PROPERTIES OF ANISOTROPIC SUPERCONDUCTORS IN SR MODEL

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1982

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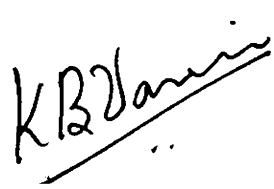
Signature. 

C E R T I F I C A T E

Certified that the work reported in the present thesis is based on the bonafide work done by Mr. K.Balakrishna Warier, under my guidance in the Department of Physics, Cochin University, and has not been included in any other thesis submitted previously for the award of any degree.

Cochin - 22
25th January 1982.

C.Purushothaman,
Supervising Teacher.

D E C L A R A T I O N.

Certified that the work presented in this thesis is based on the original work done by me under the guidance of Dr. C.Purushothaman in the Department of Physics, Cochin University, and has not been included in any other thesis submitted previously for the award of any degree.

Cochin - 22
25th January 1982.

K.Balakrishna Warier.

PREFACE

The subject matter of this report is the work done by the author in the Department of Physics of Cochin University during 1979 - 1981.

The present thesis deals with the theoretical investigations on the effect of anisotropy on various properties of magnetically doped superconductors described by Shiba - Rusinov model.

Chapter 1 is introductory. It contains a brief account of the current status of theory of superconductivity. In chapter 2 we give the formulation of the problem. Chapter 2.1 gives the BCS theory. The effect of magnetic impurities in superconductors as described by AG theory is given in chapter 2.2A and that described by SR model is discussed in chapter 2.2B. Chapter 2.2c deals with Kondo effect. In chapter 2.3 the anisotropy problem is reviewed. Our calculations, results and discussions are given in chapter 3. Chapter 3.1 deals with Josephson tunnel effect. In chapter 3.2 the thermodynamic critical field H_{c2} is described. Chapter 3.3 deals with the density of states. The ultrasonic attenuation coefficient and nuclear spin relaxation are given in chapter 3.4 and 3.5 respectively. In chapter 3.6 we give the upper critical field calculations and chapter 3.7

deals with the response function. The Kondo effect is given in chapter 3.8. In chapter 4 we give the summary of our results.

A part of these investigations has been presented in the form of the following papers.

1. K.Balakrishna Warier and C.Purushothaman
   Solid State Communications Vol.37 No.12
   pp 1001 - 1004 (1981)
2. K.Balakrishna Warier and C.Purushothaman,
   Solid State Communications (~~Communicated~~ accepted)
3. K.Balakrishna Warier and C.Purushothaman,
   Paper presented at the 51st Annual Session
   of National Academy of Sciences, India
   (Cochin) 1981.

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SYNOPSIS

The thesis deals with studies on several properties of anisotropic superconductors containing paramagnetic impurities in Shiba - Rusinov model. In the SR model, the Kondo effect is neglected. This effect essentially arises while considering the scattering of a conduction electron from a magnetic impurity exactly if (i) the exchange coupling between the electron and the impurity spins is antiferromagnetic and (ii) the non-commutativity of the spin operators is taken into account. The appearance of the impurity band within the BCS energy gap is one of the most significant results of the SR model. Further, the position of the local level ϵ (for a single isolated impurity and for s-wave scattering of the conduction electron) is independent of temperature. Under various approximations the parameter ϵ can be related to temperature T and the Kondo temperature T_K of the system. This relation is valid under the following approximations (i) the electron energies are near the Fermi surface (ii) the impurity concentration is very low and (iii) the temperature is near T_c. In the limited impurity concentration and the temperature ranges, the present calculations can be used to understand some aspects of the Kondo effect in an anisotropic superconductor. Anisotropy is introduced into the problem by assuming a separable form for the effective electron - electron matrix element [1].

The tunnelling phenomenon has been developed into one of the most sensitive techniques for studying the properties of metals and dilute alloys in the normal and superconducting states. A study of the Josephson tunnelling can be a very sensitive probe for the effect of impurities in anisotropic superconductors that constitute the junction.

Considering a junction with two identical superconductors the Josephson current for an anisotropic superconductor described by Shiba - Rusinov model, containing paramagnetic impurities has been calculated by an extension of the theory of Nagi and Upadhyaya [2]. The slope K^* of Josephson current against temperature T near T_c has been computed and shown to agree with the isotropic - limit.

The thermodynamic critical field H_c for a bulk specimen of unit volume has been dervied for an anisotropic impurity-doped superconductor on the basis of calculations of John R. Clem [3].

The density of states $N(\omega)$ of quasi-particle excitations for isotropic superconductors was obtained by Chaba and Nagi [4]. An effort has been made to include anisotropy into the problem.

The Ultrasonic attenuation coefficient and nuclear spin relaxation rate for a magnetically doped superconductor was investigated by Shukla and Nagi [5]. The behaviour of ultrasonic attenuation coefficient and nuclear spin relaxation rate for anisotropic superconductor for temperature near T_c has been

estimated. Expressions for these quantities for the two limiting cases of low and high impurity concentrations has been obtained.

The Upper critical field H_{c2} for an anisotropic superconductor doped with paramagnetic impurities was investigated for very low and very high impurity concentrations. The slope of the upper critical field against T/T_c at T_c is derived.

The response function (electromagnetic property) to a weak transverse field for an isotropic superconductor described by SR model has been studied. The calculations are done in a similar line with Skalski et al [6]. In the limit of $\omega >> \alpha$ the equation obtained agrees with equation (16) of K.Maki [7]. In the limit of $\omega \to o$ the equation reduces to that of Chaba [8].

The computer programme for the calculations has been given in the appendix.

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CHAPTER I

INTRODUCTION

Investigations on the effects of magnetic impurities in superconductors provide information regarding the interaction between magnetic spins and conduction electrons in metals. Gapless superconductivity was first proposed in the original work of Abrikosov and Gor'kov (AG) [1] on the effect of magnetic impurities on superconductivity. In this theory the interaction between individual spins and the electrons is treated by perturbation theory. The results of the theory for variety of phenomena have been reviewed by Maki [2].

The work of AG has been succeeded by extensions of the theory to take into account the interactions between the individual impurity spins and the conduction electron beyond perturbation theory. In normal metals this extension leads to an understanding of the Kondo effect. A treatment of the thermodynamic properties of superconductors that takes some account of the Kondo effect has been given by Muller-Hartmann and Zittartz [3]. It has not yet been fully compared with experimental results.

An extension of the theory of AG has been provided by Shiba [4] and Rusinow (SR) [5]. In this theory the impurity spins are treated classically, but their interaction with the electrons is calculated exactly. One striking new qualitative

feature of the model is the existence of bound states in the energy gap.

The presence of impurities in a superconductor has an interesting influence upon the effects of anisotropy of the superconducting energy gap. An important result of the addition of ordinary non-magnetic chemical or physical impurities is the reduction or 'washing out' of the anisotropy of the energy. Such a reduction of the anisotropy has been observed in specific heat, nuclear spin-lattice relaxation, tunneling, infrared absorption, and surface resistance experiments [6].

AG found first of all that the transition temperature decreases sharply with increasing impurity concentration. Furthermore, there exists a region of concentration where the gap in the excitation energy spectrum is zero even though the substance is still a superconductor in the sense of having pair correlations and a non-zero transition temperature. In contrast to the case of non-magnetic impurities, paramagnetic impurities give rise to a real life time effect. Beause of the spin-exchange scattering, the life time of a pair state is no longer infinite and this results in a rapid decrease in the ordering and therefore in the transition temperature. The problem of a superconductor containing paramagnetic impurities is very similar to the problem of an ordinary impure

superconductor in the presence of an external current or magnetic field [7].

The existence of a finite energy gap between the ground state energy and the lowest excited states of a superconducting metal plays an inportant role in the theory of superconductivity. The BCS theory was the first to show that the superconducting state at finite temperature could be described in terms of individual particle-like excitations having energy gap. The energy of one of these quasi particles is given by

$$E_p = |(\epsilon_p^2 + \Delta_p^2)^{1/2}|$$

where ϵ_p is the energy measured from the Fermi level, of a Bloch electron having a wave vector p, and where Δ_p is a quantity called the gap parameter. The minimum quasiparticle energy for a given p attained when $\epsilon_p = 0$ is equal to Δ_p. In a pure single crystal the energy gap parameter Δ_p depends upon the direction of p with respect to the crystal axes and may be regarded as a basic physical property.

The lowest order corrections arising from anisotropy which are proportional to the angular average of the square of the deviation of the gap parameter from its average value, are most conveniently expressed in terms of the mean squared anisotropy defined as

$$\langle a^2 \rangle = \langle (\Delta_p - \langle \Delta_p \rangle_{av})^2 \rangle_{av} / \langle \Delta_p \rangle_{av}^2$$

Pokrovskii theoretically obtained the effects of anisotropy upon the critical field at low temperatures and upon the specific heat jump at the critical temperature. Superconducting tunneling experiments have recently been used to obtain information about anisotropy of the energy.

The Josephson tunnel effect in isotropic superconducting alloys described by Anderson model has been investigated by Nagi and Upadhyaya [9]. The influence of magnetic impurities on Josephson current was studied by Kulik [10], Nagi and Lo [11] and Lo and Nagi [12].

In the SR model the normalized position of local states within the BCS energy gap with a single impurity, is independent of temperature and the sign of exchange interaction.

The effect of anisotropy on the order parameter and thermodynamic critical field for pure superconductors was investigated by Clem [13] in the AG model. The effect of gap anisotropy in superconductors containing paramagnetic impurities with local states within the energy gap has been investigated by the present author[14].

For a transition metal atom, the spins are due to 3d electrons, which are not so deep inside the atom with such atoms as impurities in superconductors the magnetic electrons would react strongly with the conduction electrons of the host material and the AG theory would not be appropriate to

the problem. The applicability of the SR model to understand the behaviour of the 3d magnetic impurities in superconductors was first brought out by Chaba and Nagi [15] who used this model to explain the tunneling conductance curves for superconducting quench-condensed PbMn film observed by Woolf and Rief [16].

Dick and Reif [17] have measured the real part σ, of the conductivity of rare earth impurities like Gd in Pb and transition metal impurities like Mn in Pb by measuring the infrared radiations absorbed by a film of the above alloys. In the above measurements, it is found that the experimental results in the case of rare earth impurities agree with the theoretical ones derived from the AG theory. But in the case of transition metal impurities, the agreement is not satisfactory as the observed effects in this case are more pronounced than those predicted by the AG theory. In the case of rare earth impurities, the localized spin is due to the coupling between the electrons and the impurities may be weak and the Born approximation, as used by AG, may be valid. In the case of transition metal impurities the spin of the impurities is due to the d - electrons. In this case the coupling between the electrons and magnetic impurities may not be weak and hence, the Born approximation may not be justified.

The effect of gap anisotropy in superconductors with non-magnetic impurities was investigated by Markowitz and Kadanoff [18] and Brink and zuckermann [19] and Fulde [20]. The effect of gap anisotropy in pure and impure superconductors was investigated by Clem [8,21]. The effect of resonance scattering on anisotropic superconductors was studied by Zuckermann and Singh [22]. In the present thesis, I have investigated the effect of gap anisotropy on magnetically doped superconductors described by the SR model. The Kondo effect has been included in the problem.

CHAPTER II

FORMULATION OF THE PROBLEM.

2.1 Bardeen - Cooper - Schrieffer Theory.

Bardeen - Cooper - Schrieffer (BCS) [23] in 1957 gave a successful microscopic theory of superconductivity. The theory is based on Cooper's [24] idea that in metals an attractive interaction between two fermions in the Fermi sea leads to the appearence of a bound state, now called a Cooper pair. The momenta and the spins of the two members of the pair are $(\hbar,\underset{\sim}{k},\uparrow)$ and $(-\hbar,\underset{\sim}{k},\downarrow)$, where $\underset{\sim}{k}$ is the wave -vector. As was shown by Frohlich [25], in metals, under certain conditions, the electron - phonon interaction indeed provides an attractive interaction for electrons near the Fermi surface. The BCS theory was formulated in terms of Green's function by Gor'kov [26] and is described in detail in several books [27] - [31]. Below we give a short summary.

Consider a system of interacting fermions interacting by an attractive spin - independent, contact type two body interaction,

$V(\underset{\sim}{X}-\underset{\sim}{Y}) = -g\ \delta(\underset{\sim}{X}-\underset{\sim}{Y})$, which is brought about indirectly by the electron - phonon interaction. Here g is the strength of the electron - electron intraction. The hamiltonian of the system in second quantized form is [31].

$$H = \int d^3X \psi^+_\alpha(\underset{\sim}{X},t)\ [-\nabla^2/2m]\ \psi_\alpha(\underset{\sim}{X},t)$$
$$-g/2 \int d^3X \psi^+_\alpha(\underset{\sim}{X},t)\ \psi^+_\beta(\underset{\sim}{X},t)\ \psi_\beta(\underset{\sim}{X},t)\ \psi_\alpha(\underset{\sim}{X},t) \quad .. \quad (2.1)$$

Where $\psi(\underset{\sim}{X},t)$ and $\psi^+(\underset{\sim}{X},t)$ respectively are the single electron

annihilation and creation operators, $\underset{\sim}{X}$,t denote position and time. α and β are spin indices and the energies are measured from the Fermi energy. The summation over the repeated spin indices is implied. (we use energy units $\hbar = k_B = 1$ throughout this thesis.)

The electron zero-temperature Green's function is defined by

$$\bar{G}(\underset{\sim}{X}t,\underset{\sim}{X}'t') = -i\langle T[\,\Psi_{\uparrow}(\underset{\sim}{X}t)\,\Psi^{+}_{\uparrow}(\underset{\sim}{X}'t')]\rangle \qquad ..(2.2)$$

where T is Wick's time - ordering operator $\Psi_{\uparrow}(\underset{\sim}{X}t)$ and $\Psi^{+}_{\uparrow}(\underset{\sim}{X}'t')$ respectively, annihilate and create an electron at position $\underset{\sim}{X}$ and time t, with spin $\uparrow$, and the bracket <----> denotes the quantum mechanical expectation value of the operators. We use Heisenberg picture. In this picture the field operator is governed by the following equation of motion.

$$i(\partial/\partial t)\,\Psi_{\alpha}(\underset{\sim}{X},t) = [\,\Psi_{\alpha}(\underset{\sim}{X},t),H] \qquad ..(2.3)$$

where [,] denote the poisson bracket.

Using equations (2.1), (2.2) and (2.3) we get

$$(i\partial/\partial t - \nabla^2/2m)\,\bar{G}(\underset{\sim}{X}t,\underset{\sim}{X}'o) = \partial^3(\underset{\sim}{X} - \underset{\sim}{X}')\,\partial(t) + ig\langle T[\,\Psi^{+}_{\alpha}(\underset{\sim}{X},t)\,\Psi_{\alpha}(\underset{\sim}{X},t)\,\Psi_{\uparrow}(\underset{\sim}{X},t)\,\Psi^{+}_{\uparrow}(X,o)]\rangle \qquad ..(2.4)$$

On the right hand side (r.h.s) of equation (2.4) one has an average consisting of four field operators. Using $x=\underset{\sim}{X},t$ such an average is factorized in the following way.

$$\langle T[\psi_\alpha(x_1)\psi_\beta(x_2)\psi^+_\gamma(x_3)\psi^+_\delta(x_4)]\rangle =$$
$$\langle T[\psi_\alpha(x_1)\psi^+_\delta(x_4)]\rangle \langle T[\psi_\beta(x_2)\psi^+_\gamma(x_3)]\rangle$$
$$-\langle T[\psi_\alpha(x_1)\psi^+_\gamma(x_3)]\rangle \langle T[\psi_\beta(x_2)\psi^+_\delta(x_4)]\rangle$$
$$+\langle T[\psi_\alpha(x_1)\psi_\beta(x_2)]\rangle \langle T[\psi^+_\gamma(x_3)\psi^+_\delta(x_4)]\rangle \quad (2.5)$$

The first two terms on the r.h.s of equation (2.5) are the usual Hartree - Fock terms which also come in the factorization appropriate for a normal metal. As one is interested in the deviations from normal metals, these terms are usually ignored. For handiling the last term on the r.h.s of equation (2.5), Gor'kov introduced the following Green's function [29]

$$F(x,x') = -i\langle T[\psi_\alpha(x_1)\psi_\beta(x_2)]\rangle \quad ..(2.6)$$

$$\bar{F}(x,x') = -i\langle T[\psi^+_\gamma(x_3)\psi^+_\delta(x_4)]\rangle \quad ..(2.7)$$

and the zero - temperature order parameter of a pure superconductor,

$$\Delta(0,0) = -g\langle\psi_\uparrow(x)\psi_\downarrow(x)\rangle \quad ..(2.8)$$

Equation (2.4) can now be easily rewritten in terms of $\Delta(0,0)$ and $\bar{F}$.

For a homogeneous superconductor, it is more convenient to work with the Fourier transformed Green's functions $\bar{G}(\underset{\sim}{k},\omega)$, $F(\underset{\sim}{k},\omega)$ and $\bar{F}(\underset{\sim}{k},\omega)$. In terms of these, equation (2.4) reduces to

$$[\omega - \varepsilon_k]\,\bar{G}(\underset{\sim}{k},\omega) = 1 + \Delta(0,0)\,\bar{F}(\underset{\sim}{k},\omega) \quad ..(2.9)$$

where $\mathcal{E}_k = k^2/2m$.

Now one writes the equation of motion of $\bar{F}(\underset{\sim}{X}t,\underset{\sim}{X}'t')$, does the Gor'kov factorization on the r.h.s and takes the Fourier transform to obtain

$$[\omega + \mathcal{E}_k]\, \bar{F}(\underset{\sim}{k},\omega) = \Delta^*(0,0)\, \bar{G}(\underset{\sim}{k},\omega) \qquad (2.10)$$

where $\Delta^*(0,0)$ is the complex conjugate of $\Delta(0,0)$. Taking $\Delta(0,0)$ to be real, equation (2.9) and (2.10) give

$$\bar{G}(\underset{\sim}{k},\omega) = (\omega + \mathcal{E}_k)/(\omega^2 - E_k^2) \qquad (2.11)$$

$$\bar{F}(\underset{\sim}{k},\omega) = \Delta(0,0)/(\omega^2 - E_k^2) \qquad (2.12)$$

where $E_k = \sqrt{[\mathcal{E}_k^2 + |\Delta(0,0)|^2]}$ and is called the excitation energy of a quasi - particle in a superconductor.

The density of single particle states in a superconductor can be calculated from the imaginary part of the Green's function by using the relation

$$N_s(\omega) = -(1/\pi)\, \mathrm{Im} \sum_{\underset{\sim}{k}} \bar{G}(\underset{\sim}{k},\omega) \qquad (2.13)$$

and one finds.

$$N_s(\omega) = \omega/\sqrt{[\omega^2 - \Delta^2(0,0)]} \quad \text{for} \quad \omega > \Delta(0,0)$$

$$N_s(\omega) = 0 \quad \text{for} \quad \omega < \Delta(0,0) \qquad (2.14)$$

Thus in a pure superconductor there are no single partcle states for energies less than the gap energy $\Delta(0,0)$.

For finite temperature calculations, one introduces the Green's functions [31]

$$\bar{G}(\underset{\sim}{X},\tau\ \underset{\sim}{X}',\tau') = -\langle \bar{T}[\psi_{\uparrow}(X,\tau)\,\psi^{+}_{\uparrow}(\underset{\sim}{X}';\tau')]\rangle \qquad (2.15)$$

where $\tau = it$, is the imaginary time and $\bar{T}$ is the Wick's operator which orders imaginary times. The bracket $\langle \ldots. \rangle$ now indicates the averaging of the operators in the Grand - canonical ensemble. The Gor'kov function F, $\bar{F}$ and the order parameter are defined in a similar way.

Usually one goes from the zero - temperature to the finite temperature results by replacing ω to $i\omega_n$ with $\omega_n = \pi\ T\ (2n + 1)$ where T is the temperature and n is an integer. At the same time one replaces $\int d\omega/2\pi \rightarrow iT\sum_n$, where $\sum_n$ indicates the sum over n. Now

$$\bar{G}(\underset{\sim}{k},\omega_n) = [\ i\omega_n + \xi_k]/[\ -\omega_n^2 - E_k^2] \qquad (2.16)$$

$$\bar{F}(\underset{\sim}{k},\omega_n) = -\Delta(0,T)/[\ \omega_n^2 + E_k^2] = F(\underset{\sim}{k},\omega_n) \qquad (2.17)$$

and

$$\Delta(0,T) = -g\langle \psi_{\uparrow}(x)\,\psi_{\downarrow}(x)\rangle = g\ F(\underset{\sim}{X}\tau^{+},\ \underset{\sim}{X}\tau) \qquad (2.18)$$

with $\tau^{+} = \tau + 0^{+}$

Equations (2.17) and (2.18) give the self consistency equation for the order paramater

$$\Delta(0,T) = g\ T \sum_n \sum_{\underset{\sim}{k}} \Delta(0,T)/[\ \omega_n^2 + \xi_k^2 + \Delta^2(0,T)] \qquad (2.19)$$

The sum over $\underset{\sim}{k}$ can be replaced by

$\sum_k \rightarrow N(0)\int d\xi_k$ where $N(0) = mk_f/2\pi^2$ is the density of single electron states of one spin at the Fermi surface(k_f is the Fermi momentum and m is the electron mass).

