RMS MEASUREMENTS

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THESIS SUBMITTED IN PARTIAL FULFILMENT

OF

THE REQUIREMENTS FOR THE DEGREE

OF

DOCTOR OF PHILOSOPHY

DEPARTMENT OF ELECTRONICS

UNIVERSITY OF COCHIN

1980

DECLARATION

This is to declare that, the Thesis entitled 'R M S Measurements has not previously formed the basis of award for any other degree, diploma, fellowship or any other similar title or recognition.

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CERTIFICATE

This is to certify that the Thesis entitled 'R M S Measurements' which is hereby submitted by Mrs. Deo Prabha Vasant, to the University of Cochin, in fulfilment of the requirements for the award of the Degree of Doctor of Philosophy, is a record of bona - fide research work carried out by her under my guidance and supervision.

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Abstracts

RMS measuring device is a nonlinear device consisting of linear and nonlinear devices. The performance of rms measurement is influenced by a number of factors; i) signal characteristics, 2) the measurement technique used and 3) the device characteristics. RMS measurement is not simple, particularly when the signals are complex and unknown. The problem of rms measurement on high crest-factor signals is fully discussed and a solution to this problem is presented in this thesis.

The problem of rms measurement is systematically analized and found to have mainly three types of errors: (1) amplitude or waveform error 2) Frequency error and (3) averaging error. Various rms measurement techniques are studied and compared. On the basis of this study the rms measurement is reclassified into three categories: (1) Wave-form-error-free measurement (2) High-frequncy-error free measurement and (3) Low-frequency error-free measurement. In modern digital and sampled-data systems the signals are complex and waveform-error-free rms measurement is highly appreciated.

Among the three basic blocks of rms measuring device the squarer is the most important one. A squaring technique is selected, that permits shaping of the squarer error characteristic in such a way as to achieve waveform-error-free rms measurement. The squarer is designed, fabricated and tested.

A hybrid rms measurement using an analog rms computing device and digital display combines the speed of analog techniques and the resolution and ease of measurement of digital techniques. An A/D converter is modified to perform the square-rooting operation. A 10-V rms voltmeter using the developed rms detector is fabricated and tested.

The chapters two, three and four analyse the problems involved in rms measurement and present a comparative study of rms computing techniques and devices. The fifth chapter gives the details of the developed rms detector that permits wave-form-error free rms measurement. The sixth chapter, enumerates the the highlights of the thesis and suggests a list of future projects.

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CHAPTER - ONE

Introduction

1.1) ROLE OF RMS MEASUREMENT

RMS measurement gives the root - mean - square value (rms) of the measurand. Basically it is a measure of statistical error to express stability of standards and product tolerances. Secondly, rms value is the measure of heating power of a signal. Thirdly, rms value is the conventional measure of signal amplitude. These basic measurements can be extended to measure various signal and device characteristics.

A conventional a. c. instrument however is either average or peak responding instrument calibrated to read rms of a sinusoid. The true rms measurement is a must for signals with unknown complex waveforms. In modern digital and sampled data systems, in SCR controlled power circuits and in noise control and measurement circuits, the wave form are complex. Thus, the true rms measurement is indispensible today.

1.2) DEFINITION OF RMS

Next to a mathematician an electrical engineer is interested in the term 'rms'. The rms is a mathematical function of variable 'x' defined as (Ref. 1).

$$\text{rms} = \left[\frac{1}{T} \int_{a}^{a+T} x^2 \, dt \right]^{0.5}$$

In electrical engineering, the rms value is a measure of signal amplitude. A signal's rms value is equal to the dc signal that would dissipate the same amount of power as the signal dissipates.

1.3) BASIC METHODS OF RMS MEASUREMENT

Basically there are three methods of rms measurement (1) To use a proper scale factor on conventional a. c. instruments. (2) To use a true rms instrument that consists of a heater and a converter like a thermopile. (3) To simulate the rms computation with the help of analog or digital computing circuits.

The first method is useful for sinusoidal and slightly distorted signals. It can be used for complex signal if its crest factor or form - factor is known. The amount of error introduced in the measurement without using the correction factor is found in many references (Ref. 2 to 7). As quoted by Scheingold & Counts, the rectifier type a. c. instrument reads 11% high on dc or symmetrical square waves, 4 percent low on triangular and saw - tooth waves and 11.3% low on gausian noise.

The second method is widely used. It can claim the highest accuracy because its action is directly based on the rms definition. However, there are certain limitations imposed by basic characteristics of the converter. As discussed by Baird and others (Ref. 8) a thermocouple device has (1) a sluggish response (2) inaccuracy at low level inputs, (3) susceptibility to burn out and (4) thermal problems.

The third method is also based on the rms definition and can claim the highest accuracy. This method is still in the development stage. True - rms function generators are available commercially, e. g. Teledyne Philbrick's 4370, Burr Browns 4340. Still, there is a want of true rms computing instrument which will measure the true rms value easily, quickly and accurately irrespective of the signal waveform.

Historical developments of rms instrument can be grouped in three parts. (1) Before 1960 electromechanical and thermal devices were used, (2) between 1960 and 1970, various square - law devices e. g. diode, transistor - bipolar and field effect. thermistor etc., were used.(3)Though the diode function generator was used for rms measurement as early as in 1960, the real era of the computing rms techniques has started since 1970.

1.4) THE PROBLEM

The application note on the digital voltmeter by H. P. (Ref. 9) says, It should be mentioned that there is a third type of a. c. converter known as quasi - rms converter. A quasi rms technique simulates true rms response using operational amplifiers to square input, take the average of the square then take the square root. This type of converter holds a lot of promise but is not widely used. This synthesized rms response is not mathematically perfect and for that reason is limited to symmetrical wave shapes.

Thus, the problem of simulation of true rms computation using operational amplifiers is taken up.

The problems encountered in rms measurement are systematically analyzed in the second chapter by using a novel concept of an idealized rms detector. Various rms computing techniques and their areas of applications are presented in the third chapter. The fourth chapter compared various techniques for squaring, averaging and square - rooting. Based on the results of the compartive study of devices and techniques appropriate recommendations are made for rms measurement on high crest - factor signals. The development of the rms detector for high crest - factor signals is presented in the fifth chapter.

Thus, a solution to the problem of rms measurement on high crest - factor signal is presented in this thesis.

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CHAPTER - TWO

Problems in RMS Measurements

2.1) INTRODUCTION

One comes across a variety of signals. The quality of rms measurement on these signals is decided by the quality of the measuring device as well as by the signal characteristics. The performance of rms instrument is a function of both, the instrument characteristics and the signal characteristics. For the sake of generalization, the concept of idealized rms detector is introduced. It is supposed to have the ideal rectangular amplitude and frequency response characteristics. The performance, the amplitude and frequency errors, on various signals is determined and compared.

2.2) INPUT TO RMS MEASURING DEVICE

The input to an rms measuring device is physical data or an electrical signal. The input can be classified into two broad classes (a) deterministic data and (b) random data. Deterministic data can be further classified into two subclasses. Periodic data - sinusoid and complex signals that are formed by sum of two or more commensurately related sine waves, (2) Nonperiodic data - transient data and almost periodic formed by summing two or more sine waves with arbitrary frequencies, e. g.

$$X(t) = X_1 . \text{Sin } 2t + X_2 \text{ Sin } [\sqrt{(50)} . t]. \quad \text{(Ref. 1)}$$

The discussions presented in this chapter are restricted to the rms measurement on periodic and stationary ergodic random signals only. The stationary ergodic processes are those for which the time averaged properities are equal and therefore the averages over only a single sample function can be considered.

2.3) RMS DETECTOR AS A BLACK BOX

Let rms detector be represented by a black box. The output of this box is a measureble entity - voltage, pulsewidth or deflection - which is directly related to the input rms value. Like all other devices, this box has two basic characteristics, the amplitude and frequency characteristics. The amplitude response is specified in terms of a dynamic range. It is equivalent to the d. c. measurement span and is expressed as a ratio. The frequency response is expressed in terms of the bandwidth.

Since the rms detector. essentially incorporates a nonlinear device, the amplitude response is limited both at the high and the low - level inputs. At the higher end, the limitation is imposed either by the overload capacity or by the device response at high - level inputs. For example the diode and the transistor type square law circuits loose squaring property at high level inputs. Secondly at the lower - end the limitation is imposed by noise and also by the low - level response of the device. For example, thermocouple is not satisfactory, as a square law device at low - level input. Thus, unlike a. c. conventional instrument, the amplitude response of rms instrument is important and must be considered at both high and low levels.

In true rms measurement the input is necessarily d. c. coupled to the instrument. At higher frequencies, the limitation on the frequency response is imposed by the cutoff frequencies of linear and nonlinear devices of the black box. On the lower side, the averaging property is the deciding factor.

2.4) THE DETECTOR PERFORMANCE AND SIGNAL CHARACTERISTICS

Random data are characterized by power density spectra and probability distribution function. On the other hand, complex periodic signals are characterized by discrete power spectra and by a ratio of peak to rms valve called the crest-factor (CF). The performance of rms detector on a particular signal is decided by the three factors. (1) the relationship between the detector band width and the signals power - density spectrum, (2) the relationship between the detector's dynamic range and the crest - factor or the probability distribution function of the signal and (3) the averaging property of the detector on low - frequency signal components.

The signal characteristics of commonly occuring signals are usually known but the instrument characteristics differ from instrument to instrument and are less frequently available. Hence as a first step in the analysis of the problems in rms measurement, the idea of an idealized rms detector is introduced.

2.5) THE IDEALIZED RMS DETECTOR

The two basic characteristics of rms detector are (1) frequency response- The relationship between the proportionality constant (M), between the rms input and the detector output, and the signal frequency. (2) Amplitude response - The relationship between the proportionality constant (M) and the d. c. input level. The two characteristics of the idealized rms detector are assumed to be ideal rectangles show in the figure (Fig. 2.1)

The detector, responds to only those components of a signal with frequency less than the cut - off frequency (fc). The components with frequencies higher than fc are just thrown off. Secondly, the input amplitudes which fall beyond the dynamic range (V_L to V_H) are rejected by the detector. It is assumed that the attenuator attenuates the input signal such that the peak value always lies within the dynamic range of the detector but the minimum value may or may not. It must be noted at this stage, that the attenuator is not capable of changing the crest - factor of the signal and therefore it is not useful in solving the problem of rms measurement on high crest - factor signals.

These two characteristics will help to determine the performance of the detector on a given input signal. For example, let the input signal be f(t) as shown in the figure [Fig. 2.2 (A)]. The signal as seen by the idealized rms detector is shown in the figure [Fig. 2.2 (B)] It gives out at its output rms of a signal as seen by it. The difference in rms values of the two signals, represented in the figures [Fig. 2.2 (A) and (B)], is the amplitude error' and can be determined.

Secondly, let us consider the rms measurement on a distorted sinusoidal signal with repetition frequency some what lower than the cutoff frequency (fc) of the detector. Let the distortion components be 5% third and 5% fifth harmonics. The idealized rms detector sees, only the fundamental component of signal and throws off the harmonics as these lie beyond the cut off frequency (fc). The difference between the rms values of the fundamental (i. e the signal seen by the detector) and the input distorted sinusoidal signal, is the 'frequency error' and it can be determined.

This basic concept of an idealized rms detector is used to determine the generalized performance of rms measurement on complex deterministic and random signals.

2.6) FREQUENCY ERROR

The frequency error is the error introduced by the limited frequency response of the rms detector. It is decided by those frequency components of the signal that lie outside the rectangular response characteristic. The appendix I gives a flow chart for determining the frequency error in the rms measurement of a variable duty - cycle pulse train. The parameter 'K' is the ratio of the detector's band width to the fundamental signal frequency (i. e. repetition frequency). The frequency error is plotted as a function of K (Fig. 2.3), it increases as K decreases. Similar plots can be obtained for various other signals. The formulae to determine the Fourier Coefficients for commonly occuring signals are well known (Ref. 2). A typical set of signals is tabulated in the table. (T 2.1) The derivations are given in the Appendix II. The frequency error plots for a commonly occuring periodic signal can be obtained. The flow chart of the Appendix I should be modified by using the appropriate formulae from the table (T. 2.1), for the true rms value and the Fourier Coefficients.

In case of a signal with continuous power - density spectrum, the frequency error is determined as given below:

A power density spectrum of a signal is represented in the figure [Fig. 2.4 (A)]. The power density spectrum of the signal as seen by the detector is shown in the figure [Fig. 2.4 (B)]. The power density of that portion of the signal thrown out by the detector is shown in the figure [Fig. 2.4 (C)]. The frequency error in the measured mean square value ($\hat{\delta}_{sf}$) is given by

$$\hat{\delta}_{sf} = \frac{\text{Shaded area of Fig. 2.4 (C)}}{\text{Net area of Fig. 2.4 (A)}} \qquad \text{— (1)}$$

The corresponding error (δ_{rf}) in rms measurement can be determined by applying the following rule (Ref. 3):

IF $y = f(x)$,

$$\Delta y = \frac{\partial\,[f(x)]}{\partial x} \cdot \Delta x$$

where Δx is the absolute error x and Δy is the corresponding absolute error in y.

The absolute error in measured rms value Δ (rms), corresponding to that in the mean square value Δ (rms), is given by.

$$\Delta\,(rms) = \frac{\partial\,(\sqrt{ms})}{\partial\,(ms)} \cdot \Delta\,(ms)$$

$$\frac{\partial\,(\sqrt{ms})}{\partial\,(ms)} = \frac{0.5}{\sqrt{ms}}$$

therefore,

$$\Delta\,(rms) = \frac{0.5\,\Delta\,(ms)}{\sqrt{ms}}$$

and

$$\delta_{rf} = 0.5\,\frac{\Delta\,(ms)}{ms}$$

i. e.

$$\delta_{rf} = 0.5\,\delta_{sf} \qquad \text{---} \qquad (2)$$

Thus the frequency error in case of signals with known power-density spectrum can be determined using the relationships (1) and (2). The table (T 2.2) presents the frequency error in case of rms measurement on various signals with known power - density spectrum (Ref. 4). The derivations are given in the Appendix III.

2.7) AMPLITUDE ERROR

The problem of the amplitude error has not received the same amount of attention as the frequency error has. There are brief references to it, such as:-

1) Handler and Cate (Ref.5) quote, the squaring and square-rooting operations make the output a nonlinear function of the input, therefore, the percentage error is amplitude dependent.

2) Baird and others (Ref. 6) quote, the high crest-factor performance is not easily obtained, an rms voltmeter with a crest-factor rating must have amplifier with a sufficient dynamic range to pass signals that have a peak magnitude many times larger than the full scale rms value.

A systematic study of the amplitude error is presented here. An rms detector consists of one or more nonlinear devices. The performance of a non-linear device is limited both at low and high level inputs. Therefore, the ratio of the maximum to minimum levels of a given signal plays an important role. Let us consider a pulse train (doublet) which is mathematically represented by

$$f(t) = V_a \qquad o < t < t_p$$

$$f(t) = V_b \qquad t_p < t < T$$

It is periodic signal with zero average value and a duty - cycle $D = t_p / T$. In case of low duty - cycle signals the ratio V_a / V_b may exceed the detectors dynamic range and the measured mean square value will be in error by an amount $[\, V_b^2 \,(1 - D)\,]$

The amplitude error is computed for various commonly occuring periodic signals and is tabulated in the table (T. 2.3) The error is expressed in terms of the parameter 'a' equal to the reciprocal of the detector's dynamic range. The derivations of the frequency and amplitude errors are given in the appendices (III) and (IV) respectively.

In case of the random signals, the amplitude error in rms measurement (δ_{ra}) is related to the probability density function p (x) of signal x by the relationship,

$$\delta_{ra} = \frac{0.5}{x^2_{ave}} \cdot \int_{o}^{x_{min}} x^2 \, p(x) \, dx$$

where x_{min} is the amplitude threshold of the rms detector i. e. $V_L = x_{min}$,
and x^2_{ave} is the mean square value of the signal.

2.8) COMPARISON OF AMPLITUDE AND FREQUENCY ERRORS

The amplitude error is independent of frequency, where as the frequency error increases as the signal frequency increases. The amplitude error may be insignificant in case of high frequency signals, but it must be considered for measurements on low frequency signals. For example i) The two errors are almost equal in case of a 0.1 duty cycle doublet with repetition frequency of one - tenth of the instruments bandwidth 2) the amplitude error is about 10 times the frequency error in case ot a 0.1 duty cycle doublet with repetition frequency of one-hundredth of the instruments bandwidth.

2.9) AVERAGING ERROR.

Averaging error is a typical characteristic of rms measurements. It is decided by the characteristics of the averaging device used and also by those of the computation techniques used. It is a practice to use a low pass filter for averaging. An integrator is preferred in case of low frequency signals. The averaging characteristics of both the devices are discussed by Bendat & Piersol (Ref.7). It is well known that for periodic signals, the time constant of the filter should be greater than the signal period and output attains a steady stage avarage value after the time interval of 4 to 5 time constants. In case of random signals however the output continues to fluctuate and possesses statistical as well as bias errors The bias error can be avoided by forming the sample record into a loop and recirculating the record to obtain a continuous presentation of data.

The effect of the fluctuating filter output depends upon the measurement technique used. This is therefore discussed later in the fourth chapter.

2.10) CONCLUSION

A systamatic study of errors in rms measurement is presented for the first time. Basically there are three types of errors, amplitude, frequency and and averaging. The problem of amplitude error has received lesser attention. The study has however revealed that the amplitude error plays an important role in rms measurement on high CF signals of medium and low frequency. Such type of signals appear frequently, e.g. SCR controlled circuits, acoustics, communication baseband signals, noise measurement and control.

