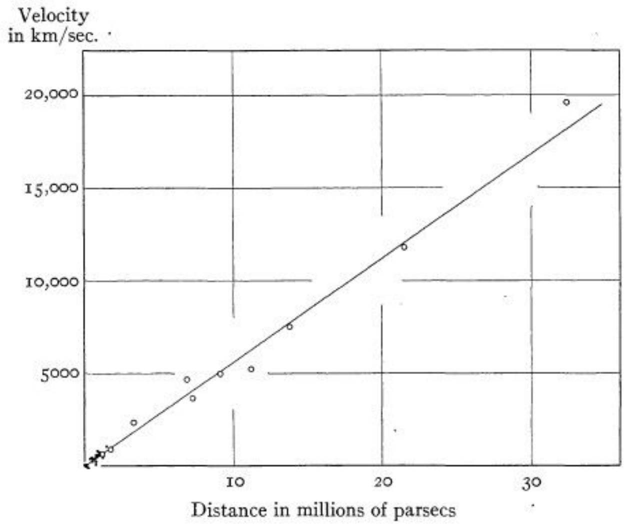

Figure 1.1: The Hubble flow velocity vs distance profile of galaxies based on Cepheid variable data in 1931 [Courtesy: Hubble E. and Humason M. L. [34]]

In due course, improvements in experimental setups led to Hubble-Humason collaboration [34] and the Hubble's theory of expanding universe became an assertion beyond doubt, as evident from the figure (1.1).

Assuming that the universe is homogeneous and isotropic in the matter distribution, he concluded that, the law is valid with respect to any galaxy in the universe. This implies that the galaxies are in fact receding from each other, which is essentially referred to as the expansion of the universe.

The discovery of Hubble is actually a verification of the theoretical speculation made around 1917 by the Russian scientist, A Friedmann[35]. From Einstein's theory of gravity and following assumption of homogeneous and isotropic distribution of matter, he argued that the universe is expanding. The geometry of a homogeneous and isotropic universe satisfying Einstein's law of gravity, can be described by the metric,

$$ds^2 = c^2dt^2 - a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \right) \tag{1.7}$$

where c is the velocity of light, t the cosmic time, $a(t)$ the scale factor of the expansion of the universe, k the curvature parameter and (r, θ, ϕ) is the comoving coordinates. In the preferred coordinate system (r, θ, ϕ) the galaxies are at rest so that the time measured by all the observers are identically same, the proper time, and is termed as the cosmic time t. One can view this as the expansion of the 3-dimensional space with time, responsible for the recession of galaxies along with it, a process termed 'Hubble flow. As a result, the coordinate separation between any two galaxies will remain the same with time and the actual separation between them becomes, $d = a(t)r$. Hence the real velocity of recession of any galaxy with respect to an observer becomes, $\dot{d} = \dot{a}(t)r$, where the over-dot represents the derivative with respect to cosmic time. This fact can be suitably put as $\dot{d} = \frac{\dot{a}(t)}{a(t)}r$ and is nothing but the Hubble law $V = Hd$ with $V = \dot{d}$ and $H = \frac{\dot{a}(t)}{a(t)}$.

The parameter, k determines the future of the universe, in such a way that, $k = +1$ corresponds to closed universe, $k = -1$ corresponds to open universe and $k = 0$ corresponds to flat universe. The recent extensive studies by Baryon Oscillation Spectroscopic Survey telescope (BOSS) and Planck survey have established the curvature parameter k as nearly zero[36, 37] and hence it is accepted that the universe is flat.

Lemaitre independently obtained the expanding universe models around the same time. Years later the model was rediscovered by Robertson and Walker, hence the model is popularly called as FLRW model. Following Friedmann and Lemaitre, George Gamow could predict a radiation dominated era in the early stages of the universe, which leaves a remnant radiation background, known as the cosmic microwave background radiation (CMBR). Gamow also predicted a hot big bang event at the origin followed by synthesis of light elements. The former assertion was verified by an empirical discovery of CMBR by Penzias and Wilson and the latter one, came true from the data of abundance of light elements in the universe.

The FLRW model is based on cosmological principle that universe in large scale is isotropic and homogeneous. It was only an intelligent guess until the end of 20^{th} century. Later on, the SDSS, and WMAP deep sky surveys were organized to the depths of the order of hundreds to thousands of Mpcs and the distribution of galaxies were studied with the type of computation power available at the time. A recent analysis with galaxies within about 500 Mpc confirms to the homogeneity and isotropy of cosmos when coarse grained in the range $70 - 100$ Mpc [38].

1.2.1 Friedmann Equations

FLRW metric given by eq(1.7) being applicable to universe in large scale, it gives way to Copernican universe in which there is no point is special in space and all points remains of equal importance too. The metric is obviously diagonal with elements $g_{00} = 1$, $g_{11} = \frac{a^2(t)}{1-kr^2}$, $g_{22} = a^2(t)r^2$, $g_{33} = a^2(t)r^2 sin^2(\theta)$ [24] . To find the respective Einstein equation, one has to know the tensor representing the matter and energy, $T_{\mu\nu}$. For a homogeneous and isotropic distribution as seen by a comoving observer, the energy-momentum tensor of matter(and energy) has all the off-diagonal elements equal to zero and the diagonal elements given by,

$$T_{\mu\mu} = \left(\rho c^2 + p\right) \frac{v_\mu v_\mu}{c^2} - p g_{\mu\mu} \tag{1.8}$$

where ρ is the density of a given cosmic component, p is the pressure of the component, v_μ is the four velocity having component $(1, 0, 0, 0)$[39]. The diagonal time-time component for energy-momentum tensor is given by

$$T_{00} = \rho c^2 \tag{1.9}$$

and radial space-space component

$$T_{11} = \frac{pa^2(t)}{1 - kr^2}. \tag{1.10}$$

The equation (1.10) for a FLRW flat space with $k = 0$ becomes $T_{11} = pa^2(t)$. Any discussion on T_{22} and T_{33} components are superfluous as they essentially cover the same physics as already discussed with the other two energy momentum tensor components.

Substituting for the energy momentum tensor components in Einstein field eq(1.3), Friedmann obtained the equations known after him as,

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = -\frac{k}{a^2} + \frac{8\pi G}{3}\rho \tag{1.11}$$

and

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = -8\pi G p. \tag{1.12}$$

Here, use is made of the fact that $\frac{\dot{a}}{a} = H$ and we took the units such that $c = 1$. From eqs (1.11 and 1.12) we obtain,

$$\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p). \tag{1.13}$$

For flat universe, k=0, one obtains a simpler form for the Friedmann equations. Subtracting eq(1.11) from eq(1.12) and rearranging for like terms we have,

$$-3a\dot{a}\left(\frac{p}{c^2} + \rho\right) = \dot{\rho}a^2 \tag{1.14}$$

Using $\frac{\dot{a}(t)}{a(t)} = H$, the above equation (eq(1.14)) leads to

$$\dot{\rho} + 3H(p + \rho) = 0 \tag{1.15}$$

The eq(1.15) is of a very general nature, accounting for any value of k and hence for any kind of curvature. Yet again, the most important aspect all about the eq(1.15) and its general nature is that the overall change in matter-energy components making the cosmos is zero. Thus, the expansion of the cosmos is adiabatic. As regards the pressure due to matter, it was proposed that, it is either zero or negligibly small with a profound reasoning that the galaxies viewed in the macroscopic universe analogous to molecules in an ideal gas, would have moved with random velocities if matter contributed to pressure just as how molecules being the source of pressure in the gas do.

To look into the evolution of the cosmos, one needs to understand the density and scale factor dependence of its various components. We see that here too the eq(1.15) is useful as we are able to obtain the equation

$$\int H(t)dt = \int \frac{\dot{a}(t)}{a(t)}dt = \int d\ln a = \int \frac{d\rho}{\rho(t)\left(1 + \frac{p(t)}{\rho(t)}\right)} \tag{1.16}$$

which finally comes down to

$$\ln a = \int \frac{d\rho}{\rho(t)\left(1 + \frac{p(t)}{\rho(t)}\right)}. \tag{1.17}$$

The equation connects the state function $\frac{p}{\rho} = \omega$ with the scale factor a. The solution becomes straight forward if pressure to density dependence for the components in the universe are known. On using $p = \omega\rho$, in eq(1.17) we arrive at the obvious conclusion that

$$\rho \propto a^{-3(1+\omega)} = (1+z)^{3(1+\omega)} \tag{1.18}$$

where we assume that ω is a slowly varying function. Though the cosmic redshift z is the observable physical quantity, we carry out the consequences of eq(1.18) in terms of a, because the data on z is not available before the recombination. Thus the radiation dominated, matter dominated and dark energy dominated situations can be described as:

i) The radiation dominated universe had relativistic hot gas. In this situation the particles in the universe underwent elastic scattering. The statistical mechanics applies here as in ideal gas and hence we have

$$p = \frac{1}{3}\rho_r \tag{1.19}$$

where ρ_r is the average density of radiation. Eq(1.19) implies that $\omega = \dfrac{1}{3}$ and substituting this into eq(1.18) we understand

$$\rho_r \propto a^{-4} \tag{1.20}$$

ii) For matter dominated era, we know that the pressure $p = 0$. Therefore, from eq(1.17) this means $\omega = 0$ and substituting this into eq(1.18) it is seen the matter density ρ_m would evolve as,

$$\rho_m \propto a^{-3} \tag{1.21}$$

and

iii) For the cosmic component dark energy, the cosmological constant satisfies the equation of state

$$p = -\rho_\Lambda, \tag{1.22}$$

which implies $\omega_\Lambda = -1$ hence ρ_Λ is positive constant. It is important to note that such a component exhibits uniform negative pressure. The experimental verification of densities of various components in relations to a, with data using light from past can be found in [43].

The visible universe, known as per HST (Hubble Space Telescope) SDSS (Sloan Digital Sky Survey) deep field surveys is comprised of galaxies to the order of 10^{13}, each accommodating stars to the order of 10^{11}, which means a total number of stars as numerous as sand grains on all beaches of earth as per a rough estimate by Carl Sagan[44]. However, it was realized in 1930s an exotic form of matter called dark matter contributes much more to the average density of the universe; the ratio of the exotic matter to baryonic matter as per a recent estimate turning out to be 5 - 6. As dark matter interacts with components of baryonic matter and radiation only through gravitation, its presence is known as well as optimum possible information extracted only through its gravitational influence on stars and galaxies forming LSS (Large Scale Structures). Another obvious way of proofs about the exotic form of matter comes through gravitational lensing effects[7, 12, 13] on distant galaxies, as dark matter falls in the way of their light coming in. The space-time curvature by the presence of dark matter deviates the light rays, thereby giving way to magnification effects and multiple image formations.

The knowledge of dark matter and its role in the universe dynamics was first abstracted by Zwicky in 1930s by collecting data on red shifts with a few thousands of galaxies in Coma cluster. As in the N-body problem dynamics as the galaxy clusters are, individual galaxies move about a common center of mass, giving way to red shifts superposing with the Hubble flow red shifts. The peculiar velocities of the individual galaxies were in thousands of kms per second. The breakthrough result came in as the overall mass was calculated using virial theorem. The ratio of the lowest estimate of virial mass to luminosity ratio of Comma cluster was around 500, whereas, in comparison, the mentioned ratio for the local Kapteyn stellar system was 3 [2]. The virial theorem used by Zwicky in this regard is valid as Subramaniam Chandrasekhar[14] and Bonazzola[15] could prove it can be extended to GR (General Relativistic) limits. The convincingly astonishing deviation from the expected mass in Coma cluster was enough evidence for the invisible matter.

Later observations led to the findings that the exotic dark matter is what largely dominates over the baryonic matter in the universe. The contributions came from Rubin and Ford [3] studying the rotation curve of Andromeda galaxy, which again, was not going as per luminous mass in the galaxy as can be predicted using virial theorem applied to the axi symmetric, approximately static space-time case discussed in reference [15]. The profile of the orbital velocities of stars at the edge of the spiral arms were tending to go flat as per radial distance from their galaxy centers, rather than fast decreasing as per inverse square of the mentioned distance. Einasto and Saar later obtained the rotation curve with galaxy IC 342 at 3.5 Mpc, which also indicates the influence of dark matter on stars along the edges of the galaxy. In this regard, figure (1.2), for instance, gives an excellent graphic account. This was followed by Einasto et al ob-

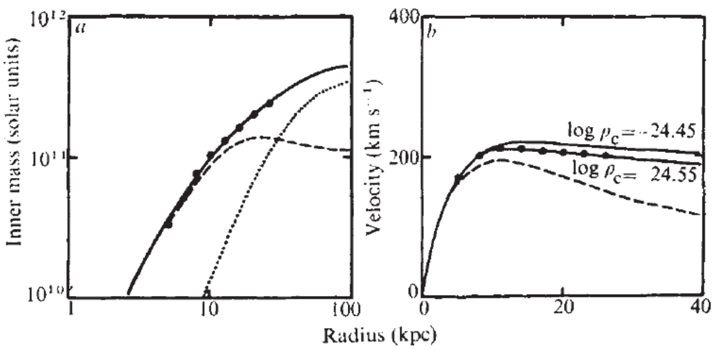

Figure 1.2: The profiles showing the mass (left figure) and the circular velocity (right figure) distributions in the galaxy IC342, as a function of the radial distance from its center. The dashed curves indicate the theoretically anticipated curves as per the stellar population, the black dots the experimentally observed data points, the dotted curves the profile in corona and the solid lines the total distribution. The deviation from the curves is a telltale sign of the exotic dark matter surrounding the galaxy. [Courtesy: Einasto J. and Saar E.[45]]

taining from observations the data on the ratios of virial masses per galaxy to their luminosities (figure1.3)which revealed that the dark matter has a density coming up to 20% of the critical density of the cosmos[45]. Similar extensive but independent studies by Ostriker, Peebles and Yahil[51] matched the figure for dark matter density in the universe as obtained by

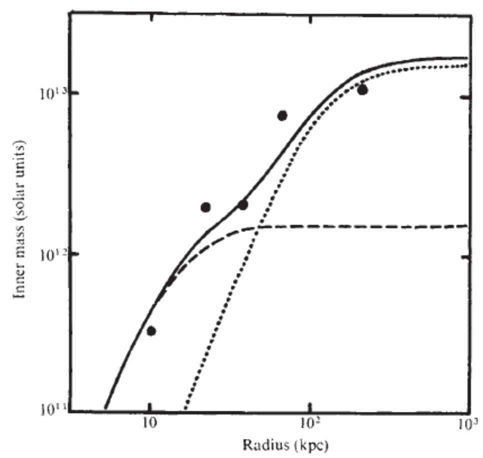

Figure 1.3: The mean mass distribution $< M(R) >$ obtained with 105 pairs of galaxies in Coma cluster. R refers to the radial distance from the center of mass of the cluster. Black dots represent the data points, dashed line the anticipated profile as per luminous mass, dotted line the one that corresponding to corona and the solid lines for total distribution. [Courtesy: Einasto J. and Saar E.[45]

Einasto et al. The CDM (Cold Dark Matter) theory that the universe has 20% of the critical density comprised of CDM and baryonic matter could predict fluctuations in CMBR (Cosmic Microwave Background Radiation) accordingly as per the figure which was falling into COBE probe observations. Planck and WMAP surveying Large Scale Structures also finally obtained the dark matter contribution to the density of cosmos more precisely as about 5 - 6 times as baryonic matter. Theoretical calculations of cosmological models, with dark matter density being approximated as matter density, solutions to Friedmann equations and behavior of phase-space trajectories close to critical points are obtained. During the matter dominating era of the universe, the dark matter influence alone being considered for gravity, the occasion of late acceleration and even the age of the universe are not deviating significantly. In our works also we follow the dark matter domination on evolution of the universe that the baryonic matter apparently has no role to play with the universe when viewed macroscopically.

Yet again, there had been doubts as regards the constitution of the universe. Friedmann also had the rare distinction of coming up with a theoretical model of universe with an accelerating expansion as a possible solution to Einstein's field equations[35]. Despite this model not being viewed seriously in his time, year 1998 onwards the course of physical cosmology took a drastic turn that there was experimental confirmation for universe subjected to accelerated expansion [16–21]. The standard model of 21st century thus had a transition as regards its counterpart of the previous century in that there was a shift from steadily expanding universe to accelerating universe.

1.2.2 Accelerating Universe

As the technology of reception of light from distant celestial objects underwent a quantum jump with very large telescopes aided by adaptive optics and HST (Hubble Space Telescope) aided by CCDs (Charge Couple Devices), the clarity of information about objects billions of light years distant improved spectacularly. Thus, two separate teams, one led by S. Perlmutter and the other led by Reiss and Schmidt were motivated to look for deceleration of the universe under gravity, which was quite natural a consequence if non-relativistic matter contributing zero pressure was dominating in the cosmos[16–21].

The successes with their projects on obtaining precise data about Hubble flow velocities in the past was made possible because type I a supernova at its peak brightness is a standard candle. The manner in which a white dwarf explodes makes the event unique and hence its peak luminosity becomes a constant. A type Ia supernova is an exploding white dwarf and comparable in brightness to its parent galaxy as a whole. This makes a type Ia supernova visible even from a distance around 6-8 billion light years, in contrast to Cepheid variable stars visible from not more than 0.1 billion light year, through the advanced optical instruments of the last decade of 20^{th} century.

A type I a supernova with a luminosity $\mathfrak{L}_{\mathfrak{s}}$ in Minkowsky space will have its flux at a point on a 2 sphere of known radius with the source at the centre as given by the inverse square law in optics. In FLRW space, the role of the distance is replaced by luminosity distance d_L and so the flux $\mathfrak{F}$ at a point with radius d_L is given by

$$\mathfrak{F} = \frac{\mathfrak{L}_{\mathfrak{s}}}{4\pi d_L^2}, \tag{1.23}$$

Therefore,

$$d_L^2 = \frac{\mathfrak{L}_{\mathfrak{s}}}{4\pi \mathfrak{F}}, \tag{1.24}$$

If ΔE is the energy emitted in time Δt and ΔE_0 that reaches the sphere of radius d_L in time Δt_0,

$$\mathfrak{L}_{\mathfrak{s}} = \frac{\Delta E}{\Delta t} \tag{1.25}$$

and

$$\mathfrak{L}_{\mathfrak{o}} = \frac{\Delta E_0}{\Delta t_0} \tag{1.26}$$

But we have, the ratio of wavelength λ_0 of light emitted from the supernova and λ that received by the observer on earth as

$$\frac{\lambda_0}{\lambda} = \frac{1}{a(t_s)} = (1+z) \tag{1.27}$$

Using (i) $\Delta E \propto \nu$ and $\Delta E_0 \propto \nu_0$ where ν and ν_0 are the frequency of radiation emitted and that received by observer on earth after the cosmic red shift respectively, (ii) $\dfrac{\lambda_0}{\lambda} = \dfrac{\nu}{\nu_0}$ and (iii) $\nu\Delta t = \nu_0 \Delta t_0$, we have

$$\frac{\Delta E}{\Delta E_0} = \frac{\nu}{\nu_0} = \frac{\Delta t_0}{\Delta t} = \frac{\lambda_0}{\lambda} = \frac{1}{a(t_0)} = 1+z \tag{1.28}$$

Combining eqs(1.26, 1.25 and 1.28), we obtain

$$\mathfrak{L}_{\mathfrak{s}} = (1+z)^2 \mathfrak{L}_{\mathfrak{o}}. \tag{1.29}$$

The light traveling from the type I a supernova to Earth, along η_s direction traces a geodesic in FLRW space and hence $c^2dt^2 - a^2(t)d\eta_s^2 = 0$; the approximation of the path as a geodesic here being reasonable as the average density of matter in the universe is only of the order of 10^{-26} kg m^{-3}. This means the distance travelled by the light to reach the observer is,

$$\eta_s = \int_{t_s}^{t_0} \frac{dt}{a(t)}, \tag{1.30}$$

where t_s is the time of emission of the light from super nova and t_0 the time at which the signal reaches the observer. Using $\dfrac{da}{a(t)H(t)} = dt$ alongside $da = d\left(\dfrac{1}{1+z}\right)$ in eqn(1.30) we obtain

$$\eta_s = \frac{1}{H_0}\int_0^z \frac{dz'}{h(z')}, \tag{1.31}$$

where we have used $z(t_s) = z$ and $z(t_0) = 0$. Here H_0 is the current value of Hubble parameter ~ 70 km S^{-1} Mpc^{-1} and $h(t) = \dfrac{H(t)}{H_0}$. Thus the flux received by the observer can alternately be written as

$$\mathfrak{F} = \frac{L_0}{4\pi\eta_s^2}. \tag{1.32}$$

Comparing RHS of eq(1.23 and 1.32) and using eq(1.29), finally we obtain the luminosity distance as

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{dz'}{h(z')}. \tag{1.33}$$

It is possible to have a simple equation connecting d_L with absolute magnitude M and apparent magnitude m by taking logarithm of eq(1.23) and using the relationships between $\mathfrak{F}$ and $\mathfrak{L}_{\mathfrak{s}}$ is as given below:

$$m - M = 5\log_{10}\left(\frac{d_L}{Mpc}\right) + 25. \tag{1.34}$$

The eq(1.33) straight away gives the theoretical expression for Hubble parameter corresponding to a given luminosity distance with a given z as,

$$H(z) = \left[\frac{d}{dz}\left(\frac{d_L}{1+z}\right)\right]^{-1}. \tag{1.35}$$

However, the critical density in the present universe due to all components in space, as obtained from the Friedmann eq(1.11) for FLRW universe with k=0 must be,

$$H_0^2 = \frac{8\pi G}{3}\rho_{critical}, \tag{1.36}$$

where H_0 is the present value of Hubble parameter, that is when $z = 0$. At any arbitrary time when $z \neq 0$, having the knowledge of $H = H(z)$ from eq(1.11) and that of total density of cosmos components from eq(1.18) we have,

$$H^2(z) = \frac{8\pi G}{3}\Sigma_i \rho_i(z). \tag{1.37}$$

But using eq(1.18), and using the normalized density symbols $\Omega(z) = \frac{\rho(z)}{3H_0^2}$, $\rho_i^{(0)}$ and $\Omega_i^{(0)}$ for present values of density and its normalized version respectively of ith component of the cosmos, by using eq(1.37) and eq(1.18) we have,

$$H(z) = H_0\sqrt{8\pi G \Sigma_i \left[\Omega_i^{(0)}(1+z)^{3(1+\omega_i)}\right]}. \tag{1.38}$$

Thus, from eqs(1.33 and 1.38) we see obtain d_L in terms of z and state functions of components as,

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{dz'}{\sqrt{8\pi G \Sigma_i \left[\Omega_i^{(0)}(1+z')^{3(1+\omega_i)}\right]}}. \tag{1.39}$$

In both radiation dominated era and the matter dominated one has $p \geq 0$ and hence $\omega \geq 0$. As baryonic matter to a far lesser extent and dark matter having $\omega_m = 1$ were the only components known at the time, it was deceleration that was expected with the expansion of the universe at the time the new accurate data was about to be interpreted.

If $z << 1$ that even z^2 term onward can be neglected, as is the case with even the remotest observable Cepheid variables, from eq(1.39) we have

$$d_L = \frac{z}{4H_0\sqrt{2\pi G\Omega_m^{(0)}}}, \tag{1.40}$$

which explains Hubble's linear limit relationship between Hubble flow velocity of an object and its distance from the observer. This also explains how observations at distances far less than possible with type I a supernovae will not carry the information about any acceleration/deceleration of Hubble flow, with peculiar velocities and their variations suppressing the information.