Then equation (2.19) can be integrated to give

$$\Delta(0,T) = g\,N(0)\,2\pi T \sum_{n \geq 0} 1/\sqrt{[1 + U_{n,o}^2]} \qquad (2.20)$$

where $U_{n,o} = \omega_n/\Delta(0,T)$

at T = 0 K, the $2\pi T \sum_{n \geq 0}$ is replaced by $\int_0^{\omega_D} d\omega_n$ and equation (2.20) gives

$$1/gN(0) = \left[1/\Delta(0,0)\right] \int_0^{\omega_D} d\omega_n/\sqrt{[1 + U_{n,o}^2]}$$

$$= \int_0^{\omega_D/\Delta(0,0)} dU/\sqrt{[1 + U^2]}$$

$$= \mathrm{Sinh}^{-1}[\omega_D/\Delta(0,0)] \qquad (2.21)$$

In the BCS theory one assumes $gN(0) < < 1$ then equation (2.21) gives

$$\Delta(0,0) \simeq 2\,\omega_D \exp\,[-1/gN(0)] \qquad 2.22)$$

At $T = T_{co}$, (the transition temperature of a pure metal) the order parameter vanishes. For temperature near T_{co}, $\Delta(0,T)$ is very small and one can expand equation (2.20) in powers of $\Delta(0,T)$ to obtain

$$\Delta(0,T) = gN(0)\,2\pi T \sum_{n \geq 0} [\Delta(0,T)/\omega_n - (1/2)\,\Delta^3(0,T)/\omega_n^3 \ldots] \qquad (2.23)$$

At $T = T_{co}$ equation (2.23) gives

$$1/gN(0) = 2\pi T_{co} \sum_{n=0}^{\omega_D/2\pi T_{co}} [1/\omega_n]$$

$$= \ln[2\gamma\omega_D/\pi T_{co}] \qquad (2.24)$$

where $\gamma = 1.781$ and we have used the mathematical identity

$$2\pi T \sum_{n>0}^{\omega_D/2\pi T_{co}} [1/\omega_n] = \ln[2\gamma\omega_D/\pi T] \qquad (2.25)$$

using equations (2.23) - (2.25) one can calculate $\Delta^2(0,T)$ near T_{co}. Then

$$\ln[T/T_{co}] = [1/4]\,\psi^2(1/2)\,[\Delta(0,T)/2\pi T]^2 \qquad (2.26)$$

where $\psi^n(z)$ is the poly-gamma function [32].
Expanding near T_{co} equation (2.26) gives

$$\Delta^2(0,T) = -[16\pi^2 T_{co}^2/\psi^2(1/2)](1 - T/T_{co}) \qquad (2.27)$$

2.2 Effect of Magnetic Impurities in superconductors.

A. Abrikosov - Gor'kov Theory.

A very successful theory for low concentration of magnetic impurities in a superconductor was given by Abrikosov and Gor'kov [1]. Their work indicates that in the magnetic impurity problem, the energy gap and the order parameter are not identical and furthermore the existence of the order parameter (or the formation of cooper pairs) is the only criterion for superconductivity. These authors predicted the phenomenon of gapless superconductivity which has been experimentally verfied[14].

AG theory is formulated in terms of the Green's functions in much the same manner as the BCS theory. The hamiltonian is composed of the BCS model hamiltonian plus an extra term which characterizes electron and the spin of an impurity atom.

AG have made the following assumptions, (i) the interaction between a conduction electron spin and an impurity spin is assumed weak and the lowest order Born approximation is used in treating the interaction, (ii) the impurities are assumed to be randomly distributed in the sample, and (iii) the impurity concentration is assumed to be low enough so that the impurity-impurity interaction is negligible.

The hamiltonian used by AG is

$$H = \int d^3X \psi_\alpha^+(\underset{\sim}{X},t)\,[-\nabla^2/2m]\,\psi_\alpha(\underset{\sim}{X},t) - (g/2)\int d^3X \psi_\alpha^+(\underset{\sim}{X},t)\psi_\beta^+(\underset{\sim}{X},t)\psi_\beta(\underset{\sim}{X},t)\psi_\alpha(X,t) + \sum_n \int d^3X \psi_\alpha^+(\underset{\sim}{X},t)\, V_{\alpha\beta}(\underset{\sim}{X}-R_n)\,\psi_\beta(X,t) \qquad (2.28)$$

where the first two terms are the usual BCS terms and the last term describes the electron impurity scattering. one has

$$V_{\alpha\beta}(\underset{\sim}{X}-R_n) = V_1(\underset{\sim}{X}-R_n)\,\delta_{\alpha\beta} + V_2(\underset{\sim}{X}-R_n)\,\underset{\sim}{S}.(\sigma_{\alpha\beta}/2) \qquad (2.29)$$

where V_1 and V_2 respectively are the non-magnetic and the magnetic scattering potentials. $\underset{\sim}{S}$ is impurity spin and $\sigma_{\alpha\beta}$ denote Pauli matrices.

It is convenient to introduce a 4 x 4 matrix temperature-Green's function (2) defined by

$$\bar{G}(x,x') = -\langle T[\psi(x)\psi^+(x')]\rangle$$

where

$$\psi(x) = \begin{bmatrix} \psi_\uparrow(x) \\ \psi_\downarrow(x) \\ \psi_\uparrow^+(x) \\ \psi_\downarrow^+(x) \end{bmatrix} \quad \text{and} \quad \psi^+(x) = [\psi^+(x)\psi_\downarrow^+(x)\psi_\uparrow(x)\psi_\downarrow(x)]$$

with $x = (\underset{\sim}{X},\tau)$, $\tau = it$

In this notation, the BCS Green's function is written as

$$\bar{G}^{o}(\rho,\omega_n) = [i\omega_n \rho_o \sigma_o - \xi_k \rho_3 \sigma_o - \Delta(0,T)\rho_2\sigma_2]^{-1} \qquad (2.30)$$

where ρ_i , σ_i (i = 1,2,3) are Pauli spin matrices operating on the ordinary spin and the electron - hole spin states and ρ_o , σ_o are unit matrices. A matrix like $\rho_2\sigma_2$ means

$$\rho_2\sigma_2 = \begin{bmatrix} 0 & -i\sigma_2 \\ i\sigma_2 & 0 \end{bmatrix}$$

Using the hamiltonian given in equation (2.28), treating the scattering within the lowest Born approximation and averaging over a random distribution of low concentration of magnetic impurities, AG obtain [1], [2].

$$\bar{G}(\rho,\omega_n) = [i\underset{\sim}{\omega}_n - \xi_p \rho_3 + \underset{\sim}{\Delta}_n \rho_2 \sigma_2]^{-1} \qquad (2.31)$$

where $\underset{\sim}{\omega}_n$ and $\underset{\sim}{\Delta}_n$ are the renormalized frequency and the order parameter respectively, and are given by

$$\underset{\sim}{\omega}_n = \omega_n + (1/2\tau^{1})\, \underset{\sim}{\omega}_n / \sqrt{[\underset{\sim}{\omega}_n^2 + \underset{\sim}{\Delta}_n^2]} \qquad (2.32)$$

$$\underset{\sim}{\Delta}_n = + (1/2\tau^{2})\, \underset{\sim}{\Delta}_n / \sqrt{[\underset{\sim}{\omega}_n^2 + \underset{\sim}{\Delta}_n^2]} \qquad (2.33)$$

The quantities τ^{1} and τ^{2} are the scattering life times and are defined by

$$1/\tau^{1} = 2\pi C_i N(0)[V_1^2 + [s(s+1)/4]V_2^2]$$

and

$$1/\tau^{2} = 2\pi C_i N(0)[V_1^2 - [s(s+1)/4]V_2^2]$$

where C_i is the impurity concentration, V_1 is the interaction potential due to non-magnetic impurities, and V_2 is the interaction potential due to magnetic impurities. Defining $U_n = \underset{\sim}{\omega}_n / \underset{\sim}{\Delta}_n$ equation (2.32) and (2.33) can be combined to give

$$\omega_n/\Delta = U_n[1 - (\alpha/\Delta)(1/\sqrt{[1+U_n^2]})] \qquad (2.34)$$

where

$$\alpha = 1/\tau_s = (1/2)[1/\tau' - 1/\tau^2]$$
$$= (\pi/2)\, C_i\, N(0) s(s+1) V_2^2$$

It may be noted that in equation (2.34) the deviation from BCS value arises when α is different from zero.

In the 4 x 4 matrix Green's function formulation, the order parameter is given by [3].

$$\Delta(C_i,T) = -gT \sum_n \int d^3k/(2\pi)^3\, Tr[\rho_1 \sigma_2\, G(k,\omega)]$$
$$= gN(0)\, 2\pi T_{n>\delta} \sum 1/\sqrt{[1 + U_n^2]} \qquad (2.35)$$

B. The Shiba - Rusinov (SR) model.

Shiba [4] and Rusinov [5] have generalized the AG theory to the when the scattering of a conduction electron from a magnetic impurity is strong. Following Shiba [4], first we may use the one particle Green's function for the alloy.

The scattering of a conduction electron from a magnetic impurity is described by the so called s - d exchange interaction hamiltonian.

$$H_{sd} = -(J/2)\sum_{k,k'}[(a^{+}(k)_{\uparrow}a(k')_{\uparrow} - a^{+}(k)_{\downarrow}a(k')_{\downarrow})S_z + a^{+}(k)_{\uparrow}a(k')_{\downarrow}S_{-} + a^{+}(k)_{\downarrow}a(k')_{\uparrow}S_{+}] \quad (2.36)$$

where J is the strength of the exchange interaction (which is assumed to be constant) S_z, S_-, S_+ are the components of the spin operator associated with the impurity.

Using the above hamiltonian and considering only one magnetic impurity, the 4x4 matrix Green's function is given by

$$\bar{G}(\underset{\sim}{k},\underset{\sim}{k}',\omega) = \bar{G}^{o}(\underset{\sim}{k},\omega)\,\delta_{\underset{\sim}{k},\underset{\sim}{k}'} + \bar{G}^{o}(\underset{\sim}{k},\omega)\,t(\omega)\,\bar{G}^{o}(\underset{\sim}{k}',\omega) \quad (2.37)$$

where we have averaged over the directions of the spin, $\bar{G}^{o}$ is given by equation (2.30), ω is real frequency and $t(\omega)$ is the non-spin-flip part of the scattering amplitude given by [4]

$$t(\omega) = [(JS/2)^2 F(\omega)][1 - [JSF(\omega)/2]^2]^{-1} \quad (2.38)$$

with $F(\omega) = \sum_{k} \bar{G}^{o}(\underset{\sim}{k},\omega)$. In writing equation (2.38), one has used the so-called 'classical limit': $J \to 0$, $S \to \infty$, JS = finite.

for $|\omega| < \Delta(0,T)$

$$F(\omega) = -\pi N_o[\omega + \Delta(0,T)\,\rho_2\sigma_2][\Delta^2(0,T) - \omega^2]^{-1/2} \quad (2.39)$$

Using equation (2.38) and (2.39) one notes that $t(\omega)$ has poles at

$$[\omega/\Delta(0,T)] = \pm[\omega_B/\Delta(0,T)] = \pm\xi = \pm[1 - (JS\pi N_o/2)^2][1 + (JS\pi N_o/2)^2]^{-1} \quad (2.40)$$

when $(JS\pi N_o/2)^2 << 1$, $\xi \to 1$. When $(JS\pi N_o/2)^2 = 1$, $\xi = 0$.

when $(JS\pi N_o/2)^2 >> 1$, $\xi \to -1$. The magnitude of ξ lies between 0 and 1. The pole in $t(\omega)$ signifies a bound state and $0 \lesssim |\xi| \lesssim 1$ means that the bound state lies within the BCS gap at $\Delta(0,T)$. For the finite impurity concentration problem, one assumes that the magnetic impurities are randomly distributed in the superconductor and that their concentration is low enough so that the impurity - impurity interaction is negligible. Then the thermal Green's function, averaged over the positions and the spin direction of the impurities is given by

$$[\bar{G}(\underset{\sim}{k},\omega_n)]^{-1} = [\bar{G}^o(\underset{\sim}{k},\omega_n)]^{-1} - L(\underset{\sim}{k},\omega_n) \qquad (2.41)$$

where $L(\underset{\sim}{k},\omega_n)$ is the self energy. We assume $L(\underset{\sim}{k},\omega_n)$ is independent of $\underset{\sim}{k}$, then

$$L(\omega_n) = C_i[(JS/2)^2\bar{F}(\omega_n)][1 - [JS\bar{F}(\omega_n)/2]^2]^{-1} \qquad (2.42]$$

where C_i is impurity concentration and $\bar{F}(\omega_n)$ should be obtained self consistently by

$$\bar{F}(\omega) = \sum_k \bar{G}(k,\omega_n)$$

one finds that

$$\bar{G}(k,\omega_n) = [i\omega_n - \xi_k + i\underset{\sim}{\Delta}_n \rho_1\sigma_2]^{-1} \qquad (2.43)$$

where

$$\underset{\sim}{\omega}_n = \omega_n + \Gamma_1[U_n/[1 + U_n^2]]/[\xi^2 + U_n^2] \qquad (2.44)$$

$$\underset{\sim}{\Delta}_n = \Delta(\bar{c},T) + \Gamma_2/[1 + U_n^2]/[\xi^2 + U_n^2] \qquad (2.45)$$

$$\Gamma_1 = [C_i/2\pi N(0)](1 - \xi) \qquad (2.46)$$

$$\Gamma_2 = [-C_i/2\pi N(0)]\xi(1 - \xi) \qquad (2.47)$$

and

$$U_n = \underset{\sim}{\omega}_n / \underset{\sim}{\Delta}_n \qquad (2.48)$$

equations (2.44) and (2.45) can be combined to give

$$\omega_n/\Delta(\bar{c},T) = U_n[1 - (\alpha/\Delta(\bar{c},T))\sqrt{[1 + U_n^2]/[\xi^2 + U_n^2]}] \qquad (2.49)$$

where $\alpha = \Gamma_1 - \Gamma_2 = [C_i/2\pi N(0)](1 - \xi^2)$ (2.50)

Physically, the quantity $[1/\alpha]$ represents the time it takes for a conduction electron spin to flip during scattering from the impurity. Rusinov [5] has shown that

$$\xi = \cos(\delta_o^+ - \delta_o^-) \qquad (2.51)$$

where $\delta_o^{\pm}$ are the s - wave ($\ell = 0$) phase shifts of an electron of spin $\pm$ (1/2) scattering from a magnetic impurity. Thus the spin-flip scattering of the conduction electron plays a crucial role in the SR model.

It may be noted that the results of the Abrikosov - Gor'kov approximation can be recovered from the above equations by the following argument. In the AG theory the interaction between the conduction electron spin and impurity spin is weak and the approximation $[(1/2)JS\pi N_o]^2 << 1$ is valid. Then equation (2.40) gives $(1 - \xi^2) \rightarrow 4[(1/2)JS\pi N_o]^2$. Substituting this in equation (2.50) gives α_{AG}. Then taking $\xi \rightarrow 1$ in equation (2.49) gives $[U_n]_{AG}$ in the limit of large S.

C. Kondo Effect.

It was discovered that at low temperatures the resistivity versus temperature plot for a normal metal with low concentration of transition metal impurities exhibited a 'resistance minimum'. Kondo [33] has provided explanations for this phenomenon. Using the s - d exchange interaction hamiltonian given in equation (2.36), Kondo showed that to the third order in J, the resistivity contributed by the scattering of the conduction electron from a low concentration of magnetic impurities is given by

$$\rho_{spin} = C_i \rho_n [1 + (3zJ/\varepsilon_F) \log(T)] \qquad (2.52)$$

where

$$\rho_n = 3\pi m J^2 S(S+1)/2e^2\varepsilon_F \qquad (2.53)$$

'z' is the number of conduction electrons per atom, ε_F is the Fermi energy, 'e' and 'm' are the electronic charge and mass respectively and C_i is the impurity concentration.

Equation (2.52) contains a singular term involving log(T) which increases towards low temperature, if J is negative. In that case equation (2.52) added to the phonon contribution to resistivity gives the total resistivity which shows a resistance minimum.

Because of the presence of the singular term in equation (2,52), the Kondo effect has attracted considerable theoretical

attention. For normal metals theories has been proposed by by Abrikosov [34], Suhl [35] and Nagaoka [36]. These theories are reviewed by Kondo [33].

Nagaoka [36] gave the Green's function approach for the Kondo effect in normal metals. He used retarded double-time Green's functions [37] which are defined by

$$\begin{aligned} \langle A/B \rangle_t &= -i \langle [\, A(t), B(0) \,]_+ \rangle \qquad \text{for } t > 0 \\ &= 0 \qquad\qquad\qquad\qquad\quad \text{for } t < 0 \end{aligned} \tag{2,54}$$

where $\langle \ldots\ldots \rangle$ denotes the statistical average, A and B are Fermi operators in Heisenberg representation, and $[A,B]_+$ means AB + BA. The Green's functions involved (for a local spin S = 1/2) are

$$\bar{G}_{kk'}(\omega) = \langle C_{k'\uparrow} \Big| C^+_{k\uparrow} \rangle \tag{2.55}$$

$$\Gamma_{kk'}(\omega) = \langle C_{k'\uparrow} S_z + C_{k'\downarrow} S_- \Big| C^+_{k\uparrow} \rangle \tag{2.56}$$

using s - d exchange hamiltonian given in equation (2.36) and after some approximations, Nagaoka showed that

$$G_{kk'}(\omega) = (1/2\pi)[\delta_{kk'}/(\omega - \varepsilon_k) + t(\omega)/[(\omega - \varepsilon_k)(\omega - \varepsilon_{k'})] \tag{2.57}$$

where $t(\omega)$ is the spin - independent part of the scattering t - matrix.

Hamann [38] showed that

$$t(\omega) = [1/2\pi i N(0)][1 - X/\sqrt{[X^2 + \pi^2 S(S+1)]}] \tag{2.58}$$

with

$$X = X(\omega,T) = \ln[(\omega + iT)/iT_K] \tag{2.59}$$

$$T_K = D.\exp[(1 - S(S + 1)(\pi\gamma/2)^2)/\gamma] \tag{2.60}$$

$$\gamma = JN(0)$$

In the above equation, D is the cut-off energy of the order of Fermi energy and T_k is called the Kondo temperature. For high temperature $T >> T_K$, t(0) has a term which goes as log(T). For $T \rightarrow 0$, t(0) is found to be a well behaved.

The theory of Kondo effect in superconductors is extremely complicated [39], [40]. However under certain approximations, the Kondo effect can be incorporated in the SR model.

The quantity $\alpha = [C_i/2\pi N(0)](1 - \mathcal{E}^2)$ which plays a very crucial role in the SR model can be related to the spin-dependent part $\mathcal{T}$ of the scattering t - matrix in a normal metal by

$$\alpha = C_i[2\pi N(0)S(S+1)|\mathcal{T}|^2] \tag{2.61}$$

Further, the spin-independent and spin-dependent parts of the t - matrix are related by

$$\text{Im } t(\omega) = -\pi N(0)[|t(\omega)|^2 + |\mathcal{T}(\omega)|^2 S(S+1)] \tag{2.62}$$

using equations (2.38) and (2.61) one gets

$$\pi N(0)S(S+1)|\mathcal{T}(\omega)|^2 \simeq [1/4\pi N(0)][1 - X/\sqrt{[X^2+\pi^2 S(S+1)]}] \tag{2.63}$$

equation (2.59) gives

$$X(0,T) = \ln[T/T_K]$$

then equation (2.63) gives $|\mathcal{T}(0)|^2$.

For temperatures near T_c, the superconducting gap is small and α may be calculated by using $\mathcal{T}(0)$ in equation (2.61). Then combining with equation (2,50), one gets

$$\mathcal{E}^2 = \mathcal{T}^2/\mathcal{V}[\mathcal{T}^2 + \pi^2 S(S+1)] \qquad (2.64)$$

$$\mathcal{T} = \ln[T_K/T].$$

More accurate treatment [40] gives

$$\mathcal{E} = -\mathcal{T}/\mathcal{V}[\mathcal{T}^2 + \pi^2 S(S+1)] \qquad (2.65)$$

2.3 The Anisotropy Problem.

In a pure superconductor the effective electron - electron interaction via phonons, which is responsible for pairing, depends upon the directions of electronic momenta relative to the crystal axes and the electrons take maximum advantage of this anisotropy in forming pairs. This anisotropy is neglected in the BCS formalism. When non - magnetic impurities are added to the superconductor, the scattering of electrons from impurities will smear the electronic states over the Fermi surface and this gives a reduction in the transition temperature. Now, if magnetic impurities are added, the transition temperature is further decreased because of the depairing caused by spin-flip scattering.

A general electron - electron interaction may be written as

$$H_{int} = [1/Vol] \sum_{p_1', p_2', p_1, p_2, \sigma_1, \sigma_2} \langle p_1' p_2' g p_1 p_2 \rangle \delta_{p_1+p_2, p_1'+p_2'} a^+_{p_1'\sigma_1} a^+_{p_2'\sigma_2} a_{p_2\sigma_2} a_{p_1\sigma_1} \quad (2.66)$$

where Vol. is the volume of the system. The part of this interaction responsible for superconductivity couples pairs of particles with opposite momenta, involving

$$g_{\underset{\sim}{p}\underset{\sim}{p}'} = \langle \underset{\sim}{p}', -\underset{\sim}{p} | g | \underset{\sim}{p}, -\underset{\sim}{p} \rangle \quad (2.67)$$

The anisotropy can be included by assuming a separable potential [41] and assuming $g_{\theta\theta'}$ with

$$g_{\theta\theta'} = g \sum_{\ell=0}^{\infty} p_\ell(\cos\theta) \sum_{\ell'=0}^{\infty} p_{\ell'}(\cos\theta')$$

$$= g[1 + \sum_{\ell=1}^{\infty} p_\ell(\cos\theta)][1 + \sum_{\ell'=1}^{\infty} p_{\ell'}(\cos\theta)]$$

$$= g[1 + a(\underset{\sim}{\Omega})][1 + a(\underset{\sim}{\Omega}')] \quad (2.68)$$

where $\underset{\sim}{\Omega}$ and $\underset{\sim}{\Omega}'$ are unit vectors along the directions of $\underset{\sim}{p}$ and $\underset{\sim}{p}'$ respectively and $p_\ell(\cos\theta)$ is Legendre polynomial. The quantities $a(\underset{\sim}{\Omega})$ describes the anisotropy of the interaction [18]. They are chosen so that their average

$$\langle a \rangle = \int d\Omega/(4\pi)\, a(\underset{\sim}{\Omega}) = 0 \quad (2.69)$$

but $\langle a^2 \rangle \neq 0$. Further 'a' is assumed sufficiently small so that averages of higher powers of 'a' are negligible.

The present author described the scattering of conduction electrons from a low concentration of magnetic impurities exactly by using the SR model, as given below.

