STUDIES ON THE PROPAGATION OF ERRORS IN PHYSICAL OCEANOGRAPHIC COMPUTATIONS

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By

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CERTIFICATE

This is to certify that this thesis bound herewith is an authentic record of the research carried out by Mr. K.S. Neelakandan Namboodiripad, M.Sc., under my supervision and guidance in the Department of Marine Sciences, in partial fulfilment of the requirements of the Ph.D. Degree of the University of Cochin and no part thereof has been presented before for any other degree in any University.

Cochin - 682 016,
March, 1984.

Dr. P. G. Kurup
(Supervising Teacher)

DECLARATION

I hereby declare that the thesis entitled, 'STUDIES ON THE PROPOGATION OF ERRORS IN PHYSICAL OCEANOGRAPHIC COMPUTATIONS', is an authentic record of research carried out by me under the supervision and guidance of Dr.P.G.Kurup in partial fulfilment of the requirements of the Ph.D. Degree of the University of Cochin and that no part of it has previously formed the basis for the award of any degree, diploma or associateship in any University.

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CONTENTS

Page

CHAPTER I

INTRODUCTION

Systematic procedure of physical oceanographic research involves collection of data on the independent parameters of the physical properties of sea water and the computation of the dependent parameters, leading ultimately to information on the dynamics of the oceans. The data collected at sea include, among others, values of temperature and salinity at surface and subsurface levels. Physical oceanographic measurements made at sea are subjected to careful examinations, corrections and conversions which require considerable amount of practical experience to judge the reliability of the records from instruments which operate blindly below the sea surface. Corrections are made for instrumental errors and for errors inherent in the methods of obtaining the data. The oceanographer is thus equipped with the basic data of temperature and salinity for different depths at various stations in the sea. The processing procedures do not ultimately provide perfectly accurate information on these parameters including temperature, salinity, depth, station position, etc. These procedures provide the data within certain error limits.

The basic data are then converted to standard depths to facilitate comparison with other oceanographic data. This is followed by a series of calculations required to derive the dependent quantities like specific volume, density and currents. These calculations have been highly systematised through practice by the physical oceanographers. This branch of physical oceanography is known as dynamical oceanography. Dynamical oceanography discusses the water properties and water movements and their temporal and spatial variations in the world oceans.

Dynamical oceanography has almost always neglected to consider as to what happens to the random errors, inherent in the basic data in the process of the series of calculations involved in the computational practices. It is conveniently assumed that the final results are not much affected by these errors. The fact that even small errors can, at times, lead to highly erraneous results through the propagation of errors in the computational procedures is often overlooked by oceanographers. Very few authors have made any serious mention on the error component of the derived results in dynamical oceanography. Jakhelln (1936), Thompson (1939) and Dobrovol'skii (1949) have mentioned this possibility, while Fomin (1964) has given serious consideration to this problem. The present studies aim to examine the limits of errors contained in the basic data of physical oceanography and the magnitude of the error component in the results derived in dynamical oceanography. The study also suggests a graphical method of smoothening of the derived parameters within the limits of errors to increase the reliability of the derived results.

1.1. Physical Oceanographic Practices

Customarily and scientifically, the practical procedures in Physical Oceanography include collection of the various data during oceanographic cruises. Of these, those that are relevant in the context of errors and their propagation will be discussed in this section.

1.1.1. Station position

The place where an oceanographic vessel is stopped to carry out observations and collection of data and samples is known as an oceanographic station. The geographic location of the station is a primary requirement in oceanography. When land is visible and if recognisable features are accurately located on land, the position of the vessel is determined by means of horizontal angles and bearings on shore features. Out of sight of land, the ship's position is determined by astronomic sights or by radio direction-finder bearings. Between positions established in these ways, the location at any time is obtained by dead-reckoning, from the course steered and distance run. More recently, satellite navigation systems are available which provide more accurate information on the location of oceanographic stations. The values of latitudes are of particular interest as it appears in the dynamic computation as a term in the Coriolis force component. The station positions also determine the values of distance between stations which again is a term in the dynamic computation for estimating relative currents.

1.1.2. Depth

Depth of the bottom at the station is obtained in shallow regions by the use of the classical method of lead sounding. More accurate and easy method is provided by echosounding in which the time interval between the generation of a sound impulse and the reception of its echo is used as a direct measure of the depth, using a constant sounding velocity. Depth of the oceanographic station does not directly appear in the dynamic computations and therefore does not require detailed considerations in the present study.

1.1.3. Temperature

Measurement of surface temperature is carried out using accurate mercury-in-glass thermometers. Thermometers used for measuring temperatures at subsurface levels are of the reversing type and are generally mounted upon water sampling bottles so that temperature and the water for salinity and other physical and chemical tests are obtained at the same level. Serial data on temperature and salinity are thus obtained from a hydrographic cast of water samplers arranged in series on a wire rope, to each of which are attached the protected and unprotected reversing thermometers.