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TABLE 2.1

Complex Periodic wave forms

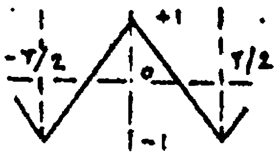
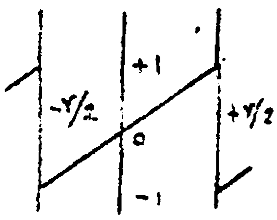
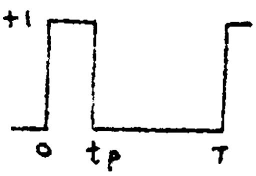
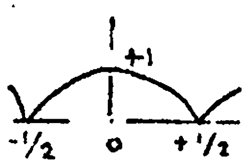
Signal	Wave form	Fourier Coefficients	RMS Value
Triangular		$a_0 = 0$, $b_n = 0$ $a_n = 4 \times \frac{1}{\pi^2 n^2} \times (1 - \cos \pi n)$	$(1/3)^{0.5}$
Sawtooth		$a_0 = 0$ $a_n = 0$ $b_n = -2 \times \frac{1}{\pi n} \cos \pi n$	$(1/3)^{0.5}$
Pulse train		$a_0 = D = t_p/T$ $a_n = \frac{1}{\pi n}(1 - \cos\theta)$ $b_n = \frac{1}{\pi n} \sin\theta$ where $\theta = 2\pi n D$	$(D)^{0.5}$
Rectified Sinusoid		$a_0 = 2/\pi$ $b_n = 0$ $a_n = \frac{2}{\pi}\left[\frac{1}{2n+1} \sin(2n+1)\pi/2 + \frac{1}{2n-1} \sin(2n-1)\pi/2\right]$	$(1/2)^{0.5}$

TABLE T. 2.2

Frequency error for random signals

Signal	Power density function	Frequency error in mean square measurement
Band - pass white noise	with $(fo - \frac{B}{2}) \leqslant f \leqslant (fo + \frac{B}{2})$ $G\ (f) = a$ $G\ (f) = o$ otherwise	$\delta_{sf} = [\ 0.5 - \frac{fc - fo}{B}\ ]$
Exponential	$G\ (f) = \frac{4a}{a^2 + 4\pi^2 f^2}$	$\delta_{sf} = 1 - \frac{2}{\pi} \tan^{-1} (\frac{2\pi fc}{a})$
Exponential cosine	$G\ (f) = 2a \left[\frac{1}{a^2 + 4\pi^2 (f + fo)^2} + \frac{1}{a^2 + 4\pi^2 (f - fo)^2} \right]$ $\delta_{sf} = \left[1 - \frac{1}{\pi} \tan^{-1} [\ 2\pi (fc + fo) / a\ ] - \frac{1}{\pi} \tan^{-1} [\ 2\pi (fc - fo) / a\ ] \right]$	

TABLE 2.3

Amplitude error in RMS measurement on Complex Periodic Signals

Periodic signal	δ_{sa}	δ_{ra}
Triangular	a^3	$0.5\ a^3$
Saw tooth	a^3	$0.5\ a^3$
Doublet pulse train (Law duty cycle)	D	0.5 D
Sinusoid or rectified sinusoid	$\frac{2}{\pi}[\sin^{-1} a - a\sqrt{(1-a^2)}]$	$\frac{1}{\pi}[\sin^{-1} a - a\sqrt{(1-a^2)}]$

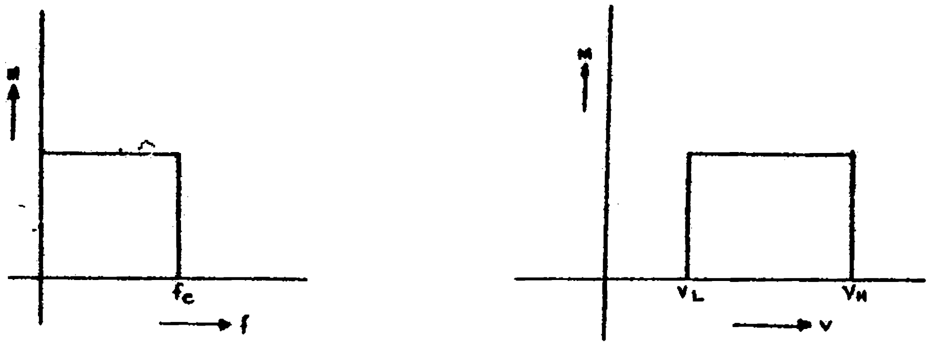


FIG: 2·1 [A]

FIG: 2·1 [B]

FREQUENCY AND AMPLITUDE CHARACTERISTICS OF IDEALISED RMS DETECTOR

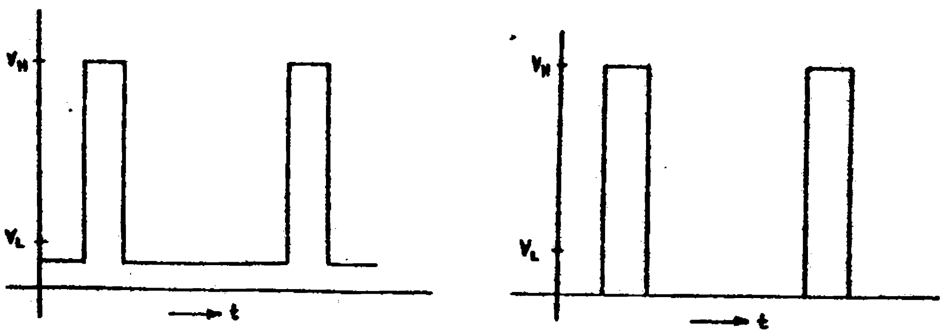


FIG: 2·2 [A]

FIG: 2·2 [B]

INPUT SIGNAL [A] AND SIGNAL SEEN BY THE RMS DETECTOR [B]

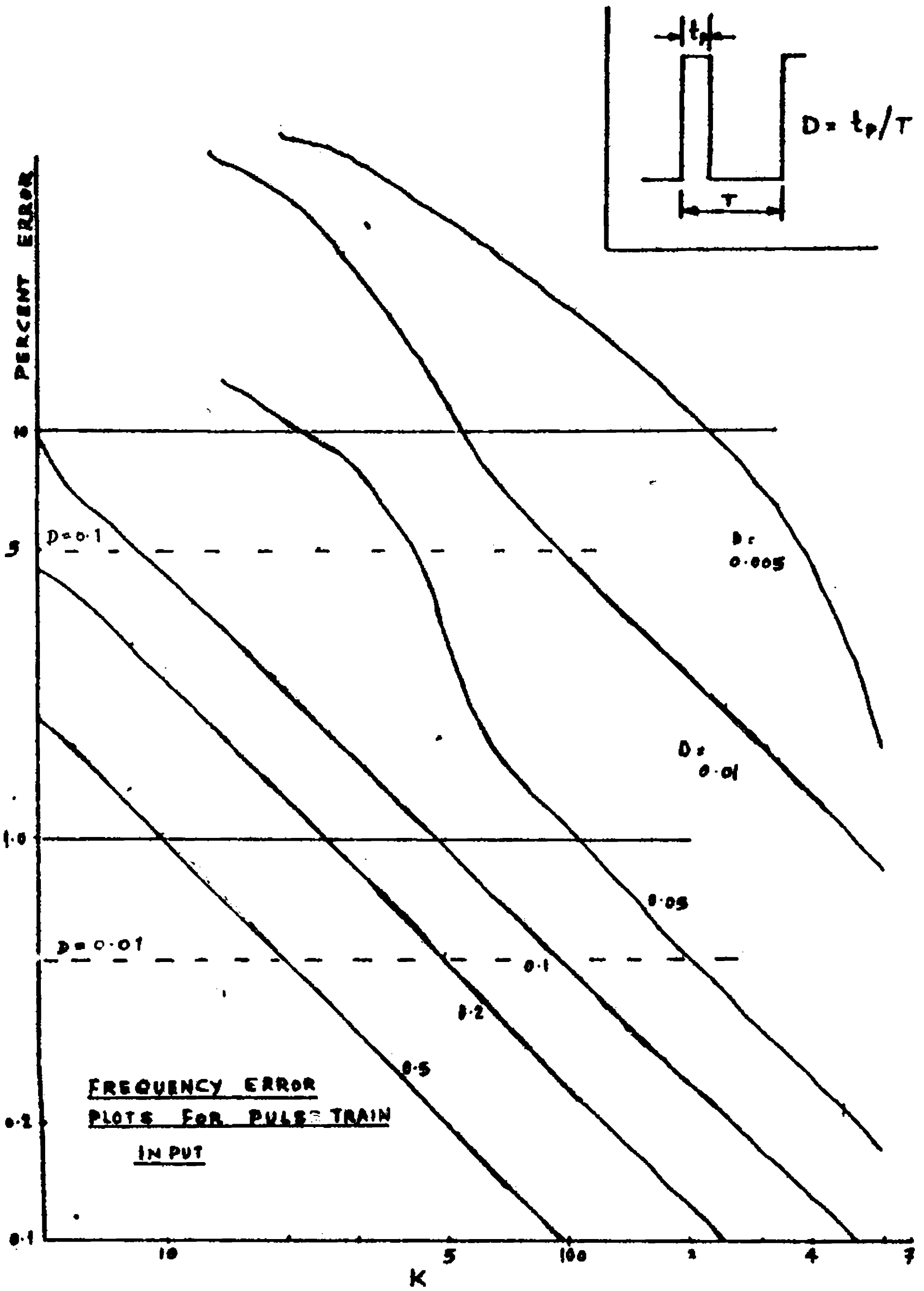

D= t_p/T
t_p
T
PERCENT ERROR
10
5
1.0
0.2
0.1
D=0.1
D=0.01
D= 0.005
D= 0.01
0.05
0.1
0.2
0.5
FREQUENCY ERROR PLOTS FOR PULSE TRAIN INPUT
10
5
100
2
4
7
K

FIG. 2.3

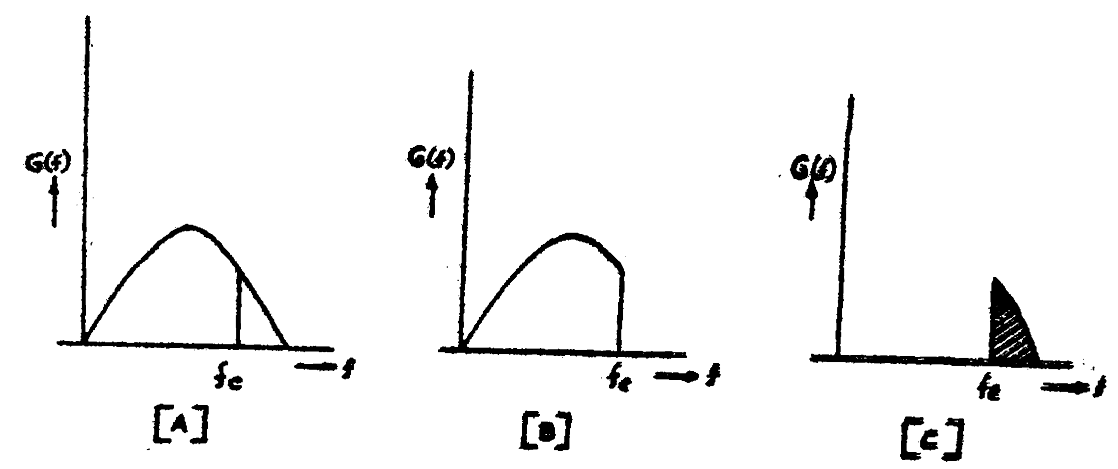


FIG: 2·4

POWER DENSITY SPECTRA OF [A] INPUT SIGNAL

[B] SIGNAL SEEN BY THE DETECTOR

[C] SIGNAL-PART REJECTED BY THE DETECTOR

CHAPTER - THREE

RMS Measurement Techniques and Applications

3.1) INTRODUCTION

The latest among the basic rms measuring methods is the rms computing technique The rms function is simulated with the help of analog or digital computing devices. Since the instrument is based on the mathematical definition of rms value, it can claim the highest accuracy. However, only a few truely computing rms instruments, without using a thermal converter, are commercially produced.

In this chapter various rms computing techniques are discussed. Their characteristics are compared in order to recommend a computing technique for rms measurement on high CF signals. The applications of rms measurement also are presented in this chapter.

3.2) DIGITAL RMS COMPUTATION

A general scheme of digital rms computation is shown in the figure (Fig. 3.1) The test signal is sampled, quantized, encoded and fed to the computing device which uses a certain algorithm to simulate the rms function. The sampler plays an important role. The fundamental question to ask about the number of samples is, 'How often must a given signal be sampled and how many samples must be processed so that the computed rms value is close to the true rms value within the specified limit ?' The solution to this problem is discussed in the fourth chapter.

A microprocessor can be used to compute rms value by using an appropriate algorithm (Ref. 1). At the same time the microprocessor can also be programmed to select the sampling frequency, number of samples etc. The Hewlett Packard's surveying equipment, HP 3805 A - a distance meter, incorporates a microprocessor that computes standard deviation and provides automatic atomspheric correction by averaging over the selected number of measurements (Ref. 2)

Alternately, rms value can be computed by using hard - ware techniques. Kitai (Ref. 3, 4, 5) gives the following algorithms:

(1) Integral square value (IS):

$$IS = \frac{2}{n^2} \sum_{r-1}^{n-1} r\, t_r - \frac{2}{n^2} \sum_{m} t_s \left(\sum_{ri=1}^{ri} r_i \right)$$

Where n is the number of quantization levels, t_r is the time during which the test voltage lies above the positive or below the negative r' th level, m is the number of samples, t_s is the sampling period and r_i is the quantized level of the i'th sample

(2) Square - root Algorithm:

If $S < N$

$$X_{new} = X_{old} + 1$$

$$S_{new} = X_{new}^2 = S_{old} + 2\, X_{old} + 1$$

If $S > N$

$$X_{new} = X_{old} - 1$$

$$S_{new} = S_{old} - 2\, X_{old} + 1$$

Thus, the rms computation by digital methods is a very interesting topic on the other hand as stated by Rizenman (Ref.6) : Until the prices of microprocessor-based devices are reduced considerably, these can be used primarily in applications where they can perform several other functions and where the cost is justified.

3.3) ANALOG SIMULATION OF RMS FUNCTION

1) Direct method : simplest possible simulation, one can think of is the direct rms computation as shown by the block diagram in FIG 3.2. It requires three ncnlinear blocks; squarer, averager and square-rooter. This technique is comparatively less frequently used (Ref.7)

2) Comparison technique: In principle, the rms instrument works equally accurately on both d.c and a.c. signals. In comparison technique, the rms of a test signal is measured in terms of its d.c. equivalant. In the figure [FIG 3.3 (A)], the d.c. reference (V_{DC}) is adjusted to get the mean square indication equal to that for the complex a.c. signal. The d.c. reference can be automatically adjusted by using the feedback as shown in the figure [Fig 3.3. (B)]. Like any other comparison methods this rms measurement technique is also popular Ref. 8, 9, 10).

Alternately, a fixed d. c. reference is used and the test signal is attenuated or amplified such that its rms value is brought to the level of d. c. reference level (Ref. 10, 11, 12) In Fluke's rms digital voltmeter (Ref. 11) the amplifier gain is automatically adjusted by controlling the feedback factor (β) of the amplifier. The feedback factor (β) is then a direct measure of the signal's rms value. Nelson (Ref. 13) uses the same technique of adjustment of the feedback factor of a feedback amplifier to raise the signal's rms level to the reference d. c. value.

One is tempted to conclude that this technique is free of amplitude error as the signal level is always brought to a reference level. Unfortunately, this conclusion is not correct because of the fact that the waveform and its crest-factor are not changed by attenuation.

In Russia (Ref. 13) a.c. reference voltage is used for the comparison. A special reference generator is used which develops a reference a.c. voltage with waveform characteristics same as those of the test signal.

3) Implicit method : The third analog computing technique is termed as 'Implicite method'. It is called 'implicite', because it neither includes a square-rooter, not does it contain an ac-dc comparator. The block diagram is shown in the figure (FIG 3.4). A spacial functional block called a^2 /b block is used. The output of this block is related to the input by the following relationship

$$V_1 = V^2_{in} / b$$

As seen from the figure (FIG. 3.4) the 'b' input of the a^2 /b block is the averager output (Vo) With a perfect averaging, the output (Vo) equals the rms value of the input V_{in}.

The a^2 /b operation can be implimented in a number of ways: i) Variable transconductance method (Ref. 14), 2) Logarithmic amplifiers (Ref. 15) and 3) Diode function generator with sliding break points (Ref. 16).

This computation technique introduces a new class of circuits wherein the feedback is not conventional. The two essential parts Viz the reference input and the error detector, of a conventional feedback system are absent. The feedback is present however, and it controls the parameter (1/b), i.e the transfer coefficient of the squarer block. The effects of the unconventional parametric feedback are discussed in the next section.

3.4) UNCONVENTIONAL FEEDBACK IN IMPLICIT RMS COMPUTATION

In the figure (FIG. 3.4) the averager output is fed back to form 'b' input of the a^2 /b block. This feedback circuit is different from the conventional one as the reference signal and the error detector are absent. Therefore, the standard methods of feeback circuit analysis are not useful. In the analysis presented below, it is assumed that the multiplier is the ideal one. Two averaging techniques are separately considered. It is also assumed that the output voltage reaches a steady state value (V_{os}) and its d.c component is a measure of the input rms value.

R. C. low pass - filter as an averager

From the figures Fig. 3.4 & 3.5,

$$v_o = \frac{1}{C} \int \frac{v_a - v_o}{R} \, dt ,$$

where $v_a = v_{in}^2 / v_o$.

Substituting for Va and rearranging,

$$v_o = \frac{1}{C} \int (v_{in}^2 - v_o^2) / R v_o \, dt.$$

Differentiating on both sides and rearranging,

$$RC \; v_o \cdot \dot{v}_o = v_{in}^2 - v_o^2$$

Let $v_o^2 = x; \quad v_o \cdot \dot{v}_o = 0.5 \dot{x}$

Substituting and rearranging,

$$\dot{x} + (2 / RC) \; x = (2 / RC) \; v_{in}^2$$

The solution of this differential equation is

$$x = Ae^{-2t / RC} + e^{-2t / RC} \int (2 / RC) \cdot e^{2t / RC} \cdot v_{in}^2 \, dt.$$

The steady state solution X_s is given by the second term. Thus the steady state output voltage V_{os} is given by,

$$V_{os}^2 = X_s = e^{-(2t / RC)} \int (2 / RC) \; e^{2t / RC} \; v_{in}^2 \, dt.$$

One cannot do any approximation at this stage because Xs does not appear as a voltage drop across any element in the actual circuit. The measured rms value is ($\overline{V_{os}}$) and is given by

$$\overline{V_{os}} = A_{ve} \left[(e^{-(2t/RC)} \cdot \int (2/RC) \cdot e^{2t/RC} \cdot v_{in}^{2}\, dt)^{0.5} \right] \quad — \quad (3)$$

It is interesting to note that the measured rms value $\overline{V_{os}}$ is not equal to the square root of the mean square value ($\overline{X_s}$) but it is the average of the square-root of Xs. Though $\overline{X_s}$ is the true mean square value $\overline{V_{os}}$ may or may not give the true rms value.