Now, the single - electron Green's function for the alloy, averaged over the positions and spin directions of a low concentration of randomly distributed non - magnetic and magnetic impurities is given by [14]

$$\bar{G}(k,\omega_n) = [i\underset{\sim}{\omega}_n(\underset{\sim}{\Omega})\rho_3 - \xi_k + i\underset{\sim}{\Delta}_n(\underset{\sim}{\Omega})\rho_1\sigma_2]^{-1} \qquad (2.70)$$

with

$$\underset{\sim}{\tilde{\omega}}_n(\underset{\sim}{\Omega}) = \omega_n + \Gamma_1 \int [d\Omega'/4\pi]\, U_n(\Omega')\sqrt{[1 + U_n^2(\Omega')]/[\xi^2 + U_n^2(\Omega')]} + \Gamma_3 \int [d(\Omega')/4\pi]\, U_n(\Omega')/[\xi^2 + U_n^2(\Omega')] \qquad (2.71)$$

and

$$\underset{\sim}{\tilde{\Delta}}_n(\Omega) = \Delta(C_i,T,\Omega) + \Gamma_2 \int [d\Omega'/4\pi]\, \sqrt{[1 + U_n^2(\Omega')]/[\xi^2 + U_n^2(\Omega')]} + \Gamma_3 \int [d\Omega'/4\pi]\, 1/\sqrt{[1 + U_n^2(\Omega')]} \qquad (2.72)$$

where $\Gamma_3 = C_i \pi N(0) V_1^2$

V_1 being the non - magnetic scattering potential. Further $\Delta(C_i,T,\Omega)$ denotes the temperature and the impurity dependent anisotropic order parameter given by

$$\Delta(C_i,T,\Omega) = gN(0)2\pi T[1\ a(\Omega)] \sum_{n=0}^{\infty} \int [d\Omega'/4\pi]. [1 + a(\Omega')]/[1 + U_n^2(\Omega')]^{1/2} \qquad (2.73)$$

CHAPTER III

3. Calculations, Results and discussions.

3.1 Josephson tunnel effect.

Consider a Josephson tunnel junction consisting of two impure anisotropic superconductors which are separated by a thin oxide layer (insulator). When the superconductors are different the Josephson current can be calculated only numerically. However, for a junction made up of two identical superconductors, one can do analytic calculations. Further, all important properties of the Josephson effect are still manifested in this case. For such a junction, the amplitude of the Josephson current at a temperature T can be written as

$$J_s(\bar{C},T,\Omega) = R_N^{-1}(2\pi T) \sum_{n=0}^{\infty} \int \frac{d\Omega'}{4\pi} [1 + U_n^2(\Omega')]^{-1} \quad \ldots(3.1)$$

R_N is the normal - state resistance of the junction and

$$U_n(\Omega) = \omega(\Omega)/\Delta(\Omega)$$

$\omega(\Omega)$ and $\Delta(\Omega)$ are given in equations (2.71) and (2.72). Near T_c $\Delta(\Omega)$ is small and $U_n(\Omega)$ is large. Hence equation (3.1) can be rewritten as

$$J_s(\bar{C},T,\Omega) = R_N^{-1}(2\pi T) \sum_{n=0}^{\infty} \int \frac{d\Omega'}{4\pi} [1/U_n^2(\Omega')] \quad \ldots(3.2)$$

We condiser terms containing Δ^2 only. We have

shown in ref [14] that

$$1/U(\Omega) = L + a(\Omega)\ M \qquad (3.3)$$

where

$$L = [\Delta/(\omega+\alpha)] + (\Delta^3/2)[\alpha(2\xi^2 - 1)/(\omega+\alpha)^4] \qquad (3.4)$$

and

$$M = [\Delta/(\omega+\beta)] + [(\Delta^3/2)(2\ ^2 - 1)\Gamma_i]/[(\omega+\beta)^2(\omega+\alpha)^2] \qquad (3.5)$$

Substituting equation (3.3), (3.4) and (3.5) in equation (3.1) we obtain

$$J_s(\bar{C},T,\Omega) = [(2\pi T)/R_N] \sum_{n=0}^{\infty} [[\Delta^2/(\omega+\alpha)^2] + \langle a^2\rangle \Delta^2/(\omega+\beta)^2]$$

$$= [\Delta^2/(2\pi T R_N)][\psi'((1/2)+(\alpha/(2\pi T))) + \langle a^2\rangle \psi'((1/2)+(\beta/(2\pi T)))] \qquad (3.6)$$

Hence

$$J_s(\bar{C},T,\Omega) = \Delta^2(\bar{C},T,\ )\ B_2(\bar{C},T)/(2\pi T R_N) \qquad (3.7)$$

Where

$$B_2(\bar{C},T) = \psi'((1/2)+(\alpha/(2\pi T))) + \langle a^2\rangle \psi'((1/2)+(\ /(\pi 2T))$$

Also we know that [14] that

$$-\ln(T/T_{co}) = B_o(\bar{C},T) + B_1(\bar{C},T)\Delta^2(\bar{C},T,\Omega)/(8\pi^2T^2) \qquad (3.8)$$

Near T_c equation (3.8) can be expanded to give

$$\Delta^2(\bar{C},T,\Omega) = [8\pi^2 T_c^2/B_1(\bar{C},T)][1+T_c \frac{\partial}{\partial T} B_o(\bar{C},T)\Big|_{T_c}][1-T/T_c] \quad (3.9)$$

where

$$B_o(\bar{C},T) = [1+\langle a^2\rangle]^{-1}\{[\psi^o((1/2)+\rho) - \psi^o(1/2)] + \langle a^2\rangle[\psi^o((1/2)+\sigma) - \psi^o(1/2)]\} \quad (3.10)$$

$$B_1(\bar{C},T) = [1 + \langle a^2\rangle]^{-1}[b_o(\bar{C},T) + \langle a^2\rangle\, b_1(\bar{C},T)] \quad (3.11)$$

$$b_o(\bar{C},T) = -(1/2)\,\psi^2((1/2)+\rho) - \frac{\rho\,(2\varepsilon^2-1)}{6}\,\psi^3((1/2)+\rho) \quad (3.12)$$

$$\begin{aligned} b_1(\bar{C},T) = {} & [3/(\sigma-\rho)][\psi^1((1/2)+\rho) - \psi^1((1/2)+\sigma)] \\ & -[C_i/4\pi^2 N(0)T][V_1^2 + (1-\varepsilon)(2\varepsilon^2-1)] \times \\ & [1/(\sigma-\rho)]^2[\psi^1((1/2)+\rho) + \psi^1((1/2)+\sigma)] \\ & -2[1/(\sigma-\rho)]^3[\psi^o((1/2)+\sigma) - \psi^o((1/2)+\rho)] \end{aligned} \quad (3.13)$$

$$\psi^n(z) = (-1)^{n+1}\, n! \sum_{k=0}^{\infty} 1/(k+z)^{n+1}$$

$$\rho = \alpha/2\pi T = \bar{C}\,(T_{co}/T)\,(1-\varepsilon^2)$$

$$\sigma = \beta/2\pi T = \bar{C}\,(T_{co}/T)\,(1-\varepsilon+\bar{V}_1^2)$$

For small impurity concentration and $T \simeq T_c$

$$\Delta^2(\bar{C},T,\Omega) = [8\pi^2 T_c^2/B_1(\bar{C},T_c)(1-T/T_c) \times [1 - (\alpha/2\pi T_c)\psi^1((1/2)+\rho)] \quad (3.14)$$

for a pure superconductor the Josephson current is obtained as

$$J_s(0,T,\Omega) = \Delta^2(0,T)\,\psi^1(1/2)[1 + \langle a^2\rangle]/R_{N(2\pi T)} \quad (3.15)$$

The normalized Josephson tunneling current can be given by

$$\frac{J_s(\tilde{C},T,\Omega)}{J_s(0,T,\Omega)} = [\Delta(\tilde{C},T)/\Delta(0,T)]^2[1/\psi^1(1/2)[1+\langle a^2\rangle]]\times [\psi^1((1/2)+\rho) + \langle a^2\rangle[\psi^1((1/2)+\sigma) - \psi^1((1/2)+\rho)]] \quad (3.16)$$

where

$$[\Delta(\tilde{C},T)/\Delta(0,T)]^2 = [T_c/T_{co}]^2[B_1(0,T)/B_1(\bar{C},T)](1-T/T_c)\times [1-4.935\bar{C}T_{co}/T_c[1-\xi^2 + \langle a^2\rangle \times [\xi^2 \simeq \xi + \bar{V}_1^2]]]/(1 - T/T_{co}) \quad (3.17)$$

For small impurity concentration and for $T \simeq T_c$

$$B_1(\tilde{C},T_c) = 8.414 - 32.47\,\bar{C}\,(T_{co}/T_c)\,(1-\xi^2) + \langle a^2\rangle[42.07 - \bar{C}\,(T_{co}/T_c)\,16.235[15 - 8\xi - 5\xi^2 - 2\xi^3 + 10\,\bar{V}_1^2]] \quad (3.18)$$

The zero temperature order parameter for a pure superconductor $\Delta(0,0)$ can be obtained as [8]

$$1 = N(0)V\langle(1 + a(\Omega))^2 \int \frac{d\xi_p \tanh(\beta E_p/2)\rangle}{2E_p}$$

$$= N(0)V\langle(1 + a(\Omega))^2 \int_0^{\omega_D} \frac{d\xi_p}{E_p}\rangle$$

$$E_p = \sqrt{[\xi_p^2 + \Delta_p^2]}$$

$$\frac{1}{N(0)V} = [1 + \langle a^2\rangle] \ln[2\omega_D/\Delta(0,0)] - (3/2)\langle a^2\rangle \quad ..(3.21)$$

$$\Delta(0,0) = 2\omega_D[1 + \langle a^2\rangle[(1/N(0)V) - (3/2)]] \exp[-1/(N(0)V] \quad (3.22)$$

From equation (2.10) of Clem [8] we obtain

$$\Delta(o,0) = 3.528(T_{co}/2)[1 - 1.5\langle a^2\rangle] \quad (3.23)$$

Combining equations (3.14) and (3.23) one gets

$$\frac{\Delta^2(\bar{C},T,\Omega)}{\Delta^2(0,0)} = \frac{32\pi^2 T_c^2 (1\ T/T_c)[1 - 4.935\ell_c]}{B_1(\bar{C},T)T_{co}^2(3.528)^2(1 - 3\langle a^2\rangle)} \quad (3.24)$$

For pure superconductor on finds

$$\frac{\Delta^2(0,T,\Omega)}{\Delta^2(0,0)} = 3.016\ [1 - 2\langle a^2\rangle](1 - T/T_c) \quad (3.25)$$

Equation (3.25) is in agreement with equation (2.14) of Clem [8] in the limit of $T \simeq T_c$. Figure 1. shows the variation of $\Delta(\bar{C},T,\Omega)/\Delta(0,T)$ with T/T_c near T_c. Significant deviation of the normalized order parameter from that without anisotropy is observed.

From equation (3.8) and (3.14) we find the slope of $J_s(\bar{C},T,\Omega)$ against temperature at T_c. This slope depends on the magnetic moment of the impurity. We define

the slope of $J_s(\tilde{C},T,\Omega)$ against temperature as

$$K^* = \lim_{\tilde{C}\to 0} \frac{[(\partial/\partial T)J_s(\tilde{C},T,\Omega)-(\partial/\partial T)J_s(0,T,\Omega)]/\tilde{C}(\partial/\partial T)J_s(0,T,\Omega)}{(T_c - T_{co})/\tilde{C}T_{co}} \tag{3.26}$$

while writing the above equation we have used

$$B_o(\tilde{C},T_c) = \ln(T_{co}/T_c)$$

Following the same procedure as given in Ref. [42] we obtain the slope of $J_s(\tilde{C},T,\Omega)$ against temperature at T_c as

$$K^* = 1.691 - 0.781[1 + \xi^2 + \langle a^2\rangle/(2(1+\xi)) \times (5 - \xi - 10\xi^2 - 10\xi^3)] \tag{3.27}$$

In the isotropic limit ($\langle a^2\rangle = 0$) our equation (3.27) is in agreement with equation (27) of Nagi and Upadhyaya [42]. The effect of anisotropy on K^* is represented in Figure 2.

Here we note that when $\langle a^2\rangle = 0.01$ the effect of anisotropy is to reduce K^* for $\xi \lesssim 0.54$ and increases K^* for $\xi \gtrsim 0.54$. Our result shows a significant deviation from previous calculations.

3.2 Thermodynamic Critical Field.

In this section the thermodynamic critical field for an anisotropic superconductor with local states within the gap will be derived. The critical field density for a bulk specimen is calculated from the relation

$$H_c^2(\bar{C},T,\Omega) = 8\pi F_{N-S}(\bar{C},T,\Omega) \quad .. \quad (3.28)$$

where $F_{N-S}(\bar{C},T,\Omega)$ is the difference in the Helmholtz free energy density of the alloy in the normal and superconducting phase. We have [43]

$$F_{N-S}(\bar{C},T,\Omega) = -\int_0^{|g|} \partial\left(\frac{1}{|g|}\right) \Delta^2(\bar{C},T)$$

$$= [N(0)/2]\, B_1(\bar{C},T,\Omega)\, \Delta^4(\bar{C},T) \quad ..(3.29)$$

The critical field is obtained as

$$H_c^2(\bar{C},T,\Omega) = [N(0)B_1(\bar{C},T,\Omega)\, \Delta^4(\bar{C},T)]/(2\pi T_c^2) \quad (3.30)$$

and

$$\left[\frac{H_c(\bar{C},T,\Omega)}{H_c(0,T,\Omega)}\right]^2 = \left[\frac{B_1(\bar{C},T,\Omega)}{B_1(0,T,\Omega)}\right]\left[\frac{\Delta(\bar{C},T)}{\Delta(0,T)}\right]^4\left[\frac{T_{co}}{T_c}\right]^2 \quad (3.31)$$

Near T_c and for low impurity concentration equation (3.30) can be simplified further. In the limit of $\bar{C}\to 0$ and $\xi = 1$ (AG approximation) we have

$$\frac{H_c^2(C,T,\Omega)}{8\pi\gamma T_c} = 0.71306[\ 1.4\langle a^2\rangle][\ 1 - (T/T_c)\ ]^2 \quad (3.32)$$

where $\gamma = [2N(0)/(3\pi^2)]$ (in energy units)

Near T_c , $[1 - (T/T_c)]^2$ is very small. Hence equation (3.32) is in agreement with equation (2.25) of J.R.Clem [8] in the appropriate limit. Figure 3 represents the normalized thermodynamic critical field versus T/T_c.

3.3 Density of States.

It is customary to work with the density of states in K - space, but here it is much more convenient to consider the density of states for single particle excitations, $N(\omega)$ in ω - space. The reason is this: because of collision broadening, the energy of a given momentum state is spread throughout a region in K - space whose width in K - space is Γ_1/v_F. On the other hand, the energy ω is conserved in each collision so that there is no broadening. We there fore calculate the density of states $N(\omega)$ as[46],

$$N(\omega) = -\int \frac{d^3k}{(2\pi)^3} \frac{1}{\pi} \text{Im } G_{11}(k,\omega+i\eta)$$

$$= -\int \frac{d^3k}{(2\pi)^3} \frac{1}{\pi} \text{Im } \frac{\tilde{\omega} + \xi}{\tilde{\omega}^2 - \xi^2 - \Delta^2}$$

$$= N(0) \text{ Re } \frac{\tilde{\omega}}{\sqrt{(\tilde{\omega}^2 - \Delta^2)}}$$

$$= N(0) \text{ Re } U/\sqrt{[U^2-1]} \qquad (3.33)$$

where Im stands for the imaginary part and Re for real part. Here $N(0)$ is the density of states at the Fermi surface. The path of integration can be deformed so that one encircles the positive real axis in a negative sense. $\tilde{\omega}$ and Δ are given by equations (2.71) and (2.72). Anisotropy can be introduced into the problem by assuming

$$U = A + a(\Omega).B \qquad (3.34)$$

Hence

$$\tilde{\omega} = \omega + \Gamma_1 \int \frac{d\Omega}{4\pi} \frac{(A+a(\Omega).B)\sqrt{[1-(A+a(\Omega).B)^2]}}{[\xi^2 - (A+a(\Omega).B)^2]} \qquad (3.35)$$

$$\tilde{\Delta} = \Delta + \Gamma_2 \int \frac{d\Omega}{4\pi} \frac{\sqrt{[1-(A+a(\ ).B)^2]}}{[\xi^2 - (A+a(\Omega).B)^2]} \qquad (3.36)$$

expanding in terms of $a(\Omega)$ we get, by equating the isotropic part

$$\Delta_0 A = \omega + \Gamma_1 \frac{\sqrt{[1-A^2]}.A}{(\xi^2 - A^2)} - \Gamma_2 \frac{A\sqrt{[1-A^2]}}{(\xi^2 - A^2)} \qquad (3.37)$$

Or

$$A = (\omega/\Delta_0) - \left[(\Gamma_2-\Gamma_1)/\Delta_0\right] \frac{A.\sqrt{[1-A^2]}}{(\xi^2 - A^2)} \qquad (3.38)$$

Equating the anisotropic part we obtain

$$\Delta_0(A+B) + \Gamma_2 \frac{B.\sqrt{[1-A^2]}}{(\xi^2 - A^2)} = C \qquad (3.39)$$

Using equation (3.38)we obtain,

$$B = -\phi A)/[1+\Gamma_2((\omega/\Delta_0)-A)/[(\Gamma_2 - \Gamma_1).A]] \qquad (3.40)$$

A and B can be determined from the above equations (3.38) and (3.40). Hence using equation (3.33) $N(\omega)/N(0)$ is calculated. The results of these numerical computations are shown in the figure(7,8), where $N(\omega)/N(0)$ is plotted against $\omega/\Delta(\tilde{C},T)$. The asymptotic curve is $N(\omega)/N(0) = 1$, achieved when $\omega/\Delta(\tilde{C},T) \to \infty$. We have taken $\xi = 0.6$ and

the different curves correspond to different impurity concentrations. When a single impurity is added, a level is introduced at $\omega/\Delta(\bar{C},T) = 0.6$ (ie inside the BCS energy gap) but for $(\omega/\Delta(\bar{C},T) > 1$, $N(\omega)$ consists of two separate parts, one inside and one outside the BCS energy gap. We may note that the present $N(\omega)$ is very much different from the one obtained in the AG theory ($\Gamma \to 1$). We observed that the density of states of single particle excitation is great for $(\omega/\Delta(\bar{C},T)) < 1$ and small for $(\omega/\Delta(\bar{C},T)) > 1$ for anisotropic superconductor.

3.4 Ultrasonic attenuation coefficient.

As an extension to the theory of Shukla and Nagi [44], the anisotropic effect on the ultrasonic attenuation coefficient for a magnetically doped superconductor is derived in this section.

The normalized value of the longitudinal ultrasonic attenuation coefficient (α_S/α_N) near T_c is obtained as

$$[\alpha_S/\alpha_N] = 1 - [\Delta^2(\tilde{C},T,\Omega)/(8\pi^2T^2)][(2\pi T/\alpha)\,\psi^1((1/2)+(\alpha/2\pi T)) - \psi^2((1/2)+(\alpha/2\pi T))] \quad (3.41)$$

where $\Delta^2(\tilde{C},T,\Omega)$ is given by equation (3.9)

when $(\alpha/2\pi T) >> 1$ (high impurity concentration)

$$\psi^1((1/2)+(\alpha/2\pi T)) = (2\pi T/\alpha) \quad (3.42)$$

and

$$\psi^2((1/2)+(\alpha/2\pi T)) = -[2\pi T/\alpha]^2 \quad (3.43)$$

Using equations (3.42) and (3.43) equation (3.41) can be written as

$$[\alpha_S/\alpha_N] = 1 - [\Delta^2(\tilde{C},T,\Omega)/\alpha^2] \quad (3.44)$$

The order parameter $\Delta(\tilde{C},T,\Omega)$ can be rewritten by considering various quantities near T_c.

The transition temperature for an impure anisotropic superconductor can be written as [14]

$$T_c^2 = (6\alpha^2/\pi^2)[\ 1 + \langle a^2\rangle(\alpha^2/\beta^2)]^{-1}[\ln(\pi T_{co}/2\gamma\alpha) + \langle a^2\rangle \ln(\pi T_{co}/2\gamma\beta)] \qquad (3.45)$$

Also

$$\ln(T/T_{co}) = -[(\Delta^2(\bar{C},T,\Omega)B_1(\bar{C},T)/(8\pi^2T^2))] - B_o(\bar{C},T) \qquad (3.46)$$

where

$$B_o(\bar{C},T) = [\ 1 + \langle a^2\rangle]^{-1}[\ \ln(4\gamma\rho) + (\pi^2T^2/6\alpha^2) + \langle a^2\rangle [\ \ln(4\gamma\sigma) + (\pi^2T^2/6\beta^2)]] \qquad (3.47)$$

$B_1(\bar{C},T)$ is given by equation (3.12). $b_o(\bar{C},T)$ for large impurity concentration can be given by

$$b_o(\bar{C},T) = -(1/2)[2\pi T_c/\alpha]^2 - (\alpha/6\pi T)(2\xi^2-1)(2\pi T_c/\alpha)^3 = (5 - 4\xi^2)(2\pi T_c)^2/6\alpha^2 \qquad (3.48)$$

Near T_c polygamma function can be expanded as

$$\psi^1((1/2)+\rho) = \psi^1((1/2)+\rho_c) + (T - T_c)\,\psi^2((1/2)+\rho_c)\frac{d\rho}{dT} \qquad (3.49)$$

$$\frac{d\rho}{dT} = -\rho_c/T_c$$

Hence

$$\psi^1((1/2)+\rho) = \psi^1((1/2)+\rho_c)+(1 - T/T_c)\,\rho_c\psi^2((1/2)+\rho_c) \tag{3.50}$$

$$\ln(T_{co}/T) = \ln(T_{co}/T_c) + (1 - T/T_c) \tag{3.51}$$

$$B_o(\bar{C},T_c) = \psi((1/2)+\rho_c) - \psi(1/2) + \langle a^2\rangle[\psi((1/2)+\sigma_c) - \psi((1/2)+\rho_c)] \tag{3.52}$$

$$b_1(\bar{C},T_c) = (3/\sigma_c\rho_c) - [\bar{C}(1-\varepsilon)\,(2\varepsilon^2-1)(T_{co}/T_c)]\,[[1/(\sigma-\rho)^2][(\sigma_c+\rho_c)/\sigma_c\rho_c][T/T_c]] - [2/(\sigma_c-\rho_c)^3][\ln\sigma_c - \ln\rho_c]] \tag{3.53}$$

Using equations (3.45) and (3.46) it can be shown that

$$\ln(T/T_{co}) = -[\Delta^2(\bar{C},T,\Omega)B_1(\bar{C},T)/8\pi^2T^2] + [1+\langle a^2\rangle]^{-1}x\,[(\pi^2T_c^2/6\alpha^2)[1+\langle a^2\rangle(\alpha^2/\beta^2)] - (\pi^2T^2/6\alpha^2) - \langle a^2\rangle(\pi^2T^2/6\beta^2)] - \ln(T_{co}/T_c) - \langle a^2\rangle\ln(T_{co}/T_c) \tag{3.54}$$

Equations (3.46) - (3.53) give

$$\Delta^2(\bar{C},T,\Omega) = (8\pi^2T^2/[b_o(\bar{C},T)+\langle a^2\rangle b_1(\bar{C},T)])\,x\,[(\pi^2T_c^2/6\alpha^2) - (\pi^2T^2/6\alpha^2) + \langle a^2\rangle[(\pi^2T_c^2/6\beta^2) - (\pi^2T^2/6\beta^2)]] \tag{3.55}$$

For isotropic case ($\langle a^2 \rangle = 0$) equation (3.55) will be reduced to

$$\Delta^2(\bar{C},T,\Omega) = [8\pi^2T^2 6\alpha^2/(5-4\,\xi^2)(2\pi T)^2] \times [(\pi^2T_c^2/6\alpha^2) - (\pi^2T^2/6\alpha^2)]$$

$$= (2\pi^2T_c^2/(5-4\,\xi^2))[1 - (T/T_c)^2] \qquad (3.56)$$

Equation (3.56) is in agreement with equation (3.11) of Shukla and Nagi [44].