Type Ia supernovae at different redshifts were studied by Perlmutter's team and also by Schmidt's and Reiss's team, and it was found that they are fainter than expected for a flat decelerated universe. The basic reason could be an acceleration in the expansion of the universe. The simplest cosmic component which can produce acceleration in the expansion is cosmological constant. They created the fitting plots by including the cosmological constant Ω_Λ as an additional cosmic component. The $H_0d_L - z$ profile marked as (a) in fig (1.4) is the one that was expected for a deceleration of universe. The observational results implies profiles deviating from the expected one. For instance the profile marked as (d) corresponds to a universe in which cosmological constant is the only constituent, i.e. $\Omega_\Lambda = 1$, which means universe subjected to acceleration alone. Any profile among (b) and (c) has both non-relativistic matter and cosmological constant as cosmic components and hence it has an early decelerating phase and a late accelerating epoch. So in such cases the universe might have the switching from a prior deceleration to a late acceleration. The switching over from deceleration to acceleration will eventually result in larger values of d_L than expected of the case with deceleration alone (profile(a)). Profile (c) corresponds to $\Omega_m^{0)} = 0.7$ and $\Omega_\Lambda^{(0)} = 0.3$ and (b) that of $\Omega_m^{0)} = 0.3$ and

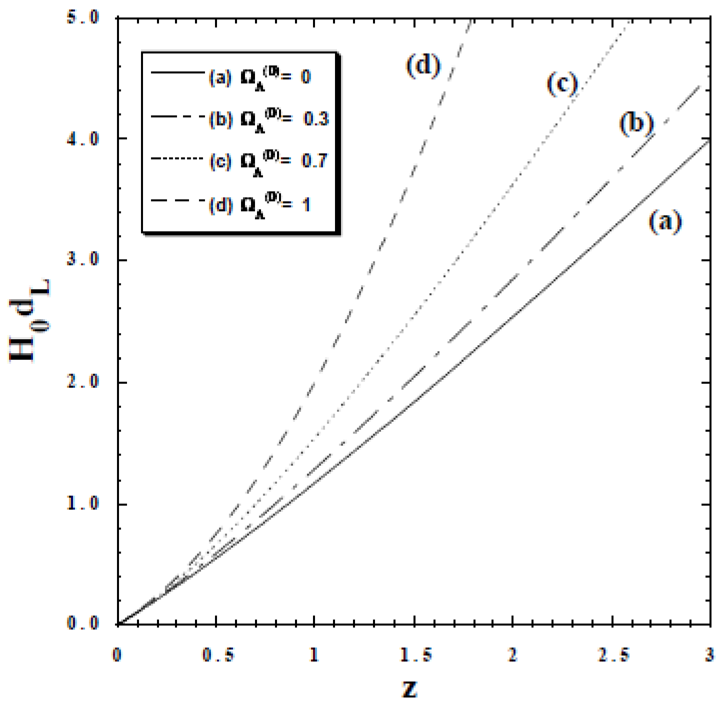

Figure 1.4: Profile of luminosity distance in the units of H_0^{-1} vs Hubble flow redshift for various values of $\Omega_m^{0)}$ and $\Omega_\Lambda^{(0)}$. [Courtesy: Copeland E.J. et al[52]]

$\Omega_\Lambda^{(0)} = 0.7$. The mentioned teams studying supernova data arrived at (b) as the best fitting situation. They found that the universe enter a phase of accelerated expansion at about a redshift of around $z \sim 0.7$, which is equivalently some five billion years ago in the past.

1.2.3 Standard ΛCDM model of the universe

> *"... we stand at a truly remarkable time in the history of our subject, largely (but clearly not exclusively) by virtue of a growth in the observational capabilities...most importantly a standard model has emerged which, through detailed numerical*

simulations, is capable of detailed predictions and interpretation of observables," - Ellis R. S.

The most successful model which explain the late acceleration of the universe is standard ΛCDM model of the universe. This model considered the cosmological constant as the so called dark energy responsible for the late acceleration. Inclusion of cosmological constant consequently modifies the Einstein field equation as,

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}, \tag{1.41}$$

where Λ is a constant, means a simplest possible addition of a covariant term contributing negative pressure to energy momentum tensor. In the FLRW background with metric given by eq(1.7), modified Einstein's field equations as in eq(1.41) give the Friedmann equations as,

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{1.42}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{1.43}$$

The eq(1.43) clearly shows Λ contributes a negative pressure term and hence contributes a repulsive effect.

The term $(\rho+3p)$ in the above equation refers to the contribution from ordinary matter and radiation, and it turns out that the term is positive for both of them. Hence ordinary matter causes only deceleration in the velocity of expansion. The cosmological constant Λ, which accounting for the negative pressure as, $p_\Lambda = -\rho_\Lambda$, will give rise to the acceleration. The densities of the non-relativistic dark matter matter and the cosmological constant were constrained by the observation as, nearly 70% and 25% respectively, with baryonic matter contributing 4% and radiation, 1%. The model is also appreciated for its simplicity involving only two components in the macroscopic FLRW universe picture.

One can solve the Friedmann equations for the Hubble parameter and scale factor for the two component ΛCDM model by assuming the equation of state. With dark matter and cosmological constant as cosmic components, the solution can be obtained as,

$$H = H_0\sqrt{\Omega_\Lambda}\coth\left(\frac{3}{2}\sqrt{\Omega_\Lambda}H_0 t\right), \tag{1.44}$$

and the scale factor,

$$a(t) \sim \left(\frac{\Omega_{m0}}{\Omega_\Lambda}\right)^{1/3}\sinh^{2/3}\left(\frac{3}{2}\sqrt{\Omega_\Lambda}H_0 t\right). \tag{1.45}$$

The above solution clearly indicates that as $t \to 0$ the scale factor evolves as $a(t) \sim (H_0 t)^{2/3}$ corresponding to the prior matter dominated decelerated epoch and in the limit $t \to \infty$ the scale factor will go as $a(t) \sim exp(\sqrt{\Lambda/3})$ which represents the de Sitter epoch. Hence the universe might undergo a transition from the prior decelerated epoch to a later accelerated one.

Apart from predicitng the transition into the late accelerated epoch, the model has made successful predictions in many respects, especially regarding later phases of the universe. To list out the main among them:
i) It successfully predicts the redshifts of the observed type Ia supernovae, as already discussed.
ii) The Age of the universe predicted by the model is very close to the one obtained from the observations of oldest globular clusters,
iii) The model predicts in an almost successful way the formation of large scale structures[53, 70] such as clusters of galaxies and superclusters of galaxies,
iv) Presence of BAO (Baryon Acoustic Oscillation).

Let us now consider the age prediction by the model. The basic theoretical prediction for the age of the universe, t_0, can be obtained using eq(1.11) as

$$t_0 = \int_1^0 \frac{da}{Ha}. \tag{1.46}$$

In terms of red shift this equation becomes,

$$t_0 = \int_0^\infty \frac{dz}{H(1+z)}, \tag{1.47}$$

for a flat universe (the observed curvature of the cosmos from Planck data is $\Omega_k^{(0)} = 0.000 \pm 0.005$ [37, 78, 79] and hence the flat 3−space approximation as in the FLRW universe is valid) the Hubble parameter becomes of the form,

$$H(z) = H_0\sqrt{\Omega_m^{(0)}(1+z)^3 + \Omega_\Lambda^{(0)}}\ . \tag{1.48}$$

Therefore, from eqs (1.47) and (1.48)

$$t_0 = \frac{1}{H_0}\int_0^\infty \frac{dz}{(1+z)\sqrt{\Omega_m^{(0)}(1+z)^3 + \Omega_\Lambda^{(0)}}}\ . \tag{1.49}$$

The age of the universe is obtained as 13.1 GY when $\Omega_m^{(0)} = 0.3$[37] and $\Omega_\Lambda^{(0)} = 0.7$ [52]. This is astonishingly close to the age obtained from Globular cluster data[72–76]. It is also seen that if dark energy is absent in eq (1.49), the age of the universe falls less than 10 GY, which is not quite within decent error limits as per the mentioned observations with the Globular clusters.

Einasto et al [45] and Ostriker et al[51], by their extensive research into galaxies, had estimated the density of dark matter in the cosmos which is in close agreement with the COBE, WMAP and Planck observations, around 23%. This is in support of the ΛCDM model. Another independent phenomenon which goes by ΛCDM model prediction is BAO (Baryon Acoustic Oscillation) phenomenon. This occurred after the epoch of recombination. The baryon density and hence electron density falls enough to increase the mean free path of photons, which then undergo Thompson scattering. As the radiation escaped, the inward gravitational pull of

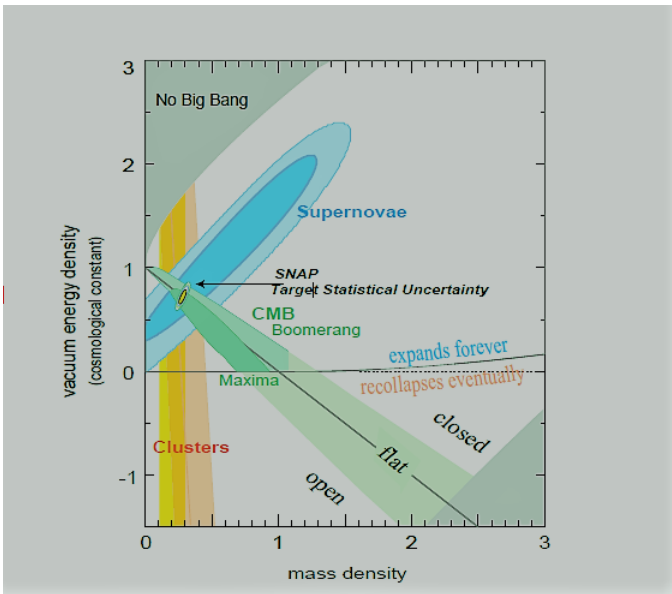

Figure 1.5: The confidence contour plots for the present dark energy density vs that of dark matter, constrained by type Ia supernova, CMBR and LSS galaxy clustering. [Courtesy: Aldering[83]]

baryons and the outward inflatory forces counteracted with each other. This resulted in acoustic oscillations within the baryonic matter sector. Theoretical predictions were done [41, 42] on the propagation of these density wavelength. Elaborate and highly precise SDSS observations on the distributions of galaxies (baryonic matter) were fitting into the theoretical predictions [37] by ΛCDM model.

1.2.4 The Shortcomings with ΛCDM model

> *"Any one of these issues (cosmological coincidence and fine tuning problems) would represent a serious challenge to physicists and astronomers; taken together, they serve to remind us how far away we are from understanding one of the most basic features of the universe"* - Carroll S. M.

In spite of the remarkable successes of the ΛCDM model, there arise some very serious draw backs. One of the most striking short coming is the cosmological constant problem. As per the observational data, the magnitude of the density of the cosmological constant dark energy is found to be,

$$\rho_\Lambda^{(0)} = \frac{\Lambda m_{pl}^2}{8\pi} = \frac{H_0^2 m_{pl}^2}{8\pi} \approxeq 10^{-47} GeV. \tag{1.50}$$

To have a theoretical estimate of this, one should rely on the field theory, according to which, the cosmological constant can be taken as the vacuum energy density. The vacuum energy density, so evaluated as the sum of zero point energies of quantum fields with mass m in units of c given by

$$\rho_{vac} = \frac{1}{2}\int_0^\infty \frac{d^3k}{(2\pi)^3}\sqrt{k^2+m^2} = \frac{1}{4\pi^2}\int_0^\infty dk\, k^2\sqrt{k^2+m^2}\ . \tag{1.51}$$

The density, though large, is not infinity as there is a cutoff at $k = k_{max}$ in quantum field theory. In the extreme case, when k_{max} corresponds to Planck mass $m_{pl} = 1.22 \times 10^{19}$ GeV, one obtains $\rho_{vac} \approxeq 10^{74} GeV$. The discrepancy by a factor of the order 10^{121} is the largest ever between theoretical prediction and experimental observation, in the history of physics. Unfortunately the standard ΛCDM model doesn't have any explanation for it.

The second important discrepancy is the problem of coincidence. Even though the evolutions of the densities of the dark energy, the cosmological constant and dark matter are different, the current densities of both these entities are found to be of the same order. This is a surprising coincidence, which has no explanation in the ΛCDM model. As we have mentioned previously, the density parameters of the dark energy and dark matter are $\Omega_\Lambda \sim 0.7$ and $\Omega_m \sim 0.3$ respectively. Since the critical density is around $\rho_c \sim 10^{-27}$ g.cm^{-3}, it is then clear that, the actual densities of these components are approximately the same in the present epoch of the universe, which needs explanation. These lead to the intriguing idea that, the dark energy which is causing the recent acceleration of the universe may not be a constant, but can be a varying one. This opened up one of the vast area of research in today's physics. There are multitude of proposals to realize the varying dark energy.

1.2.5 Attempted alternate models to ΛCDM model

. The shortcomings of the ΛCDM model, those we discussed in the previous section, invite many attempts for alternate models to alleviate mainly the cosmological constant problem and the coincidence problem. Since the fact that the late universe underwent a transition from deceleration to acceleration is convincing by the type Ia supernova observations, any model attempted must be behaving like ΛCDM model in this regards. There arised various varying dark energy models in the recent literature. We will go through the salient features of the prominent models.

Scalar field models of late accelerating universe

Quintessence

This is the case of a scale field ϕ minimally coupled to gravity, described by the action,

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}(\nabla\phi)^2 - V(\phi)\right), \tag{1.52}$$

where $\nabla\phi = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ and $V(\phi)$. The energy momentum tensor can be expressed as

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right], \tag{1.53}$$

in the quintessence limit. In the FLRW universe, the energy density of the scalar field is given by

$$\rho_{\phi q} = -T_0^0 = \frac{1}{2}(\dot{\phi})^2 + V(\phi) \tag{1.54}$$

and the corresponding pressure is given by

$$p_{\phi q} = T_i^i = \frac{1}{2}(\dot{\phi})^2 - V(\phi) \ . \tag{1.55}$$

Therefore the Friedmann equations for the uncoupled quintessence model is

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}(\dot{\phi})^2 + V(\phi)\right], \tag{1.56}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left[(\dot{\phi})^2 - V(\phi)\right] \ . \tag{1.57}$$

As obvious from e(1.57), the acceleration occurs when $(\dot{\phi})^2 < V(\phi)$. Thus it can be concluded that acceleration takes place in the limit of quintessence when $V(\phi)$ is slowly varying in time. From es(1.54) and (1.55),

$$\omega_{\phi q} = \frac{(\dot{\phi})^2 - 2V(\phi)}{(\dot{\phi})^2 + 2V(\phi)} \ . \tag{1.58}$$

As $V(\phi) \to -\infty$, $\omega_{\phi q} \to 1$, and when $V(\phi) \to \infty$, $\omega_{\phi q} \to -1$. The cosmos modeled with quintessence field coupled to matter with overall density $\widetilde{\rho} = \rho_{\phi q} + \rho_M$ has the overall state function given by

$$\omega = -1 - \frac{1}{3}\frac{d}{dx}\ln\left(\frac{\widetilde{\rho}}{3H_0^2}\right), \tag{1.59}$$

where H_0 is the current value of Hubble parameter.

The quintessence coupled matter modeled cosmos is matter dominated and decelerating when $\omega > -1/3$, at transition point to acceleration at $\omega = -1/3$, and dark energy dominating and accelerating towards de Sitter phase asymptotically in the range $-1 \leq \omega \leq -1/3$.

K-essence field

In this case the late evolution is driven by the dominant kinetic energy term of the scalar field. The approach was motivated from the work of

Armendariz Picon *et al*[46, 47], to deal with the inflation at high energies. A work by Chiba *et al* had shown that the same can be applied to dark energy too[48]. K-essence field was obtained by Armendariz Picon *et al*[49, 50] by obtaining a more general form of a Lagrangian, based on a work by Chiba *et al*. The action representing K-essence is given by

$$S = \int d^4x\sqrt{-g}\; p(\phi, X), \tag{1.60}$$

where $p(\phi, X)$ corresponds to a term with dimensions of pressure, being a function of ϕ and a non canonic kinetic energy term $X = -\frac{1}{2}(\nabla\phi)^2$. As $p = p(\phi, X)$, K-essence model can also represent quintessence model. The Lagrangian density is often represented by p taking the form[48–50]

$$p(\phi, X) = f(\phi)p(X)\ . \tag{1.61}$$

By using string theory connections[46, 47] and appropriate redefinition of the field [52], $p(\phi)$ takes the form[48]

$$p(\phi, X) = f(\phi)(-X + X^2)\ . \tag{1.62}$$

The energy density of the field is given by,

$$\rho = 2X\frac{\partial p}{\partial X} - p = f(\phi)(-X + 3X^2). \tag{1.63}$$

Using equations(1.62) and (1.63), we obtain the equation of state as

$$\omega = \frac{p}{\rho} = \frac{1 - X}{1 - 3X}\ . \tag{1.64}$$

In eq(1.64), when X is a constant $= -\frac{1}{2}$ and hence we have $\omega = -1$, we have the constant Λ situation of dark energy. Cosmos accelerates for $X < 2/3$ which corresponds to $\omega = -1/3$.

The form of $p(\phi, X)$ as in eq(1.62) meets with difficulties when it comes to dark energy dominating over matter and radiation situations. However, Armendariz-Picon *et al* obtained a more general form of $p(x)$ which he to solve the coincidence problem of dark energy [49, 50].

Phantom field

The scalar field action of the form

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}(\nabla\phi)^2 - V(\phi)\right) \tag{1.65}$$

gives way to another class of minimally coupled scalar field to gravity models known as phantom fields. The density of the field is given by

$$\rho_{\phi ph} = -T_0^0 = -\frac{1}{2}(\dot{\phi})^2 + V(\phi) \tag{1.66}$$

and the corresponding pressure[54], is given by

$$p_{\phi ph} = T_i^i = -\frac{1}{2}(\dot{\phi})^2 - V(\phi)\ . \tag{1.67}$$

and therefore the state function of the noninteracting phantom field is given by

$$\omega_{\phi ph} = \frac{\dot{\phi}^2 + 2V(\phi)}{\dot{\phi}^2 - 2V(\phi)}\ . \tag{1.68}$$

Phantom fields are when $\omega_{\phi ph} < -1$. This is possible when $\dot{\phi}^2 < 2V(\phi)$ in eq(1.68).

Using eq(1.59) with $\tilde{\rho} = \rho_{\phi ph} + \rho_M$, we obtain the overall state function. As the expansion imparted by phantom component is all too rapid, a phantom energy-cold dark matter model can apply only to late universe, suppressing the early stages of evolution of cosmos accommodated by both ΛCDM and quintessence models. The empirical data available brings in restrictions as regards the applicability of phantom phase in the matter dominated situation etc. Non canonical negative kinetic energy term also arises problems violating weak energy conditions, $T_{\mu\nu}v^\mu v^\nu \geq 0$, where $T_{\mu\nu}$ is the energy momentum tensor, v^μ and v^ν being μ^{th} and ν^{th} components of geodesic tangent vector and hence, the range of validity of a phantom field model is further limited. Phantom field based models are still a research curiosity because they support the late acceleration of the universe.

Tachyon Field Theory

Tachyon fields are described by a Lagrangian $L = -V(\phi)\sqrt{1-\partial^i\phi\partial^i\phi}$, analogous to that of a relativistic particle of mass m, the one dimensional Lagrangian of which is given by $L = -m\sqrt{1-\left(\frac{\dot{q}}{c}\right)^2}$, where $\dot{q}$ is the velocity of the particle [56]. In the field theory limit, q is upgraded to be the field ϕ. Relativistic covariance requires $\phi = \phi(\mathbf{x}, t)$, and mass m is replaced by $V(\phi)$.

The stress tensor of a Tachyon scalar field can be written as in the case of a perfect fluid as [56]

$$T^i_k = (\rho + p)u^i u_k + p\delta^i_k \tag{1.69}$$

where the density

$$\rho = \frac{V(\phi)}{\sqrt{1-\partial^i\phi\partial_i\phi}}, \tag{1.70}$$

pressure

$$p = -V(\phi)\sqrt{1-\partial^i\phi\partial_i\phi} \tag{1.71}$$

and four velocity field

$$u_k = \frac{\partial_k\phi}{\sqrt{\partial^i\phi\partial_i\phi}}\ . \tag{1.72}$$

The form of stress tensor of Tachyon field as in eq(1.69) allows it to be expressed as a sum of contributions due to the pressureless matter and a cosmological constant[57]. When the stress tensor of the Tachyon field is thus expressed in terms of the two components of matter and cosmological constant, it is seen that (i) $\omega \to -1$ and hence dark energy dominating case when $\dot{\phi} << 1$ and (ii) $\omega \approx 0$ and hence matter dominating case when $\dot{\phi} \to 1$.

Bagla, Jassal and Padmanabhan in their work as in reference[56] had shown there can be such models with tachyon field coexisting with matter and significantly contributing to energy density of the universe. By fine tuning the initial conditions in such models, the time of acceleration can be brought as in recent past. In the mentioned model, the tachyon field is observed as of non negligible density in the matter dominated era. This implies Tachyon field and their fluctuations are worthy of researching into.

Chaplygin gas

In 2001, a dark energy-matter coupled fluid model namely Chaplygin gas was proposed to explain the late acceleration of the universe by Alexander Yu Kamenshchik *et al* [61]. A generalized Chaplygin gas has the equation of state as

$$p = -\frac{A}{\rho^{\alpha}}. \tag{1.73}$$

where A and α are constants determined by the initial conditions. Thus equation of state become,

$$\omega = \frac{p}{\rho} = -\frac{A}{\rho^{\alpha+1}} \ . \tag{1.74}$$

Using conservation equation and (1.74), we at once obtain

$$\rho = \left[A + \frac{B}{a^{-3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}, \tag{1.75}$$

where B is a constant integration, to be determined from the initial conditions. From equations (1.74) and (1.75), it is seen that in the early stages of evolution of cosmos, when a is small enough that $a << \left(\frac{B}{A}\right)^{\frac{1}{3(1+\alpha)}}$, $\rho \propto a^{-3}$ and $\omega \approx 0$ indicating the matter domination, and at late times, when $a >> \left(\frac{B}{A}\right)^{\frac{1}{3(1+\alpha)}}$, $\rho \approx A^{\frac{1}{1+\alpha}}$, a constant and $\omega \approx -1$ indicating the dark energy domination, arousing the acceleration of the universe. Bean and Dore[62] and Barreiro *et al*[64] had researched into how the current data support the generalized Chaplygin gas model. The sole importance of this model is the unification of dark matter and dark energy.

f(T) models of accelerating universe

The $f(T)$ formulation is General relativity, instead of being stated in terms of a metric tensor, viewed in a teleparallel framework of orthonormal frames of tetrads. The teleparallelism was an attempt by Albert Einstein to unify electromagnetism with gravity, both being distantly connected in terms a dynamic, linear vector field called tetrad. A tetrad is a set of four linearly independent vector fields denoted by $\mathbf{e_a}(x)$, a=0,1,2,3 of the vectors defined in the local space-time [63]. The index 0 indicates a time like and indices $\{1, 2, 3\}$ those of space like vectors. In terms of the components e_a^μ their orthonormality condition can be written as

$$\epsilon_{ab} = g_{\mu\nu} e_a^\mu e_b^\nu = diag(1, -1, -1, -1) \tag{1.76}$$

The metric $g_{\mu\nu}$ can be obtained in terms of components of a co-frame $\mathbf{e}^a(x)$ as,

$$g_{\mu\nu} = \epsilon_{ab} e_\mu^a e_\nu^b, \tag{1.77}$$

where the components of co-frame is defined by

$$e_\mu^a e_b^\mu = \delta_b^a.$$

Eq(1.77) $\implies$ $\sqrt{-g} = det\ [e_\mu^a]$. Teleparallel equivalent of General relativity can be constructed with dynamical equations using tetrad, as the tetrad is connected to metric through eq(1.77).

$f(T)$ theories also can have equivalents of dark energy and gravity modifying $f(R)$ theories and so, in principle generate any possible evolution of $H(t)$, and hence describes any possible cosmic expansion history, including the recently accelerating universe. Yet again, $f(T)$ formulation has a difference that the Lagrangian using $f(T)$ does not contain a second derivative.

Holographic dark Energy

Holographic dark energy is based on holographic principle (HP) by G 't Hooft[95–97] which states that the information contained in a region of space can be represented as information on its boundary, though restricted by the fact that the boundary must contain at most one degree of freedom per Planck area.

In the context of the cosmos, according to HP, the energy density of dark energy as a physical quantity inside the universe, can correspond to some physical quantities at the particle horizon[95, 99]. Planck mass M_p and a cosmological length scale L, are the appropriate quantities which may decide the quantity.

Miao Li, had observed the distance from an observer to the future event horizon can be the appropriate L, to be given by

$$L = a\int_t^\infty \frac{dt'}{a} = a\int_a^\infty \frac{da'}{Ha'^2}, \tag{1.78}$$

to obtain the recently accelerating cosmos and a proper expression for equation of state[99, 100].