The value of $\overline{V_{os}}$ can be calculated for various commonly occuring signals like d.c, ramp and sinusoidal signal. The calculations using the equation (3) are given in the appendix V. For a d.c. signal the true rms value is measured. For a sinusoidal signal the d.c. error is (1 / 16 [1+ (W^2 R^2 C^2)]) and it is small if RC $>> \frac{1}{\omega}$. The results are interesting for a ramp signal The measured value approaches the true mean value if (RC / T) << f (t). Since f (t) $\leq$ 1, the time constant RC must be smaller than the periodic time T. This requirment is contradictory to that for the perfect averaging of sinusoidal signals.

Thus a simple R.C. low pass filter is not an appropriate choice in the implicit rms computation technique.

<u>Integrator as an averager</u>

With integrator as an averager in the figure (FIG. 3.4) the output voltage v_o is given by

$$v_o = [1/T] \int [1/RC] [v_{in}^2 / v_o] \, dt.$$

Let the integration constant (1/RC) be equal to one. Differentating on both sides of the equation and rearranging it,

$$v_o \cdot \dot{v}_o = (v^2_{in} / T)$$

Substituting $x = v_o^2$ and $v_o \cdot \dot{v}_o = 0 \cdot 5 \cdot \dot{x}$,

$$\dot{x} = 2 \ (v^2_{in} / T)$$

The Solution of this differential equation is

$$x = \int^{t} (2 / T) \ v^2_{in} \ dt + k,$$

Where k is the arbitrary constant.

Now, the output voltage is given by

$$v_o = X^{0.5}$$

$$= [\int (2 / T) \ V_{in}^2 \ dt + k]^{\frac{1}{2}}$$

The measured rms value is the output voltage at $t = T$ ie. V_{ot} say,

$$V_{ot} = [\int^{t} (2 / t) \ v_{in}^2 \ dt + k]^{\frac{1}{2}}$$

Since the value of k is decided by the initial conditions; for consistant results, it is essential to set k equal to zero by fully discharging the integrating capacitor prior to any measurment or computation.

Secondly, with k = O the measured value is given by

$V_{ot} \sqrt{2}$ (rms value)

Thus, one must be careful while using the integrator as an averager in the implicit RMS computation technique.

3.5) COMPARISON OF THE THREE RMS COMPUTING TECHNIQUES

In principle, all the three methods compute the exact rms value, and their performance is then decided by the characteristics of the individual blocks used. All the three methods need a squarer and an averager. The influence of squarer and averager characteristics on the rms measurement is different in the three cases. In the paragraphs to follow, an attempt is made to analyse the three methods to study i) the effect of squarer error and 2) the effect of the averager ripple.

3. 5-1) THE ROLE OF THE SQUARER ERROR

A squarer is assumed to possess an absolute squaring error (v^2). It is interesting to study how the rms measurement error, (ϵ) expressed as the percentage of full scale, is related to v^2 . It is a practice in error analysis to assume that only a particular error or error source is present; therefore the squarer output only is in error whereas all other blocks are assumed to be ideal. For simplicity 1 : 1 correspondance is assumed.

a) Direct Computation method

The ideal output is given by,

$$V_{oi} = \left[\overline{V^2_{in}} \right]^{0.5}$$

The rms output in presence of the squarer error (v^2) is given by,

$$V_o = \left[[V^2_{in} + v^2]_{ave} \right]^{0.5}$$

The error in the mean square value

$$\delta_s = v^2 / (V^2_{rms})$$

The corresponding relative error in rms value is approximately given by

$$\delta_r = 0.5\ \delta_s$$ (Please see. p. 9)

The corresponding percentage error ϵ in rms output is given by

$$\epsilon = [\,0.5\ \delta_s\ V_{rms}\,]\ /\ V_{fs}\ .$$

substituting for δ_s,

$$\epsilon = 0.5\ (\ \overline{v^2}\ /\ V_{rms}\ \cdot\ V_{fs}\ )$$

where V_{fs} is the fullscale output and V_{rms} is the rms input. In the worst case $\overline{v^2}$ equals the specified maximum error of the squarer, equal to $(\epsilon s \cdot V_{fs}^{2}\ .)$

Thus the accuracy of rms measurement can be related to that of the squarer, ϵs, by the relationship :

$$\epsilon = 0.5\ \epsilon s\ V_{fs}\ /\ V_{rms}\ .$$

Thus ϵ is inversely proportional to V_{rms} and therefore the lower side of the dynamic range of rms measurement is limited by the squarer error. If permissible value of ϵ is same as ϵs, the rms output at low level is limited to 50% fullscale. Thus the dynamic range is only 2 : 1.

2) COMPARISON METHOD

In this case the transfer error (v_t^2) of the squarer should be considered. The relationship can be derived similarly and is given by,

$$\epsilon = (\ 0.5\ \epsilon st\ \ V_{fs}\ )\ /\ V_{rms}\ ;$$

where ϵst is the specified maximum percentage transfer error of the squarer.

In the modified comparison method an adjustable - gain amplifier is used to raise the input rms value to a fixed reference level. The measured rms value is then proportional to the reciprocal of the adjusted amplifier gain (A), and therefore for small errors,

$$\delta_r = -0.5 \cdot \Delta (A^2) / A^2 = -\frac{\Delta A}{A}$$

The value of δ_r in terms of the transfer error v_t can be determined from the basic equation,

$$[A^2 V_{in}^2]_{ave} = V_{dc}^2 + v_t^2$$

The relative error in rms measurement δ_r is then given by,

$$\delta_r = -\frac{\Delta A}{A} = 0.5 \, V_t^2 / V_{dc}^2$$

Since V_{dc}^2 is constant, δ_r is directly proportional to V_t^2

3) IMPLICIT METHOD

The basic relationship governing the rms measurement in this case is,

$$\frac{[V_{in}^2]_{ave}}{V_o} = V_o$$

In the presence of the squarer error V^2 the equation can be written as

$$[V_{in}^2 + v^2]_{ave} = V_o^2$$

Thus the relative error in rms measurement

$$\delta_r = 0.5 \, \delta_s = 0.5 \left(\overline{v^2} / V_{rms}^2 \right),$$

The percentage error in rms measurement ($\in$) can then be related to the maximum possible percentage error of the squarer ($\in s$):

$$\in = 0.5 \in s \ V_{fs} \ / \ V_{rms}$$

In conclusion, the percentage error ($\in$) in rms measurement is inversly proportional to the input rms value in all the cases except the modified comparison method using the adjustable gain amplifier. The squarer percentage error and the corresponding error in rms measurement are plotted against the d.c. input (FIG. 3.6) . If the squarer with one percent constant error is used to measure the rms value, the dynamic range, for one percent accuracy, is limited to 2 : 1. Thus, there is a need of a squarer with adjustable error characteristic.

3.5.2) THE ROLE OF AVERAGER RIPPLE

In practice, the averager output consists of a d. c and a low frequency ripple caused by stray or by input signals. The ripple introduces fluctuations in the indication by the final output device. These fluctuations can be removed by providing heavy damping. In case of circuits, with the averager in the feedback loop, the problem is more complicated. Additional a. c. feedback is used by Folsom (Ref. 8) and also by Cox & Kusters (Ref. 9) to compensate for the ripple. Thus, only two techniques-direct method and manually operated comparison method-are free of the effects of the averaging ripple and therefore are suitable for low frequency signals.

3.6) APPLICATIONS OF RMS MEASUREMENT

RMS measurement is basically a measure of signal power, signal amplitude and statistical error. These basic measures can be extended to measurements of other related properties like distortion, noise figure, noise bandwidth etc.

It is a practice to use either conventional a. c. measurement-peak or average responding detector, calibrated for a sine wave signal-or a thermal a. c. to d. c. converter. The true rms measurement is replacing the conventional a. c. measurement; because (i) the complexity of signals in modern systems does not permit the waveform error inherent in other a. c. measurements ; and (2) the difficulties encountered in rms computation and the cost and complexity of rms detector are reducing with the advent of modern technology.

Typical rms measurement applications are tabulated in the table (T 3.1). For each of these applications, the measured signals are classified as in column 4 of the table. On the basis of the study it is revealed that for each class of signals there is an optimum rms measurement. RMS measurement can be classified into three categories for this purpose :

1) High-frequency - error - free measurement — The rms detectors with a sensor like thermocouple or thermistor with a heater can be grouped in this class.

2) Low - frequency - error - free measurements - Non-feedback rms computing techniques like direct computation method or manually operated comparison method fall under this group.

3) Amplitude - error - free measurement or waveform - error - free measurement — Non-feedback rms detectors incorporating a specially disigned squarer belong to this category.

The 'remarks' column of the table (T 3.1) reveals the importance of the waveform - error - free rms measurement. It is recommended for majority of signals. Signals of any complexity - sinusodial, distorted sinusoidal, triangular, ramp, pulse trains, etc. or random signals, (gaussian and nongaussian as well) - can be accurately measured by waveform - error - free - rms measurement·

3.7) CONCLUSION

The averager and the squarer are the main blocks of any rms computation technique. Their characteristics influence the rms measurement performance.

The study of the effects of squarer output error revealed that : (1) the constant squarer output error results in hyperbolic rms measurement error characteristics (2) the dynamic range of rms measurement is limited by the additive error in all the computing techniques except one, the comparison method with amplifier gain adjustment facility (3) Thus, there is a need of a squarer with controllable error characteristics.

The averager ripple introduces problems in feedback techniques. The analysis of the popular method, implicit RMS computation, has revealed the unsuitability of a simple R. C. filter as an averagor in a closed loop.

From the application point of view, the rms measurements can be classfied into three categories; high frequency - error - free, low - frequency - error - free, and waveform - error - free. The study of various applications has revealed the importance of the waveform - error - free design.

In conclusion, a waveform - error - free rms measurement based on direct rms computation method and using an appropriate squarer design is recommended for the rms measurement on high crest - factor signals.

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TABLE T. 3.1

RMS Measurement Applications

No.	Field of Application	Parameter Sensed	Class of Signal	Measuring device used	Remarks
1	Testing of devices. 4-terminal passive devices, signal generators and a.c. instruments. (Ref. 18)	Power or amplitude of the out-put signal		Conventional a.c. instrument	Waveform - error - free measurement extends the application to testing of 1) nonlinear devices, 2) Power output of function generators and 3) all types of a. c. instruments on sinusoidal as well as on complex signals.
2	Microwave Power measurement (Ref. 19)	1) micro-wave power 2) A f power in a microwave powermeter	HF AF	Bolometer Conventional a. c. instrument	1) Hf - error - free measurement is a must, waveform - error free measurement is desirable for higher accuracy. 2) Waveform - error - free and low-frequency - error - free measurement is desirable as it is free of envelop tracking and beat frequency errors.
3	Automatic Control of process variables. (Ref. 20, 21)	output power of the controller	Deterministic complex	Conventional a. c. or Thermistor instrument	Waveform - error - free rms measurement extends the application to the modern switching type (SCR) and digital controllers.

No.	Field of Application	Parameter Sensed	Class of Signal	Measuring device used	Remarks
4	Signal analysis 1) Distortion measurement 2) Power - density spectrum 3) CF and form factor measurement (Ref. 22)	1) Signal power 2) Signal power in the specified narrow band. 3) Signal amplitude	Random or Deterministic complex	conventional a.c. instrument or Thermal.	1) Waveform - error - free rms measurement is required for higher accuracy. 2) Waveform - error - free measurement extends the application to measurement on random signals. 3) Waveform - error - free measurement is needed for accurate measurement on signals of any complexity.
5	Acoustic measurements 1) VU measurement 2) Sound - level measurement. (Ref. 23)	signal Power	Random at audio freq	1) conventional a. c calibrated for CF = $\sqrt{10}$ 2) Conventional a. c. instrument.	with waveform - error - free measurement, there is no restriction on CF of the signal.
6	Statistical measurement: 1) Radar and hf communication. 2) seismology, structural vibrations and process control. (Ref. 24)	Standard deviation or rms error.	1) random at high frequency 2) Random and generally below one MHz .	Thermocouple or computing type.	1) waveform - error - free and high frequency - error - free measurement is a must. 2) Waveform - error - free measurement is necessary.

No.	Field of Application	Parameter Sensed	Class of Signal	Measuring device used	Remarks
7	Noise and related measurements. (Ref. 25)	Signal power or amplitude.	1) White noise 2) Narrow band 3) $\frac{1}{f}$ noise	Thermal or conventional a. c. instrument.	1) Hf - error - free measurement is needed in case of white noise. 2) Lf - error - free measurement is needed in case of 1 / f noise. 3) waveform - error - free measurement permits measurement of any type of noise distribution, gaussian and nongaussian.
8	Communications : Measurement of system and signal characteristics like noise figure, noise bandwidth, harmonic distortion, S/N ratio etc. (Ref. 26)	Signal or noise power	1) Base band signals. 2) Multiplexed signals 3) carrier modulated signals. 4) Pulse modulated signals.	Thermal or Conventional a. c. instrument.	Hf - error - free measurement is needed for all types of signals except the base band signals. The waveform - error - free measurement is a must, because the communication signals are always nonsinusoidal and random.

FIG. 3.1
DIGITAL RMS COMPUTATION

FIG. 3.2
DIRECT RMS COMPUTATION TECHNIQUE

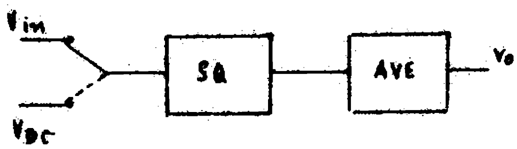


FIG. 3.3 [A]
COMPARISON TECHNIQUE

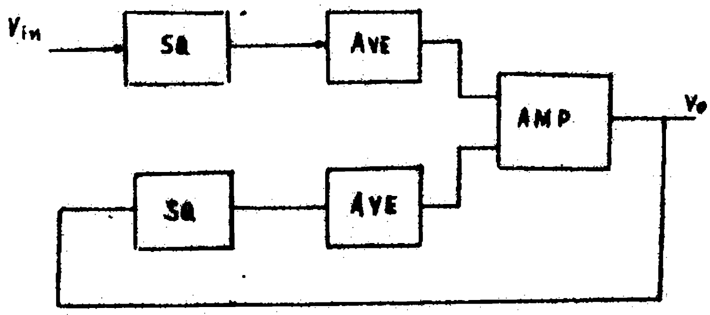


FIG. 3.3 [B]
AUTOMATIC COMPARISON

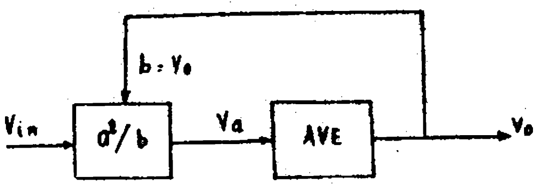


FIG 3·4

IMPLICIT RMS COMPUTATION

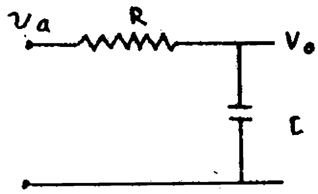
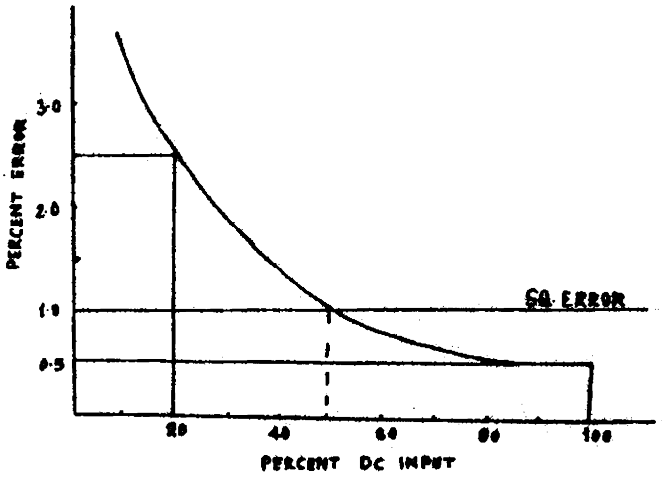


FIG 3·5

AVERAGING FILTER



FIG·3·6

HYPERBOLIC ERROR CHARACTERISTIC

CHAPTER - FOUR

Squaring Square - Rooting and Averaging Techniques

4.1) INTRODUCTION

The rms instrument consists of nonlinear circuits but possesses the linear relationship between dc or rms input and the output. The design and the selection of the nonlinear blocks is not a simple task because the relationship between the rms measurement characterisitcs and those of the individual blocks is fairly complicated. Among the various characteristics of the rms measurement the frequency characteristic is decided by the averager at the lower end and at the upper end it is decided by one of the blocks, the input amplifier or the squarer, with poorer frequency response. The dynamic range of the rms measurement is decided by the amplitude error plot of the squarer.

In this chapter, various squaring, square-rooting and averaging techniques have been discussed with a limited aim of determining their relative merits for incorporating these in an rms instrument.

4.2) SQUARING TECHNIQUES

The squarer is the heart of an rms instrument. It is a nonlinear device and influences almost all the characteristics of the instrument. it also should be noted that the squarer with a controllable amplitude - error plot is preferable for rms measurement of high crest factor signals.

4.2.1) SQUARE - LAW DEVICES

Inumerable quantity of square - law devices exist. The table (T. 4.1) lists various square - law devices used in rms measurements. These are frequently called as 'true' square function generators, because in prinicple the devices follow a square law. In practice however, the square law characteristic is true only in certain specified conditions. It is considerably affected by one or more factors like temperature, voltage variation, biasing conditions etc. In majority of the cases the true square - law characteristic is limited to a small span of input signal levels only. These devices can be grouped into several groups as shown in the table (T. 4.1).

The rms instrument using one of these devices is, no doubt, a simple device giving true rms value under the restricted conditions. On the other hand, in none of these devices the amplitude error plot is easily controllable. Though some of these devices, particularly from the thermal group, are widely used in rms measuring devices, these are comparatively less suitable for rms measurement on high crest - factor signals.

TABLE T 4.1

Square - Law Devices in RMS Measurement

No.	Group	Device	References	Remarks
1	Thermal	Thermistor	1	Wideband, CF 7 to 1, Linearity 0.02%, temperature should be maintained constant to better than 0.002°C.
2	Thermal	Thermocouple	2	AC - DC transfer standard.
3	Thermal	Differential thermocouple	3	Two thermocouples are closely associated in a heat - sink to assure that ambient temperature affects them equally.
4	Thermal	Diode tube	4	A temperature limited diode is held at a constant total emission by means of a servoamplifier.
5	Thermal	Transistor	5, 6	
6	Thermal		7	Doubly tapered foil resistor coated with liquid crystal material. A narrow colour band forms and moves on the taper. The location of the band is proportional to the rms current through the foil.
7	Electronic	Tunnel diode	8	
8	Electronic	FET	9	

No.	Group	Device	References	Remarks
9	Electronic	Cadmium Sulphide dietle-ctric diode	10	
10	Electronic	Nonlinear amplifiers	11	
11	Electromagnetic	Hall Generator	12	
12	Electromagnetic	Magneto resistive transducer	12	
13	Electrome-chanical	Electrostatic	13	Emperical Calibration is necessary.
14	Electrome-chanical	Electrodynamic	13	Secondary transfer standard at low audio frequency.
15	Electrome-chanical	Moving Iron	13	Lowcost, low precision instrument Bandwdith 25 to 125 Hz.