Combining equations (3.44) and (3.55), (3.45) we get

$$[\alpha_S/\alpha_N] \quad 1 - [8\pi^2T_c^2/[b_o(\bar{C},T)+\langle a^2\rangle b_1(\bar{C},T)]]^{-1} \times [[\pi^2T_c^2/6] - (\pi^2T^2/6) + \langle a^2\rangle[(\pi^2T_c^2/6) - (\pi^2T^2/6)]\times (\alpha/\beta)^2 \qquad (3.57)$$

In the case when $\langle a^2 \rangle = 0$ equation (3.57) can be written as

$$[\alpha_S/\alpha_N] = 1 - [12[1 - (T/T_c)^2]\ \ln[\pi T_{co}/2\gamma\alpha]]/(5 - 4\,\xi^2) \qquad (3.58)$$

Equation (3.58) is in agreement with equation (3.13) of Shukla and Nagi [44]. The initial slope at T_c is given by

$$C^* = \frac{\partial}{\partial t}(\alpha_S/\alpha_N)\Big|_{T \to T_c} = [8\pi^2T_c^2/\alpha^2][b_o(\bar{C},T)+\langle a^2\rangle b_1(\bar{C},T)]^{-1} \times [(\pi^2/6\alpha^2) + \langle a^2\rangle(\pi^2/6\beta^2)] \qquad (3.59)$$

It can be shown that in the isotropic limit equation (3.59) will be reduced to equation (3.14) of Shukla and Nagi [44]. C^*_{AG} is obtained by putting $\xi = 1$ in equation (3.59).

$$C^*_{AG} = 24[1 + \langle a^2\rangle 4[1 - (b_1(\bar{C},T_c)/b_o(\bar{C},T_c))] \times \ln[(\pi T_{co}/2\gamma\alpha)[\pi T_{co}/2\gamma\beta]^{\langle a^2\rangle}]/[1 + \langle a^2\rangle 4] \quad (3.60)$$

For large impurity concentration

$$\frac{C^*}{C^*_{AG}} = [5-4\xi^2]^{-1}[1 + \langle a^2\rangle[(1+\xi)^2 - (1+\xi)^2 \times (b_1(\bar{C},T)/b_o(\bar{C},T))]][1+4\langle a^2\rangle][1+\langle a^2\rangle(1+\xi)^2]^{-1} [1+4\langle a^2\rangle[1 - (b_1(\bar{c},T_c)/b_o(\bar{C},T_c)]^{-1} \quad (3.61)$$

$$\frac{C^*}{C^*_{AG}} = 1 \quad \text{when} \quad \xi = 1$$
$$= 0.283 \quad ,, \quad \xi = 0.6 \quad (3.62)$$
$$= 0.204 \quad ,, \quad \xi = 0$$

It was observed that when $\xi = 0.6$ % change in C^*/C^*_{AG} = 0.936 and when $\xi = 0$, % change in C^*/C^*_{AG} = 2.245

When $(\alpha/2\pi T) \ll 1$ (low impurity concentration equation (3.41) can be shown to be

$$[\alpha_S/\alpha_N] = 1 - [\Delta^2(\bar{C},T)/8\pi^2T^2][(2\pi T/\alpha)[\psi^1(1/2) + (\alpha/2\pi T)\psi^2(1/2) + (\alpha/2\pi T)^2(1/2)\psi^3(1/2)] - \psi^2(1/2) - (\alpha/2\pi T)\psi^3(1/2)] \quad (3.63)$$

Also

$$\Delta^2(\bar{C},T) = -[8\pi^2T^2/B_1(\bar{C},T)][B_o(\bar{C},T) + \ln(T/T_{co}) \quad (3.64)$$

Where

$$B_o(\bar{C},T) = \rho\psi^1(1/2) + \langle a^2\rangle [\sigma - \rho]\psi^1(1/2)$$

$$= 4\lambda(2)[\rho + \langle a^2\rangle(\sigma - \rho)]$$

$$b_o(\bar{C},T) = 8\lambda(3) - 32\lambda(4)(\alpha/2\pi T)(1+\xi^2) \quad (3\text{-}65)$$

$$b_1(\bar{C},T) = -(\rho/6)\psi^3(1/2)(2\xi^2-1) + 48[\lambda(3) - 3\lambda(4)(\sigma+\rho)] \quad (3.66)$$

Hence $\Delta^2(\bar{C},T)$ is obtained as

$$\Delta^2(\bar{C},T) = -[8\pi^2T^2/B_1(\bar{C},T)][4\lambda(2)[\rho + \langle a^2\rangle(\sigma - \rho)] + \ln(T/T_{co})] \quad (3.67)$$

If we put

$$A(T) = \ln(T/T_{co}) + 4\lambda(2)[\rho + \langle a^2\rangle(\sigma - \rho)]$$

Using above equations the normalized ultrasonic attenuation coefficient is given as

$$[\alpha_S/\alpha_N] = 1+(1/B_1(\bar{C},T))[(2\pi T/\alpha)\psi^1(1/2) - (\alpha/2\pi T)\psi^3(1/2)]A(T)$$

$$= 1+(A(T)/B_1(T)[(2\pi T/\alpha)4\lambda(2) - (\alpha/2\pi T)96\lambda(4)] \quad (3.68)$$

$$[\alpha_S/\alpha_N] = 1 + [8\pi TA(T)\lambda(2)/(B_1(\bar{C},T)\alpha)] [1 - (\alpha/2\pi T)^2 24\lambda(4)/\lambda(2)] \quad (3.69)$$

Since $(\alpha/2\pi T) << 1$ the second term in the bracket can be neglected. Hence equation (3.69) is in agreement with equation (3.16) of Shukla and Nagi[44]. But $A(T)$ and $B_1(\bar{C},T)$ are different because of anisotropy.

The slope of the normalized ultrasonic attenuation coefficient is obtained as

$$C^* = [4\lambda(2)/(\alpha/2\pi T_c)B_1(\bar{c},T_c)] \left.\frac{\partial A(T)}{\partial T}\right|_{T\to T_c} \quad (3.70)$$

$$\frac{\partial A(T)}{\partial T} = 1 - [4\lambda(2)/2\pi T_c][\alpha + \langle a^2\rangle(\beta - \alpha)] \quad (3.71)$$

$$C^* = [8\pi T_c\lambda(2)[1 - (4\lambda(2)/2\pi T_c)[\alpha + \langle a^2\rangle(\beta-\alpha)]] \times [1+\langle a^2\rangle](1/\alpha)[8\lambda(3) - 32\lambda(4)(\alpha/2\pi T_c)(1+\varepsilon^2) + \langle a^2\rangle[(\alpha/6)(2\varepsilon^2-1)\psi^3(1/2)/(1+\varepsilon) + 48[\lambda(3) - 3\lambda(4)(\sigma+\rho)]]^{-1} \quad (3.72)$$

It can be shown that in the isotropic limit equation (3.72) reduces to equation (3.20) of Shukla and Nagi[44].

$$C^*_{AG} = [1+\langle a^2\rangle](4\lambda(2)/\rho)[1-4\lambda(2)[\rho+\langle a^2\rangle(\sigma-\rho)]] \times$$
$$[8\lambda(3) - 64\lambda(4)\rho + \langle a^2\rangle[48[\lambda(3) -$$
$$3\lambda(4)(\sigma+\rho)]]]^{-1} \qquad (3.73)$$

Hence from equation (3.72) and (3,73) $[C^*/C^*_{AG}]$ can be derived.

$$[C^*/C^*_{AG}] = [8.416 - 64.96\rho_c + \langle a^2\rangle[50.49 - 227.27\rho_c]] \times$$
$$[8.416 - 32.48\,\rho_c(1+\ ^2) + \langle a^2\rangle[50.49 -$$
$$146.1\,\rho_c[(2+\varepsilon)/(1+\varepsilon)] - 16.24\,\rho_c(2\varepsilon^2-1)/(1+\varepsilon)]^{-1}$$

(3.74)

3.5 Nuclear Spin Relaxation.

In this section the behaviour of the normalized value of unclear spin relaxation rate $[R_S/R_N]$ near T_c will be discussed.

In this limit one obtains

$$[R_S/R_N] = 1 + [\Delta^2(\bar{C},T)/8\pi^2T^2][(2\pi T/\alpha)\psi^1((1/2) + (\alpha/2\pi T)) + 3\psi^2((1/2) + (\alpha/2\pi T))] \quad (3.75)$$

For high impurity concentration $(\alpha/2\pi T) >> 1$, the above equation gives

$$[R_S/R_N] = 1 + [\Delta^2(\bar{C},T)/8\pi^2T^2][(1/\rho^2) + 3(-1/\rho^2)]$$

$$= 1 - [\Delta^2(\bar{C},T)/\alpha^2] \quad (3.76)$$

The initial slope of R_S/R_N can be shown to be of the same form as C^* in equation (3.72)

For $(\alpha/2\pi T) << 1$ (low impurity concentration) we get

$$[R_S/R_N] = 1 - [4\lambda(2)/\rho][A(T)/B_1(\bar{C},T)][1 - 16\lambda(3)/\lambda(2)] \quad (3.77)$$

The initial slope of $[R_S/R_N]$ against T/T_c near T_c is

$$S^* = \left[\frac{d}{dt}(R_S/R_N)\right]_{T\to T_c}$$

$$S^* = -[4\lambda(2)/B_1(\bar{C},T_c)][1-(16\lambda(3)/\lambda(2)][1 - 4\lambda(2)[\rho + \langle a^2\rangle(\beta-\alpha)]] \quad (3.78)$$

The isotropic limit result can be obtained from equation (3.78) and found to agree with Shukla and Nagi's result [45].

$$[\, S^*/S^*_{AG} \,] = 1 - 3.86\,(1 - \xi^2) + \langle a^2 \rangle\, 7\ 72D(\rho - \xi^2) \tag{3.79}$$

where

$$D = -\,[(\bar{C}/6)(T_{co}/T_c)(1 - \xi)(2\xi^2 - 1)\,\psi^3(1/2) + 48[\lambda(3) - 3\lambda(4)\,(\sigma + \rho)]$$

3.6 Upper Critical Field. $H_{c2}(T)$

The Upper critical field for a superconductor with anisotropic coupling will be discussed in this section. If the magnetic impurities have randomly oriented spins, we have for an anisotropic superconductor [45]

$$-\ln[T/T_{co}] = \psi[(1/2)+(\alpha/2\pi T)+(DeH_{c2}(T)/2\pi T)] - \psi(1/2) - \langle a^2\rangle[\psi[(1/2)+(\alpha/2\pi T)+(DeH_{c2}(T)/2\pi T)] - [(1/2)+(1/(1+\varepsilon)[(\alpha/2\pi T)+(DeH_{c2}(T)/2\pi T)]] \quad (3.80)$$

where D is the diffusion constant given by

$$D = \tau_{tr} v_F^2/3$$

Then we can write

$$\rho_o = \rho_i + \rho_H$$

$$\rho_i = (1/2\pi T)(1/\tau_2)$$

$$\rho_H = (1/2\pi T)\, DeH^{c}_{c2}(T)$$

we know when $T = T_c$, $H_{c2}(T) = 0$. Hence equation (3.80) reduces to

$$-\ln[T_c/T_{co}] = \psi((1/2)+(\alpha/2\pi T_c)) - \psi(1/2) + \langle a^2\rangle[\psi((1/2)+(\alpha/2\pi T_c(1+\varepsilon))) - \psi((1/2)+(\alpha/2\pi T_c))] \quad (3.81)$$

since near T_c , Δ is small and Δ^2 can be neglected.

For small impurity concentration $((\alpha/2\pi T) << 1)$

$$-\ln t_c = (\alpha/2\pi T)\psi^1(1/2) + \langle a^2\rangle[(\alpha/2\pi T_c(1+\xi))\psi^1(1/2) - (\alpha/2\pi T_c)\psi^1(1/2)] \quad (3.82)$$

$$\rho = (\alpha/2\pi T) = C(1-\xi^2)/t_c$$

$$t_c = T_c/T_{co}$$

$$-\ln t_c = (C(1-\xi^2)/t_c)\psi^1(1/2) + \langle a^2\rangle[(C(1-\xi^2)/t_c(1+\xi))\psi^1(1/2) - ((C(1-\xi^2)/t_c))\psi^1(1/2)] \quad (3.83)$$

that is

$$-t_c \ln t_c = 4.935\ C\ (1-\xi)\ [\ 1 + \xi(1 - \langle a^2\rangle)] \quad (3.84)$$

considering the BCS case $((\alpha/2\pi T) = 0)$ as $T \to 0$, $\rho_H >> 1$ we get

$$-\ln[T/T_{co}] = \ln[4\gamma D e H_{c2}(T)/2\pi T)] + \langle a^2\rangle[\ln((1/(1+\xi))] \quad (3.85)$$

that is

$$4\gamma D e H^{o}_{c2}(0)/2\pi T_{co} = (1+\xi)^{\langle a^2\rangle}$$

$$\rho_{H_{red}} = D e H^{o}_{c2}(T)/2\pi T = \frac{(1+\xi)^{\langle a^2\rangle_c} H_{c2}(T)}{4\gamma t\ H^{o}_{c2}(0)}$$

Equation (3.80) will be reduced to

$$-\ln[T/T_{co}] = \psi((1/2) + \rho_i + \rho_{H_{reduced}}) - \psi(1/2)$$
$$+ \langle a^2\rangle[\psi((1/2)+(\rho_i/(1+\varepsilon)) + \rho_{H_{red}})$$
$$- \psi((1/2) + \rho_i + \rho_{H_{red}})] \qquad (3.86)$$

For small impurity concentration and upper critical field, equation (3.80) can be written for large T

$$-\ln t = [(C(1-\varepsilon^2)/t) + \rho_{H_{red}} + \langle a^2\rangle[((1/(1+\varepsilon)))$$
$$[(C(1-\varepsilon^2)/t) + \rho_{H_{red}}] - [(C(1-\varepsilon^2)/t)$$
$$+ \rho_{H_{red}}]]] \; 4.935 \qquad (3.87)$$

For small T we obtain

$$-\ln t = \ln[4\gamma(\rho_i + \rho_{H_{red}})] + \langle a^2\rangle[\ln((1/(1+\varepsilon))] \qquad (3.88)$$

that is we get the reduced upper critical field as

$$\tilde{h}_{c2}(T) = 1 - 4\gamma C(1-\varepsilon)(1+\varepsilon)^{(1-\langle a^2\rangle)} \qquad (3.89)$$

Differentiate equation (3.80) with respect to T we obtain

$$-(1/T) = \psi^1((1/2) + \rho_i + \rho_{H_{red}}) \frac{d[\rho_i+\rho_H]}{dT} + \langle a^2\rangle[\psi^1((1/2)$$
$$((\rho_i+\rho_H)/(1+\varepsilon)) - \psi^1((1/2)+\rho_i+\rho_H)] \frac{d}{dT}(\rho_i+\rho_H) \qquad (3.90)$$

where

$$\frac{d}{dT}(\rho_i + \rho_H) = -(\alpha/2\pi T) + DeH'_{c2}(T)/2\pi T - DeH_{c2}(T)/2\pi T^2 \qquad (3.91)$$

where $H'_{c2}(T) = \frac{d}{dT}H_{c2}(T)$

For $T \rightarrow T_c$, $H_{c2}(T) \rightarrow 0$. then equation (3.91) will be reduced to

$$\begin{aligned} -[1/T_c] &= \psi^1((1/2)+\rho_c)[DeH'_{c2}(T_c)/2\pi T_c) - \rho_c/T_c] \\ &+\langle a^2\rangle[\psi^1((1/2)+\rho_c/(1+\varepsilon))[(DeH'_{c2}(T)/2\pi T_c) \\ &-\rho_c/T_c] - \psi^1((1/2)+\rho_c)[(DeH'_{c2}(T)/2\pi T_c) \\ &-\rho_c/T_c]] \end{aligned} \qquad (3.92)$$

For pure superconductors $C \rightarrow 0$, $T_c \rightarrow T_{co}$, equation (3.93) reduces to

$$-[1/T_{co}] = [DeH'_{c2}(T_{co})/2\pi T_{co}]\,\psi^1(1/2) \qquad (3.93)$$

from which we obtain

$$DeH'_{c2}(T_{co}) = -1.27 \qquad (3.94)$$

Using equation (3.94) equation (3.92) can be given by

$$\begin{aligned} -1 = &-\rho_c\,\psi^1((1/2)+\rho_c) - \langle a^2\rangle\psi^1((1/2)+(\rho_c/(1+\varepsilon))\rho_c \\ &+\langle a^2\rangle\psi^1((1/2)+\rho_c)_c - 0.2026\,\tilde{h}'_{c2}(T_c) \\ &[\psi^1((1/2)+\rho_c)) + \langle a^2\rangle[\psi^1((1/2)+(\rho/(1+\varepsilon)) - \psi^1((1/2)+\rho_c) \end{aligned} \qquad (3.95)$$

where

$$\tilde{h}'_{c2}(T_c) = \frac{H_{c2}(T_c)}{H_{c2}(T_{co})}$$

For small impurity concentration ($(\alpha/2\pi T) \ll 1$) we get

$$\tilde{h}'_{c2}(T_c) = [1 - \rho_c\, 4.935]/P \qquad (3.96)$$

where

$$P = 1 - 3.4094\,\rho_c[\,1 + (\langle a^2\rangle\, \xi\,(1+\xi))]$$

and for large impurity concentration ($(\alpha/2\pi T) \gg 1$) we obtain

$$\tilde{h}'_{c2}(T_c) = -4.935\langle a^2\rangle\, \rho_c\, \xi/[1 + \xi\langle a^2\rangle] \qquad (3.97)$$

The upper critical field $H_{c2}(T)$ of $PbMO_6S_8$ was measured in a temperature region from 1.3K to superconducting transition temperature T_c by Kiichi Okuda et al [47].

The broad transition observed in the poly crystalline sample may partly be attributed to the anisotropic $H_{c2}(T)$ in this compound. Recent measurements of $H_{c2}(T)$ on single crystal by Decroux et al [48] showed that this compound has an anisotropy of about 20% with the maximum $H_{c2}(T)$ where the field is perpendicular to the ternary axis.

W.Biberacher et al.[49] measured the upper critical field $H_{c2}(T)$ in Nb_3S_4. The observed large anisotropy of $H_{c2}(T)$ in Nb_3S_4. Our graph connecting $h'_{c2}(T)$ versus T_c/T_{co} is comparable with the corresponding graps of Jun Takeuchi et al. [50] and Guertin et al [51]. $\tilde{h}_{c2}(T)$ versus T/T_{co} can be compared with the graph of Nagaoka [52].

3.7 Response Function.

In this section we derive the response function to a weak transverse field for an isotropic superconductor described by SR model. The calculations are done in a similar line with Skalski [46]. Dick and Reif [17] have measured the real part σ_1, of the conductivity of rare earth impurities like Gd in Pb and transition metal impurities like Mn in Pb by measuring the infrared radiation absorbed by a film of the above alloys.

In the above measurements, it was found that the experimental results in the case of rare earth impurities agree with the theoretical ones derived from the AG theory. But in the case of transition metal impurities, the agreement is not satisfactory. The observed effects in this case are more pronounced than those predicted by the AG theory. In the case of rare earth impurities, the localized spin is due to the inner f - electrons and, hence, the coupling between the electrons and the impurities may be weak and the Born approximation, as used by AG, may be valid. In the case of transition metal impurities the spin of the impurities is die to the d - electrons. In this case, the coupling between the electron and magnetic impurities may not be weak, hencethe Born approximation may not be justified. So here we make an attempt to derive the response to a weak

transverse magnetic field for an isotropic superconductor described by SR model ,where the coupling between the electron and the impurities is strong.

The Green's function $\bar{G}(p,\omega_n)$ of the superconducting alloy, averaged over the positions and the spin directions of the impurities, is given by [4,5]

$$\bar{G}(p,\omega_n) = \bar{G}^o(p,\omega_n) + C_i \bar{G}^o(p,\omega_n)\, \bar{L}(p,p',\omega_n)\, \bar{G}(p,\omega_n) \quad .. \quad (3.98)$$

where $\bar{G}^o(p,\omega_n)$ is the Green's function of the pure superconductor. C_i is the impurity concentration and $L(p,p',\omega_n)$ is the exact vertex part. Further

$$\bar{G}^o(p,\omega_n) = [i\omega_n \rho_3 - \xi_p + i\Delta(0,T)\rho_1\sigma_2]^{-1} \qquad (3.99)$$

$$\omega_n = (2n+1)\pi T \quad \text{and} \quad \xi_p = (p^2/2m) - E_F$$

where E_F is the Fermi emergy, σ_i and ρ_i are the Pauli matrices operating on the ordinary spin states and on the space composed of electron and hole states respectively, while $\Delta(0,T)$ is the temperature dependent order parameter of the pure superconductor. The vertex part is related to the interaction $\bar{V}(p,p')$ between the electron and the impurity by the relation [5]

$$\bar{L}(p,p',\omega_n) = \bar{V}(p,p') + \sum_{p'} \bar{V}(p,p')\bar{G}(p_1,\omega_n)\, \bar{L}(p_1,p',\omega_n)$$

where
$$\bar{V}(p,p') = \begin{bmatrix} V_{\alpha\beta}(p,p') & 0 \\ 0 & V_{\beta\alpha}(p',p) \end{bmatrix}$$

and

$$V(p,p') = U(p,p') + J(p,p')\ \underset{\sim}{S}.\underset{\sim}{\sigma} \qquad (3.100)$$

where $U(p,p')$ referes to the potential scattering, $J(p,p')$ is the strength of the exchange interaction, $\underset{\sim}{S}$ is the spin of the impurity and $\underset{\sim}{\sigma}$ denotes the pauli matrices.