The protected reversing thermometer is essentially a double ended thermometer. It is sent down to the required depth in the set position and consists of a large reservoir of mercury connected by means of a fine capillary to a small bulb at the upper end. Just above the large reservoir, the capillary is constricted and branched with a small arm, and above this, the thermometer tube is bent into a loop, from which it continues straight and terminates in the smaller bulb. In the set position, mercury fills the reservoir, the capillary and part of the bulb. The amount of mercury above the constriction depends upon the temperature of the surrounding water. When the thermometer is reversed at the required depth along with the water bottle by sending a messenger weight down the wire rope, the mercury column breaks at the point of constriction and runs down, filling the bulb and part of the graduated capillary, thus indicating the temperature at reversal. The loop in the capillary which is generally of enlarged diameter, is designed to trap any mercury that is forced past the constriction if the temperature is raised after the thermometer has been reversed. In order to correct the reading for the changes resulting from difference between the temperature at reversal and surrounding temperature at the time of reading, a small standard thermometer, known as the auxiliary thermometer, is mounted alongside the reversing thermometer. The reversing thermometer and the auxiliary thermometer are enclosed in a heavy glass tube that is partially evacuated except for the portion surrounding the reservoir of the thermometer, and this part is filled with mercury to serve as a thermal conductor between the surroundings and the reservoir. The thermometer tube eliminates the effect of hydrostatic pressure. Seawater temperature <u>in situ</u> is obtained from the reading of a protected reversing thermometer by applying corrections for instrumental error and for thermal expansion subsequent to reversal.

Reversing thermometers were first introduced by Negretti and Zambra in 1874 and have since been improved so that well made instruments are now accurate to within $\pm 0.01^{\circ}C$. Recently electronic instruments are also being used to obtain records of subsurface temperature.

1.1.4. Depth of sampling

By depth of sampling we mean the subsurface depth at which the reversing bottle along with the reversing thermometer is made to reverse collecting subsurface water sample and recording the temperature. The wire rope carrying the equipment is payed out through a meter wheel which measures the length of wire rope that has run out. The depth of reversal, as obtained from the meter wheel readings, may be erraneous due to non-vertical running out of the wire rope in the presence of ship drift or ocean currents. A correction for this can be obtained in the surface layers by measuring the wire angle. Depth of reversal is more accurately found by comparing the corrected reading of protected thermometer with the corrected reading of unprotected thermometer which is paired with a protected thermometer.

Unprotected reversing thermometers are identical with the protected reversing thermometers but have open protective tubes. Because of the difference in the compressibility of glass and mercury, thermometers subjected to pressure give a fictitious temperature reading that is dependent upon the temperature and pressure. The unprotected reversing thermometers are so designed that the apparent temperature increase due to hydrostatic pressure is about 0.01°C/m. The readings obtained from the unprotected thermometers also have to be corrected for thermal expansion and instrumental errors.

1.1.5. Salinity

Water samples for estimation of salinity are obtained in oceanographic vessels using subsurface cast of Nansen bottles in series. The Nansen bottle is a reversing bottle which can be reversed at the desired depth by sending a messenger weight down the wire rope. On reversal the bottle closes entrapping the water at that depth.

Salinity is directly proportional to chlorinity, which is determined by the titration of the water sample with silver nitrate in the presence of a suitable indicator.

Recently electrical conductivity is used as a measure of salinity and STD or CTD recorders are available for speedy collection of hydrographic data.

1.2. Oceanographic computations

There are a series of computations which the oceanographers do to derive the values of the various dependent quantities from the hydrographic data. These operations begin with the interpolation of the values of temperature and salinity at standard depths and conclude with the computation of currents from the distribution of density as obtained from the temperature and salinity data.

1.2.1. Conversion to standard depths

The first step in the processing of serial data on temperature and salinity is to prepare plots for the vertical distribution of the variables. Such plots of temperature and salinity as a function of depth are useful to detect incorrect values resulting from faulty operation of thermometers and water bottles. Another use of these plots is to scale off depths of decided values of the variables which are necessary for drawing diagrams of horizontal distributions. The main purpose of plotting variables against depth is to obtain interpolated values of temperature and salinity at 'Standard depths'. The International Association of Physical Oceanography has defined standard depths as: Surface, 10,20,30,50,75,100, 150,200,(250),300,400,500,600,(700),800,1000,1200,1500, 2000,2500,3000 and 4000 metres and intervals of 1000 metres thereafter to the greatest depth of sampling. The National Oceanographic Data Centre has accepted the following standard depths viz., Surface, 10,20,30,50,75,100,125,150, 200,250,300,400,500,600,700,800,900,1000,1100,1200,1300, 1400,1500,1750,2000,2500,3000,4000 metres and intervals of 1000 m to the greatest depth of sampling, which are currently being used.

Temperature and salinity data are also plotted on a T-S diagram. Introduced by Helland-Hansen (1916), the temperature-salinity diagrams are polled with salinity on the X-axis and temperature on the y-axis. When salinities are plotted against temperatures, the points generally lie on a well-defined curve. The T-S curve gives the temperature-salinity relationship of the sub-surface water in the area under study. Surface data have to be omitted because annual variations and local modifications lead to discrepancies. The T-S diagram helps to detect errors and to bring out watermass characteristics of the data. After the T-S curve has been drawn for the observed data, corresponding interpolated values of temperatures and salinity read from the vertical curves are also plotted. If these data do not fall on the T-S curve, certain adjustments must be made in the construction of the vertical distribution curves.

1.2.2. Computation of specific volume and specifiv volume anomaly

After obtaining the values of depth, temperature and salinity as detailed above, certain calculations are necessary to derive the values of the various dependent variables commonly used to describe the field of mass in the sea. These variables include: specific volume, anomaly of specific volume from a standard value, density *in situ* and σ_t, which represents density at surface pressure.

Specific volume is volume per unit mass. Specific volume *in situ* in the sea is expressed by the symbol $\alpha_{s,t,p}$ where the subscripts indicate salinity, temperature and pressure of the sample. Specific volume is computed by expressing it as a known specific volume under given conditions plus a series of correction terms for the dependent variables of temperature, salinity and pressure. These terms may be grouped, computed, and added as follows to give specific volume <u>in situ</u>.

$$\alpha_{s,t,p} = (\alpha_{35,0,0} + \delta_p) + (\delta_s + \delta_t + \delta_{s,t}) + \delta_{s,p} + \delta_{t,p} + \delta_{s,t,p}$$

In the first term $\alpha_{35,0,0}$ is a constant (0.97264) and δ_p represents the effect of pressure at standard salinity (35‰) and at 0°C.