Dimensional analysis favour an expression of the form,

$$\rho_{de} = C_1 M_p^4 + C_2 M_p^2 L^{-2} + C_3 L^{-4} + ... \tag{1.79}$$

where C_1, C_2 and C_3 are constant parameters[101]. The first term in RHS of eq(1.79) is not compatible with HP as it is 10^{120} times higher than the observed value and so identified with the fine tuning problem[102]. Cohen *et al* also noted that the mentioned term is not compatible with HP[103]. The contribution to dark energy density comes from second term onward and yet again, third term in the RHS of eq(1.79) can be treated as negligibly small and therefore, holographic dark energy density can be represented in the form

$$\rho_{de} = CM_p^2 L^{-2}. \tag{1.80}$$

The value of C is fitted as per the details of experimental observations on the cosmos. C appropriately chosen accounts for the early inflation and recent acceleration. $C > 0$ keeps $\omega < -\frac{1}{3}$, leading to acceleration, choosing the appropriate values for C in which range gives way to constant Λ, ($\omega = -1$) and quintessence ($\omega > -1$) and phantom ($\omega < -1$)

like behaviors[54, 55, 58, 99]. The questions on coincidence problem is assumed in certain holographic dark energy models as can be obtained by looking into the extremely low dark energy density to radiation density ratio[59, 100]. During the early inflation epoch, it is assumed there were only dark energy and inflation energy components, the latter mentioned remaining a constant. According to this holographic dark energy theory, after the inflation the second mentioned component decayed into radiation. If the phenomenon of inflation is appropriately calibrated in the theoretical holographic dark energy model, the coincidence problem is explained. Kim *et al* also came up with the same explanation for the coincidence problem[60].

2

Cosmology with Zel'dovich Fluid

The present thesis is based upon the cosmology of a universe dominated by bulk viscous Zel'dovich fluid[104, 105]. In the cosmological scenario, the recent observations on the type Ia supernovae have revealed that, the major component of the current universe is dark energy, causing the recent acceleration of the inverse, which constitute nearly 70% of the total density of the universe. Even before this discovery, a variety of observational data, including weak[7] and strong[6] lensing, large scale structure[8], cosmic microwave background[11], etc, led to the conclusion that, about 23% of the total density is comprised of a weakly interacting matter called dark matter. In spite of all these data in support of dark energy and dark matter, the existence of other components like stiff fluid, which has an equation of state, $p = \rho$, has not been ruled out.

Many have speculated about the possibility of a stiff nature of the matter component in the early stages of the universe[107–110]. The ideal Zel'dovich fluid, the stiff fluid, was first introduced by Zel'dovich Ya B.[106] to tackle the equation of state problem of matter in heavy stars condensing to ultra high densities, till the point of neutron degeneracy pressure[111, 112]. In stellar models with neutron degeneracy pressure, it was pointed out that the pressure reached such high values so that the speed of sound in the matter became greater than c, the speed of light[106]. To avoid the paradox, the first attempt was to put special relativistic constraint to the pressure p as $3p \leq \epsilon$ in an adhoc fashion[113], where ϵ is the energy density reaching the maximum value $3p = \epsilon$ when the fluid is radiation dominated. Zel'dovich considered the model with baryons other than neutrons too, which interact through vector fields. The field equations incorporating interaction energy of charged baryons with the degeneracy pressure were obtained and this led to what was later known as the Zel'dovich fluid, with pressure $p = \epsilon$. This in the units of c, as followed here, becomes $p = \rho$ which means the speed of sound in this medium is equal to speed of light. As speed of light is the maximum possible limit and speed of sound having reached the mark, Zel'dovich fluid is the stiffest possible fluid and hence, very often referred to as 'stiff fluid. Zel'dovich, on the basis of assumption that early universe was dominated by cold baryons, hypothesized the stiff fluid like behaviour in the early phase of the cosmos. However, with the advent of hot big bang theory, Zel'dovich's reasoning for the presence of stiff fluid in the early universe was discarded.

The proposal on an early Zel'dovich fluid like behaviour of the constituent fluid was revived with relativistic Scalar Field (SF) modeling of cosmos. This is because the early kination epoch of the SF's evolution leads to stiff fluid behaviour. A fully relativistic SF Bose Einstein Condensate model was developed by Li *et al* [108] which had stiff fluid era when SF oscillations were slower than Hubble expansion, followed by radiation and matter dominating eras in succession, when SF oscillations became faster than the Hubble expansion.

A theoretical model by Stiele *et al*[109] with dark matter components, self interacting via exchange of vector mesons through minimal coupling, have shown that the self interaction energy will resemble the presence of a stiff fluid in the early universe.

Apart from cosmology with Einsteinian gravity, the presence of Zel'dovich fluid in the early epoch appears in Horava - Lifshitz gravity based cosmology too. For a convenient simplification, a 'detailed balancing condition' was imposed, a discussion on the merit of the 'balancing condition' was discussed in references[114–116]. When this detailed balancing condition is relaxed, such models undergo a stiff fluid phase[117–122].

Having considered the Zel'dovich fluid as the one with an equation of state $p = \rho$ the continuity equation (1.15), then implies,

$$\frac{d\rho}{\rho} = -6\frac{da}{a}, \tag{2.1}$$

which in turn implies that, the density of the Zel'dovich fluid follows as,

$$\rho \sim a^{-6}. \tag{2.2}$$

This drastic decrease in the density of the stiff fluid with cosmic expansion implies that, its effect will dominate in the very early stages of the universe and become negligible in the later stages. One of the prominent scenario in which one can find the effect of such a fluid might be the primordial synthesis of light elements, which happened during the first week since the beginning of the universe. Dutta and Scherrer have shown that, the observed primordial abundance implies a constraint on the density of the stiff fluid as, $\frac{\rho}{\rho_s} < 30$. This categorically denies any effect of the Zel'dovich fluid in the later evolution of the universe. But the situation will change once the Zel'dovich fluid become viscous. We have found that a viscous Zel'dovich fluid does affect the later evolution of the universe also.

The viscosity in the matter sector can be arised due to the restoration of thermal equilibrium whenever the universe undergoes a fast expansion or disturbances. The advantage of taking account of the viscosity is that, it may cause even late acceleration of the universe, since it can produce negative pressure like cosmological constant and therefore one can omit requirement of any exotic form for dark energy. The viscosity can be associated with any of the cosmic component. We study the bulk viscous stiff fluid in the context of the late acceleration of universe. Since many have speculated the existence of stiff fluid in the early universe, associating viscosity with it, is of great importance in the context of the late acceleration of the universe. Eventhough the standard ΛCDM model is matching almost well with the current observational results, the model is in slight tension regarding the prediction of the age of the universe[38] . It is interesting to check, whether the consideration of the viscous Zel'dovich (stiff) fluid can alleviate this problem. We will first consider the effect of viscous Zel'dovich fluid on the expansion of a single component universe. Following this we consider the two component universe, with bulk viscous Zel'dovich fluid and decaying vacuum as dark energy[88], which results in an improved prediction of age of the universe, closer to the Globular cluster related observations [72–76].

2.1 Bulk viscous Zel'dovich fluid

The ideal Zel'dovich fluid obeying the equation of state $p = \rho$, being irrelevant at the later stages of the cosmos. Incorporating bulk viscosity in the Zel'dovich fluid [104] results into a pressure, different from the pure fluid and might have sensible effects on the later evolution of the universe also. Following Eckart's formulation [124] the effective pressure of the bulk viscous Zel'dovich fluid can be written as,

$$p' = p - 3\zeta H \tag{2.3}$$

where ζ stands for viscosity and H for Hubble parameter. Eckart's expression for pressure includes the effect of dissipative processes occurring when there are deviations from local equilibrium. As it is seen from the expression, viscosity introducing the negative pressure for restoration of the equilibrium. Alternately Landau and Lifshitz also had arrived at an equivalent treatment[123] for local disturbances. Eckart's theory had a short coming that the resulting equilibria will be unstable, leading to the generation of signals with superluminal velocities, as pointed out by

Israel[125, 126], which violate causality. A more general theory to overcome this inconsistency was developed later by Israel and Stewart [127], to which Eckart's treatment arised as a first order limit. However, the simple form of Eckart's theory was worth exploiting to several researchers for bulk viscous fluid modeling of the universe who were able to explain the recent acceleration of the universe[128–132]. Besides, Hiscock *et al* pointed out that Eckart's theory can be preferred over the Israel-Stewart model to generate the negative pressure which can give way to the inflation [133]. Thus, in the context of the late acceleration of the universe too, Eckart formulation is the more favourable candidate and so the works on bulk viscous Zel'dovich fluid modeling of cosmos were based upon the same [104, 105]. In the flat FLRW bulk viscous universe, the Friedmann equations are

$$3H^2 = \rho \tag{2.4}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = p'. \tag{2.5}$$

Here we have adopted the units $8\pi G = 1$ and $c = 1$. Differentiating (2.4), along with the conservation equation, one arrives at,

$$\dot{H} = \frac{3H}{2}(\zeta - 2H). \tag{2.6}$$

For convenience we change variable to x such that $x = \ln a$ so that $\dfrac{dx}{dt} = H$. Then using $\dot{H} = \dfrac{dH}{dx}\dot{x} = H\dfrac{dH}{dx}$ the above equation become,

$$\frac{d^2H}{dx^2} = 3H(\zeta - 2H). \tag{2.7}$$

On solving this we get the evolution of Hubble parameter as,

$$H = \frac{H_0}{6}[\bar{\zeta} + (6 - \bar{\zeta})a^{-3}] \tag{2.8}$$

where $\bar{\zeta} = \frac{3\zeta}{H_0}$, is the dimensionless viscous parameter brought in for convenience. The asymptotic behaviour of the Hubble parameter are follows. In the early period as $a \to 0$ the Hubble parameter evolves as, $H \sim a^{-3}$ representing the prior decelerated era. In the later era as $a \to \infty$ the Hubble parameter tends to a constant, $H \to H_0\zeta/6$ corresponding a de Sitter epoch of late accelerated expansion. The corresponding density evolution can then easily be obtained as,

$$\rho = \frac{H_0^2}{36}[\bar{\zeta}^2 + 2\bar{\zeta}(6-\bar{\zeta})a^{-3} + (6-\bar{\zeta})^2 a^{-6}]. \tag{2.9}$$

An interesting consequence of the above equation is the natural appearance of a constant term and a term corresponding to a^{-3} apart from the conventional term a^{-6} corresponding to the evolution of the ideal Zel'dovich fluid. Eq(2.9) clearly indicates that for very early stages of the universe, the a^{-6} term dominates over the other two and for an intermediate era, the term corresponding to a^{-3} will dominate and in the extreme future the constant term will dominate over other two varying terms. This subsequent dominance are arised due to the viscous nature of the Zel'dovich fluid.

We obtained the corresponding solution for the scale factor of expansion as,

$$a = \left(\frac{(\bar{\zeta}-6) + 6\exp\left(\frac{\bar{\zeta}}{2}H_0(t-t_0)\right)}{\bar{\zeta}}\right)^{\frac{1}{3}}. \tag{2.10}$$

It is easy to see that the asymptotic behaviour of the scale factor is compatible with that of Hubble parameter. At prior stage, as $t \to 0$ the scale factor evolve as, $a \sim (1+3H_0(t-t_0))^{\frac{1}{3}}$, while in the future stage it evolves as $a \sim \exp\left(\frac{\bar{\zeta}}{2}H_0(t-t_0)\right)$, which is the de Sitter epoch and this guarantee the transition from a prior decelerated epoch to a late accelerated epoch. So the model predicts a transition to the late accelerating epoch as warranted by the observational data.

2.1.1 Case with zero viscosity

Here we briefly check the effect of switching off of the viscosity. A similar result in the case of the early stage can be obtained by assuming zero viscosity limit. For instance, $\underset{\bar{\zeta}\to 0}{Lt}\left(\frac{(\bar{\zeta}-6)+6\exp\left(\frac{\bar{\zeta}}{2}H_0(t-t_0)\right)}{\bar{\zeta}}\right) = (1+ 3H_0(t-t_0))$
which implies, in the absence of viscosity the scale factor behaves as,

$$a = (1 + 3H_0(t-t_0))^{\frac{1}{3}} \tag{2.11}$$

and the corresponding acceleration is,

$$\ddot{a} = -\frac{2H_0^2}{(1+3H_0(t-t_0))^{\frac{5}{3}}}. \tag{2.12}$$

The above equation shows that the model predicts eternal deceleration in the absence of viscosity.

An important thing to be noted is the behaviour of the density of the Zel'dovich fluid as $a(t) \to 0$, at which it becomes singular as evident from eq(2.9). This is an indication of a possible big-bang at the origin of the universe. The existence of the singularity can be further verified by calculating the curvature scalar for the flat FLRW universe, using the equation

$$R = \left(\frac{\ddot{a}}{a} + H^2\right). \tag{2.13}$$

A good reference for an understanding for the curvature scalar is[134]. Substituting for $\frac{\ddot{a}}{a} = \dot{H} + 2H^2$, then for $\zeta = 0$, eq(2.6) implies that $\dot{H} = -3H^2$, the curvature becomes $R \sim -H^2$ and hence in this case $|R| \to \infty$ as $a \to 0$ at the origin, thereby leading to a singularity.

The age predicted by the model can be obtained by setting $a = 0$ corresponding to a time t_B, the time of big bang. From eq(2.11) it can then be obtained, for zero viscosity, the age as

$$t_0 - t_B = \frac{1}{3H_0}. \tag{2.14}$$

2.1.2 The case, $0 < \bar{\zeta} < 6$

From the evolution of the Hubble parameter given in (2.8), it is clear that $\bar{\zeta} = 6$ marks the border of the two distinct situations in this model. For $\bar{\zeta} = 6$ the Hubble parameter becomes a constant and it corresponds to an everlasting de Sitter epoch. Hence for $\bar{\zeta} > 6$ the evolution would correspond to an eternal acceleration. It can be concluded that for a transition from a prior decelerated epoch to a late accelerated epoch the viscous parameter must lie in the range $0 < \bar{\zeta} < 6$.

The profile of density of viscous Zel'dovich fluid for various values of viscosity in the range $0 < \bar{\zeta} < 6$ as per eq(2.9) is as given in fig(2.1). The plot shows that density, $\rho_s \to \infty$ as $a \to 0$ for $\bar{\zeta} < 6$ and it implies

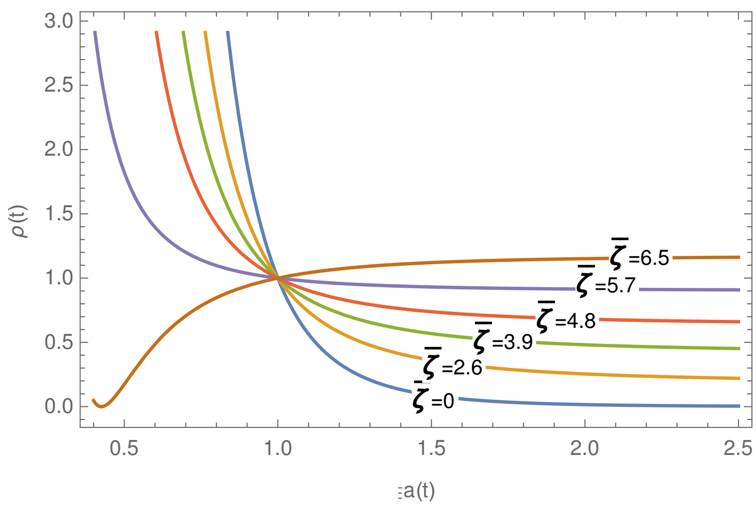

Figure 2.1: The profile of bulk Viscous Zel'dovich fluid density vs scale factor for various $\bar{\zeta}$.

an initial singularity. The singularity as $a \to 0$ can be confirmed from curvature scalar in this case as per eq(2.13) and it turns out to be,

$$R = \frac{3\zeta H}{2} - H^2 = H\left(\frac{3\zeta}{2} - H\right) \tag{2.15}$$

and it goes to infinity as $H \to \infty$ when $a \to 0$.

The age of the universe in this case is obtained from Eq(2.10) by applying the initial condition, $a(t) = 0$ when $t = t_B$, the time of the big bang. We arrived at the age as,

$$t_0 - t_B = \frac{2}{H_0\bar{\zeta}} \ln\left(\frac{6}{6-\bar{\zeta}}\right). \tag{2.16}$$

For a Hubble parameter $H_0 = 72.25$ km s^{-1} Mpc^{-1}[98], setting $\bar{\zeta} = 5.66$, the age of the universe is found to be around 13.73 GY, which is close enough to that obtained from Globular cluster data and CMB anisotropy data[135]. It is to be noted that, for $\bar{\zeta} > 6$ the age is not defined in this model due to the absence of big-bang, which justifies constraint on the value of the viscous parameter with an upper limit as $\bar{\zeta} < 6$.

The evolution of scale factor as per eq(2.10) for various values of $\bar{\zeta} < 6$ is depicted in fig(2.2). The plot also contains the evolution corresponding to $\bar{\zeta} > 6$, in which case there is no big-bang. It is clear that, at relatively

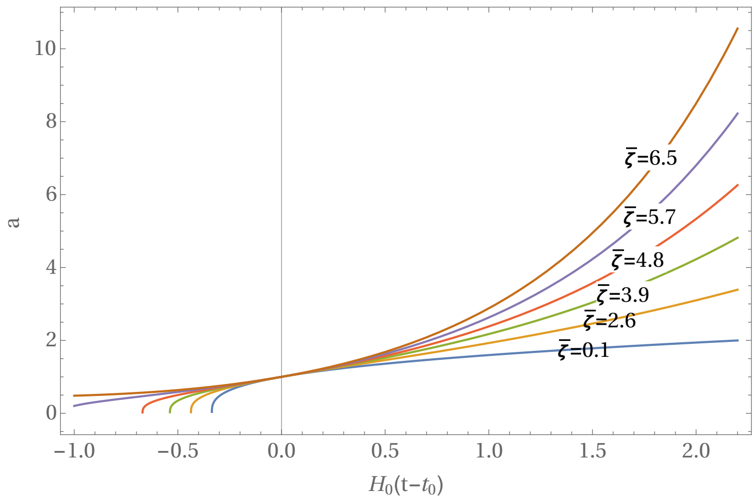

Figure 2.2: The profile of evolution of scale factor for various values of $\bar{\zeta}$.

low values of $\bar{\zeta}$ the scale factor evolves slowly in conformation with the earlier result, $a(t) \approx (1+3H_0[t-t_0])^{1/3}$, implying a prior decelerated epoch. But as time increases, the curvature of plot changes in such a way that, the scale factor evolves to represent an acceleration as, $a(t) \rightarrow exp\left(\frac{\bar{\zeta}H_0(t-t_0)}{6}\right)$. So there occurs a switch over from deceleration to acceleration for $0 < \bar{\zeta} < 6$. This switching over would be shifted to the deep past as the value of $\bar{\zeta}$ increases. This means the bulk viscous Zel'dovich fluid resembles a cold

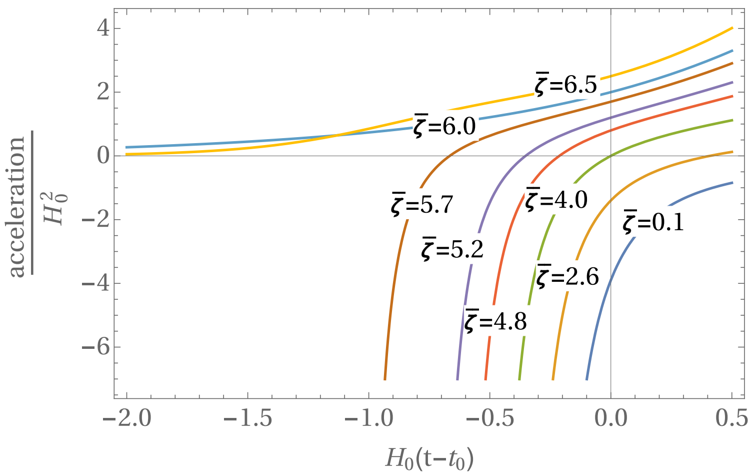

Figure 2.3: The evolution of $\ddot{a}$ for various values of $\bar{\zeta}$. The two limits, one of $0 < \bar{\zeta} < 6$ having an initial decelerated phase switching over to acceleration and the other of $\bar{\zeta} \geq 6$ with eternal acceleration are obvious in the profile. It is also seen that for $\bar{\zeta} = 4$ the transition point is at present, for $\bar{\zeta} < 4$ in future and for $\bar{\zeta} > 4$, in the past.

dark matter nature in the past and a dark energy behaviour in the later stage, thus unifying the dark matter and dark energy in the background of the cosmos.

In the fig(2.2), each of the curves with $0 < \bar{\zeta} < 6$ has the slopes of the tangents at various points decreasing progressively first till a transition point, passing which, they keep on increasing and hence the $a(t)$ vs t graphs also is indicative of early deceleration and then transition to the acceleration. It is seen from the plot that that at $\bar{\zeta} = 4$ the switch over from deceleration to acceleration takes place at the present. For $0 < \bar{\zeta} < 4$ the transition point from deceleration to acceleration is in future and for

$4 < \bar{\zeta} < 6$, the transition in the past. This important fact can be better shown by obtaining the double derivative of a, the acceleration. For this let us first recast the Hubble parameter equation eq(2.8) as,

$$\dot{a} = \frac{H_0}{6}[\bar{\zeta}a + (6 - \bar{\zeta})a^{-2}]. \tag{2.17}$$

Now taking derivative of this with respect to a leads to,

$$\frac{d\dot{a}}{da} = \frac{H_0}{6}[\bar{\zeta} - 2(6 - \bar{\zeta})a^{-3}]. \tag{2.18}$$

The transition scale factor $a = a_T$ is the one corresponding to $\frac{d\dot{a}}{da} = 0$ and is obtained as,

$$a_T = \left(\frac{2(6 - \bar{\zeta})}{\bar{\zeta}}\right)^{1/3}. \tag{2.19}$$

In terms of redshift the above equation becomes,

$$z_T = \left(\frac{\bar{\zeta}}{2(6 - \bar{\zeta})}\right)^{1/3} - 1. \tag{2.20}$$

From the eqs(2.19) and (2.20) it is clear that the transition would have occurred at present corresponding to $a_T = 1$ or $z_T = 0$ for $\bar{\zeta} = 4$. This implies the transition will occur in the future for $\bar{\zeta} < 4$ and would be in the past for $\bar{\zeta} > 4$. These things are geometrically clarified in figure 2.3. Hence the value of the viscous parameter must be in the range $4 < \bar{\zeta} < 6$.

A further confirmation of the above fact is depicted in fig(2.4), where we have plotted the evolution of the deceleration parameter. The deceleration parameter can be defined as,

$$q = -1 - \frac{\dot{H}}{H^2}. \tag{2.21}$$

Deceleration parameter $q > 0$ implies deceleration in expansion and $q < 0$ an acceleration in the expansion. Using eqs(2.6) and (2.8), the equation for decelerating parameter becomes

$$q = -1 + \frac{3(6 - \bar{\zeta})}{\bar{\zeta}a^3 + (6 - \bar{\zeta})}, \tag{2.22}$$

which alternately, in terms of red shift becomes,

$$q = -1 + \frac{3(6-\bar{\zeta})(1+z)^3}{\bar{\zeta} + (6-\bar{\zeta})(1+z)^3}. \tag{2.23}$$

For non-viscous Zel'dovich fluid, the deceleration parameter is q is 2, the

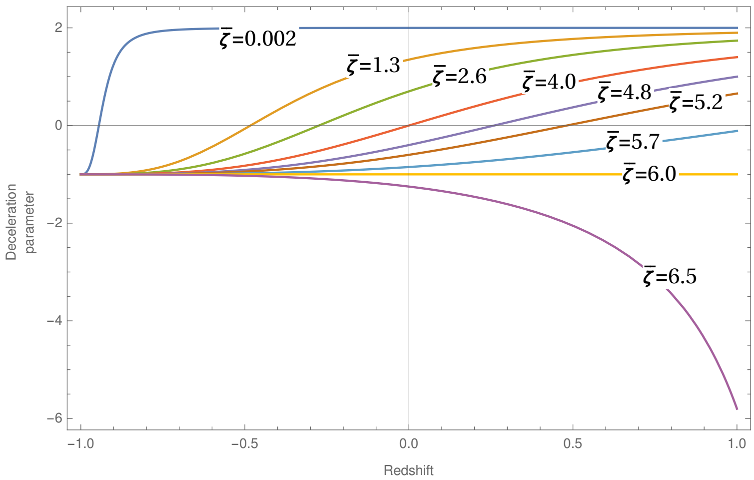

Figure 2.4: The profile of deceleration parameter vs cosmic redshift for various values of $\bar{\zeta}$. The negative values of z, despite not being physical, are included here as they will indicate the future. This graph confirms to the fact that the deceleration to acceleration transition point will be in future for $\bar{\zeta} < 4$, at present when $\bar{\zeta} = 4$ and in the past when $4 < \bar{\zeta} < 6$. It is also visible that for $\bar{\zeta} \geq 6$ the cosmos is eternally accelerating.

case of a constant deceleration. At the maximum value $\bar{\zeta} = 6$, the deceleration parameter $q = -1$ and is corresponding to an ever accelerating de Sitter universe. At present, $z = 0$, we have from eq(2.23) that

$$q_0 = 2 - \frac{\bar{\zeta}}{2}, \tag{2.24}$$

which implies that, the transition will occurred at present for $\bar{\zeta} = 4$. But observational results indicate that the transition occurred in the past. Therefore the value of the viscous parameter must lie in the range

$4 < \bar{\zeta} < 6$. The current observational value on deceleration parameter is $q_0 \sim -0.64 \pm 0.03$, [135, 136] which therefore sets the value of the viscous parameter as per the above equation as, $\bar{\zeta} > 4$. This means the value of the bulk viscosity is in the range as per $4 < \bar{\zeta} < 6$.