Using the above equations, and neglecting Kondo effect Rusinov [5] has calculated the Green's function for a superconducting alloy. According to him $\tilde{G}(p,\omega_n)$ may be written as

$$\tilde{G}(p,\omega_n) = [i\tilde{\omega}_n \rho_3 - \varepsilon_p + i\tilde{\Delta}_n \rho_1 \sigma_2]^{-1} \qquad (3.101)$$

where

$$\tilde{\omega}_n = \omega_n + (\pi C_i/mp_o) \sum_{\ell} (2\ell + 1)(\sin^2\delta^+ + \sin^2\delta^-)\ x$$
$$U_n/[U_n^2 + 1][U_n^2 + \cos^2(\delta^+ - \delta^-)]^{-1}$$

$$\tilde{\Delta}_n = + (2\pi C_i/mp_o) \sum_{\ell} (2\ell + 1)\ \sin\delta^+.\sin\delta^-.$$
$$\cos(\delta^+ - \delta^-)/[U_n^2 + 1][U_n^2+\cos^2(\delta^+-\delta^-)]^{-1}$$

while

$$U_n = \tilde{\omega}_n/\tilde{\Delta}_n. \quad \text{and} \quad \varepsilon_\ell = \cos(\delta_\ell^+ - \delta_\ell^-)$$

Various physical quanties can be calculated in a closed form when $\ell = 0$ (isotropic scattering). Hence we get

$$\tilde{\omega}_n = \omega_n + \Gamma_1\ [U_n/[1+U_n^2][U_n^2 + \varepsilon^2]^{-1} \qquad (3.102)$$

$$\tilde{\Delta}_n = \Delta + \Gamma_2 U_n \sqrt{[1+U_n^2][U_n^2+\xi_o^2]^{-1}} \qquad (3.103)$$

$$\xi_o^2 = \cos^2(\delta_o^+ - \delta_o^-)$$

We investigate the effect of magnetic impurities on the electrodynamic properties of superconductors in SR model. As a first problem we calculate the response function.

The response to a weak transverse field is described in terms of the wave-number and frequency-dependent kernel $Q(q,q_o)$ (q - wave number and q_o frequency).

$$j(q,q_o) = Q(q,q_o)\, A(q,q_o) \qquad (3.104)$$

where $A(q,q_o)$. $q = 0$

We expect that most samples of the alloys will have short mean free paths so that we can assume an essentially local relation between field and current, and therefore have neglected the dependence of the response function on the wave number q. Hence

$$Q(0,q_o) = \sigma^N_{q_o} + K(0,q_o) \qquad (3.105)$$

$$\sigma^N = Ne^2\tau/m\,, \qquad \tau = 1/2\Gamma_2$$

Following the calculations of Skalski [46] the zero-temperature response function is obtained as

$$K(0,q_o) = (Ne^2 i/2mC)\int d\omega \;\; [1/\sqrt{[\Delta_+^2 - \omega_+^2]}] + \int_{-\infty}^{+\infty} \frac{dt}{\pi} \frac{[\omega_+\omega_- + \Delta_+\Delta_- + \omega_-^2 - \Delta_-^2]}{(\xi^2 + \Delta_+^2 - \omega_+^2)(\xi^2 + \Delta_-^2 - \omega_-^2)} \qquad (3.106)$$

C

The second integration can be performed and can be shown to be equal to $[\sqrt{(\Delta_+^2 - \omega_+^2)(\Delta_-^2 - \omega_-^2)}]^{-1}[\sqrt{(\Delta_+^2 - \omega_+^2)} + \sqrt{(\Delta_-^2 - \omega_-^2)}]^{-1}$

Hence from equation (3.106) we get

$$K(0,q_o) = \frac{Ne^2}{2mc}\int_C d\omega \frac{[1 - (1+U_+U_-)[\sqrt{(U_+^2 - 1)(U_-^2 - 1)}]^{-1}}{[\Delta_+\sqrt{(U_+^2 - 1)} + \Delta_-\sqrt{(U_-^2 - 1)}]} \tag{3.107}$$

Near T_c $\int_{-\infty}^{+\infty} d\omega \longrightarrow 2\pi T \sum_{n=0}^{\infty}$

Finite temperature response function (near T_c) is obtained as

$$K(0,q_o) = (Ne^2/mc)(2\pi T)\sum_{n=0}^{\infty}[1 - \frac{[U_+U_- - 1]}{\sqrt{(U_+^2+1)(U_-^2+1)}}] \times$$

$$[\Delta_+\sqrt{(U_+^2+1)} + \Delta_-\sqrt{(U_-^2+1)}]^{-1} \tag{3.108}$$

where

$$\tilde{\omega}_+ = \omega + (q/2) + \Gamma_1[\sqrt{U_+^2 + 1}]U_+/[U_+^2 + \xi^2] \tag{3.109}$$

and $$\tilde{\Delta}_+ = \Delta + \Gamma_2[\sqrt{(U_+^2+1)}]/[U_+^2 + \xi^2] \tag{3.110}$$

$$\tilde{\omega}_- = \omega - (q/2) + \Gamma_1[U_-\sqrt{(U_-^2+1)}]/[U_-^2 + \xi^2] \tag{3.111}$$

$$\tilde{\Delta}_- = \Delta + \Gamma_2[\sqrt{(U_-^2+1)}]/[U_-^2 + \xi^2] \tag{3.112}$$

$$U_+ = \tilde{\omega}_+/\tilde{\Delta}_+$$

$$U_- = \tilde{\omega}_-/\tilde{\Delta}_-$$

Near T_c equations (3.110) and (3.112) become

Equation (3.108) reduces to

$$K(0,q_o) = (Ne^2/mc\Gamma_2)(2\pi T)\sum_{n=0}^{\infty}\left[1 - \frac{[U_+U_- - 1]}{[\sqrt{(U_+^2+1)(U_-^2+1)}]}\right] x$$

$$[[(U_+^2+1)/(U_+^2+\xi^2)] + [(U_-^2+1)/(U_-^2+\xi^2)]]^{-1} \qquad (3.113)$$

Near T_c, Δ is small and U will be very large. So we can expand various quanties as following

$$\frac{U_+^2+1}{U_+^2+\xi^2} = 1 - (\xi^2/U_+^2) + (1/U_+^2) \qquad (3.114)$$

Equation (3.113) can be shown to be

$$K(0,q_o) = (Ne^2/mc\Gamma_2)(2\pi T)\sum_{n=0}^{\infty}(1/4)[[(1/U_+^2) + (1/U_-^2)$$

$$+(2/(U_+U_-)) + (\xi^2-2)[(1/U_+^3U_-) + (1/U_+U_-^3)]$$

$$+(\xi^2-(3/2))(1/U_+^2U_-^2) + (\xi^2-(5/4)) x$$

$$[(1/U_+^4) + (1/U_-^4)]] \qquad (3.115)$$

But U_+ and U_- are obtained from Ref. [53] as given below

$$[1/U_+] = [\Delta/(\omega_+ + \alpha)] + [(2\xi^2-1)\alpha\,\Delta^3/(2(\omega_+ + \alpha)^4)$$

$$[1/U_-] = [\Delta/(\omega_- + \alpha)] + [(2\xi^2-1)\alpha\,\Delta^3/(2(\omega_- + \alpha)^4)$$

$$[1/U_+U_-] = \Delta^2[(\omega+(q/2)+\alpha)(\omega-(q/2)+\alpha)]^{-1} + (2\xi^2-1)\alpha\Delta^4 x$$

$$[2(\omega+(q/2)+\alpha)^4(\omega-(q/2)+\alpha)]^{-1} + (2\xi^2-1)\alpha\Delta^4 x$$

$$[2(\omega+(q/2)+\alpha)(\omega-(q/2)+\alpha)^4]^{-1}$$

where we use

$$\omega_+ = \omega + (q/2)$$

$$\omega_- = \omega - (q/2)$$

$$\frac{1}{U_+^2} = \Delta^2[\omega+(q/2)+\alpha]^{-2} + \alpha\Delta^4(2\xi^2-1)[\omega+(q/2)+\alpha]^{-5}$$

$$[1/U_-^2] = \Delta^2[\omega-(q/2)+\alpha]^{-2} + \alpha\Delta^4(2\xi^2-1)[\omega-(q/2)+\alpha]^{-5}$$

$$[1/U_+^2U_-^2] = \Delta^4[(\omega+(q/2)+\alpha)^2(\omega-(q/2)+\alpha)^2]^{-1}$$

$$[1/U_+^3U_-] = \Delta^4[(\omega+(q/2)+\alpha)^3(\omega-(q/2)+\alpha)]^{-1}$$

$$[1/U_+U_-^3] = \Delta^4[(\omega+(q/2)+\alpha)(\omega-(q/2)+\alpha)^3]^{-1}$$

After doing partial fraction and converting sum into polygama function equation (3.115) becomes

$$K(0,q_0) = [\sigma^N\pi T/2C][(\Delta^2/(2\pi T)^2)[\psi^1((1/2)+\rho+q/4\pi T) + \psi^1((1/2)+\rho-q/4\pi T)] - [(2\xi^2-1)\rho\Delta^4/(24(2\pi T)^4)[\psi^4((1/2)+\rho+q/4\pi T) + \psi^4((1/2)+\rho-q/4\pi T)] + (\Delta^2/q\pi T)[\psi((1/2)+\rho+q/4\pi T) - \psi((1/2)+\rho-q/4\pi T)] + (2\xi^2-1)\rho\Delta^4[(1/(q^3 2\pi T))[\psi^1((1/2)+\rho-q/4\pi T) - \psi^1((1/2)+\rho+q/4\pi T)] + (1/2q^2(2\pi T)^2)\ x [\psi^2((1/2)+\rho+q/4\pi T) + \psi^2((1/2)+\rho-q/4\pi T)]$$

contd.

$$+(1/6q(2\pi T)^3)[\psi^3((1/2)+\rho-q/4\pi T)-\psi^3((1/2)+\rho+q/4\pi T)]]$$

$$+(\xi^2-2)\Delta^4[(1/q^3\pi T)[\psi((1/2)+\rho+q/4\pi T)-$$

$$\psi((1/2)+\rho-q/4\pi T)]-(1/q^2(2\pi T)^2)[\psi^1((1/2)+\rho+q/4\pi T)$$

$$+\psi^1((1/2)+\rho-q/4\pi T)]-(1/2q(2\pi T)^3)\ x$$

$$[\psi^2((1/2)+\rho-q/4\pi T)-\psi^2((1/2)+\rho+q/4\pi T)]$$

$$+(\xi^2-(3/2))[(2\Delta^4/q^3(2\pi T))[\psi((1/2)+\rho-q/4\pi T)$$

$$-\psi((1/2)+\rho+q/4\pi T)]+(\Delta^4/q^2(2\pi T)^2[\psi^1((1/2)+\rho+q/4\pi T)$$

$$+\psi^1((1/2)+\rho-q/4\pi T)]]+(\xi^2-(5/4)\Delta^4/6(2\pi T)^4)\ x$$

$$[\psi^3((1/2)+\rho+q/4\pi T)+\psi^3((1/2)+\rho-q/4\pi T)]] \qquad (3.116)$$

where

$$\sigma^N = Ne^2/2m\Gamma_2$$

For small q_o we get

$$K(0,q_o) = \frac{\sigma^N}{C\pi T}[\Delta^2\psi^1((1/2)+\rho)+(\Delta^2/12)(q/2\pi T)^2\psi^3(1/2+\rho)$$

$$-(\Delta^4/(2\pi T)^2)[(2-\xi^2)/6)\psi^3((1/2)+\rho)$$

$$+(2\xi^2-1)\rho\psi^4((1/2)+\rho)/24]] \qquad (3.117)$$

as $q_o \rightarrow 0$ equation (3.117) agrees with equation (9) of K.Maki[54] when $\omega >> \alpha$.

$$Q(0,q) = \sigma^N q + K(0,q)$$

and from Skalski [46] the conductivity is given by

$$\text{Re}\ \sigma(0,q) + i\text{Im}\ \sigma(0,q) = (C/iq)\ K(0,q) \qquad (3.118)$$

3.8 Kondo Effect in the Problem

In the Shiba - Rusinov model, the Kondo effect is neglectd. The effect is essentially arises while considering the scattering of a conduction electron from a magnetic impurity exactly if, (i) the exchange coupling between the electron and the impurity spins is antiferromagnetic and (ii) the non-commutativity of the spin operators is taken into account. The role of the Kondo effect in superconductors is considerably clarified by the work of Muller - Hartmann and Zittartz [3]. Within certain approximations one can include the Kondo effect in the SR model by assuming that

depends on $\nu = \ln[T_K/T]$, where T_K is the Kondo temperature). These approximations are (i) the electron energy is near the Fermi energy (ii) the temperature T is near the transition temperature T_c and (iii) the impurity concentration is very low. The actual relation between and is more complicated but several authors have used the simpler relation

$$\varepsilon^2 = \nu^2[\nu^2 + \pi^2 S(S+1)]^{-1}$$

where S is the impurity spin. We take $\nu = \ln[T_K/T_c]$

$$\nu_c = \ln[T_K/T_{co}] + \ln[T_{co}/T_c]$$

Taking S = 1/2 we get

$$\varepsilon^2 = \frac{0.1351 \ln^2(T_c/T_K)}{[\,0.1351 \ln^2(T_c/T_K) + 1\,]}$$

We have calculated the slope K^* of the Josephson current versus T/T_c, K^* of the Josephson current versus T_c/T_{co} for various values of T_K/T_{co} and are tabulated in the tables.

We have also observed that ξ is anisotropic. We have tabulated the different values of the local states within the gap ξ for $\langle a^2 \rangle = 0$ and $\langle a^2 \rangle = 0.05$. We observed significant change in ξ due to anisotropy.

CHAPTER 4

SUMMARY

I have investigated the effect of gap anisotropy on the Josephson current for an impure superconductor described by SR model. It is observed that as the impurity concentration is increased, the Josephson current gradually decreased and finally goes to zero for a particular value of the impurity concentration. For a given impurity concentration if we increase ξ the Josephson current also increases. The values of the normalized Josephson current for various values of impurity concentration and for two values of the local states within the gap ξ are given in Table 1. Figure 1 shows the nature of variation of the normalized Josephson current with T/T_c for $\xi = 0.95$. We noticed that when $\bar{C} = 0.2$, the effect of anisotropy is to decrease Josephson current. As we increase the impurity concentration the Josephson current will be increased due to anisotropy.

When $T_K/T_{co} > 1$ as we increase (T/T_{co}) we found that the percentage change in the slope of Josephson current versus (T/T_{co}), K^* gradually decreases where as when (T_K/T_{co}) is less that one the above quantity incrrases as we increase (T/T_{co}).

The slope of Josephson current versus (T_c/T_{co}), K^* is

tabulated for various values of T_K/T_{co}. For $(T_K/T_{co}) > 1$, as more and more impurities are added, the effect of anisotropy is to increase the percentage change in K^* due to anisotropy. For $(T_K/T_{co}) < 1$, increase of impurity will decrease the percentage change in K^* due to anisotropy.

When $\langle a^2 \rangle = 0.01$, $K^* = 0.16$ giving a 24.03% deviation from previous calculations. $K^*_{AG} = 0.129$ (for $\xi = 1$ and $\langle a^2 \rangle = 0$). As the strength of interaction increases ($\xi < 1$) the effect of anisotropy on $\Delta(\tilde{C},T)$, $H_c(\tilde{C},T,\Omega)$ and $J_s(\tilde{C},T,\Omega)$ is small, but if the strength of interaction decreases ($\xi \simeq 1$) the effect of anisotropy is more prominent. For a given value of ξ, for small impurity concentration, the effect of anisotropy is to increase $\Delta(\tilde{C},T)$, $H_c(\tilde{C},T,\Omega)$ and $J_s(\tilde{C},T,\Omega)$ but for large impurity concentrations the effect of anisotropy is to reduce the above quantities.

I have calculated the anisotropic density of electronic states, $[N(\omega)/N(0)]$. Outside BCS energy gap ($\omega/\Delta > 1$) density of states will be decreased due to anisotropy and inside BCS energy gap ($\omega/\Delta < 1$) it is increased for anisotropic superconductors.

For a given impurity concentration as we increase the strength of interaction (decrease ξ) we observe that the percentage change in the normalized slope of ultrasonic attenuation coefficient (C^*/C^*_{AG}) will be more.

The normalized upper critical field $\tilde{h}_{c2}(T)$ increases due to anisotropy as we increase the impurity concentration. For a given impurity concentration as we increase the strength of interaction, anisotropy will increase the percentage change in $\tilde{h}_{c2}(T)$.

The normalized slope of upper critical field versus T/T_c, $\tilde{h}'_{c2}(T)$ also behaves the same way as $\tilde{h}_{c2}(T)$ with anisotropy.

I have observed that the local states within the BCS energy gap ξ, is anisotropic.

The response to a weak transverse electromagnetic field is studied for an isotropic superconductor. The response function agrees with equation (16) of K.Maki [54]. In the limit of $q \to 0$, the equation (3.117) reduces to that of Chaba [55].

The Kondo effect of some of our problems is also investigated.

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APPENDIX

Computer Programme.

SUBROUTINE FOR COMPLEX DIGAMMA FUNCTION

```
COMPLES FUNCTION DIGAM*16(Z)
IMPLICIT COMPLEX*16(A-H,O-Z)
REAL*8 TEST.CDABS,DIGAM?YTEST
DIMENSION B(6)
PI=3.141592653589793
B(1)=8.33333333333333D-2
B(2)=-8.33333333333333D-2
B(3)=3.96825396825397D-3
B(4)=-4.16666666666667D-3
B(5)=7.575757575757576D-3
B(6)=-2.10927960927960D-2
V=Z
YTEST=DIMAG(Z)
IF(YTEST.LT.O.OODO)V=DCONG(Z)
TEST =CDABS(Z)
H=O.ODO
IF(TEST.GE.7.ODO)  Go to 3
IV=TEST
N=6-IV
H=1.ODO/V
IF (N.EQ.O)  GO TO 2
DO 1 I=1,N
V=V+1.ODO
```

```
      R=1.0D0/(V*V)
      DIGAM=CDLOG(V)-0.5/V-R*(B(1)+R*(B(2)+R*(B(3)+R*(B(4)+R*
     *(B(5)+R*(B(6)+R*B(1))))))-H
      IF(YTEST.LT.0.D0)  DIGAM =DCONJG(DIGAM)
      RETURN
      END
      FORTRAN
      FORTRAN STARTED
      PROGRAM COCHIN
      COMPLES A.B.Y.Z
      R=0.55
      T=0.03
      DO 12 I=1,2
      C=0.0
      DO 13 J=1,2
      X=2
      DO 14 K=1,20
      A=(1.0,1.0)
      DO 15 L=1,500
      Y=X+T*A*CSQRT(1.0-A**2)/(R*R-A*A)
      Z=A-Y
      IF((ABS(REAL(Z)).LE.0.001).AND.(ABS(AIMAG(Z)).LE.0.001))
                                                    GO TO 8
      A=Y
   15 CONTINUE
    8 CONTINUE
      B=-A*(1.D+(R*(X-A)/((R+1.0)*A)))
```

```
      WRITE(8.10) A,Y,B
10    FORMAT(15X,2E12.5,2E12.5,2E12.5)
      A=A*A+C*B*B
      D=REAL(CSQRT(A/(A-1)))
      WRITE(8.17)X,D
17    FORMAT(15Z,F6.2,5X,F8.4)
      X=X-0.1
14    CONTINUE
      WRITE(8.20) C
20    FORMAT(15X,F8.4)
      C=C+0.05
13    CONTINUE
      WRITE(8.21) T
21    FORMAT(15X,F8.4)
      T=0.15
12    CONTINUE
      STOP
      END
      IMPLICIT COMPLEX*16(A-H,O-Z)
      REAL*8 DREAL,DIMAG
      Z=(1.5,0.0)
      DO 10 J=1,2
      A=DIGAM(Z)
      B=TIGAM(Z)
      C=PGI(Z)
      D=PGII(Z)
      E=PGII(Z)
      PRINT 20,DREAL(A),DIMAG(A),DREAL(B),DIMAG(B),DREAL(C),
```

```
     *DIMAG(C),DREAL(D),DIMAG(D),DREAL(E),DIMAG(E)
20   FORMAT('O',10F10.7)
     Z=(1.6,1.9)
10   CONTINUE
     STOP
     END

     SUBROUTINE FOR COMPLEX DIGAMMA FUNCTION

     TC/TCO VS CBAR = FOR ANISOTROPIC SUPERCONDUCTOR
     IMPLICIT REAL*8(A-H,O-Z)
     E=0.6DO
     DO 15 K=1.2
     A=0.0DO
     DO 14 I=1,2
     C=0.0DO
     DO 10 L=1,10
     Z=T(E,A,C)
     IF(Z .LE. 0.01DO) GO TO 9
     PRINT16,E,A,Z,C
16   FORMAT('O',F4.2,4X,F4.2,4X,F5.3,4X,F4.2)
 9   CONTINUE
     A=A+0.05DO
10   CONTINUE
     C=C+0.03DO
14   CONTINUE
     E=0.95DO
     STOP
     END
```

```
      SUBROUTINE FOR TC

      REAL FUNCTION T*8(E,A,C)
      IMPLICIT REAL*8(A-H,O-Z)
      PSI=-1.963510026021430O
      TC=1.ODO
      DO 8 K=1,500
      TI=TC
      Q=C*(1.OBO-E)/TC
      P=C*(1.ODOE**2)/TC
      X=O.5DO+P
      Y=O.5DO+Q
      F=DIGAM(X)
      G=DIGAM(Y)
      TC=DEXP((PSI-F+A*(PSI-G))/(1.ODO+A))
      IF(DABS(TC-TI) .LE. 1.OD-8*TI) GO TO 7
8     CONTINUE
7     T=TC
      RETURN
      END

      SUBROUTINE FOR TIGMA

      REAL FUNCTION TIGMA*8(Z)
      IMPLICIT REAL*8(A-H,O-Z)
      TIGMA=O.ODO
      DO 8 K=1,500
      I=K-1
      TIGMA=TIGMA+1.ODO/(Z+1)**2
8     CONTINUE
      I=500
```

```
      TIGMA=TIGMA+(1.0D0/(1.0D0/(1+Z))+(1.0D0/2.0D0)*(1.0D0/
     *     (1+Z)**2)
      RETURN
      END
      APPROXIMATE FORMULA FOR PGIII
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSIONB(7)
      Z=1.05D0
      B(1)=-6.0D0
      B(2)=-12.0D0
      B(3)=-10.0D0
      B(4)=7.0d0
      B(5)=-12.0D0
      B(6)=33.0D0
      B(7)=-130.0D0
      PGII=B(1)/Z**4+B(2)/Z**5+B(3)/Z**6+B(4)/Z**8+B(5)/Z**10
     *+B(6)/Z**11+B(7)/Z**14
      PRINT,PGIII
      STOP
      END
```