4.2.2) ANALOG SQUARE - FUNCTION GENERATORS.

A diode function genrator is widely used in curve tracing. The curve is shaped segment by segment being approximated by a step, linear or a polynomial function. In case of the square - function generation the linear approximation is obviously the best. Though it is a practice in linear approximation to design for a constant value of the maximum absolute segment error; there is a lot of freedom for the designer to shape the error characteristics. It can be shaped by appropriately selecting (1) number of segment. (2) break points and (3) slopes of the approximating straight lines. Secondly the nonlinear devices are used in switiching mode only, to control the break points; hence the variations in their characteristics have no influence on major part of the generated function. On the other hand, the unstable switiching characteristic poses a problem in building up the function generator. Larger the number of segments, more difficult is the building up process.

The reduction of the number of segments is one of the problems before the designers. In the method used by S. Marjanovic (Ref. 14) the error of a two-segmented square function approximator is drastically reduced by adding the correcting function, generated by using a one-percent accurate multiplier. Alternately, the number of segments can be reduced drastically by using the principle of 'sliding break points' (Ref. 15) The break points are controlled by the filtered output signal. This technique improves the squarer performance at low input levels. Both these techniques require further analysis to justify their application in rms measurement.

It is well known that in case of a linearly segmented square function generation the approximation error plots are identical for all segments of approximation. Hence the error $\hat{\epsilon}_r$ can be expressed in terms of a normalized input (r) as given below.

$$\hat{\epsilon}_r = r\ (r - 1)\ (L^2 / 10),$$

where $0 \leqslant r \leqslant 0.5$ and r is defined as

$$r = \frac{x - x_{n-1}}{L} \qquad x_{n-1} \leqslant x \leqslant \left(x_{n-1} + \frac{L}{2} \right)$$

$$= \frac{x_n - x}{L} \qquad \left(x_{n-1} + \frac{L}{2} \right) \leqslant x \leqslant x_n$$

[x_n — the break point of the (n + 1) 'th segment x_{n-1} — the break point of the n'th segment. and L — the segment length (x_n — x_{n-1})]

This function can be easily generated by using a multiplier (Ref. 14).

Generally the error is small in comparison with the actual output and the accuracy of correcting function is not important. Let us consider a two - segmented square function generator with the segment breakpoints at $x = 0$, and $x = 5$. The segment error at mid - points ($x = 2.5$ and $x = 7.5$) is 0. 625. It is equal to the square function output ($x^2 / 10$) at $x = 2.5$. Thus the accuracy of the error function generation directly decides the accuracy of the function generation in the lower segment.

The relative errors of a two - segmented square function generator alone are 100% and 11% at the mid points of the lower and the upper segments respectively. With the help of the one percent accurate error - function generator these squarer errors are reduced to one percent and 0.1% respectively. Thus, the errors are reduced drastically. It should be noted, however that the ratio of the two maximum segment - errors is almost unchanged.

Secondly, a multiplier itself can be used as a square function generator (Sect. 4. 2. 3) . One percent error in the multiplier results in 0. 5 percent error in the rms measurement.

Thus, this technique for reduction of the number of segments is not useful in achieving the desired rms measurement error characteristics.

The sliding break - point technique is found both in English and Russian literature (Ref. 16, 17) . If the break points of the segments (x_1, x_2, x_3) are made proportional to the input rms value (x_{rms}) , the output will approximate the function (x_{in} / x_{rms}) . Thus, the functlon generator wlth sliding break-points controlled by the rms output acts as ' a^2 / b ' functional block of 'implicit RMS ' computation ' method. The merits and demerits of this computation techniques have already been discussed (Sect. 3.4)

Secondly, let us consider the measurement of a low duty - cycle - doublet type pulse train signal. Since the break points and therefore the approximation range are decided by rms value, the pulse - height may lie beyond the approximation range. Thus, in this technique the low - level performance is improved at the cost of the high level performance and therefore this technique is not selected for waveform - error - free rms measurement.

The author has developed (Ref. 18) a square function generator wherein the percentage error in rms measurement is a design criterion. The segments are so designed that the maximum percentage error in rms measurement is constant for all the segments. The details of this design are given in the fifth chapter. This square function generator solves the problem of the amplitude error in rms measurement by avoiding the hyperbolic nature of the amplitude error characteristic.

4.2.3) ANALOG MULTIPLIERS

Two types of analog multiplier chips are used widely for squaring and square - rooting operations in rms measurement. The multiplier output is given by (yz / x). By selecting $y = z =$ input signal, and $x = 10$, or a suitable constant, squaring with a proper scale factor is achieved.

With log - antilog type multipliers, all the input voltages must be positive and may not even fall to zero. A variable transconductance type multiplier, on the other hand, is a four quadrant multiplier. It needs four initial adjustments. Its scale factor is sensitive to supply voltage changes, as the emitter current is controlled by an adjustable resistance (Ref, 19)

Multipliers have a rigid amplitude - error characteristic and therefore the additive constant error limits the dynamic range of rms measurement. Let us consider for example, the specifications of a typical multiplier chip 4371 by Teledyne Philbricks. (Ref. 20) Multiplication mode - $E_o = YZ/10$, the output error (for $Y = Z = 10$mV to $+ 10$V) is $+ 5$mV $+ 0.3\%$ of the output.

Let us determine the low - level performance of the rms measurement using the multiplier as a squarer. The output error of 5mV is equivalent to

0.5% error at 0.5v input. Thus, the dynamic range of 0.5 percent rms measurement is limited to 0.5v to 10V.

As per the test results given by Lenk (Ref. 21) the rms voltmeter using Motorola's multiplier MC 1594 has a dynamic range of 2 V to 10 V (Peak) Secondly in case of the rms function generator 4370 of Teledyne Philbricks (Ref 22), the error varies from 10mV to about 100 mV for CF variation from 1.4 to 3.5.

In conclusion, the multiplier chips-may be useful as a squarer or a square-rooter in rms measurement. But, because the error characteristic is not controllable, it is not the correct choice for waveform - error - free rms measurement.

4.3) SQUARE - ROOTING TECHNIQUES

Low precision rms instruments incorporate the square-rooting operation in the layout of the instrument scale itself, with the result that the instrument has a nonlinear scale. The modern high precision instrument based on the direct rms computation method needs a square - rooting device. The design or selection of the square - rooting circuit is less difficult because (1) the input is always d. c. irrespective of the waveform of the test signal and (2) the rms measurement error is directely related to the output square - rooter error.

In principle, a square - rooting function can be easily implemented by using a multiplier as shown in the figure (Fig 4.1 A) the adjustment is done automatically and Vo is the required square - root value. Lenk (Ref 23) gives the performance of such a square - rooter circuit. The relative error varies from — 7.6% at 1V input to + 1.1% at v_{in} = 81V.

From the manufacturers data, one can conclude that the performance of a multiplier chip in the square - rooter mode is comparatively worse than that in the square mode. For example, Burr Brown's 4301 has squarer accuracy of ± 3mv for 0.1 V d.c. $\leqslant E_1 \leqslant$ 10V d.c. and square - root accuracy of 0.07% over 0.5 V d.c. $< E_1 <$ 10V. The square - rooter error increases to ± 55 mv over the range 0.02V $< E_1 <$ 0.5V (Ref. 24)

A square - rooting A/D converter does two functions at a time and hence is preferable to other method. The integrators integrate the known d.c. signal twice, providing the integrated ramp as a reference voltage for comparison. The output pulse width is proportional to the square - root of the comparator input v_{in} . This type of the square - rooting A/D converter is worked out by the author (Ch. 5).

Alternately, there is quite an interesting way of eliminating one integrator as shown in the figure (Fig 4.2) . This circuit was worked out in Russia (Ref 25) and showed a better performance. The integrator integrates a d.c. voltage which is proportional to the output pulse - width (t_p)

4.4) AVERAGING TECHNIQUES

Averagers are widely used to obtain (1) the time interval average characteristics of periodic and non - periodic signals, (2) the period - average characteristics of periodic signals, (3) the statistical average of a number of observations (N) of discrete events. In addition, an averager plays an important role in signal recovery (Ref 26)

In rms measurement. the averager has to obtain either the time - interval average or the period - average value. The application in rms measurement of both techniques, digital and analog are presented in the following sub sections.

4.4.1) DIGITAL AVERAGING TECHNIQUE FOR RMS MEASUREMENT

A signal is squared and then sampled and encoded. The 'n' samples are then processed by the digital averager to determine the mean - square value. There are two questions to answer; (1) what should be the sampling rate ? and (2) how many samples one should process ? In rms measurement, is it necessary to select the sampling rate higher than twice the bandwidth of the signal to be sampled ? Is it necessary to select 'n' so high that the statistical averaging error is within the premissible limits.

It is obvious that only one sample gives the correct result if and only if, that sample coincides with the instantaneous mean - square value. Therefore, it may be possible to make the averaging accuracy independent of the number of samples if the samples are properly positioned. The influence of the time delay of the sampling signal on the averaging error is investigated for various signals. The outcome of the investigation is presented below.

Let the periodic test signal be f (t) and the sampled signal f* (t) which can be represented as a function of a fraction of the signal period (t).

$$f^*(t) = f\left(\frac{(r + k_o)}{N} T\right)$$

where r = variable, varying in steps from 0 to (N — 1)

k_o = constant fraction, relative time delay.

T = Period of the test signal.

N = Ratio of test period to the sampling period.

Thus, T / N = sampling period and $\frac{k_o T}{N}$ = time delay.

The mean - square value (MS) is given by,

$$MS = [1 / N] \Sigma [f^*(t)]^2$$

$$\text{i.e. } MS = \frac{1}{N} \Sigma f^2 [(r + k_o) T / N] \qquad — \qquad (4)$$

The averaging error in the mean square value is given by

$$\delta_{MS} = \left[\frac{MS}{TMS} - 1\right] \times 100 \qquad — \qquad (5)$$

where TMS is the true mean square value.

The averaging error in the mean - square value (δ_{MS}) is a function of two variables; (1) N - the number of samples per period and (2) k_o - the relative time delay. It can be computed and plotted for various commonly occuring signals using the equations (4) and (5).

Typical plots for a ramp signal are presented in a tabular form in the table (T. 4.2) and shown graphically in the figure (Fig. 4.3). The zero - cross over points for all the graphs coincide and occur at the value of k_o equal to 0.52. The ramp function takes the rms value at $t / T = 1 / \sqrt{3} = 0.57$ For trapezoidal waveforms the error function δ_{MS} is tabulated in the table T. 4.3 and plotted in the figure (Fig. 4.3) the zero cross over points are 0.22 and 0.79; the function takes the rms value at two points $t / T = 0.24$ and 0.75.

In conclusion, one enjoys a freedom of selecting the sampling frequency if the value of k_o , i. e. the relative time delay or the value of t / T at which the signal takes the rms value, is known. In practice the value of k_o can be determined from the known approximate rms value and the display of the signal waveform on CRO screen. This criterion is applicable to all periodic signals which exist and are continuous at $t = k_o T$.

4.4.2) ANALOG AVERAGING TECHNIQUES

There are mainly two types of averagers used in rms measurement; (1) RC low pass filter and (2) integrator.

The filter gives the period - average value. It is difficult to use filters for low frequency signals. For perfect averaging higher - order low pass filters with sharp cutt - off are necessary. The selection of (a) the cut - off frequency and (2) filter order should be done, by compromising between the low frequency averaging characteristics, the settling time and the complexity.

An analog integrator is preferred for low frequency signals. It gives, in principle, the time - average value at a particular instant t = the averaging period.

4.5) CONCLUSION

The only technique, that provides the facility of shaping the error characteristic, is the square function generation technique. Therefore, the square function generator is recommended for rms measurement on high crest - factor signals.

The square - rooting A / D converter is preferable to analog square-rooters with multiplier, as it permits hybrid measurement providing the digital output.

The study of digital averaging technique has resulted in the development of a new sampling criterion. This criterion provides the flexibility in the selection of the sampling characteristics like frequency and the number of samples.

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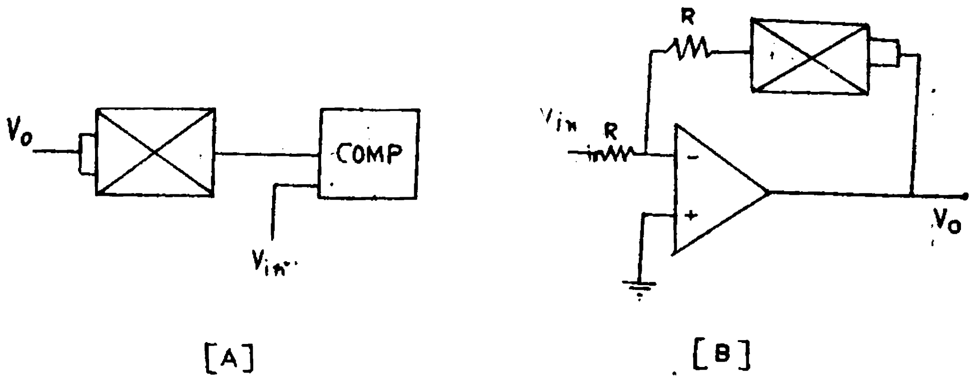

FIG 4·1 SQUARE-ROOTERS WITH MULTIPLIER

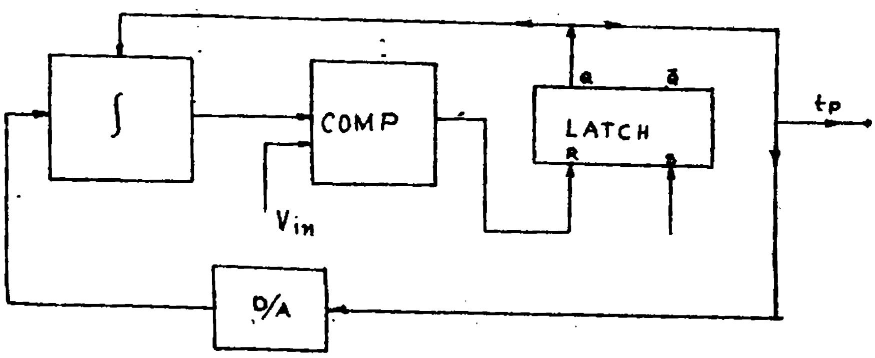

FIG 4·2 SQUARE ROOTING A/D CONVERTER

TABLE T. 4.2 (a)

$$f(t) = \frac{t}{T} \qquad o \leqslant t \leqslant T$$

$$f(t) = f(t - nT) \qquad nT \leqslant t \leqslant (n + 1)T$$

where n is an integer.

$$TMS = \frac{1}{3}$$

N	MS	δ_{ms}
	K = 1	
10	0.3850	15.5
20	0.3588	7.62
30	0.3502	5.05
40	0.3459	3.78
50	0.3434	3.02

TABLE 4.2 (b)

K_o	N = 50		N = 10	
	MS	δ_{MS} %	MS	δ_{MS} %
0	0.3234	− 2.98	0.2850	— 14.50
0.1	0.3254	— 2.39	0.2941	— 11.77
0.2	0.3273	— 1.80	0.3034	— 8.98
0.3	0.3293	− 1.20	0.3129	— 6.13
0.4	0.3313	− 0.60	0.3226	— 3.22
0.5	0.3333	− 0.01	0.3325	— 0.25
0.6	0.3353	+ 0.59	0.3426	2.78
0.7	0.3373	+ 1.19	0.3529	5.87
0.8	0.3393	+ 1.80	0.3634	9.02
0.9	0.3414	+ 2.41	0.3741	12.23
1.0	0.3434	+ 3.02	0.3850	15.5

TABLE 4.3

Trapezoidal Waveform

$$f(t) = \frac{3t}{T} \qquad o \leqslant t \leqslant T/3$$

$$= 1 \qquad T/3 \leqslant t \leqslant 2T/3$$

$$= \frac{3(T-t)}{T} \qquad \frac{2T}{3} \leqslant t \leqslant T$$

$$\text{True MS value} = \text{TMS} = \frac{5}{9}$$

K_o	N = 3		N = 4		N = 10	
	MS	δ_{MS} %	MS	δ_{MS} %	MS	δ_{MS} %
0	0.667	20.0	0.531	− 4.8	0.555	− 0.7
0.1	0.607	9.2	0.535	− 4.0	0.5526	— 0.58
0.2	0.56	1.0	0.548	− 1.4	0.5545	− 0.20
0.3	0.517	− 6.9	0.5815	+ 5.2	0.5570	+ 0.4
0.4	0.507	− 8.7	0.573	+ 3.6	0.5577	+ 0.4
0.5	0.500	−10.0	0.5702	+ 3.0	0.5575	+ 0.4
0.6	0.507	− 8.7	0.573	+ 3.6	0.5577	0.4
0.7	0.517	− 6.9	0.569	+ 2.8	0.5570	0.4
0.8	0.56	+ 1.0	0.548	− 1.4	0.5545	− 0.2
0.9	0.607	+ 9.2	0.535	− 4.0	0.5526	− 0.58