The equation of state function of the bulk viscous Zel'dovich fluid ω given by

$$\omega_s = -1 - \frac{1}{3}\frac{d}{dx}\ln h^2, \tag{2.25}$$

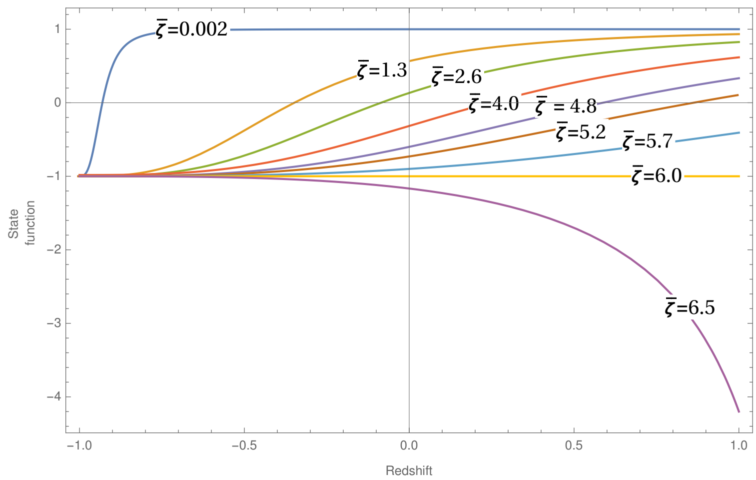

Figure 2.5: The profile of state function vs cosmic redshift for various values of $\bar{\zeta}$ for the bulkviscous Zel'dovich fluid. The deceleration to acceleration transition point according to eq(2.28) will be in future for $\bar{\zeta} < 4$, at present when $\bar{\zeta} = 4$ and in the past when $4 < \bar{\zeta} < 6$. It is also visible that for $\bar{\zeta} \geq 6$ the cosmos is eternally accelerating. (*please add name of different plots*)

where the dimensionless $h = \dfrac{H}{H_0}$, serves as a normalized Hubble parameter. Using eq(2.8) we have the equation of state in terms of redshift as,

$$\omega = -1 + \frac{2(6-\bar{\zeta})(1+z)^3}{\bar{\zeta} + (6-\bar{\zeta})(1+z)^3}, \tag{2.26}$$

The evolution of equation of state of the bulk viscous Zel'dovich fluid is as shown in fig(2.5). For $\bar{\zeta} = 0$, the non-viscous case, the equation of state become $\omega = 1$. Irrespective of the value of $\bar{\zeta}$ it is seen that $\omega \to -1$, in the extreme future corresponding to $z \to -1$, $(a \to \infty)$. For all the cases of $0 < \bar{\zeta} < 6.0$, the cosmos is subjected to initial deceleration, crossing over to later acceleration ending up in de Sitter phase, as ω starts from an initial value of $+1$ to approach -1 in the asymptotic limit as mentioned above. From Eq(2.23) it is seen that at the point of transition, at which $\omega = -1/3$, it follows,

$$\frac{(6-\bar{\zeta})(1+z_T)^3}{\bar{\zeta}+(6-\bar{\zeta})(1+z_T)^3} = \frac{1}{3} \tag{2.27}$$

where z_T is the transition redshift, which is then obtained as,

$$z_T = -1 + \left(\frac{\bar{\zeta}}{2(6-\bar{\zeta})}\right)^{\frac{1}{3}}. \tag{2.28}$$

The present value of the equation of state can be obtained by taking $z = 0$ n equation (2.26) and is,

$$\omega(present) = 1 - \frac{\bar{\zeta}}{3}. \tag{2.29}$$

This again confirms that for the transition to occur in the past, the viscous parameter must be in the range $4 < \bar{\zeta} < 6$. The observational value of $\omega(present)$ is around -0.94 ± 0.1 [135].

2.1.3 The case $\bar{\zeta} \geq 6$, eternal acceleration

From (2.23) and (2.26), it is clear that the model predicts eternal acceleration if $\bar{\zeta} \geq 6$. This is clearly seen in figs (2.3),(2.4) and (2.5). Interestingly, there is no singularity in the past for $\bar{\zeta} > 6$, and the scale factor never becomes zero but it attains a minimum value as $t - t_0 \to -\infty$, (as indicated in fig(2.2))

$$\underset{(t-t_0)\to-\infty}{Lt} a(t) = a_{min} = \left(1 - \frac{6}{\bar{\zeta}}\right). \tag{2.30}$$

However as it is seen from (2.9) the density becomes zero at $a = a_{min}$.

2.2 State finder analysis for the bulk viscous fluid.

From previous analysis especially from sect.2.1 it is seen that the viscous Zel'dovich fluid model of universe with a bulk viscosity coefficient in the range $4 < \bar{\zeta} < 6$ predicting reasonable the evolution of the late universe. However, there are several dark energy models accounting for the late stage evolution of the universe. Like the present model they also predict the various cosmological parameters like, deceleration parameter, equation of state etc in a reasonable way. So it seems difficult to contrasts between the merits various models. A sensitive diagnostic tool is essential to distinguish between different dark energy models or to distinguish a given model with the standard ΛCDM model[137].

As the acceleration of the universe is a recent phenomenon [16, 18–23] the scale factor $a = a(t)$ can be Taylor expanded as

$$a(t) = a(t_0) + a|_0 + \dot{a}|_0(t - t_0) + \frac{\ddot{a}|_0}{2}(t - t_0)^2 + \frac{\dddot{a}|_0}{6}(t - t_0)^3 + ... \quad (2.31)$$

as applicable to small $(t - t_0)$. The eq(2.31) can be examined for generating a more sensitive discriminator of expansion. Thus, apart from deceleration described in terms of second order derivative of a, it can be seen that higher sensitive diagnostic tools are obtainable using a combination of both second order and third order derivatives of a as present in eq(2.31)[137]. As the deceleration parameter is $q = -\frac{\ddot{a}}{aH^2}$, in order to incorporate both $\ddot{a}$ and $\dddot{a}$, in addition to H and q, the state finder pair $\{r, s\}$ are defined as [137, 138],

$$r = \frac{\dddot{a}}{aH^3} \quad (2.32)$$

and

$$s = \frac{r - 1}{3(q - \frac{1}{2})}. \quad (2.33)$$

A good account of the sensitivity of the state finder pair $\{r, s\}$ to non dark energy brane world models and dark energy models of quintessence

($\omega \neq -1$, but again a constant) and kinessence (ω is a function of time, having quintessence one of the simplest form besides Chaplygin gas too one of its kind) is given in [137]. These are degenerate models to the present time values of q and ω. Differentiating twice the Hubble parameter with respect to time, we have,

$$\ddot{H} = \frac{\dddot{a}}{a} - \frac{\ddot{a}}{a}H - 2\dot{H}H. \tag{2.34}$$

Writing RHS of eq(2.34) in terms of r and $q = -1 - \frac{\dot{H}}{H^2}$, we get,

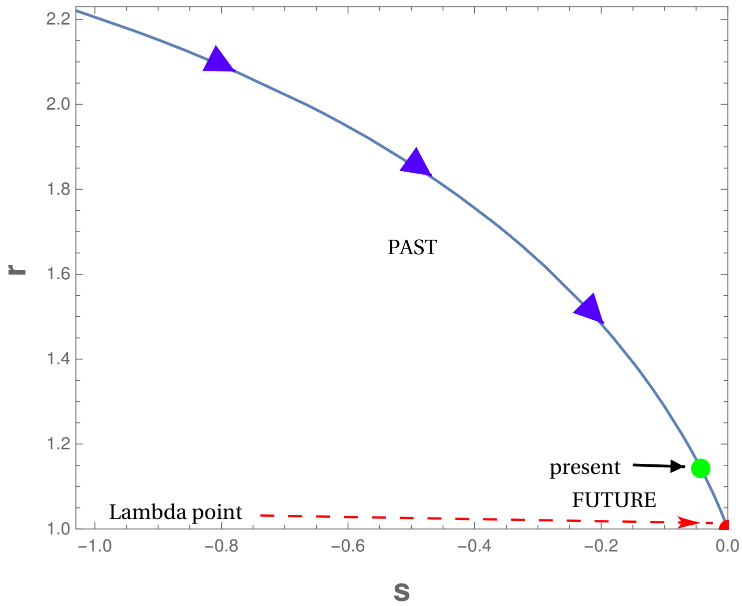

Figure 2.6: The r-s plot for the bulk viscous fluid with $\bar{\zeta} = 5.25$. The trajectory lies in the region $r > 1$ and $s < 0$. In the asymptotic limit as $a \to \infty$, $\{r, s\} \to \{1, 0\}$ which resembles the case of the ΛCDM model. It is seen that when $a = 1$, at present, $\{r, s\} = \{1.14, -0.04\}$. For other values of $\bar{\zeta}$ in the range $4 < \bar{\zeta} < 6$, the profiles remain close enough that they appear different only in the high resolution limit.

$$\ddot{H} = H^3\left(r + q - 2\frac{\dot{H}}{H^2}\right) = H^3\left(r + 3q + 2\right) \tag{2.35}$$

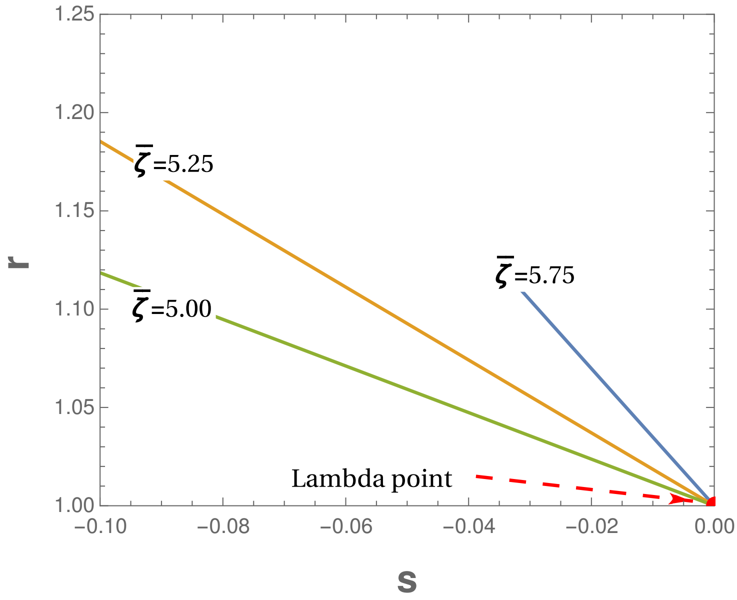

Figure 2.7: The r-s plots for the bulk viscous fluids in high resolution, close to Lambda point, for $\bar{\zeta} = 5.75$, 5.25 and 5.00. It is seen that for different values of $\bar{\zeta}$, the paths are different for $\{(r > 1), (s < 0)\}$ and they merge to $\{r, s\} = \{1, 0\}$ as $a \to \infty$.

and therefore,

$$r = \frac{\ddot{H}}{H^3} - 3q - 2. \tag{2.36}$$

We have from eq(2.6),

$$\ddot{H} = \frac{3\dot{H}}{2}(\zeta - 4H) \ . \tag{2.37}$$

From eqs 2.36, 2.37, 2.6, 2.8 and 2.22,

$$r = 1 + \frac{9(6-\bar{\zeta})^2 a^{-6}}{(\bar{\zeta} + (6-\bar{\zeta})^2 a^{-3})^2} \tag{2.38}$$

and

$$s = \frac{2(6-\bar{\zeta})^2 a^{-6}}{(6-\bar{\zeta})^2 a^{-6} - \bar{\zeta}^2} \ . \tag{2.39}$$

The equations (2.38) and (2.39) show that as $a \to \infty$, the state finder set $\{r, s\} \to \{1, 0\}$, and hence the bulkviscous Zel'dovich fluid model resembles the ΛCDM model of the universe in remote future. Fig(2.6) is the r-s plot of the bulk viscous Zel'dovich fluid model for the best estimate (evaluated in a later section) of the viscous coefficient $\bar{\zeta} = 5.25$. Apart from the ΛCDM behaviour in the asymptotic limit, the evolution in the $r - s$ plane indicate that $r > 1$ and $s < 0$ in the profile of the Zel'dovich fluid. This resembles the generalized Chaplygin gas model of the dark energy[139]. The present position of the model in this plane is corresponding to $\{r_0, s_0\} = \{1.14, -0.04\}$. From Eqs(2.39) and (2.38), it is seen that for $\bar{\zeta} = 0$ corresponding to the normal Zel'dovich fluid, $(r, s) = \{10, 2\}$. The $r - s$ evolution of the present model has also shown marked deviation from another standard model of dark energy, the holographic dark energy model, in which the dark energy density has been obtained using the cosmological holographic principle with the event horizon as IR cut off of the length scale involved in the problem. The holographic dark energy evolves from around $(r, s) \sim (1, 2/3)$ and ends at $(r, s) \sim (1, 0)$[140, 141].

The evolution of $r - s$ trajectory towards the ΛCDM point, is different for different values of the viscous parameters. In figure (2.7) we have plotted these evolution for different values of $\bar{\zeta}$, showing that evolution becomes more steeper as $\bar{\zeta}$ increases.

2.3 Entropy evolution and generalized second law of thermodynamics

The theoretical works on entropy of the universe were inspired by the course of thermodynamical studies on the dynamics of black holes which involved their event horizons, in 1970s. Hawking proved that in all the classical limit physical processes, the sum of areas of event horizons of interacting black holes cannot decrease, which is an assertion resembling the law of entropy with interacting systems in isolation[142]. Later, Wheeler raised a puzzle if given amount of matter crossing the horizon, over to a black hole, will lead to a reduction in entropy for an outside observer, which violates the law of entropy. The perplexity was overcome with Bekenstein[143–145], exploiting the analogy between Hawking's theory on increase of sum of areas of event horizons of classically interacting black holes with that of the law of entropy on a union of classically interacting thermodynamical systems in isolation. Bekenstein attributed an entropy to a black hole, proportional to the area of its event horizon. This led to a version of second law of thermodynamics that

$$\frac{d}{dt}(S_{BH} + S'_{univ}) \geq 0 \tag{2.40}$$

where S_{BH} = entropy of the black hole event horizon and S'_{univ} = the entropy of the universe with the exception of the black hole, and the law was termed as GSL (Generalized Second Law). The GSL finds its version in the FLRW universe, in that it leads to the conclusion, the sum of the entropies of the apparent horizon and the contents with in it is greater than or equal to zero. Therefore, for the stiff fluid,

$$\frac{d}{dt}(S_H + S_z) \geq 0 \tag{2.41}$$

where S_H = entropy of the apparent horizon and S_z = that of the Zel'dovich fluid. This view of GSL resembles the same being viewed from inside a black hole. The entropy of the Zel'dovich fluid within the apparent horizon is related to its pressure and energy density through Gibb's relation as

$$TdS_z = d(\rho_z V) + p'_z dV. \tag{2.42}$$

Here, $V = \dfrac{4\pi}{3H^3}$, the volume of a flat universe inside the apparent Horizon of radius $r = H^{-1}$ and T is the temperature of the universe. Assuming the Zel'dovich fluid within the horizon is in thermodynamical equilibrium with the apparent horizon, the temperature of the fluid is given by $T =$

$\frac{H}{2\pi}$, which is equal to the Hawking temperature of the apparent horizon. Substituting for T, V, p'_z using eq(2.3) and using $p_z = \rho_z$ in eq. (2.42),

$$dS_z = \left(\frac{16\pi^2}{H^3} - \frac{24\pi^2(2H-\zeta)}{H^4}\right) dH \ . \tag{2.43}$$

Bekenstein-Hawking formula gives the entropy of the apparent horizon[146–148] as,

$$S_H = 2\pi A \ , \tag{2.44}$$

where $A = 4\pi H^{-2}$ is the area of apparent horizon. Hence,

$$dS_H = -\frac{16\pi^2}{H^3} dH \ . \tag{2.45}$$

It follows from eqs (2.41), (2.43) and (2.45) that

$$-\left(\frac{24\pi^2(2H-\zeta)}{H^4}\right)\frac{dH}{dt} \geq 0 \ . \tag{2.46}$$

For an accelerating universe this means,

$$(\zeta - 2H)\frac{dH}{dt} \geq 0. \tag{2.47}$$

From eqs(2.47) and (2.6), we have,

$$H(\zeta - 2H)^2 \geq 0. \tag{2.48}$$

As $(\zeta - 2H)^2$ is a positive number, the inequality (2.48) requires $H \geq 0$ and hence, from eq (2.8),

$$(\bar{\zeta} + (6 - \bar{\zeta})a^{-3}) \geq 0, \tag{2.49}$$

as the required condition for validity of GSL. For $0 < \bar{\zeta} \leq 6$ the condition as in inequality (2.49) is satisfied throughout, as seen in fig(2.8). However, for $\bar{\zeta} > 6$ the condition is valid only for $a \geq a_{min}$, given by eq(2.30).

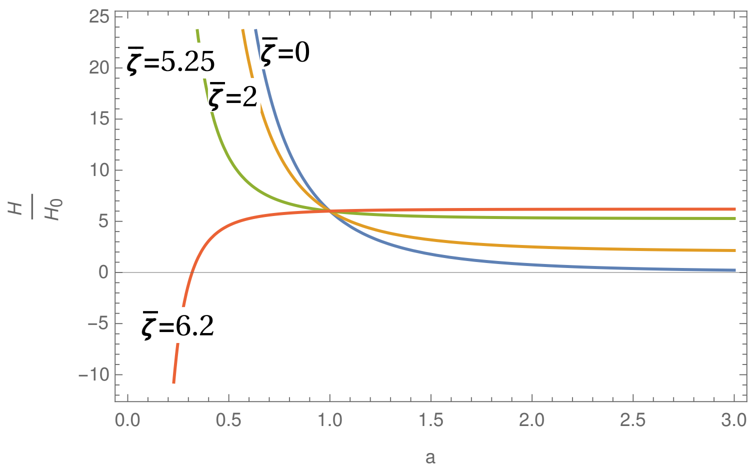

Figure 2.8: The normalized Hubble parameter vs scale factor profile.

2.4 Summary - Bulk viscous Zel'dovich fluid

Ideal Zel'dovich fluid model with $\omega = 1$ was well known as applicable to early phases of the cosmos; however, its density diluted too swiftly on being inversely proportional to sixth power of the scale factor. The ideal stiff fluid model does not explain the later matter dominated phase in which $\rho \propto a^{-3}$ and the late acceleration. Therefore, the prospects of an alternate model of bulk viscous Zel'dovich fluid with constant bulk viscosity is studied. It is found that when a constant, dimensionless bulk viscous coefficient satisfying $\bar{\zeta} \geq 0$ is introduced, the effective pressure of the fluid becomes $p_z' = p_z - 3(3H_0\bar{\zeta})H$, the model accounts for an expanding universe which accelerate in the late stage. We found that the viscous Zel'dovich fluid accounts for a universe starting with a big bang followed by deceleration and later undergoing a transition to the accelerating phase when $0 < \bar{\zeta} < 6$. For $\bar{\zeta} = 4$ the transition point from decelerating phase to accelerating phase happens to be at present. For $0 < \bar{\zeta} < 4$ the transition point will be in future, whereas for $4 < \bar{\zeta} < 6$, the same occurred in the past. Therefore it is possible for a bulkviscous Zel'dovich fluid model of cosmos to account for the recent acceleration.

We have analytically solved for the Hubble parameter and scale factor and extracted the age of the universe.

The behaviour of deceleration parameter q and the state function ω with respect to the scale factor a is also studied. It is seen that the conditions $q < 0$ and $\omega < -\frac{1}{3}$ have been sissified during the past evolution of bulk viscous fluid with $4 < \bar{\zeta} < 6$. For $\bar{\zeta} > 6$ it is always $q < 0$ and $\omega < -\frac{1}{3}$ and hence the eternal acceleration. It is also seen that both q and ω tends to -1 as $a \to \infty$ (alternately $z \to -1$), which means de Sitter epoch in the infinite future, regardless of the value of $\bar{\zeta}$.

The state finder analysis was carried out in order to contrast the model with the standard ΛCDM model and other prominent dark energy models. From the r-s plot it is also observed $\{r, s\}$ is different from that of ΛCDM model at present and coincides with that of ΛCDM model in the infinite future, as $a \to \infty$ and $z \to -1$.

The model predicts a possible big-bang as $a(t) \to 0$ as $t \to 0$. This has been verified by calculating the curvature scalar and found that it tends to ∞ as $(t - t_0) \to 0$.

The thermodynamical analysis as in section(2.3) proves Zel'dovich fluid with $0 < \bar{\zeta} < 6$ to be in concordance with GSL, although, starting with a big bang followed by deceleration progressing over to acceleration.

3

Asymptotic behaviour of the viscous Zel'dovich model

From the discussions so far, it is clear that the bulk viscous Zel'dovich fluid model can be an alternative to ΛCDM model to accommodate late acceleration and alleviate the cosmological constant and cosmological coincidence problems. It was deduced that for $4 < \bar{\zeta} < 6$, the transition to accelerating phase takes place in the past and for $\bar{\zeta} \sim 5.7$ the predicted age of the universe by the model is very near to that predicted by ΛCDM theoretical model and Globular cluster empirical data. The evolution of scale factor, equation of state and deceleration parameter are analyzed with the best fit values of the bulk viscosity coefficient and the present Hubble parameter. In the present chapter we study the asymptotic properties of the model by carrying out a phase space analysis.