The next term depends only on salinity and temperature and may be summed to give $\Delta_{s,t}$ or δ_T, known as the thermosteric anomaly. This term is found from values of temperature and salinity by means of tables or graphs (Sverdrup, 1933). The salinity-pressure term, $\delta_{s,p}$ and the temperature-pressure term $\delta_{t,p}$ are also found from tables or graphs. The last term, $\delta_{s,t,p}$, is so small that it is always neglected.

The sum of the terms $\Delta_{s,t}$, $\delta_{s,p}$ and $\delta_{t,p}$ constitutes the anomaly of specific volume from the standard $\alpha_{35,0,p}$ and is designated by the symbol δ. For computing currents, the variations in specific volume along isobaric surfaces are required. Since pressure is constant along any given isobaric surface, the term δ_p is a constant. It is sufficient therefore to calculate the specific volume anomaly, δ, since the standard term does not contribute to variation in specific volume along an isobaric surface.

Graphs or tables are available for the calculation of specific volume anomaly (Sund, 1926; LaFond, 1940; Callaway, 1950). LaFond (1951) presents the oceanographic tables for finding out the values of the various terms, the sum of which gives the specific volume anomaly.

1.2.3. Computation of density and sigma-t

Density in situ is the reciprocal of specific volume in situ. Another way to express the density is by the symbol $\sigma_{s,t,p}$. By definition $\sigma_{s,t,p}$ is equal to $10^3 (\rho_{s,t,p}-1)$. This expression has the advantage that the numerical value contains fewer digits and is easier to handle.

σ_t represents the density of water of given salinity and temperature at surface pressure. In oceanography, σ_t assumes significance because the motion along σ_t surfaces involves little change in energy and therefore mixing of water masses tends to take place along these surfaces. After calculating $\Delta_{s,t}$, σ_t is obtained directly from the table for σ_t for values of $10^5 \Delta_{s,t}$ (Sverdrup, 1933). Based on Knudsen's equations (Knudsen, 1901), several authors have compiled tables for σ_t for values of temperature and salinity (Mc Ewen, 1929; Mathews, 1932; Fleming, 1939; Ennis, 1944; Bumpus and Martineau, 1948).

1.2.4. Computation of currents

Dynamic computations provide information on relative currents pertaining to the distribution of mass in the sea. Such relative currents are deduced from a consideration of the balance of forces in the sea. The forces considered are those which act along an isobaric surface. When the isobaric surface is not level, a component of gravity acts downward along it. This is balanced by the Coriolis force so that the slope of the surface is maintained.

The thickness of an isobaric layer, which is the layer between two isobaric surfaces, depends upon the average specific volume of the layer. Therefore, the slope of an isobaric surface relative to another, which is assumed to be level, may be found. Since the dynamic height is a measure of the work performed against gravity in moving unit mass from one level to another, the component of the force of gravity acting down the slopping isobaric surface between two stations is the difference in dynamic height of the surface at the two stations divided by the distance between the stations. Equating this expression to the expression for Coriolis force, we can obtain the component of current normal to the line joining the two stations (Sandstrom and Helland-Hansen, 1903). This current is at the upper isobaric surface and is relative to any current which is present at the lower reference surface. In dynamic computation, the isobaric surface is assumed level at some depth where the motion is negligible and the dynamic slope of an upper isobaric surface is found from the variation of specific volume along the isobaric layer. Thus, the current at the upper surface relative to any possible current at the lower surface is determined.

For each oceanographic station, the dynamic thickness of the isobaric layer is calculated by means of the equation,

$$D_2 - D_1 = \int_{p_1}^{p_2} \alpha \, dp$$

where $D_2 - D_1$ is the dynamic thickness of the isobaric layer, α is the specific volume and dp is the pressure interval. Since $\alpha = \alpha_{35,0,p} + \delta$, the total dynamic thickness of the layer may be considered as the sum of the dynamic thickness of the layer of standard specific volume and the increment in dynamic thickness due to the anomaly of specific volume from the standard. Since the dynamic thickness of the standard layer is the same at every station, the differences in dynamic height between stations are given by the differences in the increments which can be obtained as $\int_{p_1}^{p_2} \delta \, dp$. In this, metres of depth are substituted for decibars of pressure.

The dynamic computation involves the following procedures. The specific volume anomaly, δ, at each depth is calculated. The mean specific volume anomaly, $\bar{\delta}$, for each depth interval is determined by averaging the two bounding values. This is multiplied by the depth interval to get the anomaly of the dynamic height, ΔD, for each small depth layer. Total ΔD for each station is obtained by adding the anomalies of dynamic height from the selected reference level to the level at which relative currents are to be computed. The relative current velocity normal to a line joining two stations is obtained in metres/second as

$$V = \frac{10(\Delta D_A - \Delta D_B)}{L\ 2\ \omega \sin \phi}$$

where $\Delta D_A - \Delta D_B$ is equal to the difference in the anomalies in the dynamic height at stations A and B in dynamic metres, L is the distance between the stations in metres, ω is the angular velocity of the earth (0.789×10^{-4} radians/sec.) and ϕ is the mean latitude between the stations.