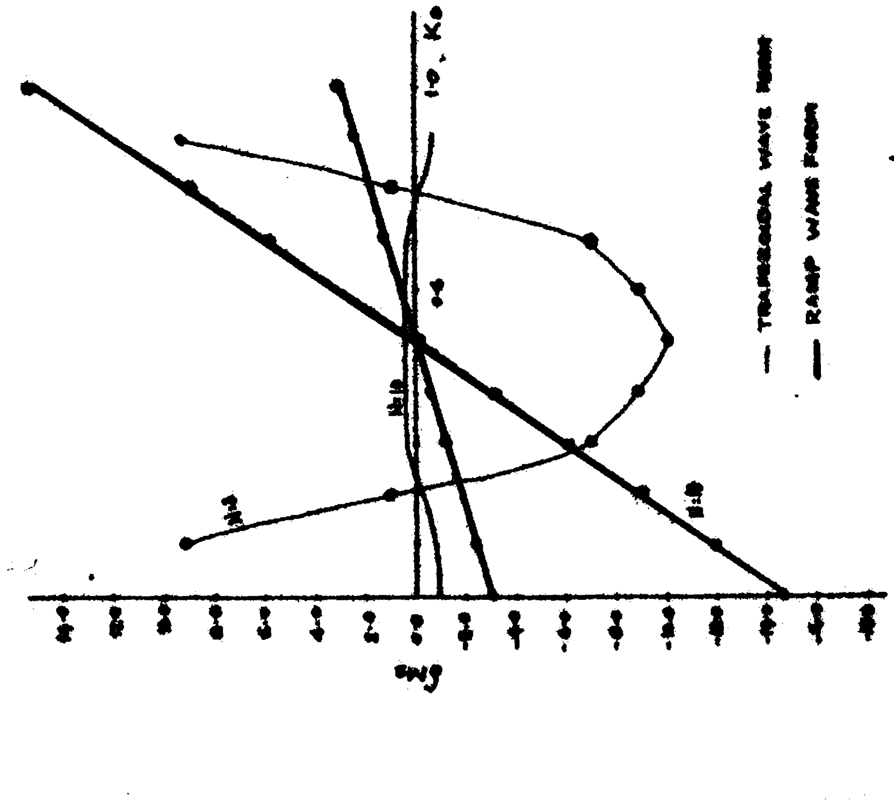

FIG. 4.3(A) δ_{Ms} Vs. K_o PLOTS

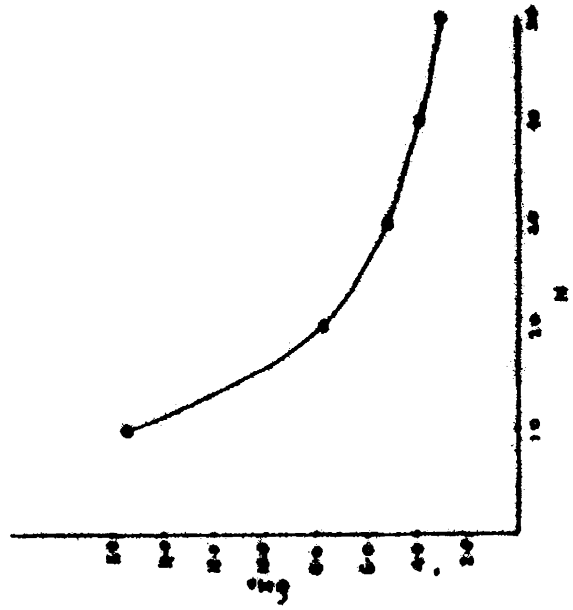

FIG. 4.3(B) δ_{Ms} Vs. N PLOT

CHAPTER - FIVE

Development of an RMS Detector

5.1) INTRODUCTION

In this chapter the details of the developed rms detector are presented. The detector has been designed for the rms measurement on low and medium frequency, high crest factor signals. The direct rms computation technique is used. A squarer, a squarerooter and an averager are designed, fabricated and tested. The squarer design is carried out specially for high crest - factor signals, however it is useful in other rms measurements also. A systematic analysis of the square - rooter is carried out and three adjustments are introduced to compensate for the square - rooter errors. These adjustments have helped in extending the the performance of the squarerooter over a wider dynamic range

5.2) SQUARER

Among the various squaring techniques, the function generator is chosen because it provides flexibility in shaping the error characteristic.

5.2.1) THE ABSOLUTE – VALUE DETECTOR

The well known absolute - value circuit as shown in (Fig. 5.1) was used along with the function generator to reduce the complexity. The absolute-value circuit needed three adjustments. (1) Zero Adjustment - by R_6 (Fig. 5.1) (2) Two adjustments were introduced to make the circuit perform identically on both positive and negative signals; low - level performance was adjusted by R_4 and high - level performance was adjustad by R_5 .

For stable operation of the absolute - value circuit as well as other blocks, using operational amplifier, it is essential that the amplifier should have sufficiently low noise and drift. One method of compensating for the difference in temperature coefficients of the input bias currents and thereby controlling the resulting drift was worked out by the Author while in Russia (Ref. 1).

The DC tests on the fabricated absolute - value circuit showed that the circuit could be trimmed to have a dynamic range of 3 decades (4mV to 4V) with a percentage error of ± 0.3 %. The frequency tests showed that the circuit had a flat response upto 10 kHz.

5.2.2) THE SQUARE – FUNCTION GENERATION

The function generator approximates the curve $y = x^2$, (curve 3, Fig. 5.2), by a series of straight lines which must lie in the region bounded by the two curves

$$y_1 = x^2 + 2 \cdot C \cdot x$$

$$y_2 = x^2 - 2 \cdot C \cdot x$$

(curves 2 and 4 in Fig. 5.2)

'C' is the maximum permissible absolute error in rms computation corresponding to the approximation error.

Different constraints, like the maximum permissible error of approximation and least mean square error, appear (Ref. 2) in the function generator designs. In the present case however, the constraint is unique; the minimum number of segments for a given squarer - error characteristic, or the peak segment error proportional to the input level.

It is a common practice (Ref. 2) to use graphical methods for approximating the well - defined functions by a set of straight lines. On the other hand, the analytical methods provide more design flexibilities, A good compromise is made between the two. The straight line giving the maximum segment length is choosen graphically; whereas its constants are determined analytically. The design is first carried out for a convenient input span of 0 to 10 units with a corresponding output span of 0 to 100 units. Suitable scale factors can then be introduced to use the design for the actual input and the corresponding output spans.

5.2.3) THE DESIGN OF THE i'th SEGMENT

It is obvious that the straight line, with its end points on one boundary curve and just touching the other boundary curve (as shown in Fig. 5.2, curve 1) is the one with maximum segment length. Assuming the starting point (a) to be known the straight line ab can be completely determined (Appendix VI). The abscissa x_{1i} of the tangent point is given by

$$x_{1i} = x_{i-1} + 2\sqrt{C \cdot x_{i-1}}$$

The slope $a_i = 2x_{1i} + 2C$

The segment length $L = 4\sqrt{(Cx_{1i} + C^2)}$

The abscissa (x_i) of the end point of the i'th segment

$$x_i = x_{i-1} + L$$

Thus the i'th segment is completely designed to have (i) maximum segment length and an error characteristic. Such that the resulting peak rms measurement error is exactly C units (the derivations of the formulae are given in the appendix VI).

This design procedure is followed for all the segments except the first one. The first segment needs a special consideration. Firstly, the squarer error must be zero at zero input, Secondly the boundary curves should be modified as per the expected squarer thereshould error C_o^2 caused by the noise and drift.

5.2.4) THE DESIGN OF THE FIRST SEGMENT

The boundary curves are (Curves 2 & 4 in Fig. 5.3)

$$y_1 = x^2 - C_o^2$$

and $$y_2 = x^2 + C_o^2 ;$$

where C_o^2 = constant threshold error, The approximating straight line, ob', is fully defined by (Please see the appendix VII)

(1) the slope $a_1 = 2x_{11}$,

(2) the abscissa of the tangent point

$$x_{11} = C_o$$

and the abscissa of the end point

$$x_1 = (1 + \sqrt{2})\ C_o$$

5.2.5) THE RELATIONSHIP BETWEEN THE NUMBER OF SEGMENTS (n) AND THE PERCENTAGE RMS MEASUREMENT ERROR (ϵ_r).

In case of a linearly segmented approximator for a square function, the relationship between (n) and the percentage squarer error (ϵ_s) is well-known. (Ref. 3). In the case of a nonlinearly segmented design the relationship is not that simple. The Author has worked out the relationship (Ref. 4) in the tabular form (table T. 5.1). A flow - chart used in determination of this table is given in the appendix VIII. A squarer design with 0.3 % rms measurement error was selected for the 0.5 % rms measurement.

5.2.6) THE CIRCUIT IMPLEMENTATION

In the actual circuit the input to the squarer (Vx) ranged from 0 to 8 volts, limited by the saturation characteristics of the absolute - value circuit used. The output of the squarer (v_y) was limited to 6 volts by the maximum input characteristics of the averager block used. The following scale factors were introduced :

$$K_x = V_x / x = 0.8$$

$$K_y = V_y / y = 0.06$$

$$K_a = a_{vi} / a_i = 0.075$$

The resulting modified version of the design was then as given below:

1) Break Point of the i'th segment

$$V_{Bi-1} = 0.8 \ x_{i-1}$$

2) Slope of the i'th segment

$$a_{Vi} = 0.075 \ a_i$$

3) The length of the first segment

$$V_{Bi} = 0.8 \ x_1$$

4) The slope of the first segment

$$a_{V1} = 0.075 \ a_1$$

The function generator was implemented by using precision feedback limiters (9 Nos.) and an adder as shown in figures (Fig. 5 - 4 & Fig. 5.5) In this design, the precision feedback limiters PFL (1) to PFL (i) were in conducting state when V_x satisfied the condition, $v_{Bi-1} \leqslant v_x \leqslant v_{Bi}$; whereas PFL (i + 1) to PFL (n) were in the cutoff state. Therefore, the slope S_i of the i'th PFL is given by : $S_i = (a_{vi} - a_{vi-1})$. The table (T. 5.2) gives the complete design values of all the components.

5.2.7) THE PERFORMANCE OF THE SQUARER

The fabricated function generator had 18 adjustments· The use of stabilized power supply to control the break points resulted in successful setting up of the squarer.

The function generator was subjected to various tests. The results of d.c. tests showed that the actual nature of the squarer error was very much similar to the designed one (Curves 1 and 2 respectively, in the figure Fig. 5.6) The square function generator had a dynamic range of 0. 1 volt to 8.4 volts (d.c.) i.e about three decades with an rms measurement error of not more than 0.4 %. the frequency tests showed that the frequency error was less than 5 % upto 10 kHz and 3 dB point was at 50 kHz. The pulse - input test, was carried out at repetition period of 20ms. The duty cycle was changed in steps and the filtered output of the squarer was found to be directly proportioned to the duty cycle (D) over the range of D equal to 0.25 % to 50 %.

This test proved that the filtered output of the squarer was directly proportional to the mean square value irrespective of the duty cycle.

It must be noted that 0.25 % duty cycle is equivalent to the crest factor of 20 thus the pulse test results proved that the squarer permitted the RMS measurement of signals with crest factors as high as 20.

5.3) THE AVERAGER

Two identical second order Butter Worth's unity gain low pass filters (Fig. 5.7) were cascaded. The cut off frequency was selected as 4 Hz.

The averager was subjected to a frequency test, by applying 6 V (peak) sinusoidal signal. The frequency was varied from 1 Hz to 100 Hz. The ripple was less than a few millivolts for signals of 10 Hz and of higher frequency. The response time of the averager was about 2 seconds on 5 V step - signal.

5.4 THE SQUARE - ROOTER

It performs the two functions ; square - rooting and AD conversion. The circuit is shown in the figure (Fig. 5.8) and the working of the circuit is presented by its time diagram in the figure (Fig. 5.9)

5.4.1) THE SQUARE - ROOTER OPERATION

The basic equation is

$$V_{in} = V_2 \quad \text{at} \quad t = t_p$$

therefore,

$$t_p = \left[\frac{2\tau_1 \cdot \tau_2}{V_R} \cdot V_{in} \right]^{\frac{1}{2}}$$

Where (i) V_R is the known constant voltage; τ_1 , τ_2 are the integration time constants of the two integrators.

thus $$t_p = K \sqrt{V_{in}}$$

where $K = (2 \tau_1 \tau_2 / V_R)^{0.5}$

The circuit was designed for 10 ms output pulse width for $V_{in} = 3$ volts.

In practice, the output pulse width will deviate from the ideal or theoritical value because of various imperfections in the actual working of the three blocks ; two integrators and a comparator. Certain adjustments are therefore required to get the desired performance.

5.4.2) THE ADJUSTMENTS ON SQUARE - ROOTER

The square - rooter is a nonlinear device consisting of two integrators and a comparator. The main sources of errors in an integrator are (Ref. 5) (1) Finite open - loop gain (A_o), (2) the d. c offset V_{os} and the bias current, and (3) the limited band width of the operational amplifier.

The expressions given by Tobey & others (Ref. 5) are reproduced below:

1) Error caused by the open - loop gain A_o in case of a step input E/s:

$$\triangle e_o = A_o E \left(1 - e^{-t / A_o RC}\right) - \frac{Et}{RC}$$

with $$\frac{t}{A_o RC} << 1$$

$$\triangle e_o = \frac{E t^2}{2A_o R^2 C^2}$$

2) Error caused by d. c. offset V_{os} and the bias current I_B :

$$\triangle e_o = \frac{1}{RC} \int V_{os} \, dt + \frac{1}{C} \int I_b \, dt + V_{os} .$$

3) Error caused by the finite bandwidth in case of a step input E / s :

$$\triangle e_o = E \left(\frac{\tau_o}{RC} + e^{-t/\tau_o} \cdot \frac{\tau_o}{RC} \right)$$

Where the open - loop frequency response is approximated by a single pole at 1 / τo.

Thus, in general one can group the integrator errors in three groups : (1) Constant errors. (2) errors proportional to the first power of t and (3) errors proportional to the second power of t.

In the square - root circuit the net error at the output of the second integrator is due to the second integrators error, and the integrated error of the first integrator. The error in V_2 can be considered to have four complements ; (1) constant error (2) error component proportional to t, (3) proportional to t^2 and (4) proportional to t^3 .

The zero adjustment as shown in the figure (Fig. 5.10) compensates for the constant error component, the fullscale adjustment compensates for the error component of V_2 proportional to t^2 (Fig. 5.10) and the half scale adjustment (at 50 % of rated rms input) as shown in the figure Fig. 5.10 for the error component proportional to the first power of time· It should be noted that the perfect compensation is difficult in practice because these error components are not entirely independent.

5.4.3) THE SQUARE - ROOTER PERFORMANCE

The square - rooter was fabricated and tested. The three adjustments were introduced and carried out. The desired performance was possible only after the proper adjustments. The square - rooter possessed an input dynamic range of 15 mv to 6 v, i.e. 400 '1, with an accuracy of 0.3 %. The d.c. input - output characteristic is shown in the figure (Fig. 5.11)

It must be noted that the dynamic range of the square - rooter limits basically the range of measurement and it has nothing to do with the crest factor specification of RMS measurements.

5.5) THE RMS VOLTMETER

A complete digital rms voltmeter was developed by the Author while in Russia (Ref. 6) by using the described squarer, the averager and a feed back type square - rooter. The suitable input amplifier and a standard pulse width measurig circuitry were also incorporated. The voltmeter had a d. c. measurement span of 0.3 V to 10 V : (extendable to 15 V i.e (50 : 1) using $3\frac{1}{2}$ digit display) ; and a.c. measurement span of 0.3 to 18 V for a 2 kHz sinusoidal signal. The error was not more than ± 0.5 % (Ref. 6). The D.C. error characteristic is plotted in the figure (Fig. 5.12)

5.6) CONCLUSIONS

The theoritical and experimental work has resulted in the successful development of RMS detector circuitry for RMS measurement on high crest factor-signals of low and medium frequency. The squarer was designed specially for high crest factor signals. The filtered squarer output was a measure of the mean square value for signals with crest factor as high as 20. The development of a square - rooting A / D converter resulted in the successful application of the simple direct rms cmputation technique in the digital rms measurement.

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1) Deo P. V., Melnikov A. G.: 'The method of Reducing the drift of d.c. differential amplifiers' Scinetific Notes : No. 8 1974, Baku (USSR) p. 101, (Russian)

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4) Deo P. V., Melnikov A. G. 'Function Generator for RMS Voltmeter' Izv. VUZ. priborostroyeniya (Russian) Vol. No. 5, 1975 p. 72 (Russian)

5) Tobey, Graeme, Huelsman, 'Operational Amplifiers' Design and Applications' McGraw Hill Kogakusha Ltd, 1971 p - 213 - 217.

6) Aliyev T. M., Melnikov A. G., Deo P. V. 'Digital Voltmeter of Effective valves' IZV. VUZ. priborostroyeniya Vol. XIX NO. 11, 1976. p. 26. (Russian)

TABLE T. 5.1

Approximation Segment VS RMS Measurement Error

ϵ_r %	0.1	0.2	0.3	0.4	0.5
n	16	11	9	8	7

TABLE T. 5.2

Squarer Design

i	V_{Bi-1}	R_{2i} kohm	S_{is}	R_{4i} kohm	s_i	R_o / R_{xi}	R_{xi} kohm
1	0	-	0.0137	1.5	0.1	0.137	15.4
2	0.1792	400	0.0482	5.0	0.333	0.145	13.8
3	0.5325	169	0.0711	5.0	0.386	0.184	10.8
4	1.0807	86	0.1071	5.8	0.386	0.277	7.2
5	1.8209	49.5	0.1431	6.8	0.454	0.320	6.25
6	2.7530	32.6	0.1791	8.2	0.548	0.328	6.1
7	3.8773	23.2	0.2151	10.0	0.667	0.324	6.2
8	5.1340	17.6	0.2511	15	1.0	0.251	8.0
9	6.7015	13.4	0.2871	51	3.4	0.085	23.7