3.1 Extraction of the model parameters using Type Ia supernovae data

The best fit values of the model parameter $\bar{\zeta}$, the viscosity coefficient, H_0 the Hubble parameter etc are extracted using Type Ia supernovae observational data. Union data which consists of 307 data points [149] in the redshift range $0.01 \leq z \leq 1.55$ were used in this regard. Since the light curve fitter used by Kowalski *et al.* is SALT which is considered to be more reliable and one can easily compare their results, notably the stretch distribution with the one obtained using other light curves, like the SNIFS acquisition light curves. The 580 data set in the redshift range $0.015 < z < 1.414$, consists of more observations on low redshifts. It has been noted that the interference with the background is high for obtaining low redshift data, hence their accuracy is relatively less compared to the high redshift data[150]. More over Kolmogorov-Smirnov[151] test of these samples shows that the 307 data set have relatively larger, 85%, probability of being originated from a common distribution compared to the 580 data set. In this regard one can rely more on the 307 data than 580 data sets for the extracting fruitful values of the cosmological parameters. Even if one uses the 580 data set for extracting the parameters, the changes in their value will reflect only in the third decimal place or so hence of not be much different from that obtained using 307 data. We hope this justification is sufficient enough in our usage of the 307 supernovae data. The basic method is to compare the observational distances of supernovae with the predicted value at various redshifts.The values of the parameters are those for which the predicted distance is in close match with the observed distance. The distance modulus of the supernova at a given red shift z, is given by

$$\mu(z) = (m - M) = 5 \log_{10} d_L(z) + 25 \tag{3.1}$$

where m is the apparent magnitude, M is the absolute magnitude and $d_L(z)$ is the luminosity distance in Mpc of a supernova in a flat FLRW universe. The luminosity distance at a given redshift as a function of the model parameters is given as,

$$d_L(z, \bar{\zeta}, H_0) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{h(z', \bar{\zeta}, H_0)} \tag{3.2}$$

where $h(z', \bar{\zeta}, H_0) = H/H_0$ is the normalized Hubble parameter. The observed distance modulus μ_{kob} of a type Ia supernova corresponding to

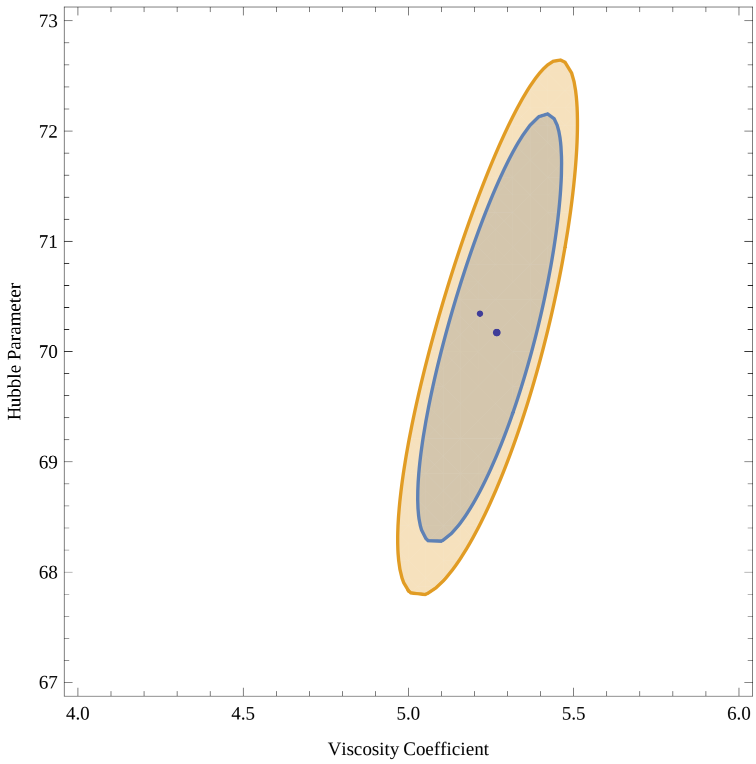

Figure 3.1: Confidence intervals for the parameters $\bar{\zeta}$ and H_0. The outer curve corresponds to 99.99% probability and the inner one corresponds to 99.73 % probability. The lower dot represents the values of the parameters corresponding to the minimum of the χ^2.

a redshift z_k is to be compared with the theoretical distance modulus μ_{kth} corresponding to the same redshift. The statistical χ^2 function for extracting the parameter can then be obtained as,

$$\chi^2 = \sum_{k=1}^{n} \frac{\left[\mu_{kth} - \mu_{kob}\right]^2}{\sigma_k^2} \tag{3.3}$$

where n is the number of data points and σ_k is the statistical error associated with the measurement corresponding to a redshift z_k. The best estimates of the parameters $(\bar{\zeta}, H_0)$ are then obtained by minimizing the χ^2 function. The minimum of the χ^2 per degrees of freedom gives measures the goodness-of-fit of the model. The confidence regions in figure 3.1 for the parameters $\bar{\zeta}$ and H_0 are then constructed for 99.73% and 99.99% probabilities respectively to find the best estimate of the parameters. The values of the parameters are shown in table 3.1 in which, a

Model	χ^2_{min}	$\chi^2_{min}/d.o.f.$	$\bar{\zeta}$	H_0
Bulk viscous model	300.264	1.011	5.25	70.20
ΛCDM model	300.93	1.013	-	70.03

Table 3.1: The best estimates of the parameters $\bar{\zeta}$ and H_0 evaluated with the supernova type-Ia union data 307 data points. But we avoided some low red shifts data, so that the net number of data used is 297.

comparison is also made of the parameters of ΛCDM model of the universe. For ΛCDM model with dark energy density around, $\Omega_{de} \sim 0.7$, gives $\chi^2_{min}/d.o.f \sim 1.013$, a value very much close to the bulk viscous Zel'dovich model. The values obtained for the present Hubble parameter by both the models are extremely close to each other. The proximity of values of cosmic parameters and minimum χ^2 mentioned imply that the present model is a good alternative to the standard ΛCDM model in predicting the background parameters of the universe. With the statistical correction the values of parameters in the present model has finally become, $\bar{\zeta} = 5.25 \pm 0.14$ and $H_0 = 70.20 \pm 0.58$. The viscous parameter value is in the range $4 < \bar{\zeta} < 6$, hence the transition from deceleration to acceleration took place in the past, as per a conclusion arrived in the previous chapter.

3.2 Evolution of cosmic parameters

The behavior of scale factor in the Zel'dovich fluid dominated universe for the best fit value $\bar{\zeta} = 5.25$, given by eq(2.10), is seen in fig(3.2). The time dependent evolution of a for a small value of $\bar{\zeta} = 0.002$ and that for

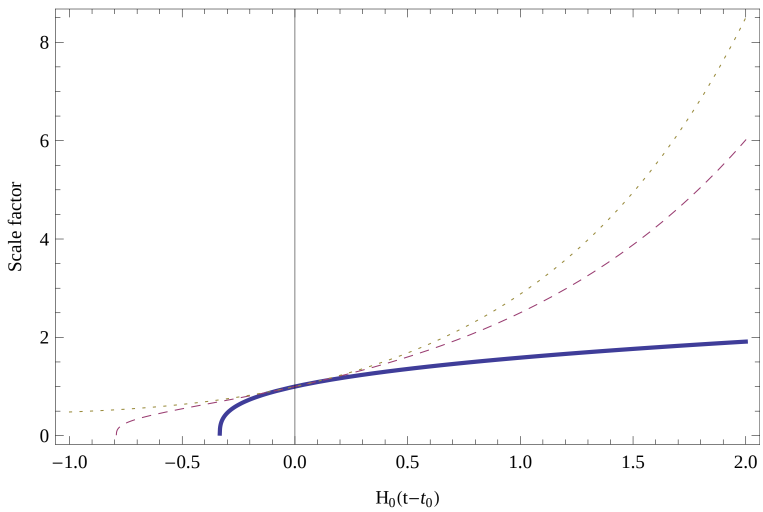

Figure 3.2: Evolution of scale factor with time. Thick line corresponds to $\bar{\zeta} = 0.002$, dashed line corresponds to $\bar{\zeta} = 5.25$ (best estimate) and dotted line corresponds to $\bar{\zeta} = 6.5$.

$\bar{\zeta} > 6$, and $\bar{\zeta} = 6.5$, too are once again plotted for contrasting with the evolution of state function corresponding to the best fit value of $\bar{\zeta}$. The initial deceleration, recent switch over to acceleration and the asymptotic de Sitter phase are exactly as per as per the discussions in section 2.1.2.

For the best estimates of the parameters $\bar{\zeta}$ and H_0, the age of the universe obtained from eq(2.16) is around 10-12 GY. This predicted age is significantly deviating from those obtained from Globular clusters data and the ΛCDM model.

The fig(3.3), as per eq(2.26) depicts the variation of state parameter against the red shift at the best fit $\bar{\zeta} = 5.25$. The time dependence of ω_z for a small value of $\bar{\zeta} = 0.002$ and that for $\bar{\zeta} = 6.5$, too are added in the figure for comparison. The equation of state of the Zel'dovich fluid $\omega_z \to -1$ as $a \to \infty$ corresponding to de Sitter Universe. The point of transition from deceleration to acceleration corresponds to $\omega_z = -\frac{1}{3}$, and it is occurs at a red shift given by eq(2.20), *ie*, at $z \sim 0.52$, close to that obtained from type Ia supernova data with standard ΛCDM model.

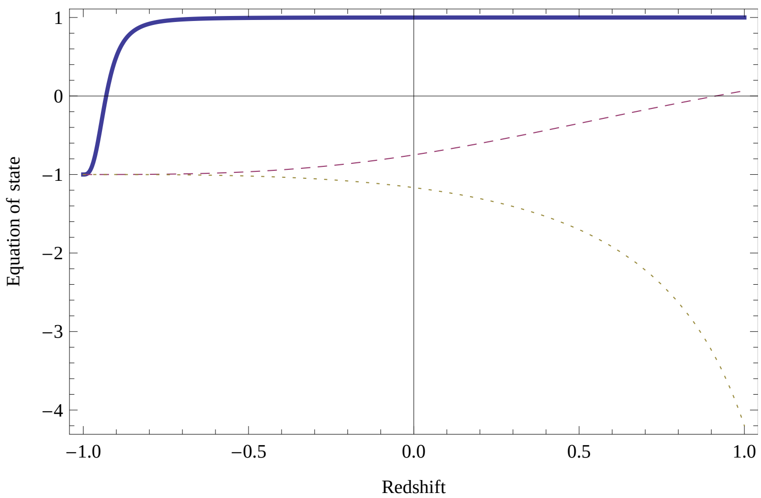

Figure 3.3: Evolution of ω_z with respect to scale factor. Thick line corresponds to $\bar{\zeta} = 0.002$, dashed line corresponds to $\bar{\zeta} = 5.25$ and dotted line corresponds to $\bar{\zeta} = 6.5$

The figure(3.4), is a profile of deceleration parameter plotted against the red shift z for best fit $\bar{\zeta} = 5.25$. In this regard too the time dependence of q for a small value of $\bar{\zeta} = 0.002$ and that for $= 6.5$, are included for comparison. The deceleration parameter $q \to -1$ asymptotically, corresponding to de Sitter Universe behaviour as seen in the distant future (*ie* when $z \to -1$). Throughout the evolution of the universe q depends on bulk viscous parameter as according to the discussions in section 2.1.2. The point where $q = 0$, corresponding to switchover from deceleration to acceleration is again at z given by eq(2.20), *ie* at $z \sim 0.52$, as confirmed.

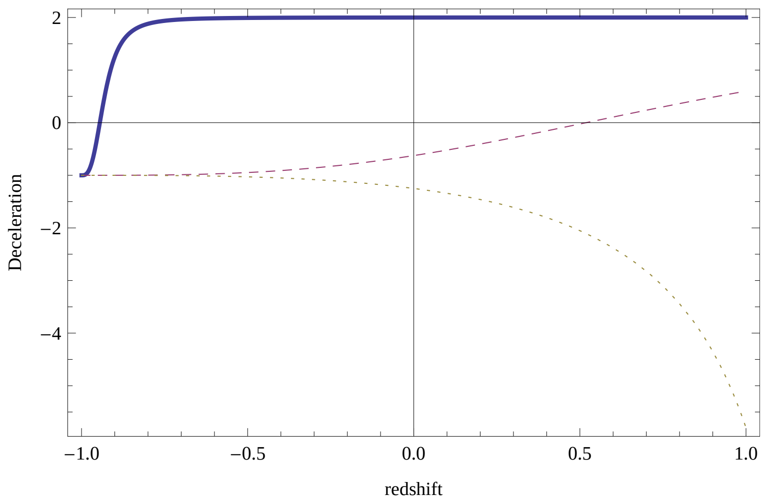

Figure 3.4: Evolution of deceleration parameter with respect to red shift. Thick line corresponds to $\bar{\zeta} = 0.002$, dashed line corresponds to $\bar{\zeta} = 5.25$ and dotted line corresponds to $\bar{\zeta} = 6.5$

3.3 Phase-space perspective

To understand the evolution of a cosmological model one have to solve the Friedmann equations fully. An alternative method to understand the general behaviour of a model is to use the tools of dynamical system analysis. This often used to understand the asymptotic properties of the model. Some times it is generally difficult to solve the cosmological field equations with more than one cosmic components. A convenient method to understand the global picture of the model is then to study the equivalent phase space. A phase-space analysis of the model would indicate whether the model is compatible with realistic evolution of the universe, i.e. the possible existence of the different stages of the universe like radiation dominated early phase, then a matter dominated phase followed by the late accelerating phase. For the phase-space analysis, one has to identify the phase space variables and be able to write down the cosmological equations as a system of autonomous differential equations. The critical points of these autonomous differential equations can then be correlated to the cosmological solutions. The stability of such critical points can be determined by examining the system obtained by linearizing about the critical point. If the critical points were a global attractor, then the trajectories of the system near the critical point will always be attracted towards it independent of the initial conditions and it will be stable one. On the other hand if the trajectories are emanating from the critical point irrespective of the initial condition then it is an unstable point. The critical point will be saddle if the emergence and starting of the trajectories near the critical point depends on the initial conditions.

3.3.1 Analysis of Zel'dovich fluid in two dimensional phase-space

In this section we analyze the phase-space behavior of flat universe dominated with bulk viscous Zel'dovich fluid. The behavior of the system in the two dimensional phase space with $h = H/H_0$ and $\Omega_z = \rho_z/3H_0^2$ as the coordinates is examined. The corresponding universe is the one with only bulk viscous Zel'dovich fluid as the component. The coupled differential equations are

$$\dot{h} = P(h, \Omega_z) \tag{3.4}$$

and

$$\dot{\Omega}_z = Q(h, \Omega_z), \tag{3.5}$$

where

$$P(h, \Omega_z) = H_0 \left(\frac{\bar{\zeta}}{2} - 3h\Omega_z \right) h \tag{3.6}$$

and

$$Q(h, \Omega_z) = -H_0 \left[\left(6 \left(\Omega_z - 1 \right) h - \bar{\zeta} \right) \Omega_z - \bar{\zeta} \ \right] \ . \tag{3.7}$$

The over-dot refers to derivative with respect to time. By setting $\dot{h} = 0$. and $\dot{\Omega}_z = 0$, we obtain the following three critical points or roots

$$(h_c, \Omega_{zc}) = (0, 1), \left(\frac{0.87667}{\Omega_z}, \Omega_z\right), (0.87667, 1). \tag{3.8}$$

The first root (0,1) has Hubble parameter as zero and hence corresponds to a static universe, while the second root depends on the instantaneous value of Ω_z, that executes a trajectory as Zel'dovich fluid density changes, and hence it is not single a fixed point rather a collection of fixed points. The third root $(h_c, \Omega_{zc}) = (0.87667, 1)$ having a positive definite value of the Hubble parameter and mass parameter of the Zel'dovich fluid corresponds to an expanding universe dominated by Zel'dovich fluid. If the system is stable in the neighborhood of a critical point, the linear perturbation in its neighborhood in phase space decays with time. The perturbations around the critical points must satisfy the following matrix equation,

$$\begin{bmatrix} \dot{\epsilon} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial P}{\partial h}\right)_0 & \left(\frac{\partial P}{\partial \Omega_z}\right)_0 \\ \left(\frac{\partial Q}{\partial h}\right)_0 & \left(\frac{\partial Q}{\partial \Omega_z}\right)_0 \end{bmatrix} \begin{bmatrix} \epsilon \\ \eta \end{bmatrix} . \tag{3.9}$$

Here ϵ and η are perturbations in h and Ω_z respectively in the neighborhood of a given critical point. The suffix $'0'$ denotes the value evaluated at the critical point, (h_c, Ω_{zc}). The corresponding Jacobian is

$$\begin{bmatrix} \left(\frac{\partial P}{\partial h}\right)_0 & \left(\frac{\partial P}{\partial \Omega_z}\right)_0 \\ \left(\frac{\partial Q}{\partial h}\right)_0 & \left(\frac{\partial Q}{\partial \Omega_z}\right)_0 \end{bmatrix} = H_0 \begin{bmatrix} \left(\frac{\bar{\zeta}}{2} - 6h\Omega_z\right) & -3h^2 \\ 6\Omega_z(1-\Omega_z) & \frac{\bar{\zeta}}{2} + 3h(1-2\Omega_z) \end{bmatrix} . \tag{3.10}$$

If the eigenvalues of the Jacobian matrix are all negative, then the critical point is stable, otherwise the critical point is generally unstable. If the eigenvalues are positive then the critical point is an unstable node and if there are positive and negative eigenvalues, then the critical point is a saddle point.

The eigenvalues corresponding to the first critical point $(h_c, \Omega_{zc}) = (0, 1)$ are found to be $(-368.2, 184.2)$. As they are of opposite signs the

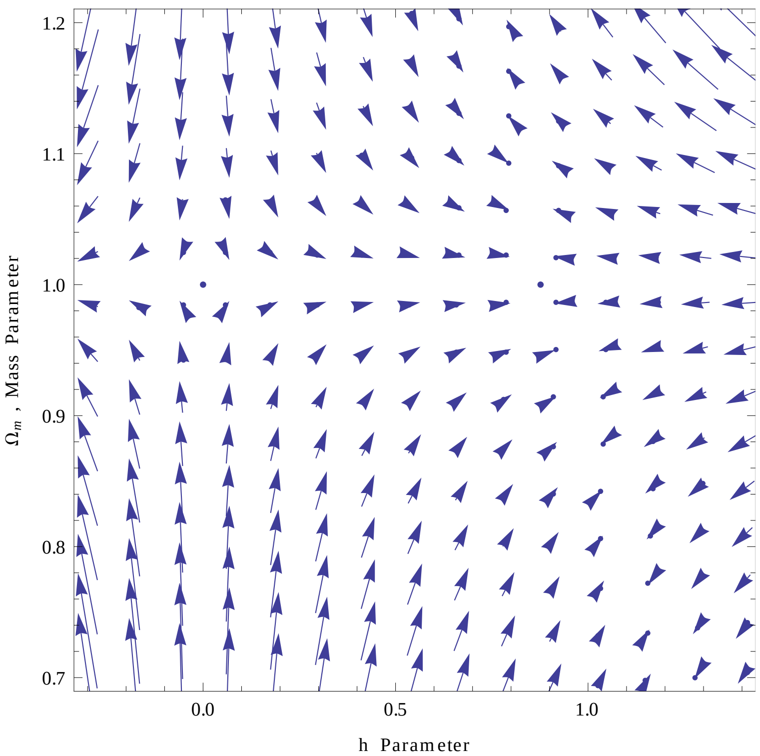

Figure 3.5: Phase space structure around the critical points. The first critical point (0,1) is a saddle point. The vector diagram clearly indicate that the trajectories approaching this point are repelled away and are finally converging on to the attractor critical point on the right.

critical point is a saddle point and hence unstable. Depending on the initial conditions the nearby trajectories around this point may approach the saddle point, but repelled by it and finally approaching a possible stable attractor in the future. As in the figure 3.5 the trajectories are turning away from the equilibrium point as and when they approach it.

The second critical point $\left(\frac{0.87667}{\Omega_z}, \Omega_z\right)$ is not an isolated point, but varies with Ω_z. As per the relationship between h and Ω_z, it represents a rectangular hyperbola with the axes $h = 0$ and $\Omega_z = 0$ as asymptotes. The eigenvalues are found to be $\left(-184.1, \frac{-161.394}{\Omega_z^2}\right)$. Since both the eigenvalues are negative and real, the neighboring trajectories will converge on to the hyperbola and hence the critical point is a stable one. The hyperbola along which the Ω_z dependent critical point moves has the coordinate axes $h = 0$ and $\Omega_z = 0$ as the asymptotes, $h = -\Omega_z$ as the directrix and $(0.9363, 0.9363)$ as the focus.

The third critical point is $(h_c, \Omega_{zc}) = (0.87667, 1)$. It is observed as in figure 3.5, this critical point is a global attractor, and physically it corresponds to a flat expanding universe dominated by bulk viscous Zel'dovich fluid. We linearize the equation (3.4), around the third critical point, and formulated the matrix equation similar to equation (3.9). The corresponding eigenvalues are found to be $(-184.2, 0.0)$. The resulting two eigenvalues clearly indicate that the model is stable for all possible initial conditions around this critical point. It appears that the second eigenvalue 0 is suggestive of the absence of any isolated critical point and rather a line segment as a continuous array of critical points. However, a close examination of the vector field plot as in figure 3.6 shows that the field directions are invariably tilted, though slightly, towards a fixed critical point as they approach. However, on low resolution, it seems to be a straight line towards the isolated critical point. This is evident from the continuous plot in the phase space structure as shown in figure 3.7. So, in reality the isolated critical point exists and the $'0'$ eigenvalue leads to a line segment as the best fit close to the critical point. This is clear from the fact that the straight line does not arise from the original procedure of setting $\dot{h} = 0$ and $\dot{\Omega}_z = 0$ without the linear approximation and rather results in an isolated point. So, as we said earlier, depending on the initial conditions the trajectories emanating from the surroundings of the saddle critical point are repelled away from it and they finally approach the stable critical point, $(0.87667, 1)$. From the eq(2.21) of the deceleration parameter , it can be easily seen that this critical point corresponds to

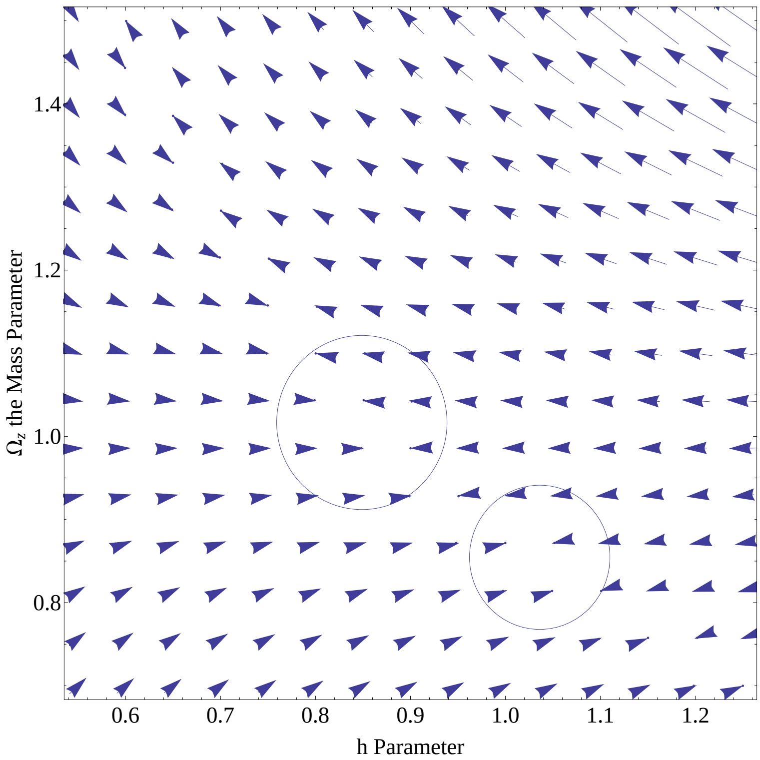

Figure 3.6: Vector field plot of the phase space around the third critical point (0,87667,1). The vectors encircled shows their continuous tilt towards the critical point.

$q < 0$ implies an accelerating phase. This, by and large implies the stability of the universe dominated by the bulk viscous Zel'dovich fluid, which accelerates the expansion in the later stage.

3.3.2 Analysis of Zel'dovich fluid in the three dimensional phase-space

In the realistic case, the universe have radiation dominated phase followed by matter dominated phase and a subsequent late accelerated epoch. In order to imply a realistic evolution of the universe by the present model, it

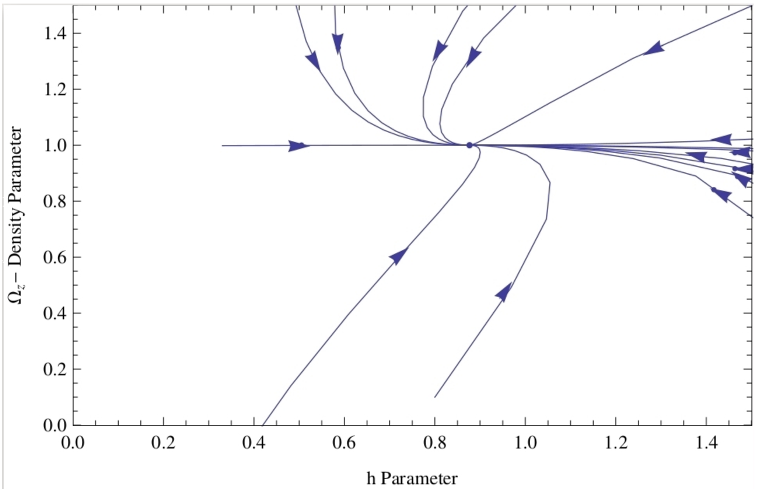

Figure 3.7: Plot of phase space trajectories around the critical point (0,87667,1). All trajectories are converging to the critical point and hence it is stable.

should predict a prior radiation dominated phase followed by matter and accelerating epochs. For knowing that, we include, besides the Zel'dovich fluid, the conventional radiation also to study the phase-space structure. On including the radiation component also, the first Friedmann equation becomes

$$3H^2 = \rho_z + \rho_\gamma \tag{3.11}$$

where ρ_γ is the radiation density. The conservation equation for the radiation component by assuming a pressure $p_\gamma = \rho_\gamma/3$, is

$$\dot{\rho}_\gamma + 4H\rho_\gamma = 0 \ . \tag{3.12}$$

.