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$\tilde{\epsilon}$	$\tilde{c}$	$\frac{\Delta(\tilde{c},T)}{\Delta(0,T)}$		$\frac{H_c(\tilde{c},T,\tilde{\Omega})}{H_c(0,T,\tilde{\Omega})}$		$\frac{J_s(\tilde{c},T,\tilde{\Omega})}{J_s(0,T,\tilde{\Omega})}$	
		$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$
	0.00	1.000	1.000	1.000	1.000	1.000	1.000
	0.02	0.740	0.737	0.569	0.561	0.522	0.516
	0.04	0.582	0.580	0.366	0.357	0.305	0.300
0.6	0.06	0.468	0.469	0.246	0.240	0.183	0.182
	0.08	0.378	0.385	0.167	0.163	0.1o9	0.110
	0.1o	0.302	0.317	0.1o9	0.107	0.061	0.064
	0.12	0.230	0.258	0.063	0.063	0.003	0.003
	0.0	1.000	1.000	1.000	1.000	1.000	1.000
	0.1	0.791	0.754	0.644	0.571	0.604	0.538
0.95	0.2	0.649	0.625	0.447	0.385	0.390	0.346
	0.3	0.546	0.550	0.326	0.283	0.263	0.247
	0.4	0.464	0.523	0.242	0.225	0.179	0.201

Table 1

$T_K/T_{co} = 16$

T/T_{co}	ξ	K^*		
		$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	% change
0.90	0.727	0.498	0.553	11.02
0.91	0.725	0.499	0.553	10.90
0.92	0.724	0.501	0.555	10.77
0.93	0.722	0.502	0.556	10.66
0.94	0.721	0.503	0.556	10.59
0.95	0.720	0.505	0.558	10.46
0.96	0.718	0.506	0.558	10.35
0.97	0.717	0.508	0.560	10.22
0.98	0.716	0.509	0.561	10.12
0.99	0.715	0.511	0.562	10.00
1.00	0.714	0.512	0.563	9.88

Table 2

$T_K/T_{co} = 1/16$

T/T_{co}	ξ	K^*		
		$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	% change
0.90	0.700	0.527	0.574	8.79
0.91	0.702	0.526	0.573	8.90
0.92	0.703	0.524	0.571	9.01
0.93	0.704	0.523	0.570	9.13
0.94	0.706	0521	0.569	9.24
0.95	0.707	0.520	0.568	9.35
0.96	0.708	0.518	0.567	9.47
0.97	0.710	0.516	0.566	9.58
0.98	0.711	0.515	0.565	9.69
0.99	0.713	0.514	0.564	9.79
1.00	0.714	0.513	0.563	9.87

Table 3

$T_K/T_{co} = 64$

T/T_{co}	ξ	K^*		
		$\langle a^2 \rangle = 0$	$\langle a^2 \rangle = 0.05$	% change
0.90	0.843	0.355	0.450	26.75
0.91	0.842	0.356	0.450	26.60
0.92	0.842	0.356	0.451	26.53
0.93	0.841	0357	0.451	26.40
0.94	0.840	0.359	0.452	26.16
0.95	0.840	0.359	0.453	26.05
0.96	0.839	0.360	0.454	25.93
0.97	0.839	0.361	0.454	25.81
0.98	0.838	0.362	0.455	25.70
0.99	0.837	0.363	0.455	25.58
1.00	0.837	0.363	0.456	25.47

Table 4

$T_K/T_{co} = 1/64$

T/T_{co}	ξ	K^*		
		$\langle a^2\rangle=0$	$\langle a^2\rangle=0,05$	% change
0.90	0.830	0.372	0.462	24.25
0.91	0.831	0.370	0.461	24.46
0.92	0.832	0.370	0.460	24.57
0.93	0.832	0.369	0.460	24.68
0.94	0.833	0.368	0.459	24.79
0.95	0.834	0.367	0.459	24.91
0.96	0.834	0.366	0.458	25.02
0.97	0.835	0.366	0.458	25.13
0.98	0.835	0.365	0.457	25.24
0.99	0.836	0.364	0.456	25.35
1.00	0.839	0.363	0.456	25.52

Table 5

$T_K/T_{co} = 16$

T_c/T_{co}		K^*		
$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	% change
1.000	1.000	0.512	0.563	9.89
0.771	0.747	0.477	0.535	12.17
0.570	0.531	0.441	0.5o6	14.84
0.402	0.356	0.405	0.477	17.81
0.269	0.224	0.369	0.450	21.82
0.172	0.134	0.336	0.424	26.20
0.108	0.078	0.308	0.403	30.86
0.068	0.046	0.284	0.387	36.13
0.043	0.028	0.265	0.374	40.95
0.028	0.18	0.250	0.364	45.50

Table 6

$T_{K/T_{co}} = 1/16$

T_c/T_{co}		K^*		
$\langle a^2 \rangle = 0$	$\langle a^2 \rangle = 0.05$	$\langle a^2 \rangle = 0$	$\langle a^2 \rangle = 0.05$	% change
1.000	1.000	0.513	0.563	+9.86
0.796	0.800	0.545	0.586	+7.54
0.655	0.660	0.577	0.607	+5.28
0.553	0.558	0.605	0.627	+3.69
0.475	0.481	0.631	0.645	+2.29
0.416	0.421	0.655	0.662	+1.07
0.368	0.373	0.677	0.678	+0.09
0.330	0.335	0.698	0.691	-1.00
0.299	0.303	0.716	0.704	-1.69
0.272	0.276	0.734	0.716	-2.41
0.250	0.254	0.749	0.726	-3.04
0.135	0.137	0.852	0.793	-6.96
0.092	0.093	0.895	0.814	-9.01
0.000	0.000	0.910	0.812	-10.73

Table 7

$T_K/T_{co} = 64$

T_c/T_{co}		K^*		
$\langle a^2 \rangle = 0$	$\langle a^2 \rangle = 0.05$	$\langle a^2 \rangle = 0$	$\langle a^2 \rangle = 0.05$	% change
1.000	1.000	0.363	0.456	25.40
0.856	0.827	0.351	0.445	26.70
0.721	0.674	0.340	0.435	28.20
0.595	0.538	0.327	0.425	29.85
0.480	0.417	0.313	0.414	32.10
0.376	0.312	0.300	0.403	34.30
0.285	0.224	0.286	0.392	36.97
0.210	0.154	0.273	0.382	39.69
0.15	0.102	0.260	0.371	42.67

Table 8

$T_K/T_{co} = 1/64$

T_c/T_{co}		K^*		
$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	% change
1.000	1.000	0.363	0.456	+25.44
0.796	0.800	0.382	0.469	+22.70
0.655	0.661	0.400	0.481	+20.27
0.553	0.559	0.416	0.493	+18.45
0.475	0.482	0.432	0.504	+16.75
0.416	0.422	0.447	0.515	+15.21
0.368	0.374	0.462	0.525	+13.70
0.330	0.336	0.475	0.535	+12.50
0.299	0.304	0.488	0.544	+11.55
0.272	0.277	0.501	0.553	+10.30
0.250	0.255	0.513	0.561	+ 9.42
0.135	0.138	0.632	0.629	- 0.53
0.092	0.094	0.677	0.676	- 0.12
0.000	0.000	0.910	0.812	-10.73

Table 9

$\alpha/\Delta = 0.03$

ω/Δ	$N(\omega)/N(0)$		
	$\langle a^2 \rangle = 0$	$\langle a^2 \rangle = 0.05$	% change
2.00	1.155	1.146	-0.788
1.90	1.176	1.165	-0.901
1.80	1.202	1.190	-1.039
1.70	1.236	1.221	-1.229
1.60	1.281	1.262	-1.491
1.50	1.341	1.316	-1.849
1.40	1.428	1.394	-2.374
1.30	1.563	1.512	-3.256
1.20	1.806	1.717	-4.955
1.10	2.397	2.168	-9.373
0.70	0.175	0.185	+0.059
0.60	0.197	0.206	+0.044
0.50	0.175	0.181	+0.034
0.40	0.119	0.123	+0.028
0.30	0.001	0.001	0.000

Table 10

$\varepsilon = 0.95$

$\bar{c}$	c^*/c^*_{AG}		
	$\langle a^2 \rangle = 0$	$\langle a^2 \rangle = 0.05$	% change
0.0	1.000	1.000	0.000
0.1	0.996	0.997	0.096
0.2	0.990	0.993	0.283
0.3	0.982	0.987	0.468
0.4	0.972	0.979	0.813
0.5	0.953	0.967	1.511
0.6	0.917	0.946	3.151
0.7	0.814	0.898	10.375

Table 11

$\xi = 0.8$

$\tilde{c}$	c^*/c^*_{AG}		
	$\langle a^2 \rangle = 0$	$\langle a^2 \rangle = 0.05$	% change
0.00	1.000	1.000	0.000
0.03	0.988	0.995	0.658
0.06	0.960	0.970	1.031
0.09	0.929	0.947	1.992
0.12	0.881	0.914	3.690
0.15	0.802	0.860	7.226
0.18	0.638	0.759	18.96

Table 12

$\xi = 0.6$

$\bar{C}$	C^*/C^*_{AG}		
	$\langle a^2 \rangle = 0$	$\langle a^2 \rangle = 0.05$	% change
0.00	1.000	1.000	0.000
0.02	0.964	0.973	1.354
0.04	0.914	0.936	2.352
0.06	0.843	0.901	6.907
0.08	0.729	0.800	9.739
0.10	0.518	0.653	26.000

Table 13

$\xi = 0.95$

$\bar{c}$	T_c/T_{co}		$\tilde{h}_{c2}(T)$		
	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	% change
0.0	1.000	1.000	1.000	1.000	0.00
0.1	0.952	0.953	0.930	0.933	0.28
0.2	0.902	0.905	0.861	0.865	0.47
0.3	0.852	0.856	0.791	0.798	0.89
0.4	0.801	0.806	0.721	0.731	1.39
0.5	0.748	0.755	0.652	0.663	1.69
0.6	0.695	0.703	0.582	0.596	2.41
0.7	0.639	0.649	0.512	0.528	3.13
0.8	0.582	0.594	0.443	0.461	4.06
0.9	0.522	0.536	0.373	0.394	5.63
1.0	0.460	0.476	0.304	0.326	7.24
1.1	0.392	0.413	0.234	0.259	10.68
1.2	0.319	0.344	0.164	0.192	17.07
1.3	0.233	0.266	0.095	0.124	30.53

Table 14

$\mathcal{E} = 0.8$

$\bar{c}$	T_c/T_{co}		$h_{c2}(T)$		
	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	% change
0.00	1.000	1.000	1.000	1.000	0.00
0.03	0.946	0.947	0.923	0.925	0.22
0.06	0.892	0.894	0.846	0.850	0.47
0.09	0.836	0.839	0.769	0.775	0.78
0.12	0.778	0.784	0.691	0.700	1.30
0.15	0.720	0.726	0.614	0.625	1.79
0.18	0.659	0.668	0.537	0.551	2.61
0.21	0.596	0.607	0.460	0.476	3.48
0.24	0.531	0.543	0.383	0.401	4.70
0.27	0.462	0.477	0.306	0.326	6.54
0.30	0.387	0.405	0.229	0.251	9.61
0.33	0.304	0.328	0.151	0.176	16.56
0.36	0.205	0.238	0.074	0.101	36.49

Table 15

$\varepsilon = 0.6$

$\tilde{c}$	T_c/T_{co}		$\tilde{h}_{c2}(t)$		
	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	% change
0.00	1.000	1.000	1.000	1.000	0.00
0.02	0.936	0.937	0.909	0.911	0.22
0.04	0.871	0.873	0.817	0.821	0.49
0.06	0.804	0.808	0.726	0.732	0.83
0.08	0.735	0.740	0.634	0.643	1.42
0.10	0.664	0.671	0.543	0.553	1.84
0.12	0.589	0.598	0.451	0.464	2.88
0.14	0.511	0.522	0.360	0.375	4.17
0.16	0.426	0.440	0.269	0.286	6.32
0.18	0.333	0.351	0.177	0.196	10.73
0.20	0.221	0.247	0.086	0.107	24.42

Table 16

$\varepsilon = 0.95$

$\tilde{c}$	T_c/T_{co}		$\tilde{h}'_{c2}(T_c)$		
	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	% change
0.0	1.000	1.000	1.000	1.000	0.000
0.1	0.952	0.953	0.984	0.985	0.090
0.2	0.902	0.905	0.964	0.966	0.207
0.3	0.852	0.856	0.941	0.944	0.351
0.4	0.801	0.806	0.911	0.916	0.560
0.5	0.748	0.755	0.872	0.880	0.860
0.6	0.695	0.703	0.820	0.831	1.330
0.7	0.639	0.649	0.744	0.760	2.220
0.8	0.582	0.594	0.624	0.650	4.250
0.9	0.522	0.536	0 399	0.449	12.370

Table 17

$\xi = 0.8$

$\bar{c}$	T_c/T_{co}		$\tilde{h}'_{c2}(T_c)$		
	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle= 0.05$	% change
0.00	1.000	1.000	1.000	1.000	0.000
0.03	0.946	0.947	0.982	0.983	0.108
0.06	0.892	0.894	0.960	0.962	0.254
0.09	0.836	0.839	0.932	0.935	0.367
0.12	0.778	0.784	0.896	0.902	0.732
0.15	0.720	0.726	0.846	0.856	1.114
0.18	0.659	0.668	0.774	0.789	1.932
0.21	0.596	0.607	0.659	0.683	3.671
0.24	0.531	0.543	0.443	0.485	9.550

Table 18

$\epsilon = 0.6$

$\bar{c}$	T_c/T_{co}		$\tilde{h}'_{c2}(T_c)$		
	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	% change
0.00	1.000	1.000	1.000	1.000	0.000
0.02	0.936	0.937	0.978	0.979	0.095
0.04	0.871	0.873	0.950	0.952	0.219
0.06	0.804	0.808	0.913	0.917	0.421
0.08	0.735	0.740	0.861	0.867	0.732
0.10	0.664	0.671	0.781	0.792	1.352
0.12	0.589	0.598	0.642	0.661	3.000
0.14	0.511	0.522	0.335	0.379	13.07

Table 19

$T_K/T_{co} = 16$

$\bar{c}$	T_c/T_{co}		$\mathcal{E}$		% change in $\mathcal{E}$
	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	
0.0	1.000	1.000	0.714	0.714	0.00
0.1	0.771	0.747	0.744	0.748	0.48
0.2	0.570	0.531	0.775	0.781	0.77
0.3	0.402	0.356	0.804	0.813	1.12
0.4	0.269	0.224	0.832	0.843	1.32
0.5	0.172	0.134	0.857	0.869	1.35
0.6	0.108	0.078	0.878	0.890	1.37
0.7	0.068	0.046	0.895	0.907	1.33
0.8	0.043	0.028	0.909	0.919	1.10
0.9	0.028	0.018	0.919	0.928	1.02

Table 20

G3194

$T_k/T_{co} = 1/16$

T_c/T_{co}		$\mathcal{E}$		
$\langle a^2 \rangle = 0$	$\langle a^2 \rangle = 0.05$	$\langle a^2 \rangle = 0$	$\langle a^2 \rangle = 0.05$	% change
1.000	1.000	0.714	0.714	0.0000
0.796	0.800	0.683	0.684	0.0012
0.655	0.660	0.654	0.655	0.0015
0.553	0.558	0.625	0.627	0.0032
0.475	0.481	0.598	0.600	0.0033
0.416	0.421	0572	0.574	0.0035
0.368	0.373	0.546	0.549	0.0055
0.330	0.335	0.522	0.525	0.0057
0.299	0.303	0.499	0.502	0.0060
0.272	0.276	0.476	0.479	0.0067
0.250	0.254	0.454	0.458	0.0088
0.135	0.137	0.272	0.277	0.0183
0.092	0.093	0.141	0.145	0.0248
0.069	0.070	0.036	0.042	0.1556

Table 21

$T_K/T_{co} = 64$

$\bar{c}$	T_c/T_{co}		$\mathcal{E}$		
	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	% change
0.0	1.000	1.000	0.837	0.837	0.000
0.1	0.856	0.827	0.846	0.848	0.236
0.2	0.721	0.674	0.855	0.858	0.351
0.3	0.595	0.538	0.864	0.869	0.579
0.4	0.480	0.417	0.874	0.880	0.686
0.5	0.376	0.312	0.884	0.890	0.679
0.6	0.285	0.224	0.894	0.901	0.783
0.7	0.210	0.154	0.903	0.912	0.997
0.8	0.150	0.102	0.912	0.921	0.987

Table 22

$T_K/T_{co} = 1/64$

T_c/T_{co}		$\mathcal{E}$		
$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	$\langle a^2\rangle=0$	$\langle a^2\rangle=0.05$	% change
1.000	1.000	0.837	0.837	0.0000
0.796	0.800	0.822	0.823	0.0007
0.553	0.559	0.795	0.796	0.0012
0.475	0.482	0.782	0.783	0.0018
0.416	0.422	0.770	0.771	0.0016
0.368	0.374	0.758	0.759	0.0019
0.330	0.336	0.746	0.748	0.0027
0.299	0.304	0.735	0.737	0.0029
0.272	0.277	0.724	0.726	0.0033
0.250	0.255	0.714	0.716	0.0034
0.135	0.138	0.621	0.625	0.0064
0.092	0.094	0.546	0.551	0.0092
0.069	0.071	0.479	0.486	0.0140
0.056	0.057	0.425	0.430	0.0108

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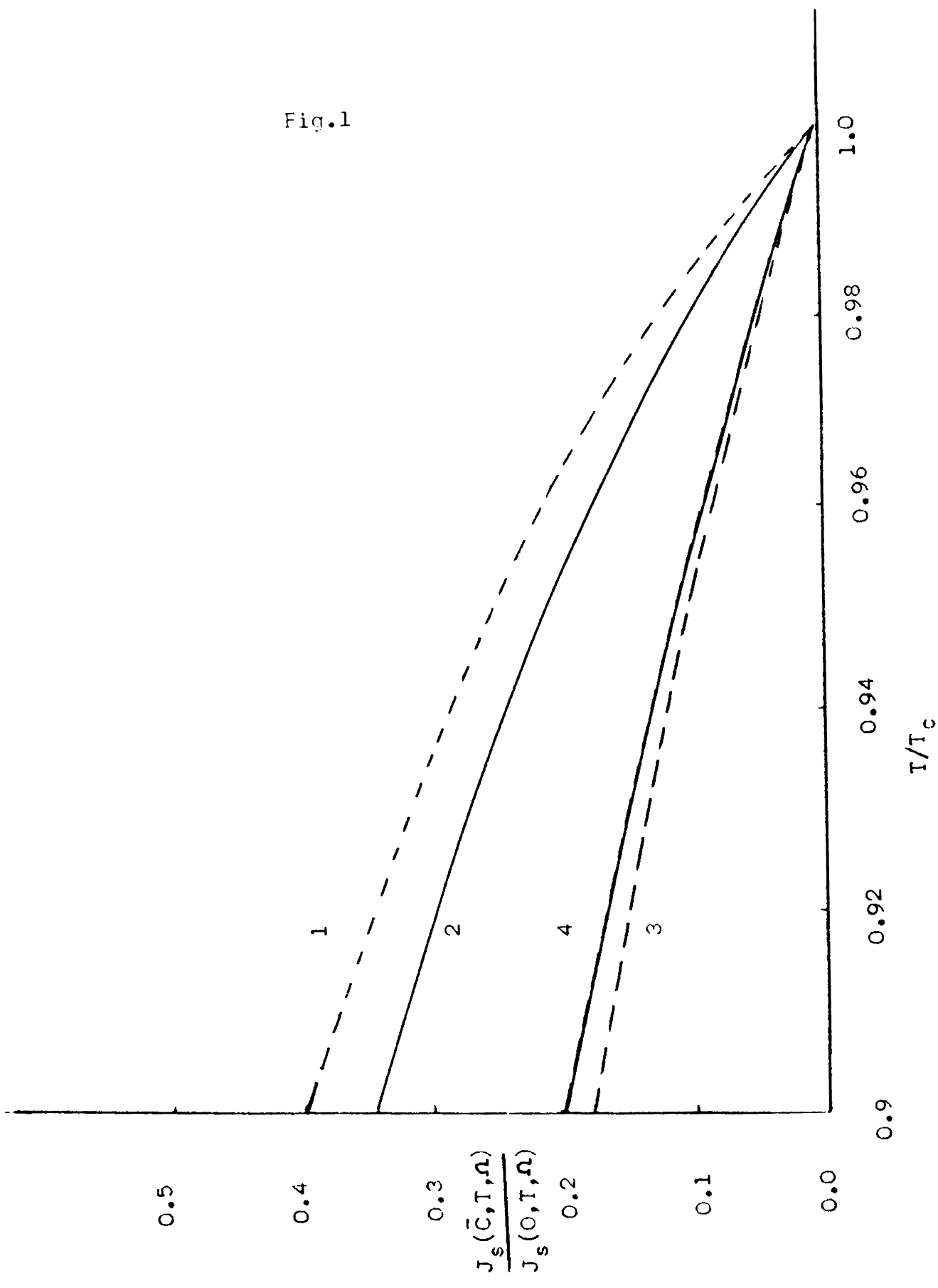
Fig.1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0.3
0.2
0.1
0.0
$\frac{J_s(\vec{C},T,\Omega)}{J_s(0,T,\Omega)}$
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0.92
0.94
0.96
0.98
1.0
T/T_c
1
2
4
3