1.3. Scheme of the present work.

The results of an investigation on the limits of the random errors contained in the basic data of Physical Oceanography and their propagation through the computational procedures are presented in this thesis. It also suggest a method which increases the reliability of the derived results. The thesis is presented in eight chapters including the introductory chapter. Chapter 2 discusses the general theory of errors that are relevant in the context of the propagation of errors in Physical Oceanographic computations. The error components contained in the independent oceanographic variables namely, temperature, salinity and depth are deliniated and quantified in chapter 3. Chapter 4 discusses and derives the magnitude of errors in the computation of the dependent oceanographic variables, density *in situ*, σ_t, specific volume and specific volume anomaly, due to the propagation of errors contained in the independent oceanographic variables. The errors propagated into the computed values of the derived quantities namely, dynamic depth and relative currents, have been estimated and presented chapter 5. Chapter 6 reviews the existing methods for the identification of level of no motion and suggests a method for the identification of a reliable zero reference level. Chapter 7 discusses the available methods for the extension of the zero reference level into shallow regions of the oceans and suggests a new method which is more reliable. A procedure of graphical smoothening of dynamic topographies between the error limits to provide more reliable results is also suggested in this chapter. Chapter 8 deals with the computation of the geostrophic current from these smoothened values of dynamic heights, with reference to the selected zero-reference level. The summary and conclusion are also presented in this chapter.

CHAPTER II

THEORY OF ERRORS

The results of experimental observations will have always inaccuracies. It is necessary to know the magnitude of these inaccuracies in order to arrive at the inaccuracy in a computed result using the observations. A knowledge of the statistical behaviour of the inaccuracies or errors of observations will be of great help in reducing the effect of these uncertainties in the final result.

2.1. Errors of observations and measurements

All observations and measurements are subject to three kinds of errors: systematic errors, accidental or random errors, and mistakes.

Systematic errors are those which affect all measurements alike. They are mostly due to imperfections in the construction or adjustment of the instrument, the 'personal equation' of the observer, etc. Such errors can be remedied by applying proper corrections.

Accidental or random errors are those whose causes are unknown and indeterminate. They are usually small.

It is found expererically that such random errors are frequently distributed according to a simple law, the law of chance. This makes it possible to use statistical methods to deal with random errors. The mathematical theory of errors deals with random errors only.

Mistakes are not, properly speaking, errors at all. They are blunders performed during reading of an instrument, recording of a result and in computations. They can be eliminated by careful work.

The word 'precision' is used in relation to random errors. A precise measurement will be free from random errors. An accurate measurement is one that is free from all kinds of errors - systematic, random and mistakes.

2.2. Errors in approximations

Numbers such as 2, 1/3, etc. are known as exact numbers because there is no uncertainty in them. On the other hand, numbers such as π, $\sqrt{3}$, etc. even though exact, cannot be expressed exactly by a finite number of digits. These numbers, when expressed in digital form, are known as approximate numbers.

Numbers of the above type and quotients of division which never terminate will have to be cut down to a manageable size to be used in practical computations. This process of cutting off superfluous digits and retaining as many as desired is known as 'rounding off'. This process, obviously, introduces an error into the number. The following rule for rounding off will cause the least possible error.

'To round off a number to n significant figures, discard all digits to the right of the n^{th} place. If the discarded number is less than half a unit in the n^{th} place, leave the n^{th} digit unchanged; if the discarded number is greater than half a unit in the n^{th} place, add one to the n^{th} digit. If the discarded number is exactly half a unit in the n^{th} place, round off so as to leave the n^{th} digit an even number' (Scarborough, 1966). When a number is rounded off according to this rule, it will be correct to n significant figures. The error introduced into the number due to rounding off will not be greater than half a unit in the n^{th} significant figure.

2.3. Errors in computations

When various quantities are used to calculate a result and when the different quantities have errors in them, then the result will also be in error by an amount which depends on the errors of the individual quantities.

2.3.1. The general formula for errors

Let

$$Q = f(a_1, a_2, a_3, \ldots\ldots, a_n) \tag{2.1}$$

be a function of several independent quantities $a_1, a_2, \ldots\ldots a_n$ which are subject to errors $\delta a_1, \delta a_2, \ldots\ldots, \delta a_n$. These errors in a's will cause an error, δQ, in the function Q according to the relation

$$Q + \delta Q = f\ (a_1 + \delta a_1,\ a_2 + \delta a_2,\ \ldots\ldots,\ a_n + \delta a_n) \tag{2.2}$$

To find an expression for δQ, we must expand the right hand side by Taylor's theorem.

$$\begin{aligned} & f(a_1+\delta a_1,\ a_2+\delta a_2,\ \ldots\ldots\ldots\ldots,\ a_n+\delta a_n) \\ &= f(a_1, a_2, \ldots\ldots, a_n) + \delta a_1 \frac{\partial f}{\partial a_1} + \delta a_2 \frac{\partial f}{\partial a_2} + \ldots\ldots \\ &\quad + \delta a_n \frac{\partial f}{\partial a_n} + 1/2\ [(\delta a_1)^2 \frac{\partial^2 f}{\partial a_1^2} + \\ &\quad (\delta a_2)^2 \frac{\partial^2 f}{\partial a_2^2} + \ldots\ldots + (\delta a_n)^2 \frac{\partial^2 f}{\partial a_n^2} + \\ &\quad 2\,\delta a_1 \delta a_2 \frac{\partial^2 f}{\partial a_1 \partial a_2} + \ldots\ldots] + \ldots\ldots \end{aligned} \tag{2.3}$$