The phase-space variables are h, Ω_z and Ω_γ among which the third parameter is $\Omega_\gamma = \rho_\gamma/3H^2$. The dynamical equations for these parameters are represented by the coupled differential equations

$$\dot{h} = P(h, \Omega_z, \Omega_\gamma) = \left((3\Omega_z + 2\Omega_\gamma)h - \frac{\bar{\zeta}}{2}\right) h, \tag{3.13}$$

$$\dot{\Omega}_z = Q(h, \Omega_z, \Omega_\gamma) = \left[2\left((3\Omega_z + 2\Omega_\gamma)h - \frac{\bar{\zeta}}{2}\right) - 6h\right]\Omega_z + \bar{\zeta} \qquad (3.14)$$

and

$$\dot{\Omega}_\gamma = R(h, \Omega_z, \Omega_\gamma) = \left[2\left((3\Omega_z + 2\Omega_\gamma)h - \frac{\bar{\zeta}}{2}\right) - 2h\right]\Omega_\gamma. \qquad (3.15)$$

The critical points are obtained by setting

$$\dot{h} = 0, \ \dot{\Omega}_z = 0, \ \dot{\Omega}_\gamma = 0\ . \qquad (3.16)$$

and they are

$$(h_c, \Omega_{zc}, \Omega_{\gamma c}) = (\frac{0.87667}{\Omega_z}, \Omega_z, 0);\ (0, 1, 0);\ (1, 0.87667, 0) \qquad (3.17)$$

out of which, the first critical point is not an isolated one, since for that, h is inversely proportional to the instantaneous value of Ω_z. The second critical point, (0,1,0), have the Hubble parameter $h = 0$, hence corresponds to static universe and third one corresponds to an expanding universe dominated by bulk viscous Zel'dovich fluid. It is to be noted that there is no critical point corresponding to a radiation dominated phase. The stability of the equilibrium points in the case of these three critical points are obtained (this time in the 3D phase-space case), once again by looking at the behavior of phase-space trajectories close to them and generated due to different initial conditions. The coupled differential equations in the linear limit in matrix representation, in the neighborhood of the equilibrium points are

$$\begin{bmatrix}\dot{\epsilon}\\ \dot{\eta}\\ \dot{\nu}\end{bmatrix} = \begin{bmatrix}\left(\frac{\partial P}{\partial h}\right)_0 & \left(\frac{\partial P}{\partial \Omega_z}\right)_0 & \left(\frac{\partial P}{\partial \Omega_\gamma}\right)_0\\ \left(\frac{\partial Q}{\partial h}\right)_0 & \left(\frac{\partial Q}{\partial \Omega_z}\right)_0 & \left(\frac{\partial Q}{\partial \Omega_\gamma}\right)_0\\ \left(\frac{\partial R}{\partial h}\right)_0 & \left(\frac{\partial R}{\partial \Omega_z}\right)_0 & \left(\frac{\partial R}{\partial \Omega_\gamma}\right)_0\end{bmatrix}\begin{bmatrix}\epsilon\\ \eta\\ \nu\end{bmatrix} \qquad (3.18)$$

where $\dot{\epsilon}$, $\dot{\eta}$ and $\dot{\nu}$ are first order perturbation terms of $P(h, \Omega_z, \Omega_\gamma) = \dot{h}$, $Q(h, \Omega_z, \Omega_\gamma) = \dot{\Omega}_z$ and $R(h, \Omega_z, \Omega_\gamma) = \dot{\Omega}_\gamma$ respectively, $\epsilon(t)$, $\eta(t)$ and $\nu(t)$

being the first order linear perturbation terms of h, Ω_z and Ω_γ respectively. The square matrix term in the equation (3.18) is the Jacobian evaluated at the critical point. We then decouple the differential equations (3.18) by means of the secular equation and in the process the eigenvalues corresponding to the equilibrium point $\left(\frac{0.87667}{\Omega_{zc}}, \Omega_{zc}, 0\right)$ are obtained as

$$\lambda_1 = \frac{0.5}{\Omega_z}\left(-368.2 + 365.57\Omega_z - \sqrt{135571 - 269206\Omega_z + 13364\Omega_z^2}\right), \tag{3.19}$$

$$\lambda_2 = \frac{0.5}{\Omega_z}\left(-368.2 + 365.57\Omega_z + \sqrt{135571 - 269206\Omega_z + 13364\Omega_z^2}\right) \tag{3.20}$$

and

$$\lambda_3 = \frac{245.466}{\Omega_z}, \tag{3.21}$$

all depending on the instantaneous value of Ω_z. The critical point in this case drifts with the variation of Ω_z along a rectangular hyperbola on the $h - \Omega_z$ plane; with the details of the hyperbola same as in the case of the second critical point in the section 3.3.1. The eigenvalues indicate that the phase space trajectories corresponding to various initial conditions, move away from the critical point and hence there is no stable situation. Even when $\Omega_z = 0$ the eigenvalues are such that, λ_1 is negative, $\lambda_2 = 0$ and λ_3 positive, and so no stable solution is implied.

The second critical point $(0, 1, 0)$ corresponding to static epoch has the eigenvalues $(-368.2, 368.2, 184.1)$ and the third critical point has the eigen values as $(-403.778, 280.0, 167.878)$. There is one negative eigenvalue and two positive eigenvalues for each critical points which again means there is no stability for the equilibrium points. This means, the phase space trajectories are not attracted by any of the critical points in the three dimensional case. For example the vector field plot as in the figure 3.8 clearly indicates how the phase-space trajectories corresponding to various initial conditions are repelled away, rather than being attracted to the

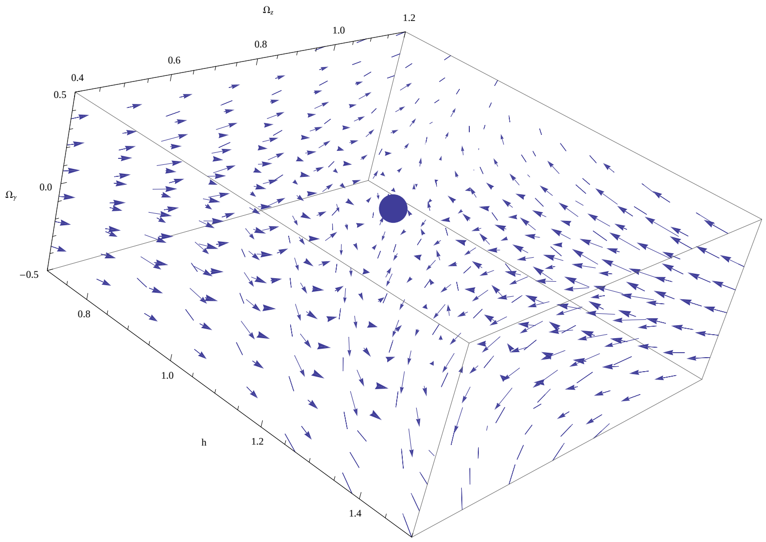

Figure 3.8: Vector field plot of the phase-space structure around the critical point (1,0.87667,0)

second critical point. So none of the critical points in this case corresponds to a radiation dominated phase and even the existing critical points are not stable also. In fact the third critical point which corresponds to a Zel'dovich fluid dominated one is unstable, it can be concluded that the inclusion of the radiation component may lead to a complete break down of the model. The bulk viscous coefficient is taken as a constant in the present study. Since it is a transport coefficient which may depend on the velocity of the fluid component also. Such a velocity dependent bulk viscous coefficient may be checked for consistency of a prior radiation dominated phase and that we reserve for a future work.

3.4 Summary - Bulk viscous Zel'dovich fluid and its asymptotic properties

By constraining with type Ia supernova data, the optimum values for present Hubble parameter H_0 and the bulk viscous parameter $\bar{\zeta}$ were obtained and the corresponding evolution of scale factor, state function and q-factor were studied. The role of $\bar{\zeta}$ is to modify the pressure in the FLRW universe as per $p = p - \bar{\zeta} H_0 H$, contributing a negative pressure and hence generating the acceleration of the universe. It is seen that the bulk viscous fluid model accounts for the late accelerating universe. For the best fit values $\bar{\zeta} \sim 5.25$ and $H_0 \sim 70.2$ km s^{-1} Mpc^{-1}, the switchover of acceleration takes place at $z \sim 0.52$, which is closely in agreement with the observed values. The asymptotic de Sitter phase also is indicated by the evolution profiles of the mentioned parameters. Thus the Zel'dovich fluid is likely to be an alternative to the ΛCDM model for the accelerating universe. χ-square minimum, the measure of accuracy too closely matches with that of ΛCDM model. However, the prediction of age of the universe is not so satisfactory in comparison with that due to ΛCDM model and the empirical data owing to globular cluster observations.

A two dimensional phase space analysis with $\Omega_z = \frac{\rho_z}{3H_0^2}$ and $h = \frac{H}{H_0}$ as co-ordinates obtains a stable critical point corresponding to accelerating universe, while having the critical point corresponding to a static universe, unstable. In the three dimensional phase-space analysis including radiation as third component, there is no stable point.

4

Bulk viscous Zel'dovich Fluid in decaying vacuum

In the previous chapter we discussed how the late acceleration of the universe can be accounted by considering bulk viscous stiff fluid as the dominant cosmic component. However, the age predicted by the model is less than the observed value. Therefore, we consider a flat universe with viscous stiff fluid and decaying vacuum energy as the cosmic components for improving the prediction of its age.

4.1 Scalar field approach to stiff fluid and interacting vacuum

It can be shown that the Zel'dovich fluid with decaying vacuum can be justified via scalar field theory approach. Let us consider the evolution of a self interacting scalar field, ϕ which is minimally coupled to gravity in flat isotropic and homogeneous universe. In describing the evolution of the scalar field we mainly follow the reference [152], so for more details see that reference. The evolution of the scalar field is governed by the equations,

$$3H^2 = 8\pi G\rho_\phi \tag{4.1}$$

and

$$\dot{\phi}\ddot{\phi} + 3H\dot{\phi}^2 = \dot{\phi}\frac{dV(\phi)}{d\phi} \tag{4.2}$$

where H is the Hubble parameter, $V(\phi)$ is the potential, dot represents a derivative with respect to cosmic time and equation(4.2) represents the dynamical evolution of the field. The equation of state of the field is,

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} \ . \tag{4.3}$$

The pressure and density of the scalar field are given by,

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) \ . \tag{4.4}$$

The self interacting scalar field can be effectively treated as the mixture of two interacting fluids, with densities ρ_1 and ρ_2 which are having the equation of state ω_1 and ω_2 respectively. Then the effective pressure is,

$$P_{eff} = \omega_{eff}\rho_{eff} = \frac{\omega_1\rho_1 + \omega_2\rho_2}{\rho_1 + \rho_2}\rho_{eff}, \tag{4.5}$$

where $\rho_{eff} = \rho_1 + \rho_2$. Hence it is now possible to have two fluids, one with equation of state, $\omega_1 = 1$, corresponding to stiff fluid and the other with equation of state, $\omega_2 = -1$ corresponding to vacuum energy, if one identifies the corresponding densities and pressures as,

$$\rho_1 = \frac{\dot{\phi}^2}{2}, \quad p_1 = \frac{\dot{\phi}^2}{2} \tag{4.6}$$

and

$$\rho_2 = V(\phi), \quad p_2 = -V(\phi). \tag{4.7}$$

One can either take these components as isolated from each other, so that each one of them satisfying separate conservation laws or can be taken as interacting components following the conservation equations,

$$\dot{\rho}_1 + 3H\left(\rho_1 + p_1\right) = Q, \ \dot{\rho}_2 + 3H\left(\rho_2 + p_2\right) = -Q, \tag{4.8}$$

for $Q > 0$ the energy flows from ρ_2 to ρ_1 and for $Q < 0$ the energy flow in the reverse direction. From equations (4.6),(4.7) and (4.8), it follows that,

$$\dot{\phi}\ddot{\phi} + 3H\dot{\phi}^2 = Q(t), \quad \dot{\phi}\frac{dV(\phi)}{d\phi} = -Q. \tag{4.9}$$

These equations are the equivalent evolution equation of the scalar field. Hence it is possible to consider a scalar field as a mixture of a stiff fluid interacting with an effective vacuum energy.

4.2 Background evolution with viscous Zel'dovich fluid and decaying vacuum $\Lambda(t)$

In reference [152], the authors have described the evolution of such a universe. Our aim is not in line with that. Instead we consider a phenomenological form for the decaying vacuum along with the bulk viscous stiff fluid (Zel'dovich fluid) and analyze both the background evolution of the universe, particularly in finding the age and also the asymptotic behavior. In this model of FLRW universe following the standard metric given by eq(1.7), the cosmic components are the bulk viscous Zel'dovich fluid and a varying cosmological constant. As the ideal, non viscous Zel'dovich fluid obeying the equation of state $p_z = \rho_z$[106] dilutes too rapidly as per $\rho \propto a^{-6}$, the viscosity is introduced in the Zel'dovich fluid in this model too, to alter the evolution of ρ_z, appropriate enough to account for late universe. This motivation behind bringing in viscosity is the same as discussed in section(2.1). The physics behind the viscosity so introduced is discussed in detail in section(2.3), as to how it arises to restore equilibrium when there are local thermodynamical disturbances. Here too, the viscosity alters the pressure as $p_z \rightarrow p'_z = p_z - 3\bar{\zeta}H$, where the component $-3\bar{\zeta}H$ generates the acceleration. The Eckart formalism, used in the single component bulk viscous Zel'dovich fluid model of cosmos (also equivalent to a formalism being discussed by Landau and Lifshitz [123]), is also extended over to the current Zel'dovich fluid and decaying vacuum two component model.

The second cosmic component in the present model is the time varying cosmological parameter given by[154],

$$\Lambda(t) = 3\alpha H^2 \tag{4.10}$$

where α is a free parameter, value of which would be less than one. Earlier introduction of this kind of decaying vacuum was considered by Carvalho and Lima[155], where the authors restricted to $\alpha \leq 1/2$. A higher value of α resulted into incompatible age for the universe as claimed by many authors like [154], so the values of α is usually restricted to below one. Since this is effectively a form of time varying vacuum energy, its equation of state is taken as, $\omega_\Lambda = (p_\Lambda/\rho_\Lambda) = -1$. Basically the $\Lambda(t)$ models have been originated from curved space quantum field theories[156]. Often there appears a constant additive term along with the time varying part in the equation for $\Lambda(t)$, which, as argued by many [157, 158] facilitate the transition from the decelerating to an accelerating epoch of the expanding universe. But in the present model such an additive constant is not needed due to the presence of viscosity in the Zel'dovich fluid component, which will otherwise guarantee such a transition from an early deceleration to a later accelerating phase of expansion.

The Friedmann metric along with the standard Einstein's field equation will give the Friedmann equation for a flat universe as,

$$3H^2(t) = \rho_z + \rho_\Lambda \tag{4.11}$$

where $\rho_\Lambda = \Lambda(t)$ (in standard units, $8\pi G = 1$, $c = 1$), is the time varying cosmological parameter, equivalent to the standard dark energy density. These components together satisfy the conservation law (in the absence of any source, i.e. $Q = 0$),

$$\dot{\rho}_\Lambda + \dot{\rho}_z + 3H\left(\rho_z + p'_z\right) = 0. \tag{4.12}$$

where a dot represents a derivative with respect to cosmic time. Combining the Friedmann equation and the conservation equation, leads to

$$\dot{\rho}_z + \dot{\rho}_\Lambda = -6H\left(\rho_z - \frac{3}{2}\zeta H\right). \tag{4.13}$$

But from Friedmann equation, $\dot{\rho}_z + \dot{\rho}_\Lambda = 6H\dot{H}$ and substituting this, the above equation becomes,

$$\dot{H} = -\left(\rho_z - \frac{3}{2}\zeta H\right). \tag{4.14}$$

Again from Friedmann equation one can substitute for ρ_z as,

$$\rho_z = 3\,(1-\alpha)\,H^2. \tag{4.15}$$

From equations (4.14) and (4.15),

$$\dot{H} + 3H\left((1-\alpha)H - \frac{\zeta}{2}\right) = 0. \tag{4.16}$$

Solving equation (4.16) we obtain the Hubble parameter as,

$$H(t) = \eta\left[1 + \coth\left(3(1-\alpha)\eta(t-t_0) + \phi\right)\right] \tag{4.17}$$

where $\eta = \dfrac{\zeta}{4(1-\alpha)}$, $\phi = \coth^{-1}\left(\dfrac{H_0}{\eta} - 1\right)$ and H_0 is the current value of Hubble parameter. Integrating the above equation, we obtain the equation for the scale factor as

$$a(t) = e^{\eta(t-t_0)}\left(\frac{\sinh[3\eta(1-\alpha)(t-t_0)+\phi]}{\sinh(\phi)}\right)^{\frac{1}{3(1-\alpha)}}. \tag{4.18}$$

Using this equation, the Hubble parameter in equation (4.17) can be recast as,

$$H = \frac{\zeta}{2(1-\alpha)} + \left(H_0 - \frac{\zeta}{2(1-\alpha)}\right)a^{-3(1-\alpha)}. \tag{4.19}$$

In the asymptotic limit $a(t) \to \infty$ the Hubble parameter becomes a constant, $H \to \dfrac{\zeta}{2(1-\alpha)}$ which corresponds to the de Sitter phase with exponential increase in the scale factor, while in the limit $a(t) \to 0$, the Hubble parameter evolves as, $H \sim a^{-3(1-\alpha)}$, which points to an earlier decelerated epoch dominated with Zel'dovich fluid with density $\rho_z \sim H^2 \sim a^{-6(1-\alpha)}$. Thus the existence of the transition from an early decelerated to a late accelerated epoch is guaranteed. In these limits the scale factor will evolve as follows. As $t \to \infty$ the scale factor will evolve as, $a(t) \to e^{2\eta(t-t_0)}$, this exponential increase corresponds to the de Sitter epoch, while in the early stage of the evolution, corresponding to $3\eta(1-\alpha)(t-t_0) < 1$ the above form of $a(t)$ almost implies that, $a(t) \sim ((1+3\eta(1-\alpha)(t-t_0))^{1/3(1-\alpha)}$, representing a decelerating epoch. What is important here is that the transition occurs without the aid of an additive constant in the $\Lambda(t)$. In non-viscous models like entropic dark energy[159] or Ricci dark energy[158], the presence of a bare constant cosmological term is essential for having a transition from the early decelerated epoch to the late accelerated epoch.

The evolution of the cosmological parameters, like Hubble parameter, scale factor etc are depending upon the numerical values of the model parameters α and viscous coefficient ζ. However it is clear from the expression of scale factor in equation(4.18) that for a constant α the beginning of the universe corresponding to $a = 0$ would have occurred earlier into the past of the universe as ζ assumes higher values. For constant ζ and increasing α, the situation will be the same too. In both the cases the age of the universe increases compared to a model with only Zel'dovich fluid as the cosmic component. However, only with an extraction of these parameters, a final conclusion regarding the age of the universe can be made.

We have considered a flat universe ($k = 0$), since observations strongly indicate that our universe is flat [153]. The inflationary models theoretically propose a very small value for curvature around $\Omega_{k0} \sim 10^{-5}$ while observations favor a value of the order of 10^{-2}. Basically for non-flat universe, the Friedmann equation becomes,

$$3H^2 = \rho_z + \rho_\Lambda + \rho_k. \tag{4.20}$$

where $\rho_k = -ka^{-2}$. Since the interaction is only between Zel'dovich fluid and the vacuum, the conservation law is,

$$\dot{\rho}_z + \dot{\rho}_\Lambda + 3H\left(\rho_z + p'_z\right) = 0. \tag{4.21}$$

Using $p'_z = p_z - 3\zeta H$ and eq(4.15) and through simple algebra we can rewrite the above equation as,

$$\dot{H} + 3H\left((1-\alpha)H - \frac{\zeta}{2}\right) = \dot{\rho}_k. \tag{4.22}$$

In our original work we took, $\rho_k = 0$ consequently the RHS of the above equation is zero. But for non-flat universe the contribution due to the RHS term, $\dot{\rho}_k = -2H\rho_k$ is extremely small especially in the late stage, first of all due to the decreasing nature of H and secondly due to the extremely low magnitude of ρ_k. Hence the solution of the above equation (our model) would be almost close to the solution of the corresponding homogeneous equation with zero curvature.

4.3 Extraction of model parameters and evolution of cosmic parameters

The best fit values for ζ, α and H_0 are estimated using type Ia supernova observational data. Union data containing 307 data points [149] in the red shift range $0.01 < z < 1.55$ has been used here. The distance modulus μ_i for i^{th} supernova at a red shift z_i, having an apparent magnitude m and absolute magnitude M being obtained from eq(3.1), the luminosity distance d_L, as in eq(1.33) in the χ-square
analysis is a function of α too so that now $dL_z = dL_z(z, \bar{\zeta}, \alpha, H_0)$, while having $h = \dfrac{H(z,\bar{\zeta},\alpha,H_0)}{H_0} = h(z,\bar{\zeta},\alpha,H_0)$, where $\bar{\zeta} = \dfrac{\zeta}{H_0}$, not same as the dimensionless bulk viscous coefficient previously used.

Equation for h in terms of z, the cosmological red shift is obtained by

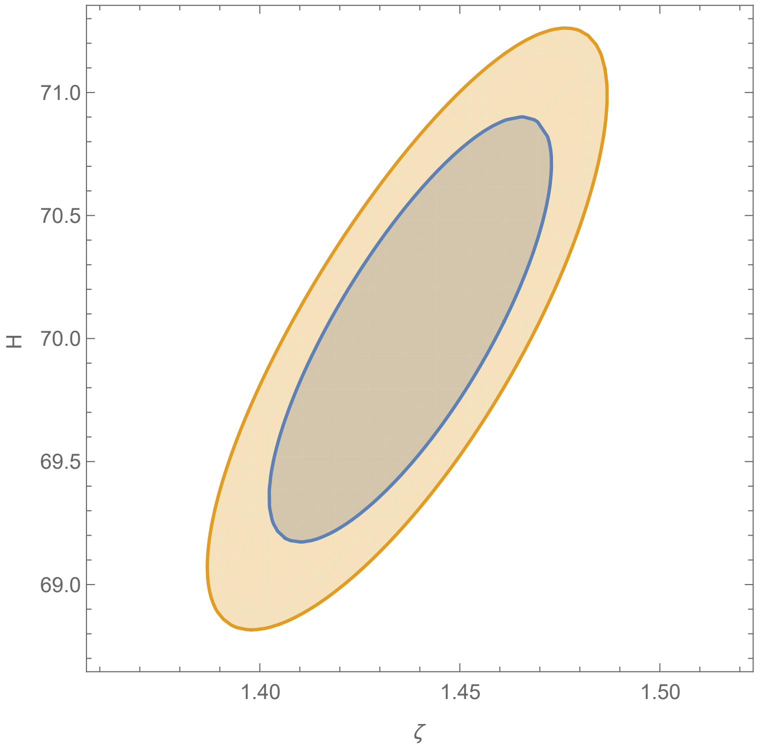

Figure 4.1: Contour plot for the parameters H and $\bar{\zeta}$ for fixed $\alpha = 0.14$.

substituting for scale factor using $a = \dfrac{1}{1+z}$ in equation(4.19) as

$$h(z', \bar{\zeta}, \alpha) = \frac{\bar{\zeta}}{2(1-\alpha)} + \left(1 - \frac{\bar{\zeta}}{2(1-\alpha)}\right)(1+z)^{3(1-\alpha)}. \tag{4.23}$$

The theoretical distance moduli for various supernova with observed red shifts z and apparent magnitudes m are obtained using eq (3.1) and eq(3.2) and are compared with the corresponding observed distance moduli, $m - M$, as in eq(3.1), where m is from the data. The statistical χ^2 function for comparing the theoretical and observational values of the distance moduli is again computed from eq(3.3) using the available 307 supernova data. By minimizing the χ^2 function, the best estimates of the parameters $(\bar{\zeta}, \alpha, H_0)$ are obtained . The minimum of the χ^2 indicates the goodness-of-fit of the model apart from giving the best estimates of the model parameters.