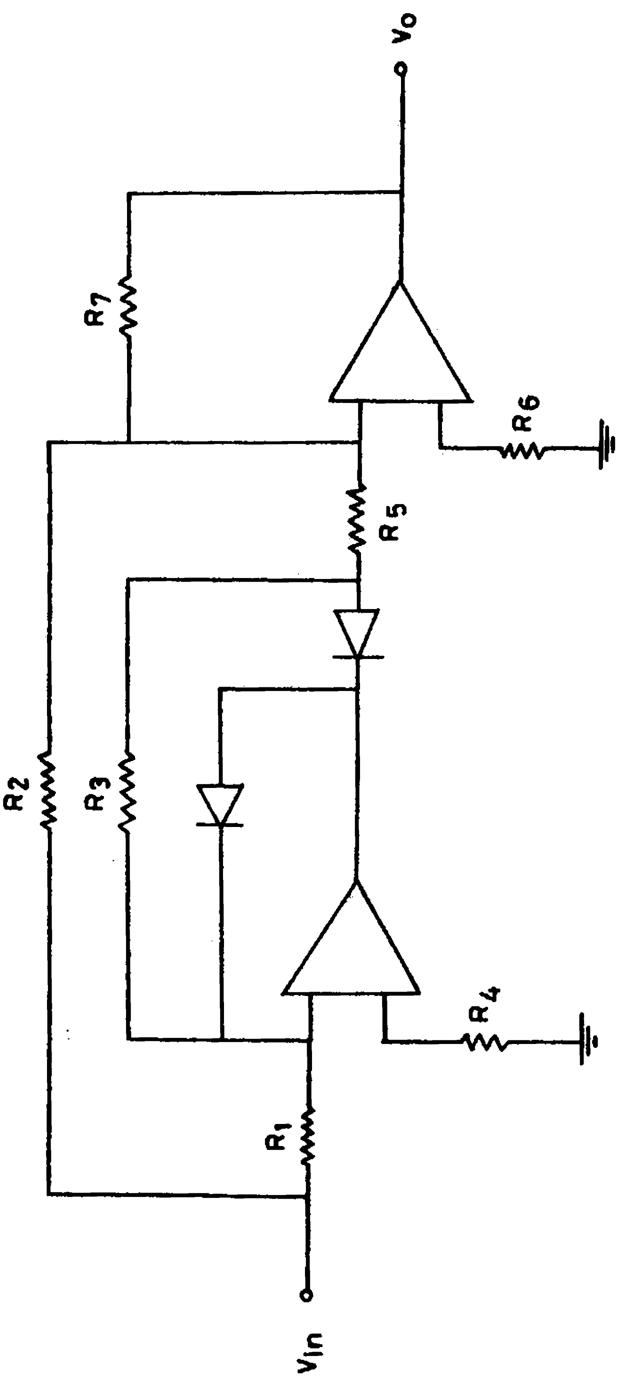


FIG.5-1 ABSOLUTE VALUE DETECTOR

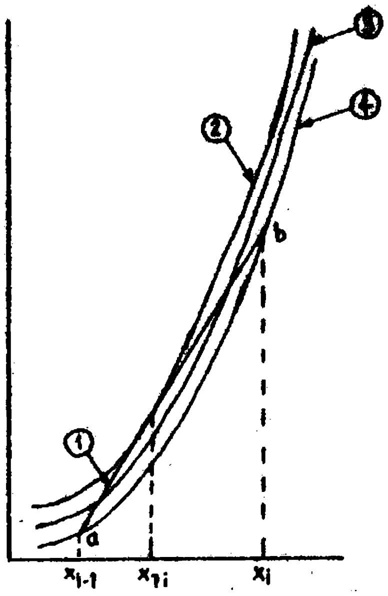


FIG. 5.2 i'TH SEGMENT

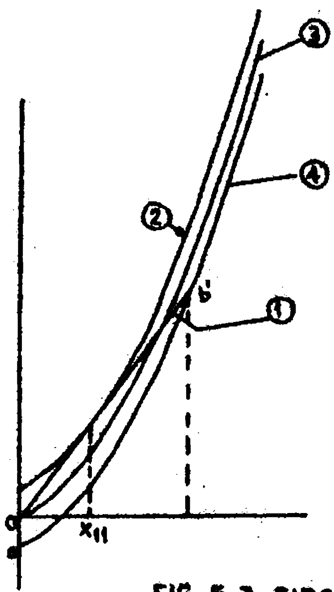


FIG. 5.3 FIRST SEGMENT

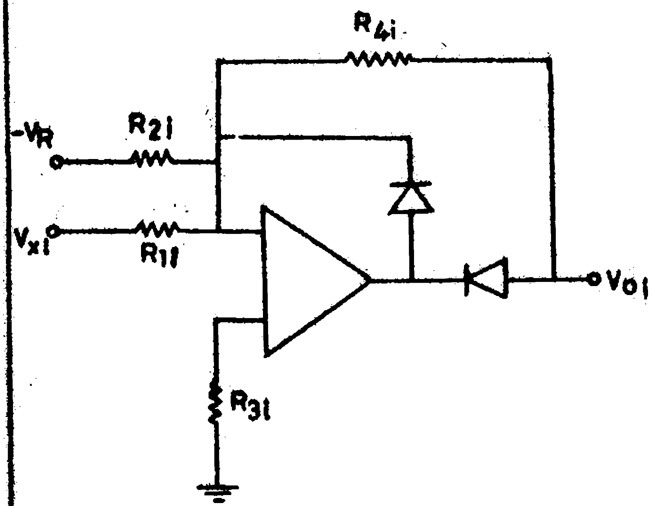


FIG. 5·4 [A]

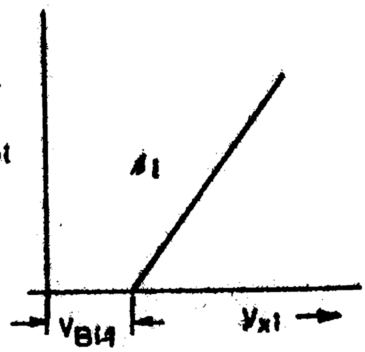


FIG. 5·4 [B]

PRECISION FEEDBACK LIMITER

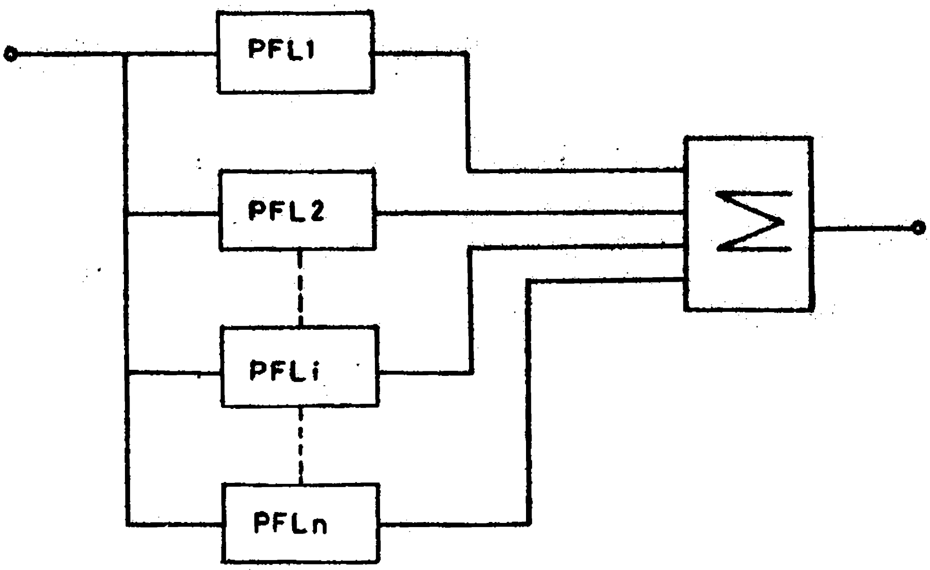


FIG. 5·5 SQ. FUNCTION APPROXIMATOR

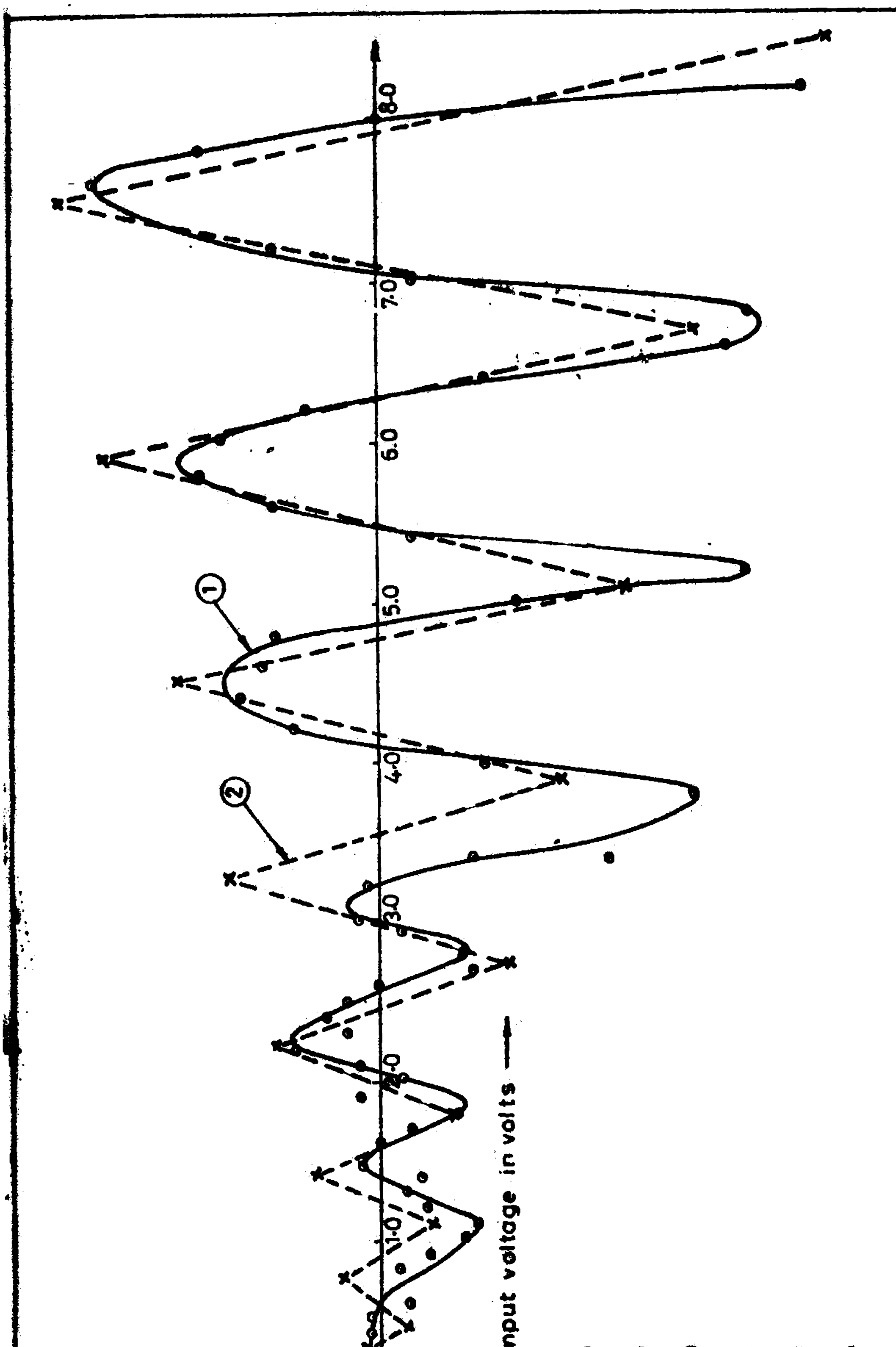


FIG. 5·6 SQUARER DC ERROR CHARACTERISTICS

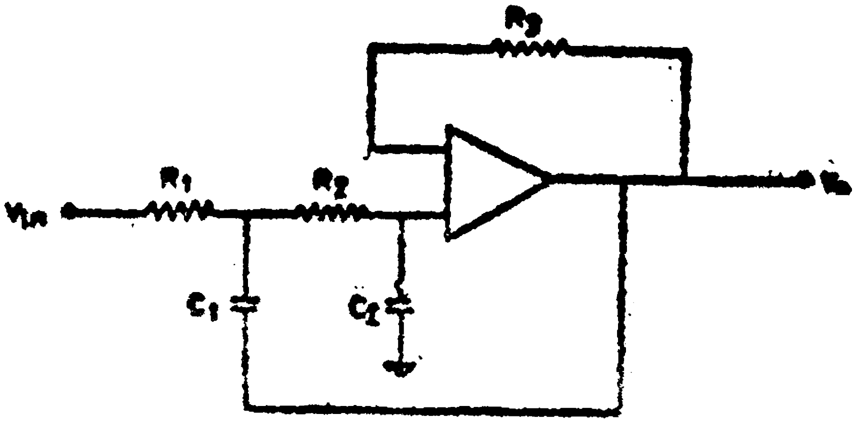


FIG. 5.7 ACTIVE LOW PASS FILTER

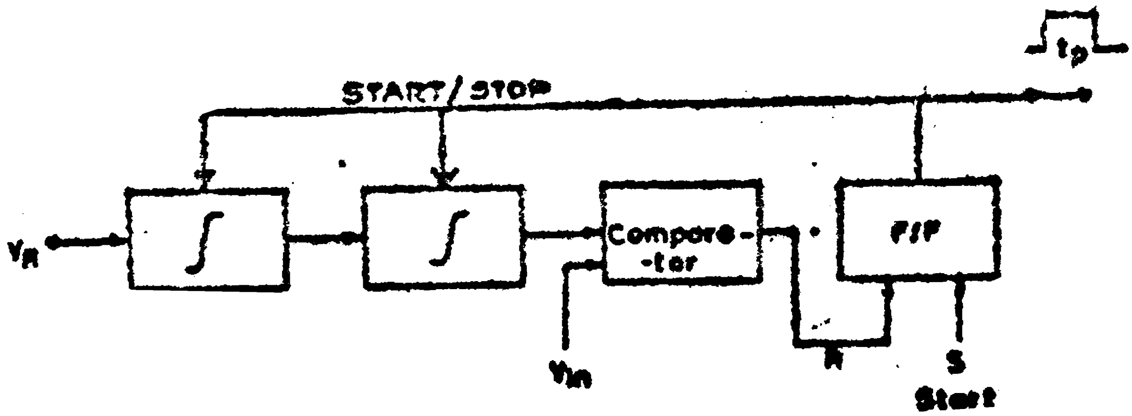


FIG. 5.8 A SQUARE ROOT A/D CONVERTER

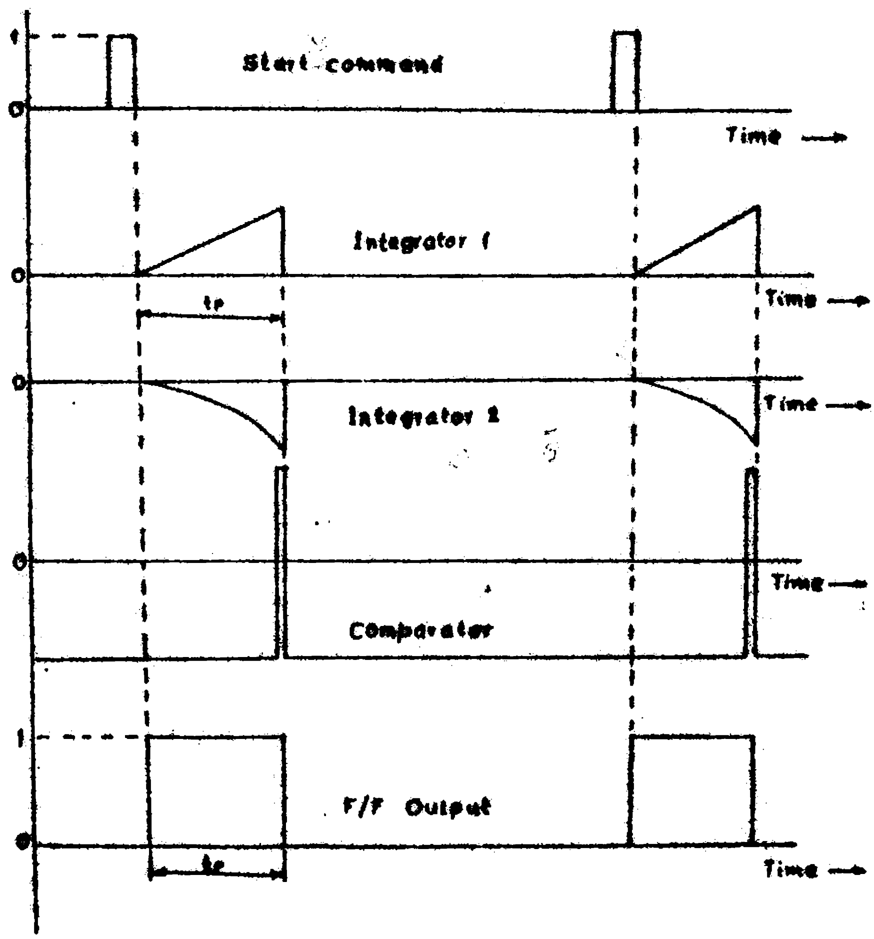
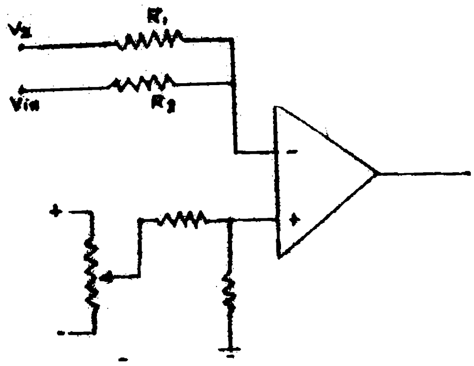
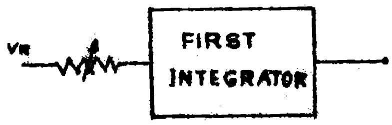


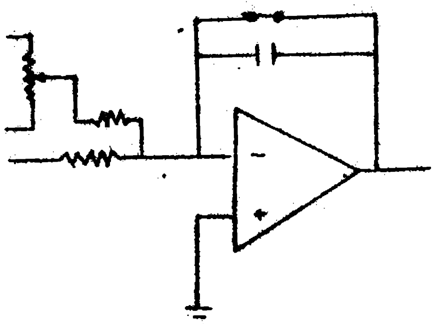
FIG. 5.5 TIME DIAGRAM OF THE SQUARE ROOT CIRCUIT