Since the errors $\delta a_1, \delta a_2, \ldots\ldots$ are relatively small we may neglect their squares, products and higher powers and so

$$Q + \delta Q = f(a_1, a_2, \ldots\ldots\ldots, a_n) + \delta a_1 \frac{\partial f}{\partial a_1} + \delta a_2 \frac{\partial f}{\partial a_2} + \ldots\ldots\ldots + \delta a_n \frac{\partial f}{\partial a_n} \qquad (2.4)$$

Substracting equation (2.1) from equation (2.4),

$$\delta Q = \delta a_1 \frac{\partial f}{\partial a_1} + \delta a_2 \frac{\partial f}{\partial a_2} + \ldots\ldots\ldots + \delta a_n \frac{\partial f}{\partial a_n}$$

$$= \frac{\partial Q}{\partial a_1} \delta a_1 + \frac{\partial Q}{\partial a_2} \delta a_2 + \ldots\ldots + \frac{\partial Q}{\partial a_n} \delta a_n \ \ldots \qquad (2.5)$$

This is the general formula for computing the error of a function and gives the absolute error. The expression, obviously, is the total differential of the function Q. The relative error in the function Q is the quotient obtained when the absolute error is divided by the true value of the quantity.

i.e.

$$\frac{\delta Q}{Q} = \frac{\partial Q}{\partial a_1} \frac{\delta a_1}{Q} + \frac{\partial Q}{\partial a_2} \frac{\delta a_2}{Q} + \ldots\ldots\ldots\ldots + \frac{\partial Q}{\partial a_n} \frac{\delta a_n}{Q} \qquad (2.6)$$

2.3.2. Application of the general formula for errors in the fundamental operations of Arithmetic

(i) Addition

Let the function Q be of the form

$$Q = a_1 + a_2 + \ldots + a_n \tag{2.7}$$

Applying the general formula for errors to the function Q, the absolute error E_a is,

$$E_a = \delta Q = \delta a_1 + \delta a_2 + \ldots + \delta a_n \tag{2.8}$$

Here each of the δa_i are just as likely to be positive as negative. In order to be sure of the maximum error in the function Q, we must take all the terms with the positive sign.

Formula (2.8) indicates that when a few numbers of different accuracies are added it would be useless and absurd to retain all the decimal digits in all the numbers because the error in the result, any way, will be greater than half a unit in the last significant figure of the least accurate number. A safe rule for addition will be to retain one more decimal digit in the more accurate numbers than is contained in the least accurate number and round off the result to the decimal digit contained in the least accurate number. The result usually will be uncertain by one unit in the last figure. Retaining one more digit in the more accurate numbers than is contained in the least accurate number eliminates the possibility of the errors due to rounding off the more accurate numbers from affecting the error in the final result.

Since errors can be just as likely to be positive as negative, their algebraic sum will never be large when a large number of approximate numbers are added. And the mean of several approximate numbers can be more accurate than the numbers from which it was obtained because the computation involves addition as well as division by the total number (Scarborough, 1966).

(ii) Subtraction

Let the function Q be of the form

$$Q = a_1 - a_2 \tag{2.9}$$

Applying the general formula for errors to the function Q

$$\delta Q = \delta a_1 - \delta a_2 \tag{2.10}$$

Since the errors δa_1 and δa_2 are as likely to be positive as negative, we should take the sum of the errors to get the maximum error in the function Q. Hence, the absolute error E_a is

$$E_a = d Q = d a_1 + d a_2 \tag{2.11}$$

A safe rule for the subtraction between two numbers of unequal accuracy is to round off the more accurate number to the same number of decimal places as the less accurate number and then subtract. The result usually will be in error by one unit in the last figure.

(iii) Multiplication

Let the function Q be of the form

$$Q = a_1 \times a_2 \times \ldots \times a_n \tag{2.12}$$

Taking the total differential and dividing by Q, the relative error E_r is,

$$E_r = \frac{d Q}{Q} = \frac{d a_1}{a_1} + \frac{d a_2}{a_2} + \ldots\ldots + \frac{d a_n}{a_n} \tag{2.13}$$

Here again, each of the $d a_i$ can be as likely to be positive as negative and in order to be sure of the maximum relative error in the function Q, we must take the sum of the relative errors.

The accuracy of a product should be investigated by means of relative error. The absolute error, can be found from the relation

$$E_a = E_r \times Q \tag{2.14}$$

While finding the product of two or more approximate numbers, the safe rule is to retain one more significant figure in the more accurate factors than that contained in the least accurate factor and to round off the result to as many significant figures as in the least accurate factor (Scarborough, 1966).

(iv) Division

Let the function be of the form

$$Q = \frac{a_1}{a_2} \tag{2.15}$$

Taking the total differential of the function and dividing by Q

$$\frac{\delta Q}{Q} = \frac{\delta a_1}{a_1} - \frac{\delta a_2}{a_2} \tag{2.16}$$

The errors δa_1 and δa_2 are as likely to be positive as negative and to get the maximum relative error in the function, we must take the sum of the relative errors. Hence, the relative error E_r is

$$E_r = \frac{\delta Q}{Q} = \frac{\delta a_1}{a_1} + \frac{\delta a_2}{a_2} \qquad (2.17)$$

The accuracy of quotients also should be investigated using relative error. The safe rule for division is to retain one more significant figure in the more accurate factor than the number of significant figures contained in the less accurate factor and to round off the result to the number of significant figures contained in the less accurate factor (Scarborough, 1966).

2.3.3. <u>The normal law of errors</u>

Since an error can be just as likely to be positive as negative, their algebraic sum will never be large in a computation involving a large number of approximate numbers. In such situations, the error in the result should be obtained using the normal law of errors, since it is generally found that the normal distribution or Gauss distribution describes the distribution of random errors (Young, 1962).