For obtaining the χ^2 function we also used Background (CMB) data from the WMAP 7-yr observation and the Baryon Acoustic Oscillation (BAO) data from Sloan Digital Sky Survey(SDSS). The BAO signal has been directly detected by SDSS survey at a scale ∼100 Mpc. The BAO peak parameter value was first proposed by Eisenstein, D. J. et al[160] and is defined as

$$\mathcal{A} = \frac{\sqrt{\Omega_m}}{h(z_1)^{\frac{1}{3}}} \left(\frac{1}{z_1} \int_0^{z_1} \frac{dz}{h(z)} \right)^{\frac{2}{3}} . \tag{4.24}$$

Here $h(z)$ is the weighted Hubble parameter, $z_1 = 0.35$ is the red shift of the SDSS sample. Using SDSS data from luminous red galaxies survey the value of the parameter $\mathcal{A}$(for flat universe) is given by $\mathcal{A} = 0.469 \pm 0.017$[160]. The χ^2 function for the BAO measurement takes the form

$$\chi^2_{BAO} = \frac{(\mathcal{A} - 0.469)^2}{(0.017)^2} . \tag{4.25}$$

The CMB shift parameter is the first peak of CMB power spectrum[161] which can be written as

$$\mathcal{R} = \sqrt{\Omega_m} \int_0^{z_2} \frac{dz}{h(z)} . \tag{4.26}$$

Here z_2 is the red shift at the last scattering surface. From the WMAP 7-year data, $z_2 = 1091.3$. At this red shift z_2, the value of shift parameter would be $\mathcal{R} = 1.725 \pm 0.018$[153]. The χ^2 function for the CMB measurement can be written as

$$\chi^2_{CMB} = \frac{(\mathcal{R} - 1.725)^2}{(0.018)^2} . \tag{4.27}$$

Considering three cosmological data sets together, i.e. (SNe+BAO+CMB), the total χ^2 function is then given by

$$\chi^2_{total} = \chi^2_{SNe} + \chi^2_{BAO} + \chi^2_{CMB} . \tag{4.28}$$

By minimizing the χ^2, we found parameter values as, $\alpha = 0.14, H_0 = 70.3$ km s^{-1}Mpc^{-1}, and $\bar{\zeta} = 1.446$ for χ^2_{min} per degrees of freedom, $\chi^2_{dof} = \frac{\chi^2_{min}}{n-m} = 1.016$, where n is the number of data points and $m = 3$ the number of free parameters. We have constructed the confidence interval plane for the parameters H and $\bar{\zeta}$ keeping $\alpha = 0.14$, its best estimated value. The confidence intervals corresponding to 68.4% and 95.4% show fairly good behavior and are given in figure.4.1. The best fit values for the parameters H_0 and $\bar{\zeta}$ with corrections for a confidence of 64.8% are $H_0 = 70.03^{+0.54}_{-0.46}$ and $\bar{\zeta} = 1.446^{+0.018}_{-0.023}$. For 95.4% probability the corrected parameter values are $H_0 = 70.3^{+0.565}_{-0.47}$ and $\bar{\zeta} = 1.446^{+0.095}_{0.032}$ for $\alpha = 0.14$. It may be noted that in reference [154], the authors have extracted an upper limit for the parameter α by constraining a model with a decaying vacuum, $\Lambda = \Lambda_0 + 3\alpha H^2$, as $\alpha \leq 0.15$.

In discussing the evolution of different cosmological parameters, it is better to start with the equation of state parameter. As it was shown in some of the earlier works[104], the equation of state of the viscous Zel'dovich fluid has natural evolution from its extreme stiff nature (corresponds $\omega = 1$) to the de Sitter type behavior through radiation (corresponds to $\omega = 1/3$) like and matter like (corresponds to $\omega = 0$) natures. First we will consider the net equation of state, comprising both the Zel'dovich fluid and the decaying vacuum, which can be obtained by the standard procedure as,

$$\omega(z) = -1 - \frac{1}{3}\frac{d}{dx}(\ln h^2) \tag{4.29}$$

From the equations (4.23)and (4.29), the equation of state can be expressed as,

$$\omega(z) = -1 + \left(2(1-\alpha) - \bar{\zeta}\right)\frac{1}{h}(1+z)^{3(1-\alpha)}. \tag{4.30}$$

For $\alpha = 0$, and $\bar{\zeta} = 0$ equation of state tends to $\omega(z) \to 1$ for very large redshift, which corresponds to the early epoch dominated with Zel'dovich

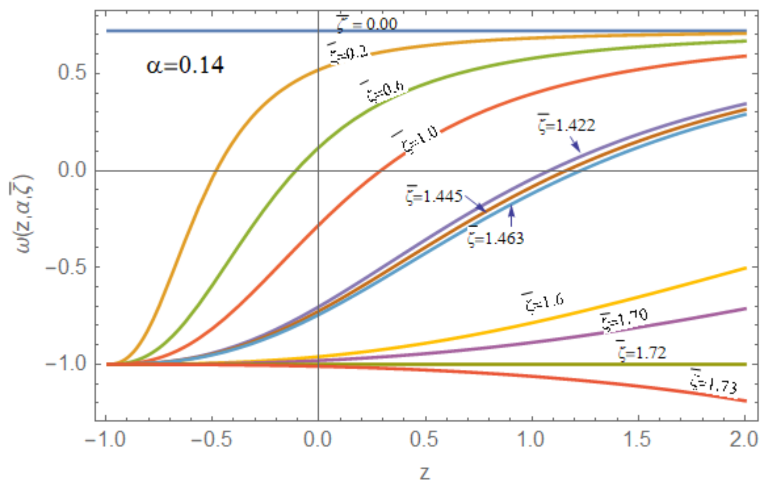

Figure 4.2: Evolution of ω with time for constant $\alpha = 0.14$ and varied values of the viscous coefficient, $\bar{\zeta}$.

fluid with negligible viscosity. For the chosen value of the parameter, $\alpha = 0.14$ a transition into the late accelerating phase would occur for a range of values of the viscous coefficient, $1.72 > \bar{\zeta} > 0$. For the best fit values of the parameters, $(\alpha, \bar{\zeta}, H_0) = (0.14, 1.445, 70.03)$ the evolution of the equation of state parameter is as expected (see figure 4.2), that is, starting with a value corresponding to stiff fluid and gradually approaching the value corresponding to de Sitter epoch. The current value of ω corresponding to the best fit values of the parameters is found to be $\omega \sim -0.72$, very much close to the range of values of the equation of state from WMAP data[153].

The equation of state of the viscous Zel'dovich fluid can be obtained using a similar approach as,

$$\omega_z = -1 - \frac{1}{3}\frac{d\Omega_z}{dx} = -1 - \frac{(1-\alpha)}{3}\frac{d\ln h^2}{dx}. \tag{4.31}$$

The difference in ω_z compared to $\omega(z)$ is that the second term on the right hand side of the above equation contains an extra term, $(1-\alpha)$, so that the final expression for ω_z becomes,

$$\omega_z = -1 + (1-\alpha)\left(2(1-\alpha) - \bar{\zeta}\right)\frac{1}{h}(1+z)^{3(1-\alpha)}. \tag{4.32}$$

The general evolution of ω_z is hence similar to that of $\omega(z)$ except in the particular numerical values corresponding to different epochs. But both will approach the de Sitter value as $a \to \infty$.

The deceleration parameter $q(z)$ is a measure of the acceleration and can be obtained from the basic equation(2.21) and eq(4.19). Deceleration

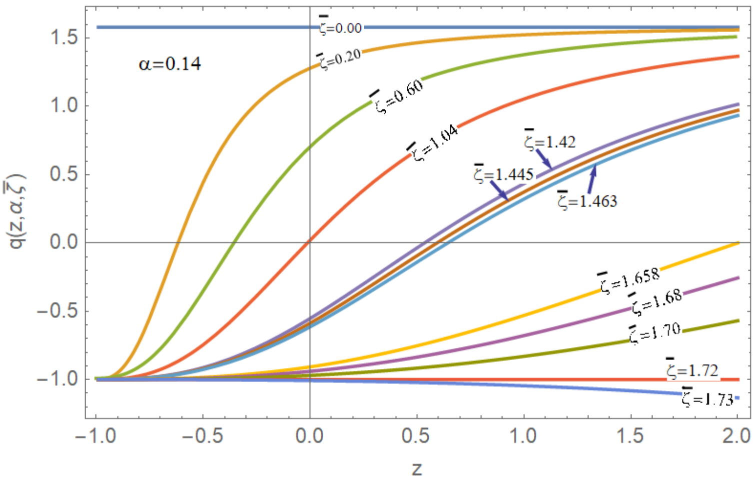

Figure 4.3: Evolution of deceleration parameter with $\alpha = 0.14$ and varying $\bar{\zeta}$.

parameter then takes the form

$$q = -1 + \left(\frac{3(1-\alpha)}{1 + (\frac{1}{\frac{2(1-\alpha)}{\bar{\zeta}} - 1})(1+z)^{-3(1-\alpha)}}\right). \tag{4.33}$$

When both the model parameters α and $\bar{\zeta}$ are equal to zero, the cosmic component becomes pure Zel'dovich fluid and $q = 2$. For the best estimated values of the model parameters, the transition is found to occur at a redshift, $z \approx 0.61$ which is again close to the observed value. Irrespective of the values of $\bar{\zeta}$ the deceleration parameter asymptotically approaches the value, $q = -1$.

Finally we will discuss about the evolution of the scale factor. The age of the universe can be directly obtained for the evolution of the scale factor. The evolution of the scale factor is given in equation(4.18). We have already shown its asymptotic behavior in a previous section, that in the early epoch it evolves as in the decelerated phase and in the extreme future it evolves as in de Sitter epoch. In general, the form of $a(t)$ indicates the presence of big-bang as $(t-t_0) \to -\infty$. The evolution of it is as shown in Figure (4.4) for the best fit values of the model parameters. But for higher values of $\bar{\zeta}$ it is found that the big-bang occur at earlier times as evident from figure (4.5). It is also clear that, for extremely high values

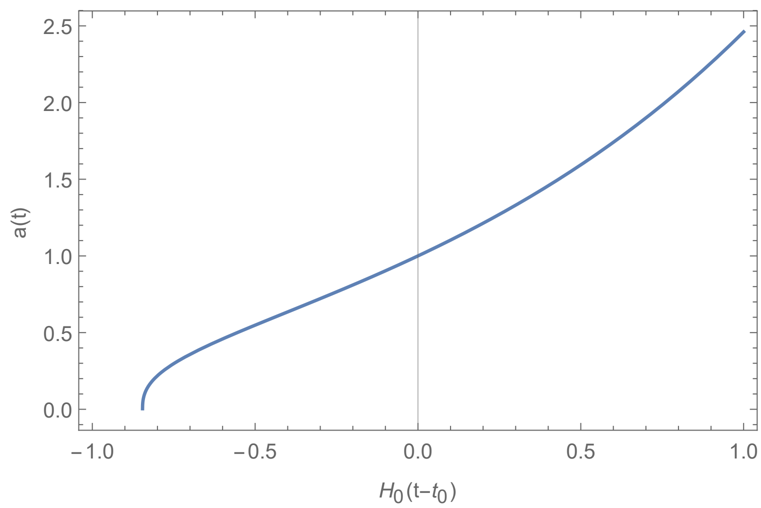

Figure 4.4: Evolution of scale factor with time. The profile corresponds to best fit values, $\alpha = 0.14$, $\bar{\zeta} = 1.445$ and $H_0 = 70.03\ km^{-1}s^{-1}Mpc^{-1}$.

of the viscous parameter, the scale factor would have no-zero values in the beginning indicating the absence of big-bang. It is found that, there is no big-bang in the model for $\bar{\zeta} > 1.72$. This means that age of the universe defined only for $\bar{\zeta} < 1.72$.

The age of the universe in the present model can be obtained by equat-

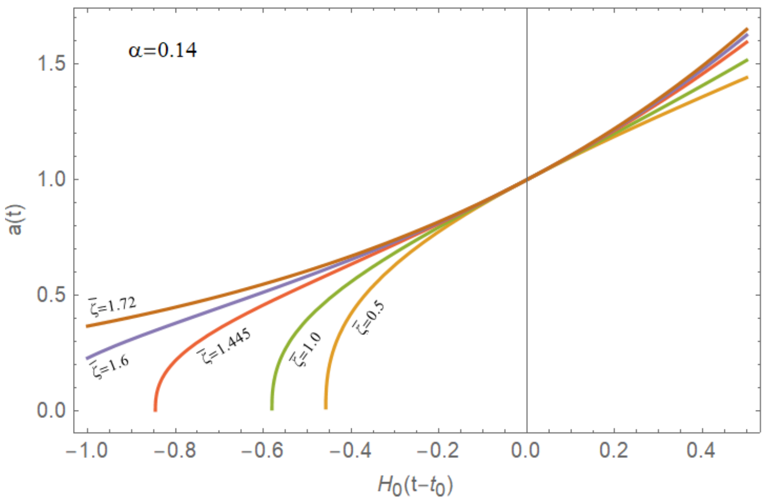

Figure 4.5: The spectrum of curves on evolution of $a(t)$ with $\alpha = 0.14$ and varying $\bar{\zeta}$. There is big-bang for $\bar{\zeta} < 1.72$.

ing the scale factor to zero, which leads to,

$$Sh\left[3\eta(1-\alpha)(t_B - t_0) + \phi\right] = 0. \tag{4.34}$$

where we took, $t = t_B$, as the big bang time. This leads to the equation of the age of the universe as,

$$t_0 - t_B = \frac{\phi}{3\eta(1-\alpha)}. \tag{4.35}$$

On substituting the expressions for ϕ and η, the above equation can be simplified into,

$$t_0 - t_B = \frac{2}{3\bar{\zeta}} \ln\left(\frac{1}{1-2\bar{\eta}}\right) H_0^{-1}, \tag{4.36}$$

where $\bar{\eta} = \eta/H_0$. For the best estimated values of the model parameters, it is found that, the above equation gives an age of the universe in the range, $11.39 - 12.18$ GY. First of all, this age is higher than the age predicted by the model in which Zel'dovich fluid is the dominant component[105]. Secondly it is near to the age of the universe obtained from the data on oldest globular clusters[73–76]. In this sense the model seems to solve the problem of age which existed in the model with Zel'dovich fluid as the only cosmic component. So it can be concluded that, the inclusion of a varying dark energy component along the bulk viscous Zel'dovich fluid is essential for the consistency with the age in this model.

The analysis so far reveals that, the inclusion of the additional component, the decaying vacuum to the viscous Zel'dovich fluid, the model gives a reasonable back ground evolution of the universe. Apart from this, the age of the resulting universe will be high compared to the model with Zel'dovich fluid alone as the cosmic component. The viscous Zel'dovich fluid component evolves in such a way that, during the very early period the matter component is a stiff fluid, compatible with many theoretical speculations[77]. But as the universe evolves, the equation of state is smoothly evolving towards that of pressureless fluid, corresponding to the non-relativistic matter. Hence the model supporting the speculation that in the early period the matter would have existed as a stiff fluid. In the late stage, the evolution is compatible with the standard ΛCDM model, in predicting the observational parameters, including the age of the universe. In next section we do a dynamical system analysis of the model, which may throw more light on the viability of the present model.

4.4 Dynamical system analysis

Dynamical system analysis is an effective method to extract the useful information about the stability of the asymptotic behavior of the model. For this, we have to express the cosmological equations governing the evolution of the model as a set of autonomous differential equations. Then the concerned information can be obtained by finding critical points and analyzing the nature of the trajectories in the neighborhood of the critical points. Eventually it will become clear from this analysis whether the model in question is consistent with the presently accepted cosmological paradigm. In the present case this means whether the model predicts a

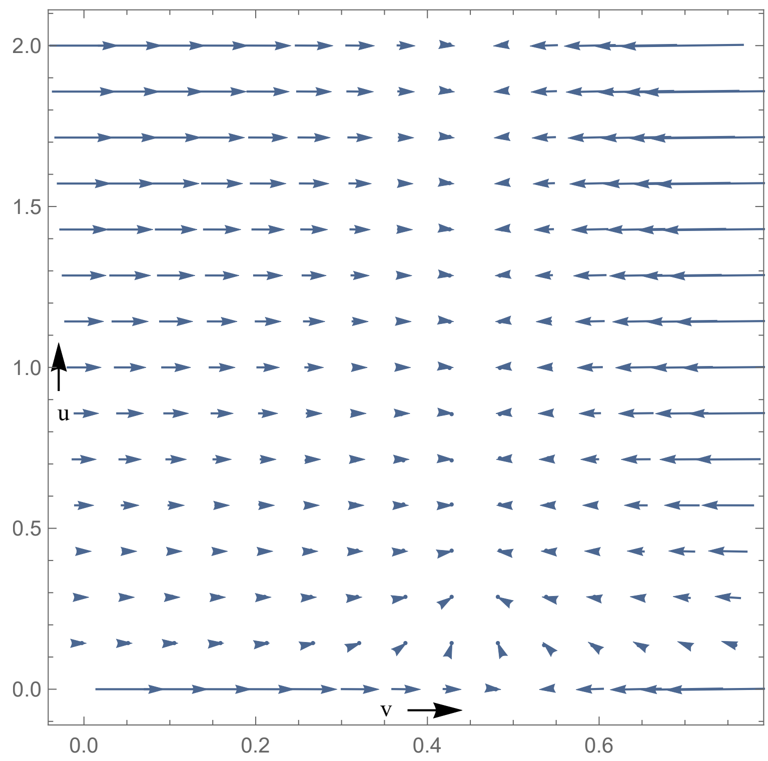

Figure 4.6: Plot of vector field of the phase space around the critical point(0.607,0.457). The arrowhead of trajectories are tilted towards the critical point.

stable evolution from a pre-decelerated epoch to a later accelerated epoch.

The first step in the dynamical system analysis is to identify proper phase-space variables. In the present case, we define,

$$u = \Omega_z = (1-\alpha)h^2, \quad v = \frac{1}{1+\frac{1}{h}} \tag{4.37}$$

as the phase-space variables where the quantities have their usual meaning. These variables will take the range, $0 \leq u \leq 1$ and $0 \leq v \leq 1$. The resulting coupled autonomous differential equations can then be formed using the Friedmann equations and they are,

$$\dot{u} = 6H_0 \frac{u}{1-\alpha}\left(\frac{\bar{\zeta}}{2} - \frac{(1-\alpha)v}{1-v}\right) \tag{4.38}$$

and

$$\frac{\dot{v}}{\sqrt{1-\alpha}} = 3H_0 v^2 \left(\frac{\bar{\zeta}}{2\sqrt{u}} - \sqrt{1-\alpha}\right). \tag{4.39}$$

The points in the phase space exhibit isomorphism with the exact solu-

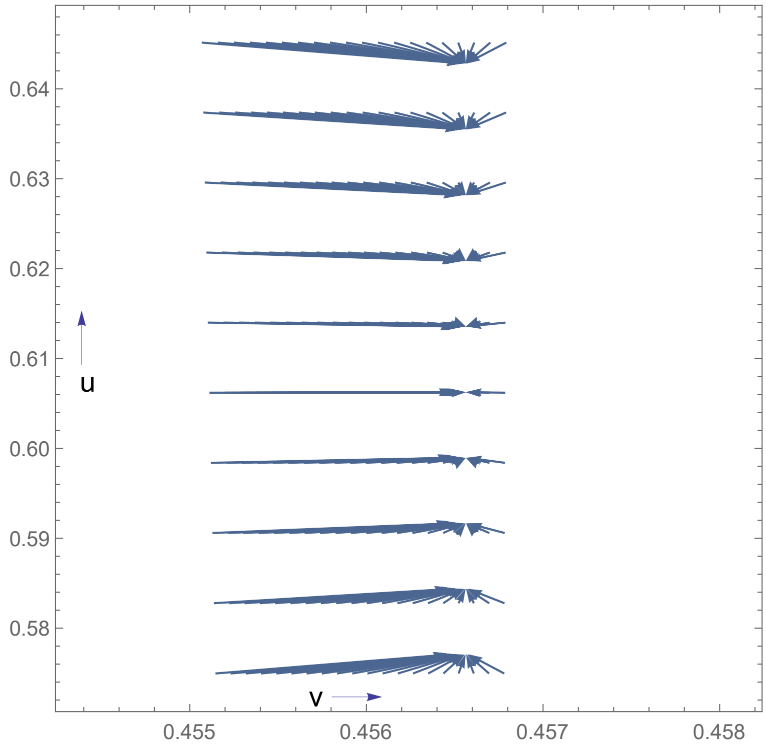

Figure 4.7: Plot of vector field of the phase space close to the critical point.

tions of cosmological field equations. The ODEs are easier to be solved when the derivatives in them are written in terms of $\tau = ln(a)$. The behavior of ODEs in the linear, closeby regions of critical points can be expressed in terms of a matrix equation accommodating $u' = \dfrac{du}{d\tau}$ and

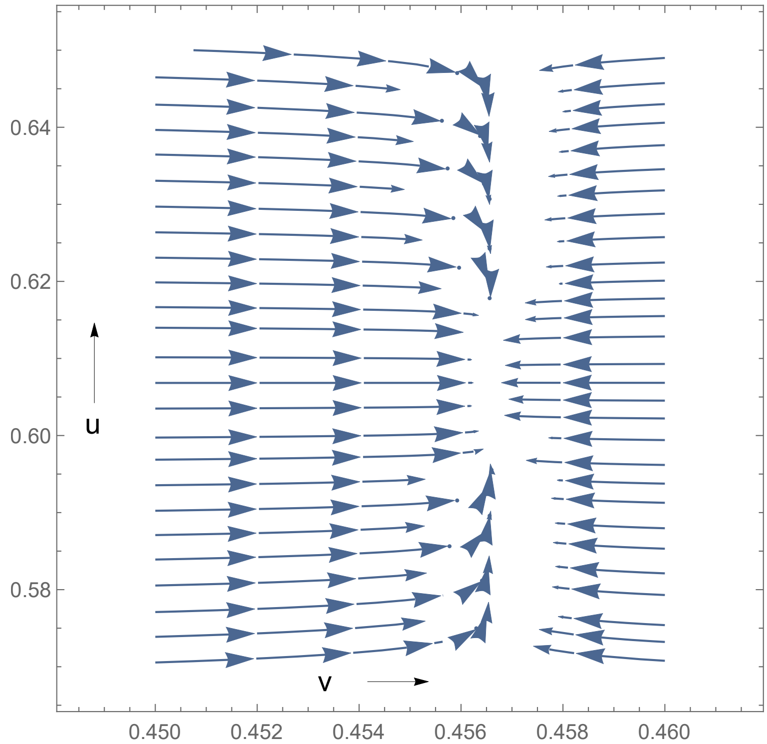

Figure 4.8: Stream plot of the trajectories around the critical point (0.607,0.457)

$v' = \dfrac{dv}{d\tau}$ enabling flux analysis in terms of τ parameter of the equivalent autonomous ODE system. We then establish the correspondence of the dynamics of $\rho_z = \rho_z(t), \rho_\Lambda = \rho_\Lambda(t)$ and $H = H(t)$ with the mentioned simplified flux analysis in phase space for small perturbations in the linear limit $u \to u + \delta u(\tau)$ and $v \to v + \delta v(\tau)$, where δu and δv are the perturbations. The critical points are the solutions of the algebraic equations $P(u,v) = 0$ and $Q(u,v) = 0$ where $P(u,v) = 6H_0 \dfrac{u}{1-\alpha}\left(\dfrac{\bar{\zeta}}{2} - \dfrac{(1-\alpha)v}{1-v}\right)$ and $Q(u,v) = 3H_0 v^2 \left(\dfrac{\bar{\zeta}}{2\sqrt{u}} - \sqrt{1-\alpha}\right)$. The perturbations around the critical points satisfy the equation,

$$\begin{pmatrix} \delta u' \\ \delta v' \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial P}{\partial u}\right)_0 & \left(\frac{\partial P}{\partial v}\right)_0 \\ \left(\frac{\partial Q}{\partial u}\right)_0 & \left(\frac{\partial Q}{\partial v}\right)_0 \end{pmatrix} \begin{pmatrix} \delta u \\ \delta v \end{pmatrix} \tag{4.40}$$

where the suffix,$'0'$ denotes the value at the critical point and 2×2 matrix in the above equation is the Jacobian. The nature of the eigen values of the Jacobian matrix determine the behavior of the system near the critical points.

The critical point of interest corresponding to the equations (4.38) and (4.39) is $(u_c, v_c) = (0.607, 0.457)$. It can be seen that the critical value v_c corresponds to the end de Sitter epoch, where the Hubble parameter becomes, $h \to \frac{\bar{\zeta}}{2(1-\alpha)}$. Then using the relation $v = \frac{1}{1+\frac{1}{h}}$ and the best fit values of the model parameter, it can be seen that $v_c = 0.457$ for the end de Sitter phase. The value $u_c = 0.607$, the mass parameter of the bulk viscous Zel'dovich fluid also corresponds to the end de Sitter epoch. That is, in the end de Sitter epoch, $u \to \frac{\bar{\zeta}^2}{4(1-\alpha)}$, which for the best estimated values of the model parameters will become equal to 0.607 and is u_c. The eigen values corresponding to this critical point is $(0, -4.11)$. The first eigenvalue, 0 apparently suggests the absence of any isolated critical point. But it is seen that such a situation does not arise originally from the ODEs by setting $\dot{u} = 0$ and $\dot{v} = 0$. So the apparent discrepancy suggestive of lack of an isolated critical point arises from the errors of linear approximation of trajectory flux of the system in its immediate neighborhood. A low resolution view of vector field in phase-space is depicted in the figure (4.6). Higher resolution vector field plot as in figure (4.7) makes the view of the critical point as an attractor. The high resolution stream plot as in figure(4.8) also is a clear indicator that the critical point is an attractor.