[A] ZERO ADJUSTMENT ON COMPARATOR



[B] FULL SCALE ADJUSTMENT



[C] HALF SCALE ADJUSTMENT ON THE SECOND INTEGRATOR

FIG 5·10 SQUARE-ROOTER ADJUSTMENTS

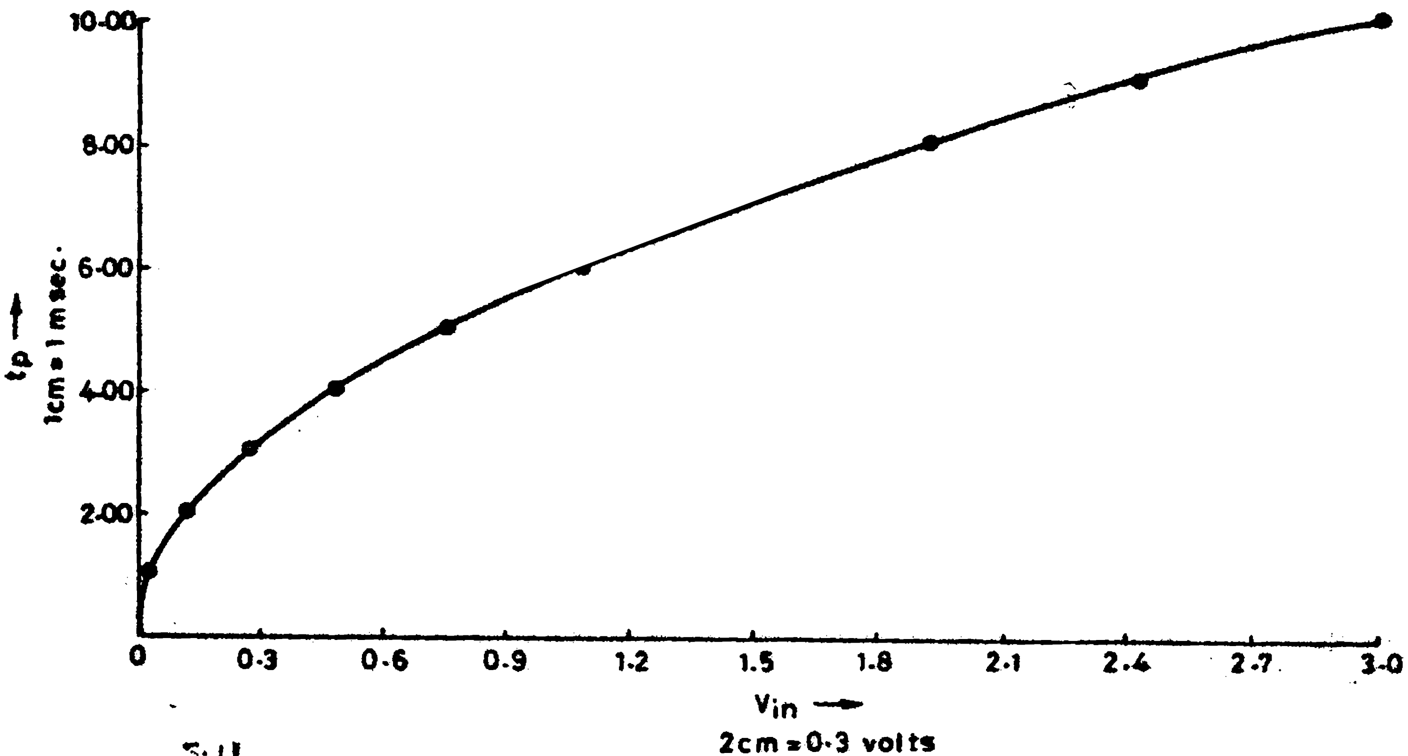
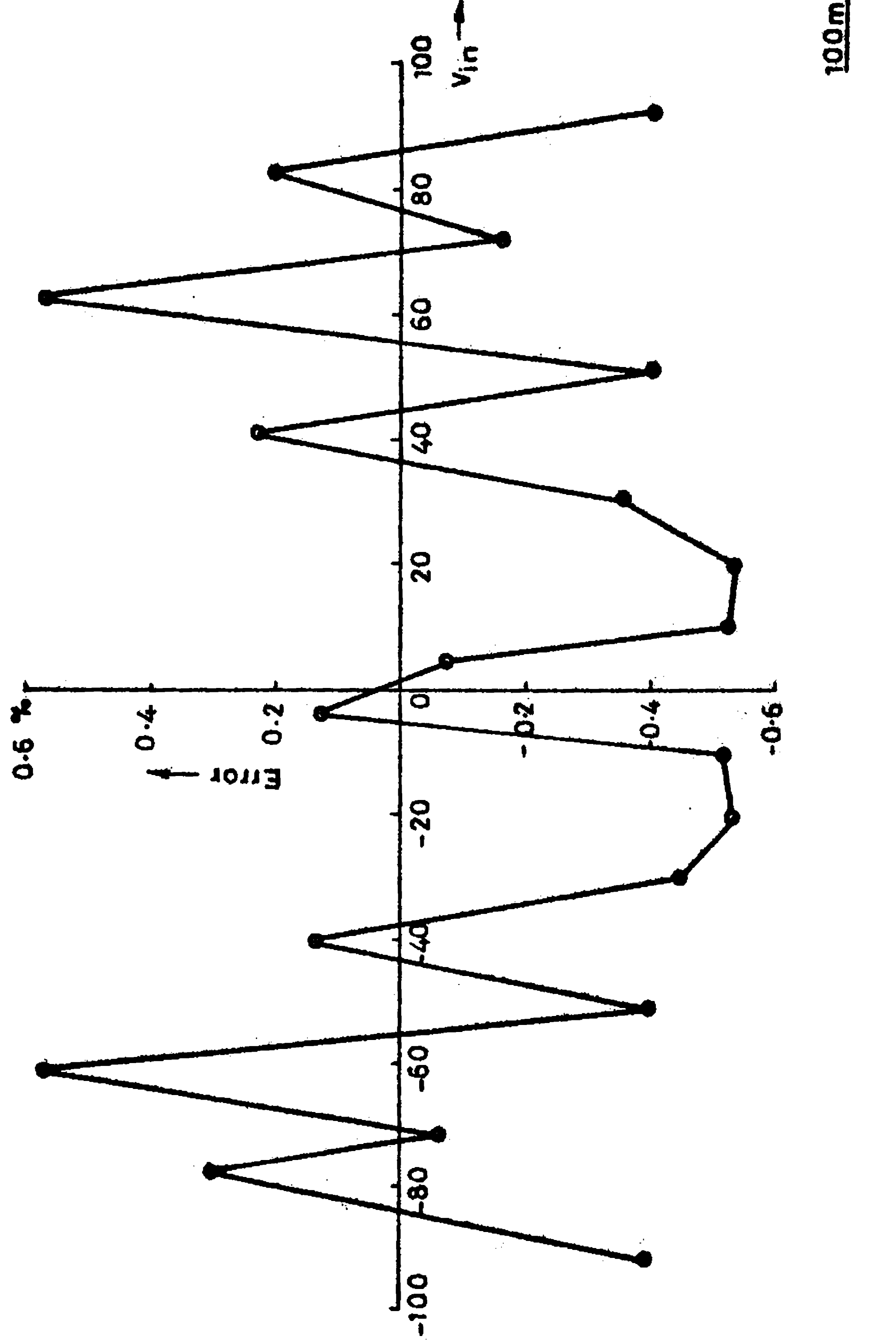


FIG 5.11 INPUT/OUTPUT CHARACTERISTICS OF THE SQUARE ROOTER



FIG·5·12 VOLT METER DC TEST

CHAPTER - SIX

Discussions and Conclusions

6.1) THE TOPIC

At first sight the topic may not be attractive because of its peculiar nature. At present a conventional a.c. instrument is used in majority of a.c. measurement. The study of applications of rms measurement has however revealed the fact that an apprcpriate rms measurement is a must for accurate a.c. measurement on signals of varying complexity.

The topic is interesting on the other hand, because of its following aspects : a) the rms detector is a nonlinear device consisting of linear and non-linear devices. b) the topic combines in it the typical problems from other fields like statistics, circuit theory, electronic circuits and devices.

6.2) THE WORK DONE

The work done can be summarized as follows : 1) Theoritical work consists of four parts: a) Analysis of problems in rms measurement. b) Comparative study of devices and techniques used in rms measurement c) Design of the squarer for amplitude - error - free (waveform - error - free) rms measurement and d) Evaluating the rms detector as regards its applications.

2) Practical work consists of (1) fabrication and testing of the squarer, square-rooter and averager blocks of a direct rms computing method, (2) construction and testing of 10 V voltmeter using the developed rms detector, the suitable input device and a digital pulse width measuring device.

6.3) HIGHLIGHTS

1) A systematic analysis of errors in rms measurement is presented for he first time. The novel concept of ' idealized rms detector ' has permitted the

generalization of characteristics and performance of rms measurement (Sec. 2.5)

2) The importance of the amplitude error (waveform error) is revealed (Sect. 2.10)

3) The relationship between the squarer error and the rms measurement error is investigated (Sect. 3.5.)

4) It is revealed that the constant squarer error results in hyperbolic rms measurement error characterietic (Fig. 3 6, Sect. 3.7)

5) The squarer error is so shaped (Fig. 5.6) that the resulting rms measurement error is inpependent of the signal level. Thus a flat error characteristic is achieved (Fig. 5.12). Thus the main hurdle in the rms measurement on high crest - factor signals is removed.

6) A wave - form - error - free rms detector is developed which can be used for measurement on signals of any complexity - high CF signals, sinusoidal signals, gaussian and non - gaussian random signals - provided the significant highest frequency component is within limits.

7) A successful development of a square - rooting A / D converter (Sect. 5.4) permits hybrid rms measurement, consisting of analog computation and digital display.

8) The effects of unconventional feedback in ' implicit RMS computation ' are investigated (Sect. 3.4) and its limitations are revealed.

9) A new criterion is proposed for digital averager. It provides a lot of flexibility in selecting the sampling signal (Sect. 4.4.1)

10) The original work presented in this thesis has to its credit three publications. (Ref. 1,2,3)

6.4) SUGGESTIONS FOR FUTURE DEVELOPMENTS

A number of future research projects can be based on the investigations presented in this thesis. A list of typical projects is given below.

1) Development of an appropriate averager for implicit RMS computation.

2) Microprocessor - based rms detector utlizing the proposed averaging criterion.

3) Development of sampling criteria for averaging of nonperiodic signals.

4) Fabrication of the developed rms detector using LSI techniques.

5) Hybrid systems to yield better accuracies.

6) Highly precise rms measurement with flat relative error characteristics.

6.5) CONCLUSION

In conclusion, the theoritical and the practical work presented in this thesis has solved the problem of rms measurement on high crest - factor signals by providing a waveform - error - free rms detector design. The detector is useful for testing, measurement and control of various properties of signals, systems and devices in the fields of electronic communication, acoustics, statistics, and automatic process control.

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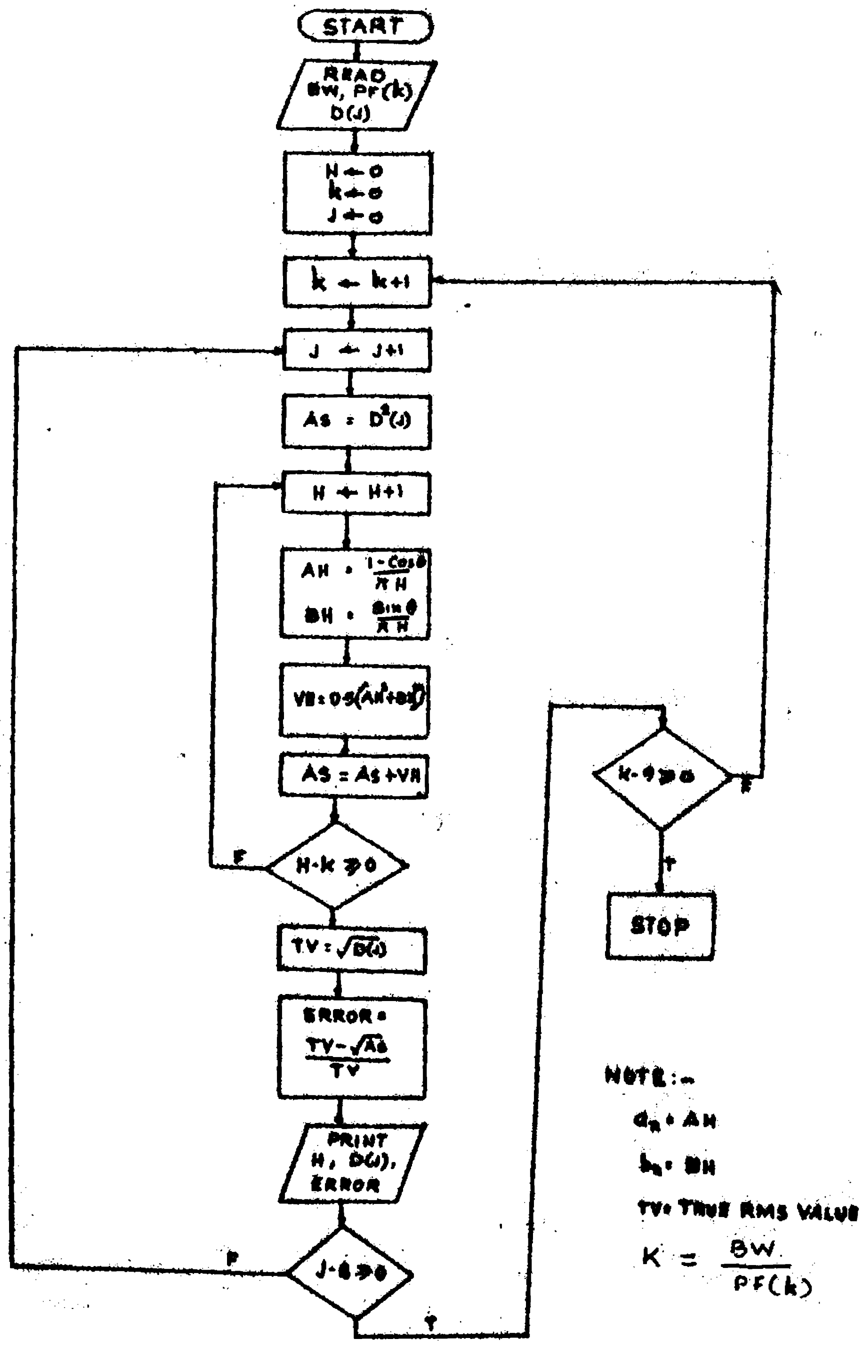
APPENDIX I

FLOW CHART FOR FREQUENCY ERROR



APPENDIX II

Fourier Coefficients of Complex Periodic Waveforms

a) TRIANGULAR WAVEFORMS

$$f(t) = 1 + (4t/T); \qquad -\frac{1}{2}T \leqslant t \leqslant o$$

$$f(t) = 1 - (4t/T); \qquad o \leqslant t \leqslant T/2$$

The average value $a_o = o$

Since the function is even $b_n = o$

$$a_n = \frac{2}{T}\int_{-T/2}^{+T/2} f(t)\ \mathrm{Cos}\ [2\pi nt/T]\ dt.$$

$$a_n = \frac{2}{T}\int_{-T/2}^{+T/2} \mathrm{Cos}\ [2\pi nt/T]\ dt + \frac{2}{T}\int_{-T/2}^{o}\frac{4t}{T}\ \mathrm{Cos}\ (2\pi nt/T)\ dt$$

$$-\frac{2}{T}\int_{o}^{T/2}\frac{4t}{T}\ \mathrm{Cos}\ (2\pi nt/T)\ dt$$

$$a_n = \frac{8}{T^2}\int_{T/2}^{o} (-t)\ \mathrm{Cos}\ [2\pi n(-t)/T]\ d(-t)$$

$$-\frac{8}{T^2}\int_{o}^{T/2} t\ \mathrm{Cos}\ (2\pi nt/T)\ dt$$

$$a_n = -\frac{16}{T^2}\int_{o}^{T/2} t\ \mathrm{Cos}\ (2\pi nt/T)\ dt$$

$$a_n = \frac{4}{\pi^2 n^2} (1 - \cos \pi n)$$

b) SAWTOOTH WAVEFORM

$$f(t) = 2t / T; \qquad -\frac{T}{2} < t < +\frac{T}{2}$$

$$f(t + T) = f(t)$$

The function is odd, therefore $a_n = o$

The average value $a_o = o$

$$b_n = \frac{2}{T} \int_{-T/2}^{+T/2} f(t) \sin [2 \pi n t / T] \, dt$$

$$b_n = \frac{2}{T} \int_{-T/2}^{+T/2} (2t/T) \sin [2 \pi n t / T] \, dt$$

$$b_n = \frac{2}{\pi n T} \Big[\left(-t \cdot \cos [2 \pi n t / T] \right) \Big|_{-T/2}^{+T/2}$$

$$+ \int_{-T/2}^{+T/2} \cos (2 n \pi t / T) \, dt \Big]$$

$$b_n = \frac{2}{\pi n T} \left[-2 \cdot \frac{T}{2} \cos \pi n \right]$$

$$b_n = \frac{-2}{\pi n} \cos \pi n.$$

c) PULSE - TRAIN

$$f(t) = 1; \qquad o < t < t_p$$

$$f(t) = o; \qquad t_p < t < T$$

$$f(t+T) = f(t)$$

$$a_o = \frac{1}{T} \int_o^T f(t)\, dt = \frac{t_p}{T} = D$$

$$a_o = D$$

$$a_n = \frac{2}{T} \int_o^{t_p} f(t) \operatorname{Sin} [2 \pi n t / T]\, dt$$

$$= \frac{2}{T} \int_o^{t_p} \operatorname{Sin} [2 \pi n t / T]\, dt$$

$$= \left[\frac{1}{\pi n} \operatorname{Cos} (2 \pi n t / T) \right]_o^{t_p}$$

$$a_n = \frac{1}{\pi n} [1 - \operatorname{Cos} (2 \pi n t_p / T)]$$

$$a_n \frac{1}{\pi n} [1 - \operatorname{Cos} (2 \pi N D)]$$

$$b_n = \frac{2}{T} \int_o^{t_p} f(t) \operatorname{Cos} (2 \pi n t / T)\, dt$$

$$= \frac{2}{T} \cdot \left[\frac{\operatorname{Sin} (2 \pi n t / T)}{2 \pi n / T} \right]_o^{t_p}$$

$$b_n = \frac{1}{\pi n} \operatorname{Sin} 2\pi n D$$

d) RECTIFIED SINUSOIDAL WAVEFORM

$$f(t) = \cos \frac{\pi t}{T} \qquad - T/2 \leqslant t \leqslant T/2$$

$$f(t + T) = f(t)$$

$$a_o = \frac{1}{T} \int_{-T/2}^{T/2} \cos(\pi t / T)\, dt$$

$$a_o = 2/\pi$$

The function is even therefore $b_n = o$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} \cos \frac{\pi t}{T} \cos \frac{2 \pi n t}{T}\, dt$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} 0.5 \left[\cos(2n+1) \pi t/T + \cos(2n-1) \pi t/T \right] dt$$

$$a_n = \frac{1}{T} \left[\frac{T}{\pi (2n+1)} \sin (2n+1) \pi t/T \right.$$

$$\left. + \frac{T}{(2n-1)\pi} \sin (2n-1) \pi t/T \right]_{-T/2}^{+T/2}$$

$$a_n = \frac{2}{\pi} \left[\frac{1}{(2n+1)} \sin \left[\frac{\pi}{2} (2n+1) \right] + \frac{1}{2n-1} \sin \left[\frac{\pi}{2} (2n-1) \right] \right]$$

APPENDIX III

Frequency Error in RMS Measurement on Random Signals

1) BANDPASS WHITE NOISE

$$G_x(f) = a; \qquad \left(fo - \frac{B}{2}\right) \leqslant f \leqslant \left(fo + \frac{B}{2}\right)$$

$$G_x(f) = o; \qquad \text{otherwise.}$$

True Mean square value $= a \cdot B$

Measured MS value $= a \left(fc - fo + \frac{B}{2}\right)$

$$\text{Relative Error in MS} = \frac{a\left[(B/2) - (fc - fo)\right]}{aB}$$

$$\delta_{sf} = \left[0.5 - \frac{1}{B}(fc - fo)\right]$$

2) EXPONENTIAL

$$G_x(f) = \frac{4a}{a^2 + 4\pi^2 f^2}$$

$$\text{Error in MS value} = \int_{fc}^{\infty} \frac{4a}{a^2 + 4\pi^2 f^2}\, df = \Delta_s$$