Fig.2

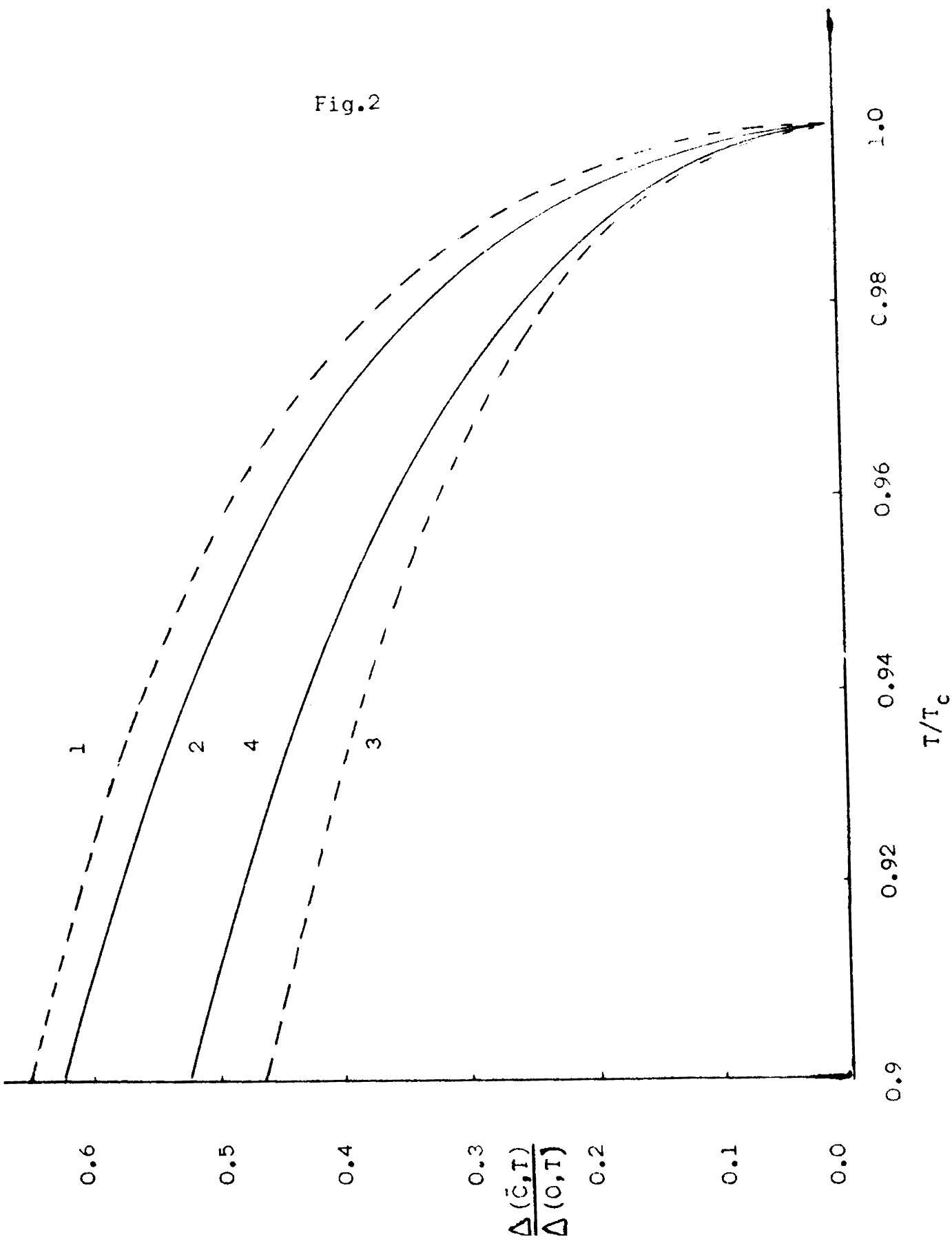


Fig.3

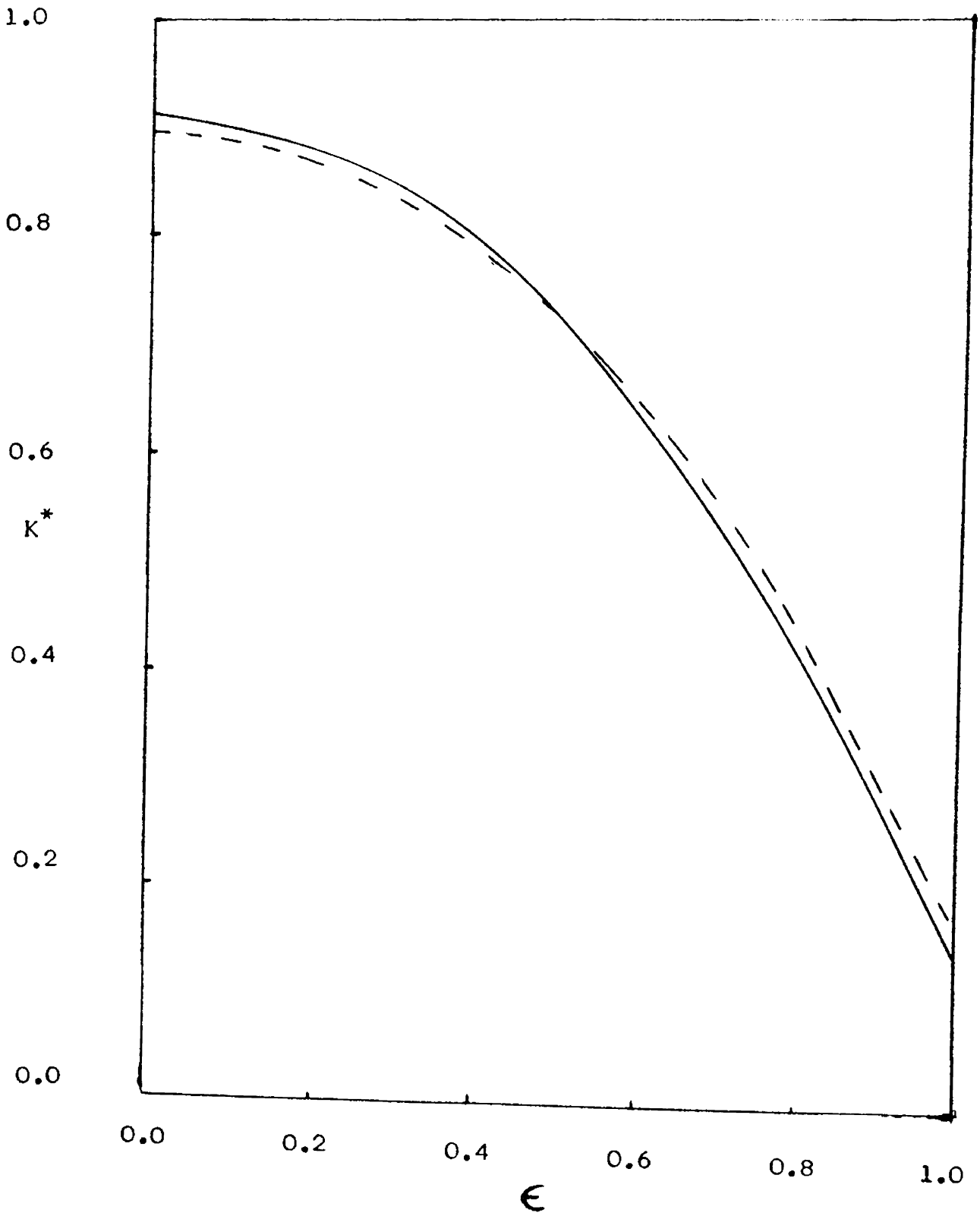
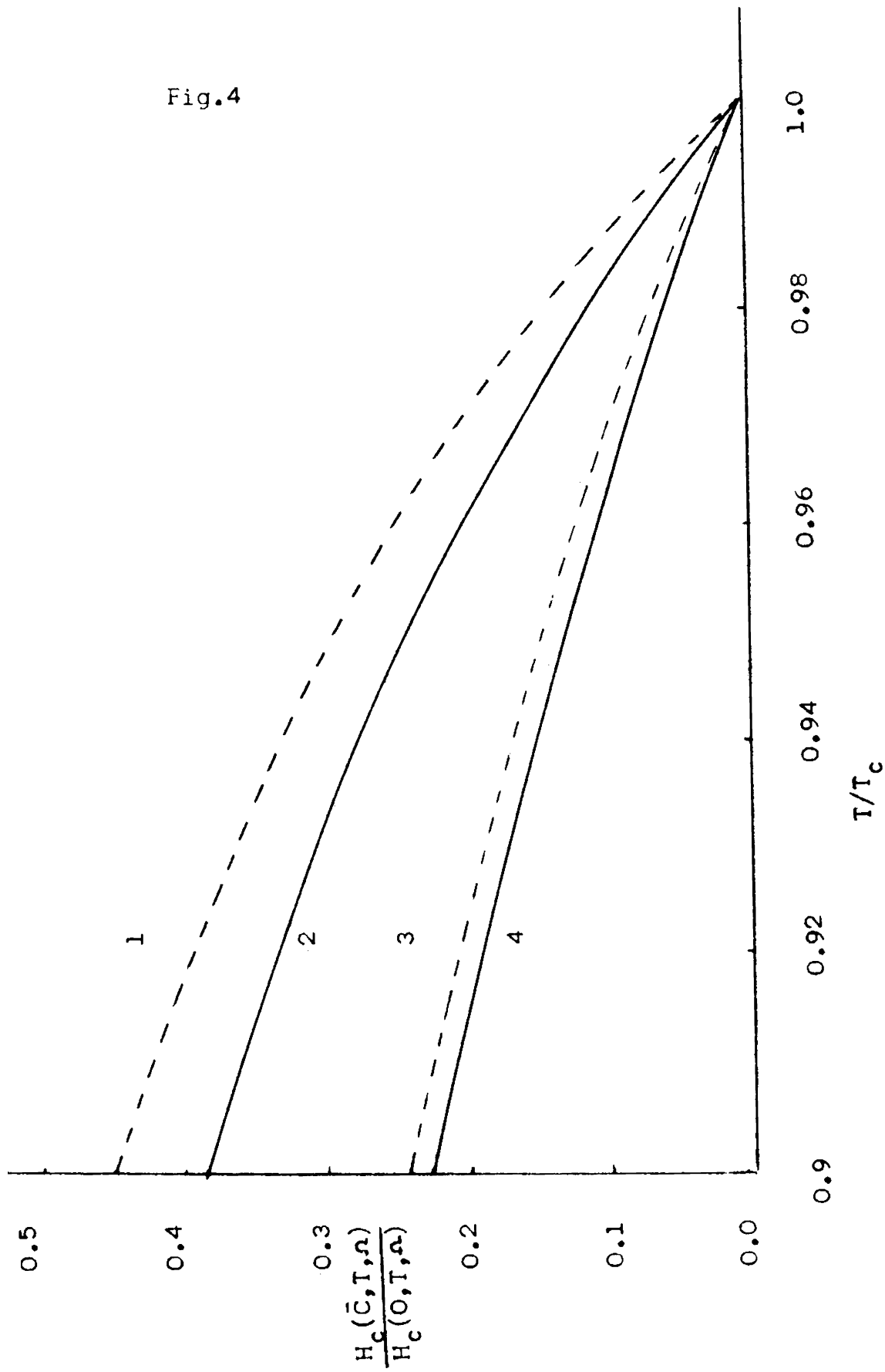


Fig.4


0.5
0.4
0.3
0.2
0.1
0.0
$\frac{H_c(\bar{c},T,\Omega)}{H_c(0,T,\Omega)}$
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0.92
0.94
0.96
0.98
1.0
T/T_c
1
2
3
4

Fig.5

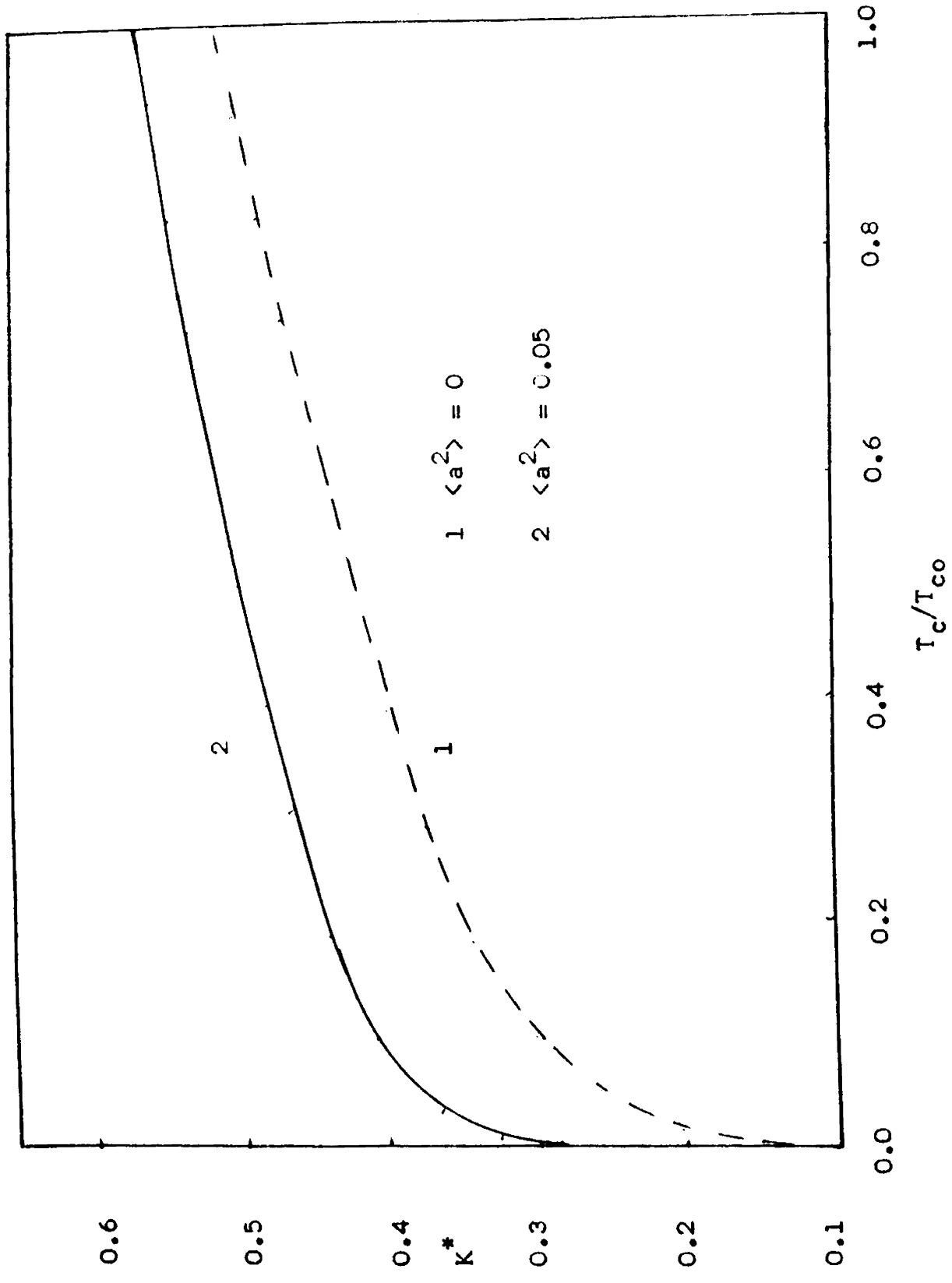
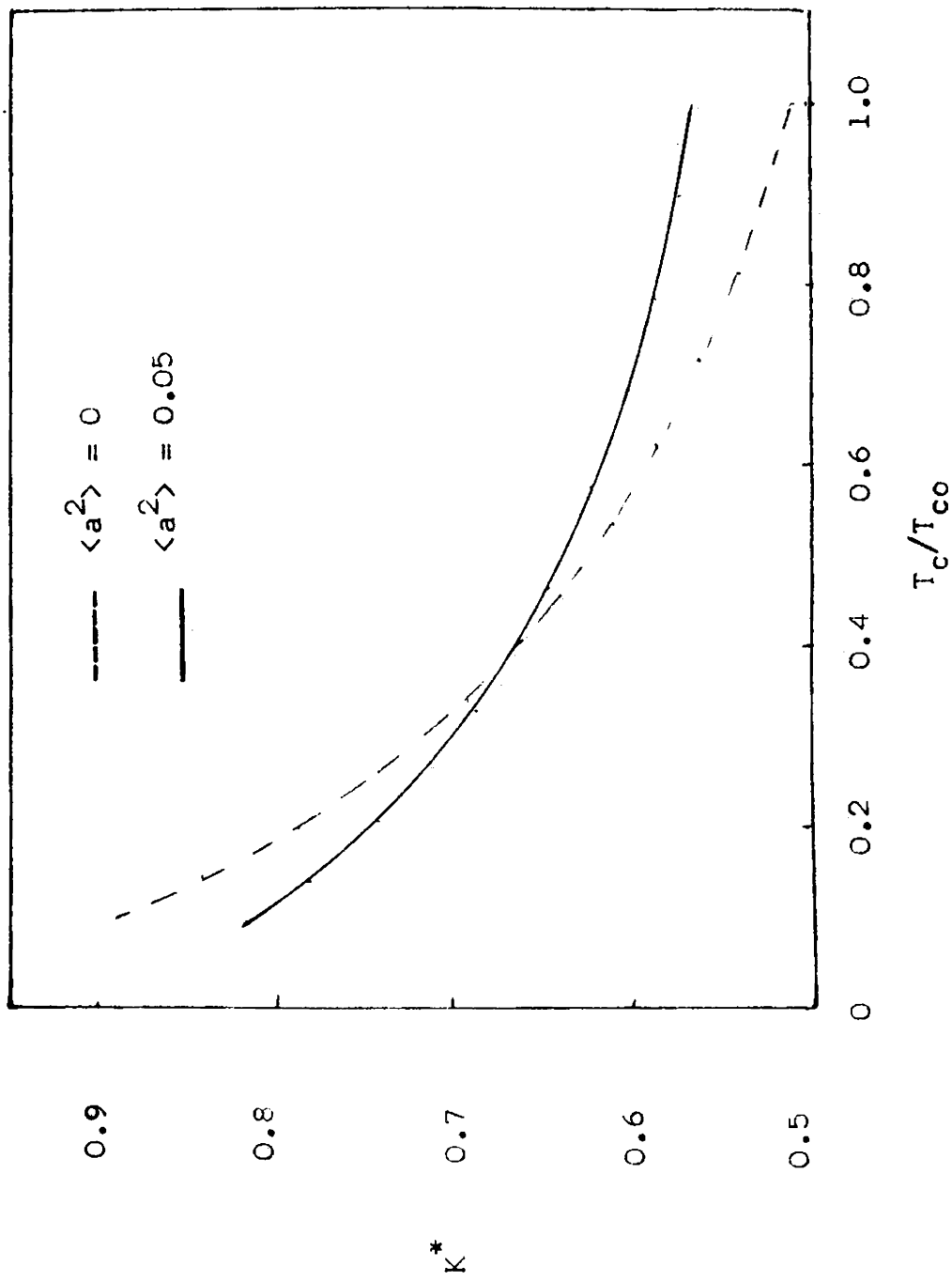

0.6
0.5
0.4
K*
0.3
0.2
0.1
0.0
0.2
0.4
0.6
0.8
1.0
T_c/T_{co}
1 $\langle a^2 \rangle = 0$
2 $\langle a^2 \rangle = 0.05$
1
2

Fig.6


$\langle a^2 \rangle = 0$
$\langle a^2 \rangle = 0.05$
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0.2
0.4
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0.8
1.0
T_c/T_{co}
0.9
0.8
0.7
0.6
0.5
K^*

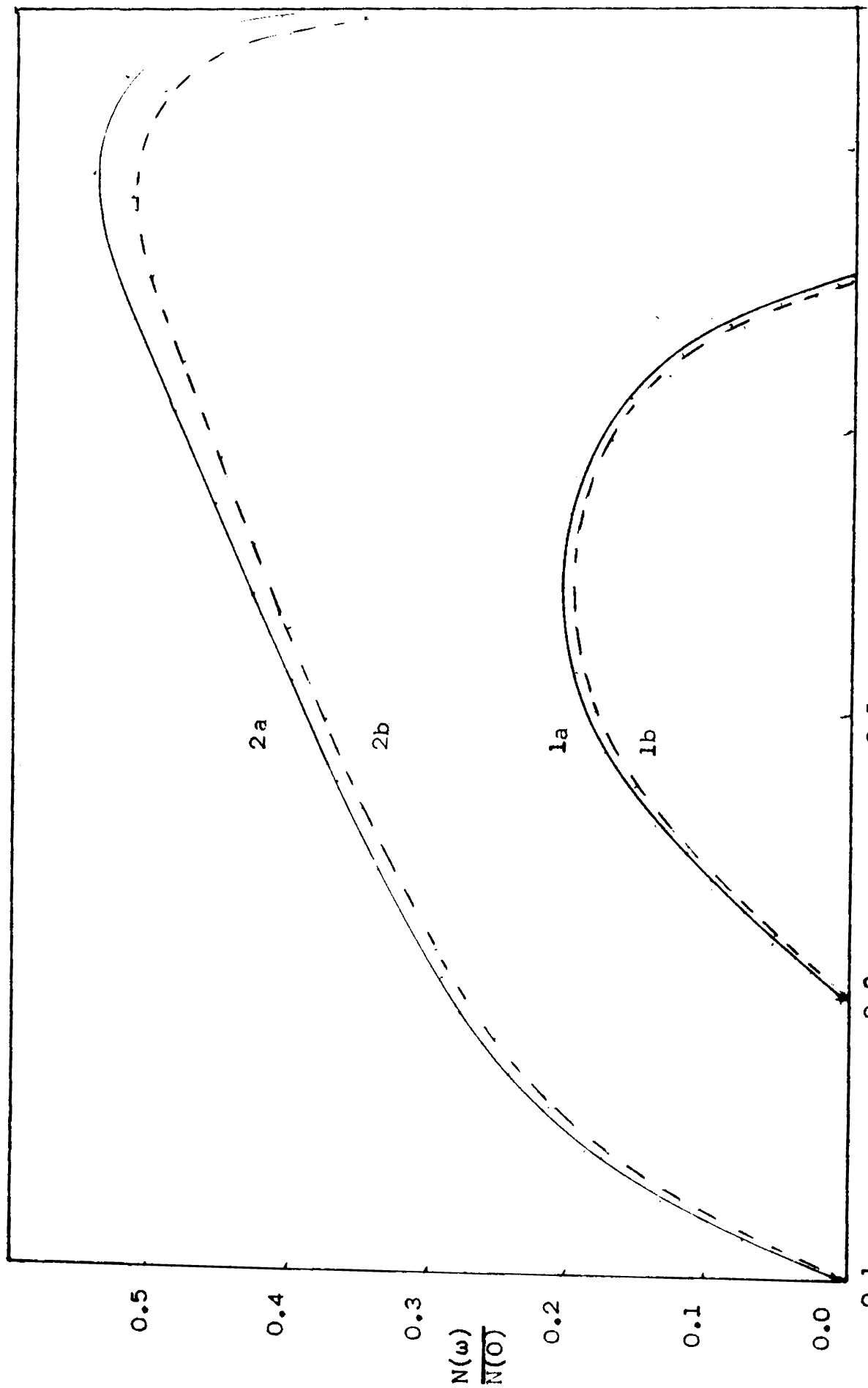
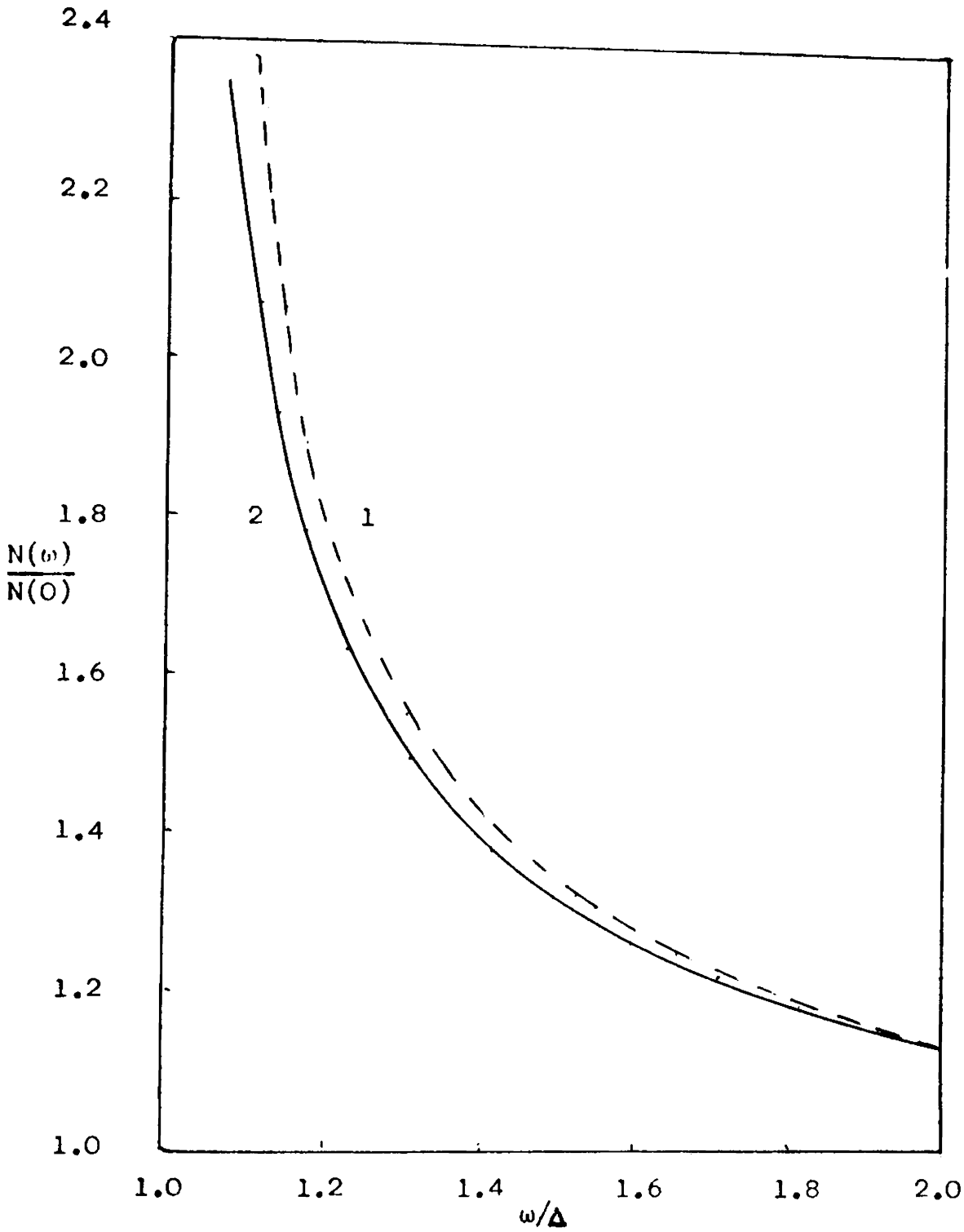