5

Summary, conclusions and future scope

Many have speculated that matter present in the early stage of the universe were of stiff nature, with equation of state, $p/\rho = 1$. But owing to the fast decrease in its energy density as $\rho \sim a^{-6}$, it would have effect only on the early stages of the universe, on processes like primordial nucleosynthesis But the conventional evolution of the universe is such that it might have undergone a matter dominated epoch and a later accelerated phase of expansion. There are no conclusive theories so far to explain a smooth transition of the early stiff fluid dominated epoch to later 'ordinary' dark matter then to dark energy dominated configurations. In the present thesis we have considered a universe dominated bulk viscous Zel'dovich fluid (stiff fluid). By comparing the model with the latest observational data on Supernovae type Ia, we have found that the resulting model predicts an earlier stiff fluid universe which subsequently make a transit over to the ordinary dark matter era and further a late accelerated epoch without the assistance of any conventional dark energy form.

In the **first chapter** we have given an introduction to the discovery of late accelerating universe and standard ΛCDM model of the universe which account for this discovery. In this model the late acceleration of the universe is caused due to the vacuum energy, Λ, the so called cosmological constant. This model is turn out to be the most successful model to explain particularly the late evolution of the universe. However the model has severe drawbacks, like fine tuning of the vacuum energy density and the problem of the coincidence between the dark matter and cosmological constant densities. We have given a sufficiently elaborate discussion on all these facts about the standard model.

In the **second chapter** we have considered a universe with Bulk viscous Zel'dovich fluid as the dominant component. The viscosity of the stiff matter was treated as per the Eckart's formalism. We have assumed the viscosity as a constant characterized by the constant coefficient $\bar{\zeta}$. We found that for $\bar{\zeta} = 0$ the model reduces to a non-viscous stiff fluid dominated universe beginning with a big bang and the density of the component is decreasing as $\rho_z \sim a^{-6}$. We have constrained the parameter value of the viscosity as $0 < \bar{\zeta} < 6$ and the model predicts a universe began with a big bang followed by a decelerated expansion and finally make a transit in to an accelerating epoch in the final stage. The time of transition in to the late accelerated epoch is strongly depending on the viscous parameter $\bar{\zeta}$. By considering the fact that for $\bar{\zeta} = 4$, the transition into the late accelerating phase would have occurred in the present time, we have further constrained the range of the viscosity parameter as $4 < \bar{\zeta} < 6$. For $0 < \bar{\zeta} < 4$ the transition would have occur only in the future. We have solved the Friedmann equation for the Hubble parameter and scale factor and obtained the age of the universe as $t_0 - t_B = -2(2/H_0\bar{\zeta}) \ln(1 - \bar{\zeta}/6)$. We have studied the evolution of deceleration parameter and the equation of state for the viscous parameter range $0 < \bar{\zeta} < 6$. We have found that the deceleration parameter starts from positive values in the prior stages and as $a \to \infty$ the deceleration parameter $q \to -1$ which corresponds to a de Sitter epoch. The equation of state behaves in such a way that, starting from a value $+1$ in the early stages it evolves finally to $\omega_z \to -1$ as $a \to \infty$. We have also executed the statefinder analysis of the model the present values of the parameters is $(r, s) = (1.25, -0.08)$, implies that the model is arguably different from the standard ΛCDM model. As $a \to \infty$ the statefinder finder parameters become $(r, s) \to (1, 0)$ a value corresponding the standard ΛCDM model. The GSL analysis shows that, the entropy always increases, if $\bar{\zeta} < 6$.

In the **third chapter** the model is constrained with SNe Ia data and evaluated the bulk viscous coefficient as $\bar{\zeta} = 5.25 \pm 0.14$ and the present value of Hubble parameter $H_0 = 70.20 \pm 0.58$. We have studied the evolution of the Hubble parameter. The behavior of the resulting scale factor shows that the model predicts a late acceleration in the expansion of the universe. Hence the bulk viscous Zel'dovich fluid can mimic the role of the conventional dark energy.

We also studied the model to analyze the stability of the solutions corresponding to various epochs using the phase space analysis method. We first analyzed the two dimensional phase-space behavior, where the contribution due to radiation is neglected (for late universe, radiation is almost irrelevant) and found that there is a past unstable saddle critical point corresponding to a static universe and a stable future fixed point corresponds to accelerated epoch. The phase-space trajectories originating from the vicinity of this saddle like point are repelled away from it and move towards the stable critical point corresponding to an expanding universe dominated by Zel'dovich fluid.

Next, we have considered a three dimensional phase-space case by incorporating the radiation component too. We obtained the analytical solution of the resulting autonomous differential equations generated from the Friedmann equations. In this case no critical points are found corresponding to a prior radiation dominated phase and more over none of the existing critical points are stable. Hence the model of the universe with bulk viscous Zel'dovich and relativistic radiation, in which bulk viscosity is characterized by a constant coefficient is found to be first of all failing to predict a prior radiation dominated phase and secondly the very inclusion of radiation makes the model unstable.

The **Fourth chapter** is devoted to the study of a two component universe, in which we have incorporated decaying vacuum as an additional component. Even though the model with bulk viscous stiff fluid alone can even cause the late acceleration of the universe, the main drawback was that, it predicts less age for the present universe. So we studied a model with Bulk viscous stiff fluid and decaying vacuum energy as cosmic components. We compared the model with thee observational data on Type-Ia supernova, CMB data from WMAP observation, BAO data which led to the value of the viscous parameter as $\bar{\zeta} \sim 1.445$, which is considerably less than the value of the parameter, for the single component minimum. We found that the model possesses reasonably good back ground evolution, so as to produce a late acceleration at about a redshift compatible with the observational results. The model also predicts a de Sitter epoch as the end phase. It is found that the transition in to the late acceleration is mainly due to the effect of bulk viscosity, because for the decaying vacuum to produce a transition into the late acceleration, there has to be an additive constant in the vacuum energy density. But the decaying vacuum we have considered doesn't have such a constant. The effect of varying vacuum energy is reflected in the age of the universe. It was found that the age of the universe was increased compared to the model with viscous Zel'dovich fluid alone as the component. Age obtained is in agreement with the age deduced from the observations of the oldest globular clusters.

It was found that the equation of state of the fluid starts from the stiff nature, but eventually reduces to that of the matter and finally goes over to that of a pure cosmological constant corresponding to the de Sitter epoch. The dynamical system analysis shows that, the end de Sitter phase is a stable one. During this stable end de Sitter phase, the density of the Zel'dovich fluid is found to be around 0.6, which itself confirms that the late phase of the universe in this model is dominantly controlled by the viscous nature of the Zel'dovich fluid rather than the decaying vacuum energy.

Let us now give some glimpses about **future scope** of our work. Being motivated by the explanation of late universe behaviour and having the improvement in the prediction of age of the universe, in going from our first model to the second one, we look forward to have considerable scope for future research. One of the immediate scope is to study the effect of the bulk viscous Zel'dovich fluid in the primordial nucleosynthesis process. Previous studies on this aspect doesn't take care of the viscous effect of the stiff fluid. We expect that the viscous effect can cause small dilution in the production of light elements, which may throw some hope on solving the overproduction problems of some light elements as per the present speculations.

Another interesting problem is effect of this model in the structure formation scenario. It will be better to study such an effect and to compare the results with the star formation data obtained observationally. Bessada and Mirando [154] did a similar work with ordinary dark matter and decaying vacuum. It is interesting the carry over such a formalism with Zel'dovich fluid and decaying vacuum. The star formation rates are straight away linked with the density fluctuations in the dark matter and the time dependent Λ background, which is responsible for the dynamical expansion of the universe. In our case, one can enquire into the prospectus of resolving the dark matter cusp and the missing satellite problem by making use of the bulk viscous Zel'dovich fluid, again, with the time dependent dark energy generating the expansion of the universe.

In the present work we have incorporated viscosity using Eckart formalism. Even though this formalism is giving reasonable results, it is not relativistically invariant. A relativistically invariant formalism for incorporating the viscosity in the cosmological scenario was developed by Israel and Stewart [167]. It is worth exploring the status of Zel'dovich fluid model using such a formalism. We expect that such work may lead to some good results in including the radiation component of the universe.

Bibliography

[1] Tisserand M.F., "Les Travaux de LeVerrier". Annales de l'Observatoire de Paris, Memoires(in French), **15**, 23(1880).

[2] Zwicky F., Ap.J. **86**, 217(1937).

[3] Rubin V. C. and Ford Jr W. K., Ap.J. **159**, 379(1970).

[4] Ostriker J.P. and Peebles P.J.E., Astrophys. J. **186**, 467(1973).

[5] Trimble V., Annu. Rev. Astron. Astrophys. **25**, 425(1987).

[6] Tyson J. A. *et al*, Astrophys. J. **498**, L107(1998).

[7] Refregier A., Ann. Rev. Astron. Astrophys. **41**, 645(2003).

[8] Allen S.W. *et al*, Mont. Not. R. Astron. Soc. **342**, 287(2003).

[9] Wright E. L., C 94-02-16 (Los Angeles Dark Matter:0033-40)Proceedings, p.0033(1994) [available as arXiv:astro-ph/9408002].

[10] Spergel D. N. *et al*, Astrophys. J. Suppl. **148**, 175(2003).

[11] Ade P.A.R *et al*(Planck Collaboration), A.&A. **571**,A1(2014).

[12] Zwicky F., Phys. Rev. **51**, 290(1937)

[13] Zwicky F., Phys. Rev. **51**, 679(1937)

[14] Chandrasekhar S., Ap.J. **142**, 1488(1965)

[15] Bonazzola S., Ap.J. **182**, 335(1973).

[16] Perlmutter S. *et al*, Astrophys. J. **517**, 565(1999).

[17] Riess A. G. *et al.*, 1998, Astron. J. **116**, 1009(1998).

[18] Riess A. G. *et al*, Astrophys. J. **607**, 665(2004).

[19] Riess A. G. *et al*, The Astrophys. J. **560**, 49(2001).

[20] Hicken M. *et al*, Ap.J. **700**, 1097(2009).

[21] Schmidt B. P. *et al*, Ap.J. **507**, 46(1998).

[22] Sahni V. and Starobinsky A. A., Int. J. Math. Phys. D **9**, 373(2000).

[23] Benitez N. *et al*, Ap.J. **577**, L1(2002).

[24] Landau L. D. and Lifshitz E. M. , Classical Theory of Fields; Course of Theoretical Physics **Vol-2**, Pergamon Press(1971).

[25] Ashok Das, Lectures on Gravitation,World Scientific Publishing Co. Pte. Ltd(2011).

[26] Padmanabhan T., Gravitation - Foundations and Frontiers, Cambridge University Press(2010).

[27] Hartle J. B., Gravity - An Introduction to Einstein's General Relativity,Addison-Wesley(2003).

[28] Pais A., Subtle is the Lord, Oxford University Press(2005).

[29] Schwarzschild K, Sitzungsber. Preuss. Akad. Wiss. Berlin, 424(1916).

[30] Einstein A., Annalen der Physik. **49**, 769(1916).

[31] Einstein A. "The Foundation of the General Theory of Relativity(1916)"; translation by Brecha R. available at Commentaries on the Exhibit's Works **14**, University of Dayton eCommons(2014).

[32] Will C.M., Living Rev. Relativ. **9**, 3(2006)

[33] Hubble E., Proc. N. A. S. **15**, 168(1929).

[34] Hubble E. and Humason M.L., Astrophysical Journal **74**, 43(1931).

[35] Belinkiy A., Physics Today, **65** (October issue), 38(2012).

[36] Anderson L.*et al*, M. N. R. A. S. **441**, 24(2014).

[37] Ade P. A. R. *et al*, Astron.Astrophys. **594**, A 13(2016).

[38] Goncalves R. S. *et al*, M. N. R. A. S. **475**, 1, L20-L24(2018).

[39] Weinberg S., Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, Wiley(1971).

[40] Weinberg S., Astrophys. J. **168**, 175(1971).

[41] Sunyaev R. and Ya B., Astrophys. & Space. Sci. **7**, 3(1970).

[42] Peeble P. J.E. and Yu T. T., The Astrophys. J. **162**, 815(1970).

[43] Wu K. K. S., Lahav O. and Rees M. J., Nature **397**, 225(1999).

[44] Carl Sagan, Cosmos episode 8, "Journeys in Space and Time." YouTube Link:[https://www.youtube.com/watch?v=sJ_tZr0D2pk]

[45] J. Einasto *et al*, Nature **250**, 309(1974).

[46] Armendariz-Picon C. *et al* Phys. Lett. B **458**, 209(1999).

[47] Garriga J. and Mukhanov V., Phys. Lett. B **458**, 219(1999).

[48] Chiba T., Phys. Rev. D **62**, 023511(2000).

[49] Armendariz-Picon. C, V. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. **85**, 4438(2000).

[50] Armendariz-Picon. C, V. Mukhanov and P. J. Steinhardt, Phys. Rev. D **63**, 103510(2001).

[51] J. P. Ostriker, P. J. E. Peebles and A. Yahil, Ap.J. **194**, L1(1974).

[52] Copeland E. J., Sami M. and Tsujikawa S., Int. J. Mod. Phys. D **15**, 1753(2006).

[53] Tegmark M. etal [WMAP Collaboration](2003), Astrophys. J. Suppl. **148**, 175(2014).

[54] Caldwell R.R., Phys. Lett. **B 545**, 23(2002).

[55] Caldwell R.R. *et al*, Phys. Rev. Lett. **91**, 071301(2003).

[56] Bagla J. S., Jassal H. K. and Padmanabhan T., Phys.Rev. D **67**, 06350(2003).

[57] Padmanabhan T. and Roy Choudhury T., Phys. Rev. D **66**, 081301(2002).

[58] Li X.-D *et al*, Sci.China Phys. Mech. Astron.G **55**, 1330(2012).

[59] Steinhardt P.J. *et al*, Phys. Rev. D **59**, 123504(1999).

[60] H.-C.Kim et al, Phys. Lett. B **661**, 67(2008).

[61] Kamenshchik Alexander Yu. *et al*, Phys.Lett. B **511**, 265(2001).

[62] Bean R. and Dore, O., Phys. Rev. D **68**, 023515(2003).

[63] Sauer T., Historia Mathematica **33**, 399(2006).

[64] Barreiro, T. *et al*, Phys. Rev. D **78**, 04353(2008).

[65] Starobinsky A.A. Phys. Lett. B **91**, 99(1980).

[66] Wang Y., Phys. Rev. D **42**, 2541(1990).

[67] Caldwell R. R. and Kamionkowski M. Ann.Rev.Nucl.Part.Sci. **59**, 397(2009).

[68] Amendola L.,Phys. Rev. Lett. **98**, 131302(2007).

[69] De Felice A.*et al* Phys. Rev. D **77**, 024017(2008).

[70] Seljak U. *et al*, 2005, Phys. Rev. D **71**, 103515.

[71] Spergel D. N. [WMAP Collaboration], Astrophys. J. Suppl. **170**, 377(2007).

[72] Jimenez P. *et al*, MNRAS **282**, 926(1996).

[73] Pont F., Mayor M., Tuton C., Vanderberg D. A., A.&A. **329**, 87(1998).

[74] Carretta E. *et al*, Astrophys. J. **533**, 215(2000).

[75] Salaris M. *et al*, Astrophys. J. **479**, 66(1997).

[76] Sukyong Yi *et al*, Astrophys. J. suppl. S. **136**, 417(2001).

[77] Dutta S., Scherrer R. J., Phys. Rev. D **82**, 083501(2001).

[78] Bucher M. *et al*, Phys. Rev. D. **52**, 3314(1995).

[79] Linde A., Phys. Rev. D **59**, 023503(1999).

[80] Weinberg, Cosmology, Oxford University Press(2008).

[81] Weinberg D. *et al*, arXiv:astro-ph/9810142v11998(1998).

[82] Amendola L. and Tsujikawa S., Dark Energy, Camebridge University Press(2010).

[83] G. Aldering G. [SNAP Collaboration], Proceedings of the SPIE **4835** 146(2002).

[84] Ellis J. *et al*, Phys. Lett. B **228**, 264(1989)

[85] Wetterich C., Astron. Astrophys. **301**, 32(1995).

[86] Damour T. and Nordtvedt K., Phys. Rev. D **48**, 3436(1993).

[87] De Felice A. *et al*, Phys. Rev. D **67**, 043509(2003).

[88] Rajagopalan Nair K. and Titus K. Mathew, Astrophys. Space Sci. **363**, 183(2018).

[89] Yun Wang, Dark Energy, WILEY-VCH(2010).

[90] Einstein A., Sitzungsber. Preuss. Akad. Wiss. Phys. Math. Kl. **217**(1928).

[91] Einstein A., Sitzungsber. Preuss. Akad. Wiss. Phys. Math. Kl. **401**(1930).

[92] Einstein A., Math. Ann. **102**, 685(1930).

[93] Boehmer C.G. *et al*, Phys. Rev. D **78**, 023505(2008).

[94] Caldera-Cabral, G. *et al*, Phys. Rev. D **79**, 063518(2009).

[95] Bousso R., Rev. Mod. Phys. **74**, 825(2002).

[96] 't Hooft G., 2000b, The holographic principle, e-print hep-th/0003004.

[97] Tavakol R. K. and G. Ellis, Phys. Lett. B **469**, 37(1999).

[98] Sebastian Pineda J. *et al*, The Astrophys. J. **826**, (1), 311(2016).

[99] Shuang Wang A, Phys. Rep. **696**, 1(2017).

[100] Li M., Phys. Lett. B **603**, 1(2004).

[101] Hartnoll S. A., Class. Quant. Grav. **26**, 224002(2009).

[102] Li M. *et al*, Commun. Theor. Phys. **56**, 525(2011).

[103] Cohen A. G. *et al*, Phys. Rev. Lett. **82**, 4971(1999).

[104] Titus K. Mathew, Aswathy M. B. and Manoj M.,Eur. Phys. J. C **74**, 3188(2014).

[105] Rajagopalan Nair K. and Titus K. Mathew, Eur. Phys. J. C **76**, 519(2016).

[106] Ya B Zel'dovich, Sov. Phys. JETP **14**, 11437(1962).

[107] Ya B Zel'dovich, Monn. Not. R. Astron. Soc. **160**, 1(1972).

[108] Li B. *e al*, Phys. Rev. D **89**, 083536(2014).

[109] Stiele R. *et al*, Phys. Rev. D **81**, 123513(2010).

[110] Chavanis P.-H., Phys. Rev. D **92**, 103004(2015).

[111] Chandrasekhar S., Mon. Notices Royal Astron. Soc. **91**, 456(1931).

[112] Landau L.D., Phys. Z. Sowjetunion **1**, 285(1932).

[113] Landau L. D. and E. M. Lifshitz E. M., Teoriya polya (Field Theory), Fizmatgiz(1960).

[114] Horava P., Phys. Rev. D **79**, 084008(2009)

[115] Calcagni G., JHEP **0909**, 112(2009).

[116] Kiritsis G. and Kofinas G., Nucl. Phys. B **821**, 467(2009).

[117] T P Sotiriou *et al*, JHEP **0910**, 033(2009).

[118] Bogdanos C. and Saridakis E. N., Class. Quant. Grav. **27**, 07500(2010).

[119] Carloni S. *et al*, Class. Quant. Grav. **27**, 045004(2010).

[120] G. Leon and E. N. Saridakis, JCAP **0911**, 006(2009).

[121] Ali A. *et al*, Gen. Rel. Grav. **44**, 657(2012).

[122] Dutta S. and Saridakis E. N., JCAP **1005**, 013(2010).

[123] Landau L. D. and Lifshitz F.M., Fluid mechanics; Addison- Wesley, Reading(1958).

[124] C. Eckart, Phys. Rev. **58**, 919(1940).

[125] Hiscock W. A., Lindblom L., Phys. Rev. D. **31**, 725(1985).

[126] W. Israel, Ann. Phys. (N.Y.) **100**, 310(1976).

[127] Israel W. and Stewart J. M., Ann. Phys.(N. Y.) **118**, 341(1979).

[128] Kremer G.M. and Devecchi F. P., Phys. Rev. D **67**, 047301(2003).

[129] Cataldo M., Cruz N. and Lepe S., Phys. Lett. B **619**, 5(2005).

[130] Fabris J. C., Goncalves V. B. and de S. Ribeiro R., Gen. Relat. Grav. **38**, 495(2006).

[131] Hu M. G. and Meng H. X., Phys. Lett. B **635**, 186(2006).

[132] Ren J. and Meng H. X., Phys. Lett. B **633**, 1(2006).

[133] Hiscock W. A. and Salmoonson J., Phys. Rev. D **43**, 3249(1991).

[134] Kolb E. W. and Turner M. S., The Early Universe. Addison Wesley California(1990).

[135] Tegmark M, Eisenstein D. J., Strauss M.A., Weinberg D. H., Blanton M. R.*et al*, Phys. Rev. D **74**, 123507(2006).

[136] Hinshaw G *et al*, Astrophys. J. Suppl. **180**, 225(2009).

[137] Alam U., Sahni V., Saini T. D. and Starobinsky A. A., Mon. Not. R. Astron. Soc. **344**, 1057(2003).

[138] Sahni V., Saini T. D., Starobinsky A. A. and Alam U., JETP Letters **77**, No.5, 243(2003).

[139] Wu Y. B.. Li S., Fu M.H. and He J., Gen. Rel. Grav. **39**, 353(2007).

[140] Huang Z. G. *et al*, Astrophys. Space Sci. **315**, 175(2006).

[141] Wang B.*et al*, Phys. Rev. D, **74**, 083520(2006).

[142] Hawking S. W., Phy. Rev. Lett. **26**, 1344(1971).

[143] Bekenstein J.D., Lett. Nuov. Cim. **4**, 73(1972).

[144] Bekenstein J. D., Phys. Rev. D **7**, 2333(1973).

[145] Bekenstein J. D., Phys.Rev. D **9**, 3292(1974).

[146] Hawking S. W., Math. Phys. **43**, 199(1975).

[147] Berkenstein J.D., Phys. Rev. D **7**, 2333(1973).

[148] Davies P. C. W., Class. Quant. Grav. **5**, 1349(1988).

[149] M Kowalsky *et al*, Astrophys. J. **686**, 749(2008).

[150] B. Schwarzschild, Phys. Today **57** (issue 6), 19(2004)

[151] F. B. Bianco *et al*, Astrophysical J. **741:20** no.1(2011)

[152] Cataldo M. *et al*, Astrophys. Space Sci. **344**, 495(2013).

[153] Komatsu E. *et al*(WMAP Collaboration), Astro. Phys. J. Suppl. **192**, 18(2011).

[154] Bessada D. and Miranda O. D., Phys. Rev. D **88**, 08353(2013).

[155] F.C. Carvalho *et al*, Phys. Rev. Lett. **97**, 081301(2006).

[156] Sola J., J. Phys.: Conf. Ser. **453**, 1(2011).

[157] Sola J, J. Phys.: Conf. Ser. **453**, 012015(2013).

[158] Paxy J. and Titus K. Mathew, Mod. Phys. Lett. A **31**, 16500075(2016).

[159] Basilakos S. and Sol J., Phys. Rev. D **90**, 023008(2014).

[160] Eisenstein D. J. The Astron. J. **142**, 72(2011).

[161] Bond J. R. and Efstathiou G., Astrophys. J. **285**, L45-L48(1984).

[162] Peebles P. J. E. and B. Ratra, Rev. Mod. Phys. **75**, 559(2003)

[163] J. F. Navarro, C. S. Frenk, and S. D. M. White, Astrophys. J. **462**, 563(1996).

[164] A. Burkert, Astrophys. J. **447**, L25(1995)

[165] G. Kauffmann *et al*, Mon.Not. R. Astron. Soc. **264**, 201(1993).

[166] Arcuri R. C. and Waga I. Phys. Rev. D **50**, 2928(1994).

[167] Israel W. and Stewart J. M., Phys. Lett. **58A**(4), 213(1976).