Let $t = \dfrac{2\pi f}{a}$, $df = \dfrac{a}{2\pi}\, dt$

$$\Delta_s = \frac{2}{\pi} \int_{fc/a}^{\infty} [1/(1+t^2)]\, dt$$

$$= \frac{2}{\pi} [\tan^{-1} t]_{(2\pi fc/a)}^{\infty}$$

$$\Delta_s = 1 - \frac{2}{\pi} \tan^{-1} \frac{2\pi fc}{a}$$

3) EXPONENTIAL COSINE

$$G_x(f) = 2a \left[\frac{1}{a^2 + 4\pi^2 (f+fo)^2} + 1/(a^2 + 4\pi^2 (f-fo)^2 \right]$$

$$\text{Error (Absolute) in MS value} = \int_{fc}^{\infty} G_x(f)\, df = \Delta_s$$

$$\Delta_s = \frac{1}{\pi} [\tan^{-1} [2\pi (fo+f)/a] + \tan^{-1} [2\pi (f-fo)/a]]_{fc}^{\infty}$$

$$\Delta_s = 1 - \frac{1}{\pi} [\tan^{-1} [2\pi (fc+fo)/a] + \tan^{-1} [2\pi (fc-fo)/a]]$$

$$\text{True MS value} = \int_{o}^{\infty} G_x(f)\, df = 1$$

The relative error

$$\delta_{sf} = 1 - \frac{1}{\pi} [\tan^{-1} [2\pi (fc+fo)/a] + \tan^{-1} [2\pi (fc-fo)/a]]$$

APPENDIX IV

Amplitude Error in RMS Measurement on Complex Periodic Signals

1) TRIANGULAR

$$f(t) = A\left(1 + \frac{4t}{T}\right); \qquad -0.5\,T \leqslant t \leqslant o$$

$$f(t) = A\left(1 - \frac{4t}{T}\right); \qquad o \leqslant t \leqslant 0.5\,T$$

$$\text{True MS} = \frac{4\,A^2}{T} \int_{-T/4}^{o} \left(1 + \frac{4t}{T}\right)^2 dt$$

$$= \frac{4A^2}{T} \left[t + \frac{8}{T} \cdot \frac{t^2}{2} + \frac{16}{T^2} \cdot \frac{t^3}{3} \right]_{-T/4}^{o}$$

$$= 1/3$$

$$\text{Error} = \frac{4A^2}{T} \int_{-T/4}^{t_1 - (T/4)} \left(1 + \frac{4t}{T}\right)^2 dt = \frac{64}{3} \cdot \frac{t_1^3}{T^3}$$

where t_1 is the time, related to V_L of the detector by the relationship.

$$f(t_1 - T/4) = V_L$$

$$\text{i. e.} \quad t_1 = \frac{V_L\,T}{4\,A}$$

Substituting in the expression for 'Error'

$$\text{Error} = \frac{64}{3} \cdot \frac{1}{T^3} \cdot \frac{V_L^3 \, T^3}{64 \, A^3} = \frac{V_L^3}{3 \, A^3}$$

$$\text{Let} \quad a = \frac{V_L}{A}$$

$$\text{Error} = a^3 \,/\, 3$$

The relative error $\delta_{sa} = a^3$

2) SAW - TOOTH

$$f(t) = \frac{2 A t}{T} \qquad - T/2 < o < T/2$$

$$f(t + T) = t(t)$$

$$\text{True MS value} = \frac{2}{T} \int_{o}^{T/2} f^2(t) \, dt$$

$$= \frac{1}{3}$$

$$\text{Error} = \frac{2}{T} \int_{o}^{t_1} f^2(t) \, dt = \frac{8 \, t_1^3}{3 \, T^3}$$

where t_1 is given by

$$f(t_1) = \frac{2At_1}{T} = V_L$$

$$\text{i. e.} \quad t_1 = \frac{V_L \, T}{2 \, A}$$

Substituting in the expression for ' Error '

$$\text{Error} = \frac{8}{3\,T^3} \times \frac{V_L^3\,T^3}{8\,A^3}$$

$$= \frac{1}{3}\,\frac{V_L^3}{A^3} = \frac{1}{3}\,a^3$$

The relative error $\varepsilon_{sa} = a^3$

3) DOUBLET

$$f(t) = V_a \qquad o < t < t_p$$

$$f(t) = V_b \qquad t_p < t < T$$

$$\text{True MS} = [\,V_a^2\,t_p + V_b^2\,(T - t_p)\,] \cdot \frac{1}{T}$$

with $D = \dfrac{t_p}{T}$ and $V_a\,D = V_b\,(1 - D)$

$$\text{True MS} = V_a^2\,\left(D + \frac{D^2}{1 - D}\right) = V_a^2\,\frac{D}{1 - D}$$

For $V_b < V_L$; the measured MS $= V_a^2\,D$

$$\varepsilon_{sa} = \frac{V_a^2\,[\,D/(1 - D)\,] - V_a^2 \cdot D}{V_a^2\;D/(1 - D)}$$

$$\varepsilon_{sa} = D$$

4) RECTIFIED SINUSOIDAL

$$f(t) = A\;\mathrm{Cos}\,(\pi\,t/T) \qquad -T/2 < t < T/2$$

$$f(t + T) = f(t)$$

$$\text{True MS} = \frac{1}{T} \int_{-0.5\,T}^{+0.5\,T} A^2 \cos^2 (\pi t / T)\, dt = \frac{A^2}{2}$$

$$\text{Error} = \frac{2}{T} \int_{-0.5\,T}^{-t_1} A^2 \cos^2 (\pi t / T)\, dt$$

$$= \frac{2}{T} \cdot \frac{1}{2} \int_{-T/2}^{-t_1} A^2 \,(1 + \cos (2 \pi t / T)\, dt$$

$$= \frac{A^2}{T} \left[t + \frac{T}{2\pi} \sin \frac{2\pi t}{T} \right]_{-T/2}^{-t_1}$$

$$\text{Error} = \frac{A^2}{T} \left[- t_1 + (T / 2) - \frac{T}{2\pi} \sin (2 \pi t_1 / T) \right]$$

t_1 is related to 'a' by

$$\cos \frac{\pi t_1}{T} = a$$

$$\text{i. e.} \quad t_1 = \frac{T}{\pi} \cos^{-1} a$$

Substituting for t_1 ,

$$\text{Error} = \frac{A^2}{T} \left[\frac{T}{2} - \frac{T}{\pi} \cos^{-1} a - \frac{T}{2\pi} \sin (2 \cos^{-1} a) \right]$$

The relative error ,

$$\varepsilon_{sa} = \frac{2}{\pi} \left[\frac{\pi}{2} - \cos^{-1} a - \frac{1}{2} \sin (2 \cos^{-1} a) \right]$$

$$\varepsilon_{sa} = \frac{2}{\pi} \left[\sin^{-1} a - a \sqrt{(1 - a^2)} \right]$$

APPENDIX V

R. C. Low-Pass Filter in Implicit RMS Measurement

1) IMPLICIT RMS COMPUTATION ON SINUSOIDAL SIGNAL

Let $v_{in} = \sin \omega t$ the equation 3.5 [P. 25]

$$\overline{V_{os}} = \text{Ave}\ \left[\left(e^{-(2t/RC)} \cdot \int (2/RC)\, e^{2t/RC} \cdot v_{in}^{2}\, dt \right)^{0.5} \right]$$

$$V_{os}^{2} = e^{-2t/RC} \int (2/RC) \cdot e^{2t/RC} \cdot \sin^{2} \omega t\ dt$$

$$= e^{-2t/RC} \int (2/RC) \cdot e^{2t/RC} (1 - \cos 2\omega t) \cdot \frac{1}{2}\ dt$$

$$V_{os}^{2} = \frac{1}{2} - e^{-2t/RC} \cdot (1/RC) \cdot I \qquad \ldots \qquad A$$

where $I = \int e^{2t/RC} \cos 2\omega t\ dt$

Let $u = \cos 2\omega t$ & $dv = \frac{2}{RC} \cdot e^{(2t/RC)} \cdot dt_{1}$

$$I = \int u\, dv = uv - \int v\, du$$

$$= (RC/2)\, e^{2t/RC} \cdot \cos 2\omega t + I_{1} \qquad B$$

where $I_{1} = \omega RC \int e^{2t/\tau} \sin 2\omega t\ dt$

with $u_1 = \sin 2\omega t$ & $dv_1 = e^{2t/RC} \cdot dt$

$$I_1 = \omega RC \left[\frac{RC}{2} e^{2t/RC} \sin 2\omega t - \omega CR\, I \right]$$

where I is given by the equation. A

Substituting this value of I_1 in the equation B

$$I = (RC/2)\, e^{2t/RC} \cos 2\omega t$$

$$+ \omega RC \left[(RC/2) \cdot e^{2t/RC} \sin 2\omega t - \omega CR\, I \right]$$

Rearranging and solving for I

$$I = \frac{RC}{2(1+\omega^2 R^2 C^2)} e^{2t/RC} (\cos 2\omega t + \omega RC \sin 2\omega t)$$

Substituting for I in equation A and simplifying ;

$$V_{os}^2 = \frac{1}{2} - \frac{1}{2} \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} \sin (2\omega t + \phi)$$

where $\phi = \tan^{-1} (1/\omega CR)$

Now,

$$V_{os} = \frac{1}{\sqrt{2}} \left[1 - \frac{1}{(1+\omega^2 R^2 C^2)^{0.5}} \times \sin (2\omega t + \phi) \right]^{1/2}$$

Expanding and neglecting higher powers of $(1/1 + \omega^2 R^2 C^2)$]

$$\overline{V_{os}} = \frac{1}{\sqrt{2}} \left[1 - \frac{1}{16(1+\omega^2 R^2 C^2)} \right]$$

2) IMPLICIT RMS COMPUTATION ON RAMP SIGNAL

Let vin = f (t) in equation 3.5 on page 25

$$\overline{V_{os}} = \text{Ave}\left[\left(e^{-2t/RC} \cdot \int (2/RC)\, e^{2t/RC} f^2(t)\, dt \right)^{0.5} \right]$$

$$V_{os}^2 = (2/RC)\, e^{-2t/RC} \cdot I \qquad \text{(a)}$$

where $I = \int e^{2t/RC} f^2(t)\, dt$

Let $u = f^2(t)$ and $dv = e^{2t/RC}\, dt$

$$I = \int u\,dv = uv - \int v\,du$$

$$= \frac{RC}{2} e^{2t/RC} \cdot f^2(t) - \int (RC/2)\, e^{2t/RC} \cdot 2f(t) \cdot f^1(t)\, dt$$

$f_1(t) = 1/T$ for a ramp with unit peak amplitude

$$I = \frac{RC}{2} e^{2t/RC} f^2(t) - \frac{RC}{T} \int e^{2t/RC} f(t)\, dt$$

$$= \frac{RC}{2} e^{2t/RC} f^2(t) - \frac{RC}{T} \left[\frac{RC}{2} e^{2t/RC} \cdot f(t) - \int \frac{RC}{2} e^{2t/RC} \cdot \frac{1}{T}\, dt \right]$$

$$I = \frac{RC}{2} e^{2t/RC} f^2(t) - \frac{R^2C^2}{2T} e^{2t/RC} f(t) + \frac{R^2C^2}{2T^2} \cdot \frac{RC}{2} e^{2t/RC}$$

Substituting this value of I in (a)

$$V_{os}^{2} = (2/RC)\ e^{-2t/RC} \cdot I$$

$$= f^{2}(t) - \frac{RC}{T} f(t) + \frac{R^{2}C^{2}}{2T^{2}}$$

$$V_{os}^{2} = \frac{f^{2}(t) + [f^{2}(t) - 2\frac{RC}{T} f(t) + R^{2}C^{2}/T^{2})]}{2}$$

$$V_{os}^{2} = \frac{1}{2}\ [f^{2}(t) + (f(t) - \frac{RC}{T})^{2}]$$

$$\overline{V_{os}} = [\{\frac{1}{2}\ [f^{2}(t) + (f(t) - \frac{RC}{T})^{2}]\}^{0.5}]_{AVE}$$

APPENDIX VI

Design of ith Segment

GIVEN : In the figure (Fig. 5.2)

Curve 1 : $y = a_i x + b_i$

Curve 2 : $y_2 = x^2 + 2Cx$

Curve 3 : $y = x^2$

Curve 4 : $y_1 = x^2 - 2Cx$

The curve 1 is a straight line tangent to the curve 2 at the point (x_{1i}, y_{1i}) and it intersects the curve 4 at the two points a and b i. e. (x_{i-1}, y_{i-1}) and (x_i, y_i)

TO DETERMINE :

(1) the constants a_i and b_i ,

(2) the length of the segment

and (3) x_i , the abscissa of the end point.

SOLUTION

(I) a_i = slope of the straight line

= slope of the curve 2 at $x = x_{1i}$

$$a_i = 2x_{1i} + 2C$$

Equating $(a_i x + b_i)$ and $(x^2 + 2Cx)$ for $x = x_{1i}$,

$$b_i = x_{1i}^2 + 2Cx_{1i} - x_{1i}(2x_{1i} + 2C)$$

$$b_i = -x_{1i}^2 \qquad \ldots\ldots\ldots$$

(II) Equating $(a_i x + b_i)$ and $(x^2 - 2Cx)$ at $x = x_i$,

$$x_i^2 - 2Cx_i = x_i(2x_{1i} + 2C) - x_{1i}^2$$

Rearranging,

$$x_i^2 - 2x_i(2C + x_{1i}) + x_{1i}^2 = o$$

Solving for x_i,

$$x_i = 2C + x_{1i} + \sqrt{(4C^2 + 4CX_{1i})}$$

Equating $(a_i x + b_i)$ and $(x^2 - 2cx)$ at $x = x_{i-1}$, and solving for x_{i-1},

$$x_{i-1} = 2C + x_{1i} - \sqrt{(4C^2 + 4Cx_{1i})}$$

Now the segment length is given by

$$L = x_i - x_{i-1}$$

$$L = 4\sqrt{(C^2 + Cx_{1i})}$$

TO DETERMINE x_{1i} :

Equating $(a_i x + b_i)$ and $(x^2 - 2Cx)$ at $x = x_{i-1}$ and solving for x_{1i}

$$x_{1i}^2 - 2x_{1i}x_{i-1} + (x_{i-1}^2 - 4x_{i-1}C) = 0$$

$$x_{1i} = x_{i-1} + 2\sqrt{[Cx_{i-1}]}$$

APPENDIX VII

Design of the first segment

GIVEN : In the figure (Fig. 5.3),

Curve 1 $\quad y = a_1 x$

Curve 2 : $\quad y_2 = x^2 + C_o^2$

Curve 3 : $\quad y = x^2$

Curve 4 : $\quad y_1 = x^2 - C_o^2$

The curve 1 is tangent to the curve 2 at $x = x_{11}$

TO DETERMINE:

(1) the slope a_1

(2) the abscissa of the tangent point x_{11}

and (3) the end point x_1

SOLUTION

(I) $\quad a_1 = $ slope of the curve 2 at $x = x_{11}$

$a_1 = 2x_{11}$

(II) $\quad a_1 x$ equals $(x^2 + C_o^2)$ at $x = x_{11}$,

therefore $\quad 2x_{11} \; x_{11} = x_{11}^2 + C_o^2$

$x_{11} = C_o$

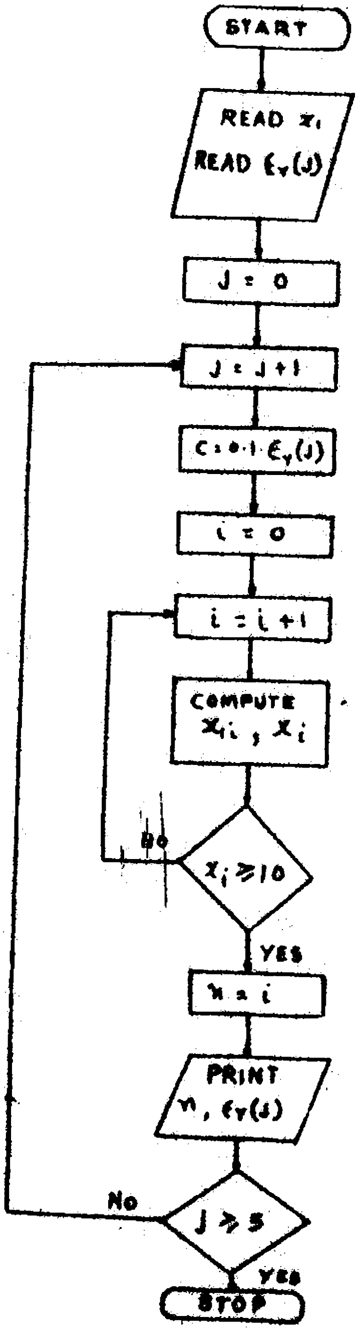
(III) $\quad a_1 x$ equals $(x^2 - C_o^2)$ at $x = x_1$

therefore $\quad 2x_{11} \; x_1 = x_1^2 - C_o^2$

$x_1 = (1 + \sqrt{2})\, C_o$

APPENDIX VIII

FLOW CHART FOR TABLE T 6·1



Data:

$\epsilon_v = 0{\cdot}1, 0{\cdot}2, 0{\cdot}3, 0{\cdot}4, 0{\cdot}5$

$x_1 = 0{\cdot}2240$

Acknowledgement

Many people made this thesis possible. It has taken the present appropriate shape under the able guidance of Dr. C. S. Sridhar, Reader, Dept. of Electronics, University of Cochin. I am greatly indebted to him for his discussions and recommendations, particularly on the theoritical part presented in the thesis.

I am also greatly indebted to Professor and Head of the Department Dr. T. M. Aliyev; Asst. Prof. Dr. A. G. Melnikov ; and other staff members of the Department of Measuring and Computing Techniques, Azerbaijan Institute of Oil and Chemistry, Baku (USSR). The major part of the practical work presented in the thesis was impossible without their keen interest, suggestions and encouragement.

I wish to thank my colleagues and friends at I. I. T. Bombay and at the university of Cochin for their sincere discussions, encouragement and criticism.

I am also grateful to those friends who are responsible for giving the thesis a pleasant appearance.