A random error may be assumed to be the result of a large number ℓ of elementary errors, all of equal magnitude ϵ, and each equally likely to be positive or negative. Now we assume the following facts to be true for the distribution of random errors.

i) Small errors are more frequent than large errors.

ii) All errors are equally likely to be positive or negative.

iii) Very large errors do not occur.

Then the probability of occurrence of an error x in the range $(-\ell\epsilon)$ to $(+\ell\epsilon)$ is given by the probability equation

$$y = P(x) = \frac{h}{\sqrt{\pi}} \; e^{-h^2 x^2} \qquad (2.18)$$

where h is called the measure or index of precision of the distribution represented by the equation and known as the normal error distribution. The measure of precision h and the standard deviation σ of the distribution is related as

$$h = \frac{1}{\sqrt{2}\,\sigma} \qquad (2.19)$$

Hence the probability equation can be written in terms of σ as:

$$Y = P(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \; e^{-\frac{x^2}{2\sigma^2}} \qquad (2.20)$$

(Scarborough, 1966)

Equation (2.18) shows that zero error has maximum probability of occurrence and the probability decreases as the magnitude of the error increases. The quantity h is called the 'index of precision' because, for larger values of h the probability of occurrence of zero error is larger and also the probability decreases faster as the magnitude of the error increases. This means that small errors are more frequent than large errors which indicates precision of the data.

For the normal error distribution, the probability of an error to fall within 1,2 and 3 standard deviations from the mean, which is zero error, is given in Table I (Young, 1962).

Table I

σ	P(x)	% P(x)
1	0.683	68.3
2	0.954	95.4
3	0.997	99.7

If M_1, M_2, M_3, ..., M_n are n independent normal error distributions having standard deviations σ_1, σ_2, σ_3 ... σ_n respectively, then their sum

$$F = M_1 + M_2 + M_3 \ \ldots\ldots + M_n \tag{2.21}$$

will also be a normal error distribution whose standard deviation is given by (Scarborough 1966)

$$\sigma_F = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots\ldots\ldots + \sigma_n^2} \qquad (2.22)$$

The standard deviation of the normal error distribution which results from the summation of S values, each of which may, with equal probability, contain an error equal to one of the values of the finite sequence

$$-l\epsilon,\ -(l-1)\epsilon,\ -(l-2)\epsilon,\ \ldots\ldots\ldots,\ -2\epsilon,\ -\epsilon,\ 0,$$
$$\epsilon,\ 2\epsilon,\ \ldots\ldots\ldots\ldots,\ (l-2)\epsilon,\ (l-1)\epsilon,\ l\epsilon$$

may be obtained as follows. The standard deviation of the above sequence is given by

$$\sigma = \frac{\sqrt{S\sum_{-l}^{l}(n\epsilon)^2}}{\sqrt{\sum_{-l}^{l} n}} = \frac{\epsilon\sqrt{S.2\sum_{0}^{l} n^2}}{\sqrt{2l+1}}$$

$$= \frac{\epsilon\sqrt{S\cdot 2l(l+1)(2l+1)}}{\sqrt{6(2l+1)}} = \frac{\epsilon\sqrt{Sl(l+1)}}{\sqrt{3}}$$

Assuming ℓ to be large, the above may be approximated to

$$\sigma = \epsilon \frac{\sqrt{S\ell^2}}{\sqrt{3}}$$
$$= \frac{\sqrt{S}\,\ell\epsilon}{\sqrt{3}} * \qquad (2.24)$$

The formula for the standard deviation in this form will be useful when only the maximum value of the individual error, $\ell\epsilon$, is known

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*Formula (2.24) is derived in the same way as was done by Fomin (1964) except for his assumption that each of the S values of the summation contains an error, with equal probability equal to one of the intigers in the range $-\ell$ to ℓ. With this assumption he got the formula for the index of precision as

$$h = \frac{\sqrt{6}}{2\sqrt{\ell(\ell+1)S}} \qquad (2.25)$$

The formula for the standard deviation can easily be obtained from the above as

$$\sigma = \frac{\sqrt{S\ell(\ell+1)}}{\sqrt{3}} \qquad (2.26)$$

There is a conceptual error in the above assumption that the error can assume only values of the intigers. Formula (2.26) will also result in different magnitudes for the standard deviation for different units used for the error ℓ.

2.4. The law of propagation of errors

Consider a quantity Q which is calculated from several observed quantities a_1, a_2, a_3 ......

$$Q = f(a_1, a_2, a_3 \ .....) \qquad (2.27)$$

Suppose that $a_1, a_2, a_3,$ .... are all measured N times. Then the law of propagation of errors is expressed by the equation (Young, 1962)

$$\sigma_Q^2 = \left(\frac{\partial Q}{\partial a_1}\right)^2 \sigma_{a_1}{}^2 + \left(\frac{\partial Q}{\partial a_2}\right)^2 \sigma_{a_2}{}^2 + \left(\frac{\partial Q}{\partial a_3}\right)^2 \sigma_{a_3}{}^2 + \ .... \qquad (2.28)$$

where σ_{a_1}, σ_{a_2}, σ_{a_3}, ... are the standard deviations of the quantities $a_1, a_2, a_3,$ .... Where the observations are not repeated, as in the case of field sciences like oceanography, meteorology, etc., the equation expressing the law of propagation of errors reduces to the general formula for errors expressed by the equation (2.5).

CHAPTER III

ERRORS IN THE MEASUREMENT OF THE INDEPENDENT OCEANOGRAPHIC PARAMETERS

Measurements of independent oceanographic parameters, namely, temperature, salinity and pressure, are subject to errors. In this chapter the magnitude of such errors are discussed assuming that only random errors are committed in the measurements.