Fig.7

Fig.8



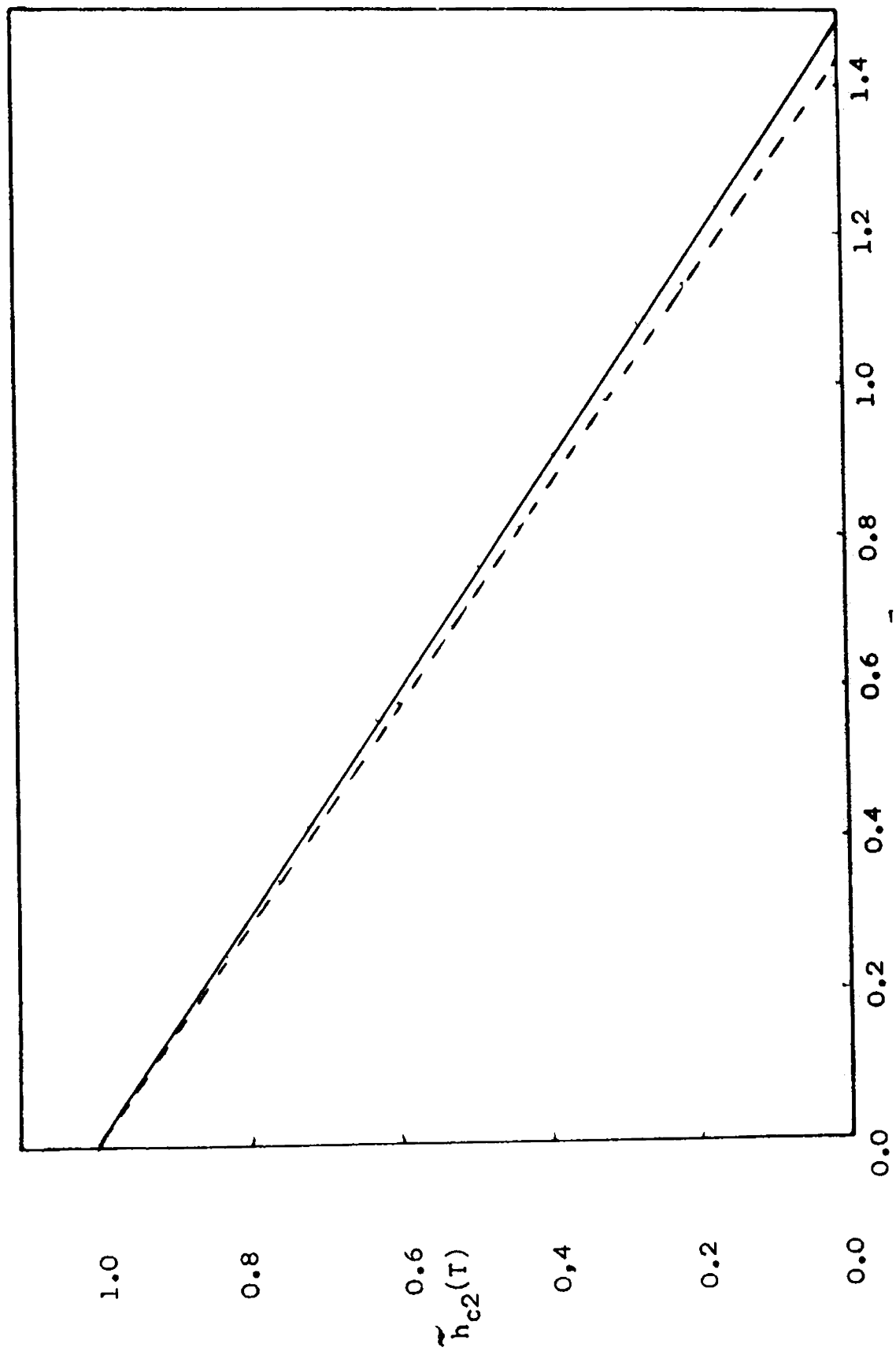


Fig.9

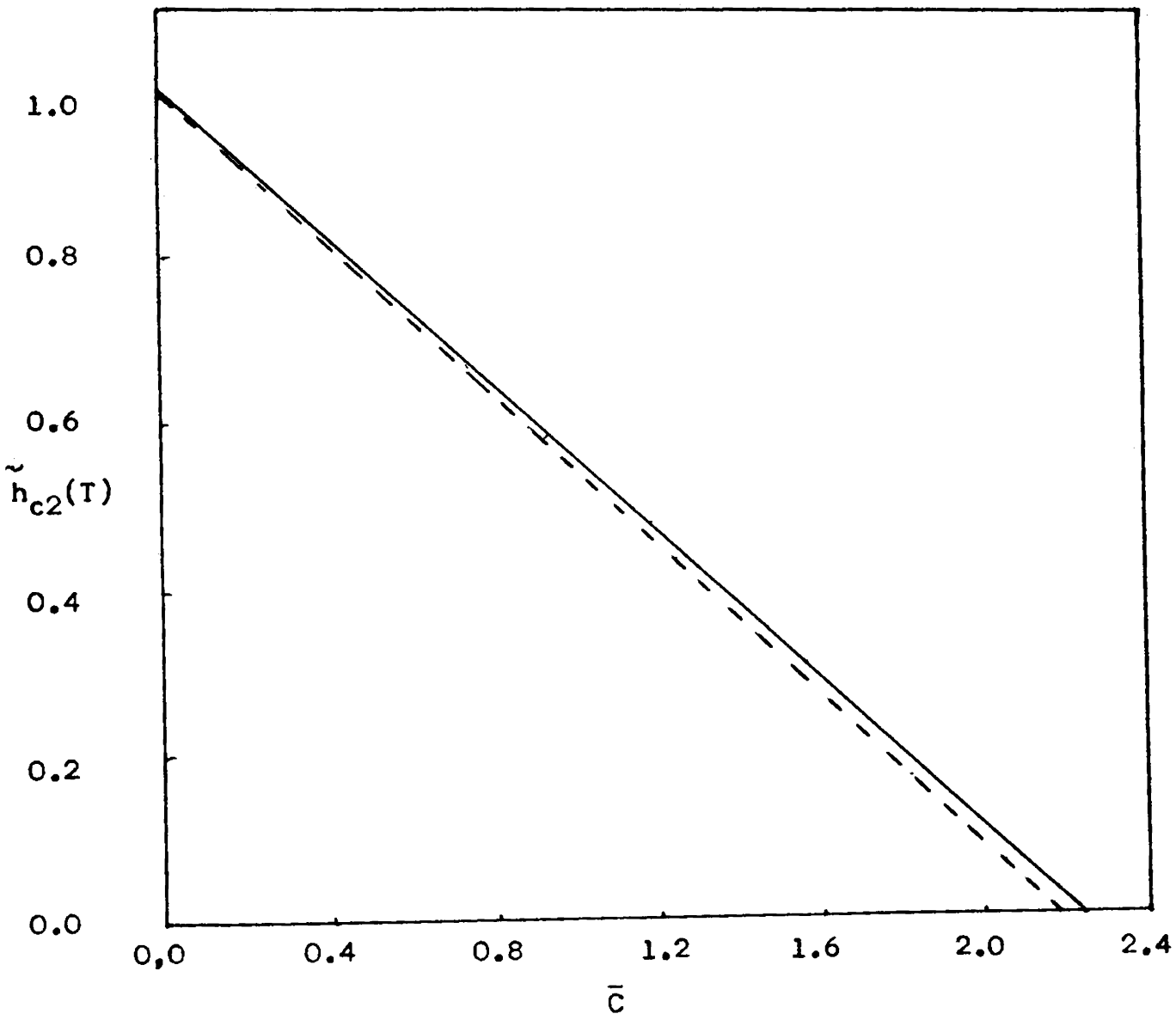


Fig.10

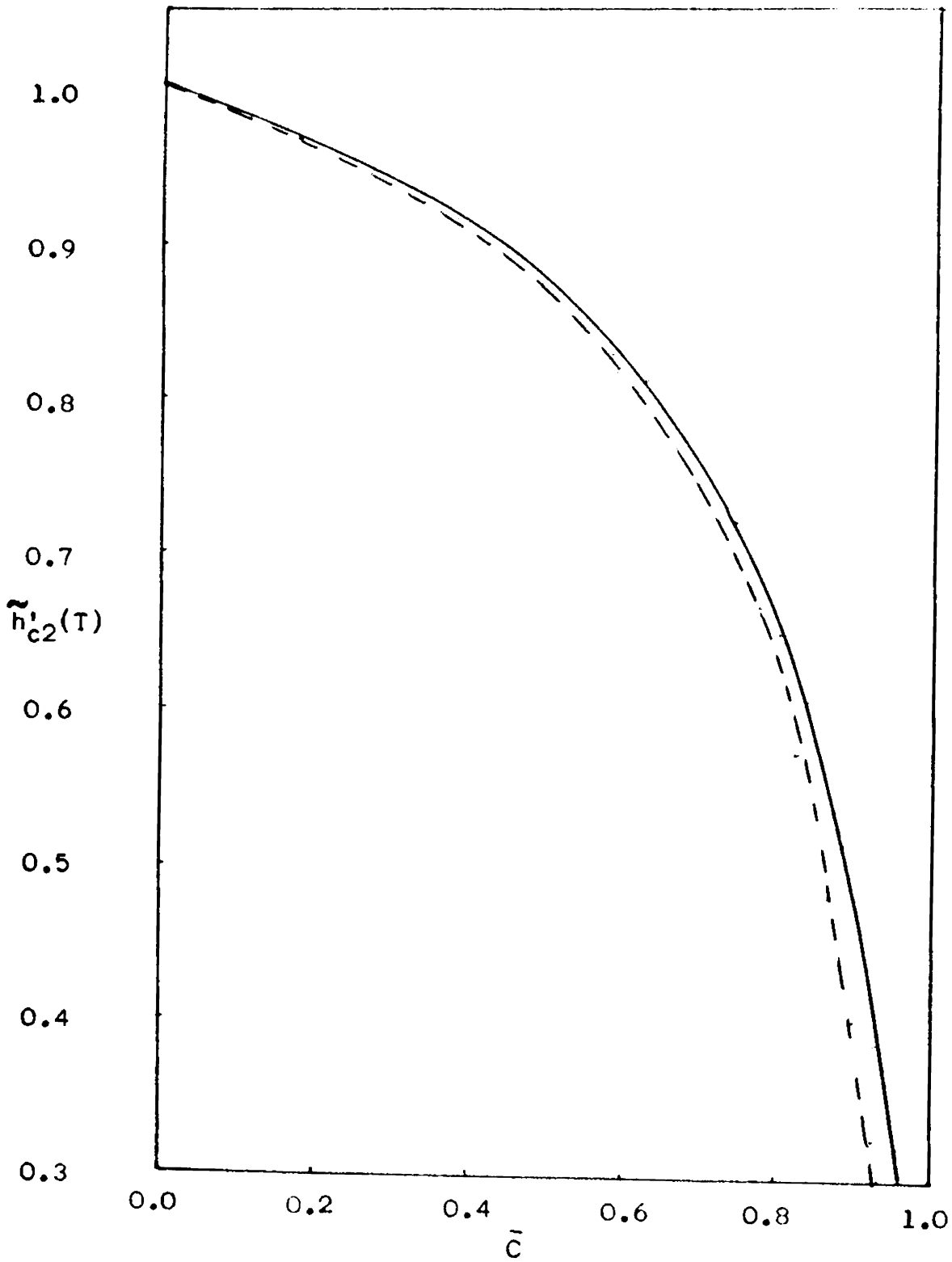
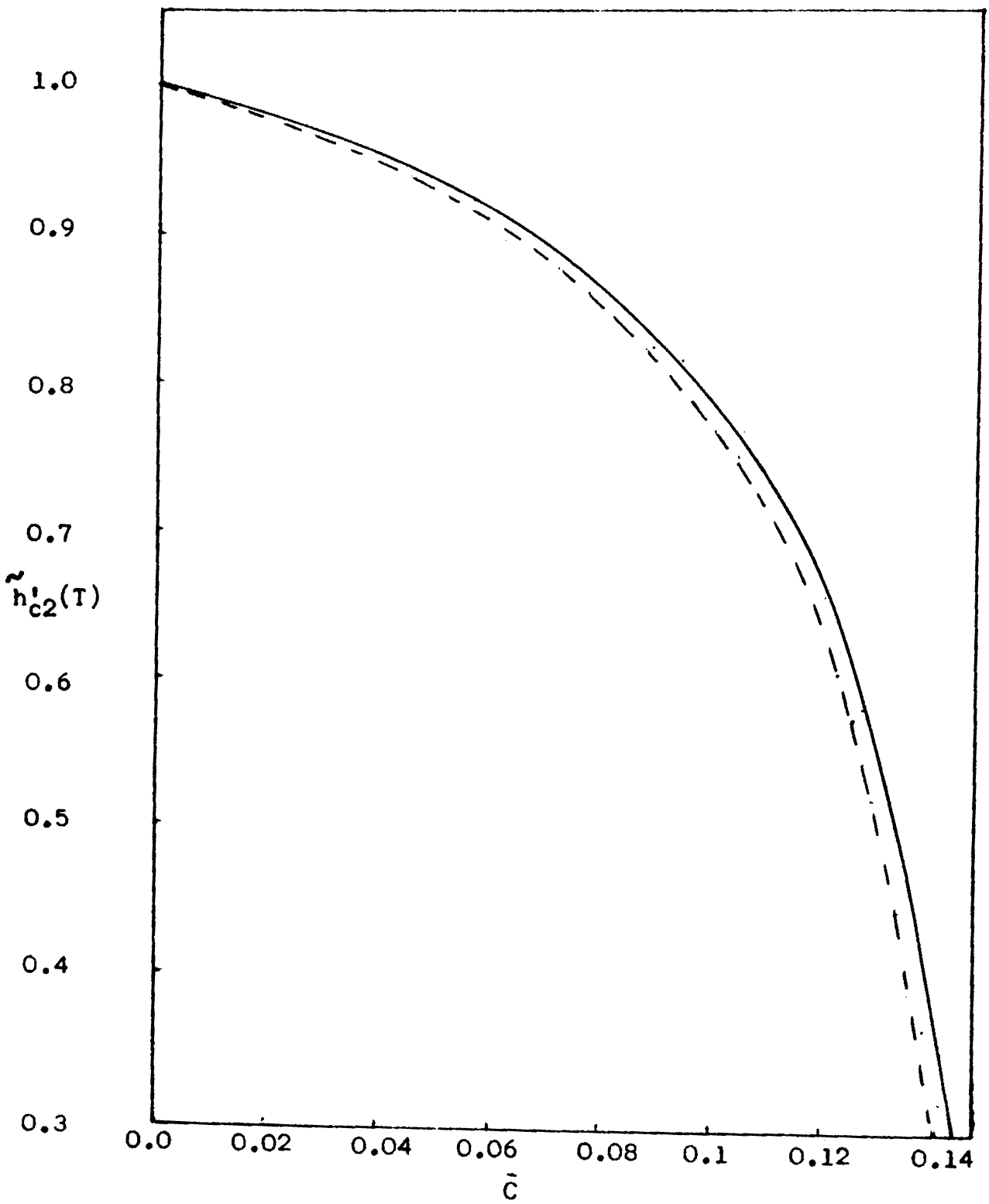


Fig.11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0.8
0.7
$\tilde{h}'_{c2}(T)$
0.6
0.5
0.4
0.3
0.0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
$\tilde{c}$

Fig. 12

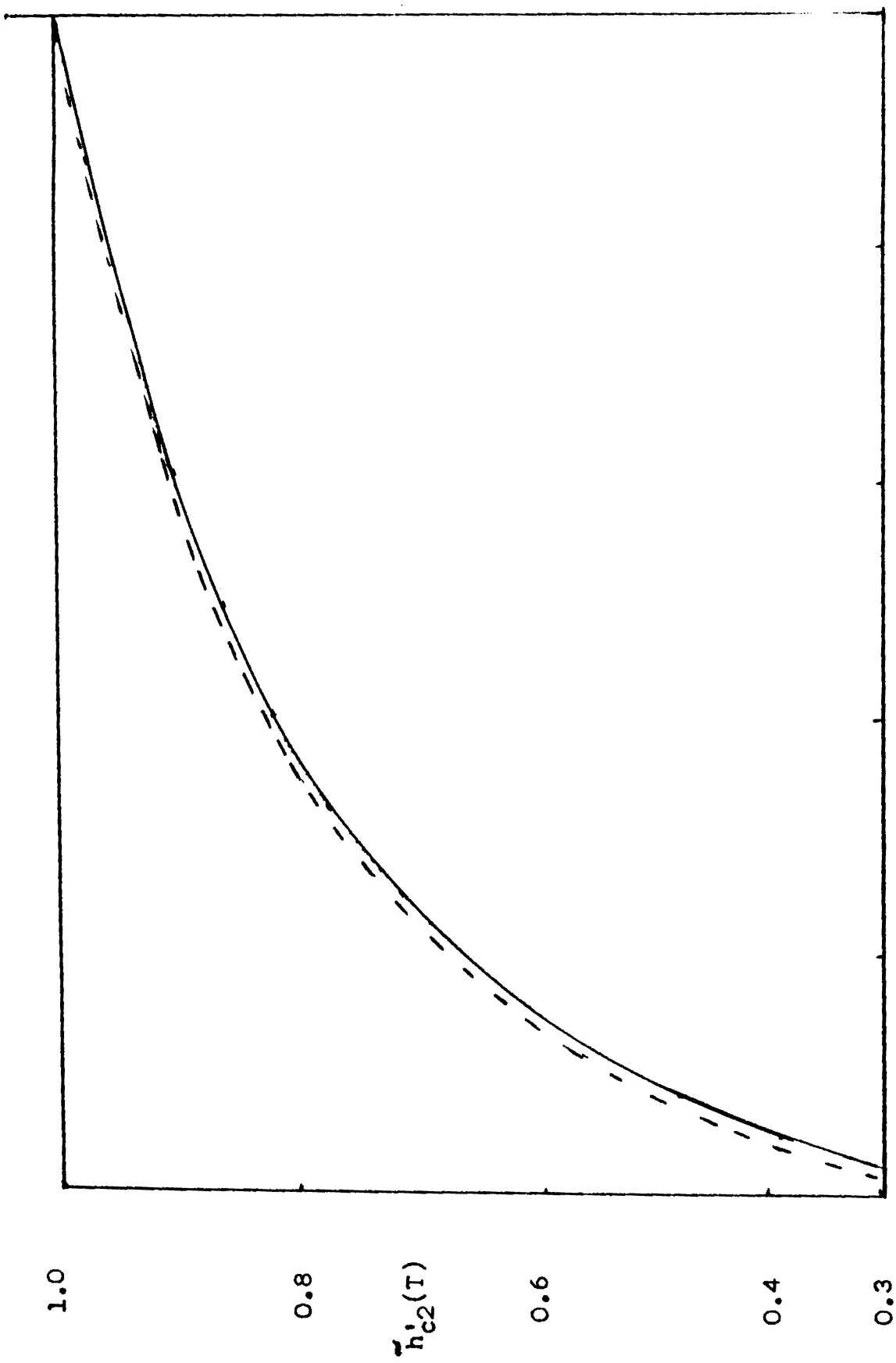

Fig. 13