3.1. Temperature

The temperature of sea water at various depths is measured using a protected reversing thermometer (Sverdrup *et al.*, 1942). To get the true *in situ* temperature, two corrections are to be added algebraically to the main thermometer reading. The first correction is for the relative expansion of mercury and glass subsequent to reversal, since the ambient temperature at which the thermometer is read may be quite different from that at which it was reversed. Schumacher's formula (Schumacher, 1923) is made use of for this purpose which gives the correction as

$$C = \frac{(T'-t)\ (T'+V_o)}{K} \left[1 + \frac{(T'-t) + (T'+V_o)}{K}\right] \qquad (3.1)$$

where the symbols have the following meanings:

C - expansion correction to be added algebraically to the main thermometer reading, expressed in ^{o}C.

t - auxiliary thermometer reading in ^{o}C.

T'- main thermometer reading in ^{o}C.

Vo- Volume of mercury below ^{o}C mark of the main thermometer, expressed in ^{o}C of the main thermometer scale.

K - reciprocal coefficient of the relative expansion of mercury and thermometer glass. This value for most of the modern thermometers in $6100^{o}C$.

The second correction, namely the index correction, represented by I, is for the errors caused by the irregular cross section of the capillary and for the irregularities of the scale etching. Hence the total correction to be added algebraically to the protected main thermometer reading is

$$\Delta T = C + I \qquad (3.2)$$

and the corrected reading of the protected main thermometer, T_w, is given by

$$\begin{aligned} T_w &= T' + \Delta T \\ &= T' + C + I \qquad (3.3) \end{aligned}$$

Since all the different quantities in the expression (3.2) are subject to measurement errors, addition of the expansion correction C and the index correction I to the uncorrected main thermometer reading introduces errors into the corrected main thermometer reading and this can be obtained by taking the total differential of the above expression (3.3)

$$d\,T_w = d\,T' + d\,C + d\,I \tag{3.4}$$

The error $d\,I$ introduced by the addition of the index correction is neglected in the present discussion assuming it to be small since modern reversing thermometers have very uniform capillary bore and also extremely uniform scale etching. An expression for $d\,C$ is obtained by taking the total differential of the expression for C and is given by

$$\begin{aligned} dC = \Bigg[& -\left\{ \frac{1}{T'-t} + \frac{1}{K+(T'-t)+(T'+V_0)} \right\} dt \\ & + \left\{ \frac{1}{T'-t} + \frac{1}{T'+V_0} + \frac{2}{K+(T'-t)+(T'+V_0)} \right\} dT' \\ & + \left\{ \frac{1}{T'+V_0} + \frac{1}{K+(T'-t)+(T'+V_0)} \right\} d\,V_0 \\ & - \left\{ \frac{1}{K+(T'-t)+(T'+V_0)} - \frac{2}{K} \right\} dK \Bigg] C \end{aligned}$$

Now, since each of the errors t, T', V_o, and K are just as likely to be positive as negative, we should take all the terms with a positive sign to be sure of the maximum error in C. The final expression for C is:

$$\dot{d}C = \Bigg[\left\{\frac{1}{T'-t} + \frac{1}{K+(T'-t)+(T'+V_o)}\right\} dt$$

$$+ \left\{\frac{1}{T'-t} + \frac{1}{T'+V_o} + \frac{2}{K+(T'-t)+(T'+V_o)}\right\} d$$

$$+ \left\{\frac{1}{T'+V_o} + \frac{1}{K+(T'-t)+(T'+V_o)}\right\} dV_o$$

$$+ \left\{1 + \frac{(T'-t)+(T'+V_o)}{K+(T'-t)+(T'+V_o)}\right\} \frac{dK}{K}\Bigg] C$$

(3.6)

Modern reversing thermometers have a minimum scale division of $0.1^\circ C$ for its main thermometer. With the use of a magnifying glass, an experienced oceanographer is able to read the main thermometer to an accuracy of $\pm 0.01^\circ C$. The minimum scale division of an auxiliary thermometer is $0.5^\circ C$ and it can be read to an accuracy of $\pm 0.1^\circ C$. The accuracy of V_o value, which is a constant for an individual thermometer may be assumed to be $\pm 0.5^{\circ}C$. The accuracy of K may be taken as $\pm 50^{\circ}C$ assuming only two significant figures in 6100. Using the above magnitudes of errors in the different independent quantities, the error due to the addition of expansion correction may be computed using the expression (3.6). Since K is assumed to be correct only to two significant figures, C and therefore δC will, in general, be correct only to two significant figures.

Table II(A) gives values of δC correct to 3 decimal places for different values of $(T'-t)$ and $(T'+V_0)$. Fig. 1(A) shows the graph of δC against $(T'-t)$ for two extreme values of $(T'+V_0)$, namely $70^{\circ}C$ and $200^{\circ}C$. The graph shows that δC increases as both $(T'-t)$ and $(T'+V_0)$ increases. The following generalisations can be made from this graph.

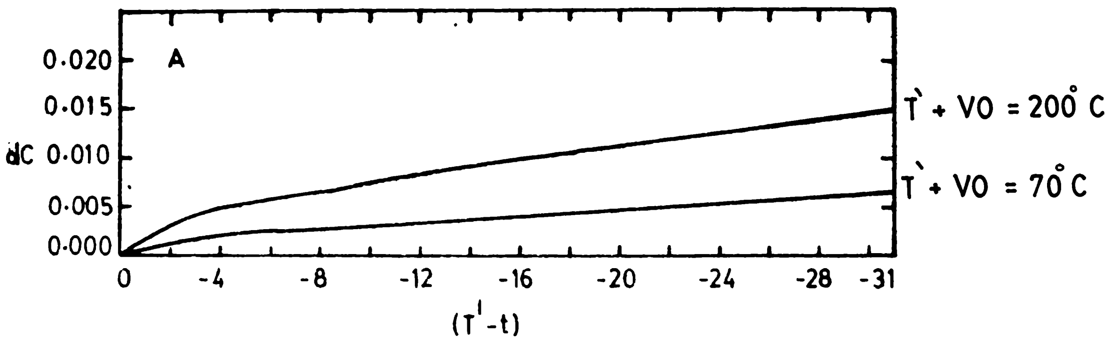
i) When $(T'-t) < -4^{\circ}C$, the value of $\delta C < 0.005^{\circ}C$ for all values of $(T'+V_0)$. δC may be neglected in such situations, and the maximum error in the corrected reading of the main thermometer, δT_w, is

$$\begin{aligned} \delta T_w &= \delta T' + \delta C \\ &= 0.01^{\circ}C + 0 \\ &= 0.01^{\circ}C \end{aligned}$$

Table II(A)

Values of δC (in 10^{-3} degrees C) for different values of $(T'-t)$ and $(T'+V_o)$

$(T'-t)$	$(T'+V_o)$ 70°C	80°C	90°C	100°C	110°C	120°C	130°C	140°C	150°C	160°C	170°C	180°C	190°C	200°C
1°C	1	1	2	3	3	3	3	3	4	4	4	4	4	4
4°C	2	2	3	3	3	3	4	4	4	4	5	5	5	5
7°C	3	3	3	4	4	4	4	5	5	5	6	6	6	6
10°C	3	3	4	4	4	5	5	6	6	6	7	7	7	8
13°C	4	4	4	5	5	5	6	6	7	7	7	8	8	9
16°C	4	5	5	5	6	6	7	7	8	8	8	9	9	10
19°C	5	5	6	6	7	7	7	8	9	9	9	10	10	11
22°C	5	6	6	7	7	8	8	8	9	10	10	11	11	12
25°C	6	6	7	7	8	8	9	10	10	11	11	12	12	13
28°C	6	7	7	8	9	9	10	10	11	12	12	13	14	14
31°C	7	7	8	9	9	10	11	11	12	13	13	14	14	15



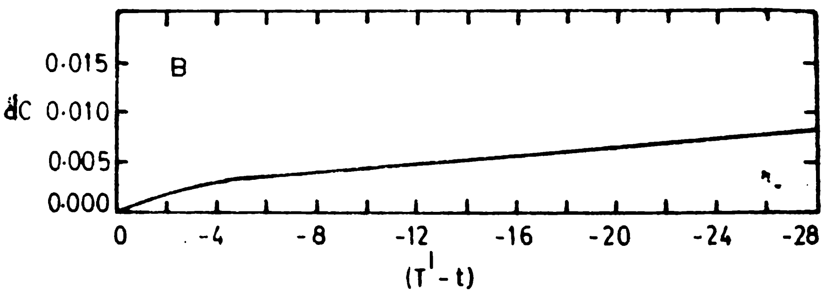


Fig. 1 (A): Curves of errors due to addition of expansion correction in a protected reversing thermometer for two extreme values of $(T'+V_o)$

Fig. 1 (B): Error due to addition of expansion correction for an individual protected reversing thermometer with $V_o=105^oC$ at $t=27^oC$.

ii) When $(T'-t) > -22^{\circ}C$, the value of $\delta C > 0.005^{\circ}C$ for all values of $(T'+V_o)$. In this case, the maximum error in the corrected reading of the main thermometer, δT_w, should be obtained as

$$\begin{aligned}\delta T_w &= \delta T' + \delta C \\ &= 0.01^{\circ}C + 0.01^{\circ}C \\ &= 0.02^{\circ}C\end{aligned}$$

iii) When $(T'-t)$ is in the range from $-4^{\circ}C$ to $-22^{\circ}C$, δT_w may be either $0.01^{\circ}C$ or $0.02^{\circ}C$ depending on the value of $(T'+V_o)$.

As an example of finding the graph of δC against $(T'-t)$ (and T') for an individual thermometer, for a specified value of t, let us assume that the V_o value of the thermometer is $105^{\circ}C$ and $t = 27^{\circ}C$. Now using Table II(A) and interpolating for the in-between values of $(T'+V_o)$, we get the required data for the above graph which is given in Table II(B). The corresponding graph is shown as Fig. 1(B). We get more or less the same graph for V_o values in the range from $95^{\circ}C$ to $115^{\circ}C$, the common range of V_o for modern thermometers, and for t in the range from $25^{\circ}C$ to $29^{\circ}C$, the common range of temperature in the tropical seas. The graph shows that for values of $(T'-t)$ below about $-13^{\circ}C$ (and for values of T' above

Table II(B)

Values of α_C (in 10^{-3} degrees C) for different values of $(T'-t)$, when $V_o = 105^\circ C$ and $t = 27^\circ C$

T'-t	$1^\circ C$	$4^\circ C$	$7^\circ C$	$10^\circ C$	$13^\circ C$	$16^\circ C$	$19^\circ C$	$22^\circ C$	$25^\circ C$	$28^\circ C$
α_C	3	4	4	5	5	6	7	7	8	8

about $14 \pm 2^{\circ}C$), the error introduced due to the addition of expansion correction $d C < 0.005^{\circ}C$, and so is negligible. For values of $(T'-t)$ above $-13^{\circ}C$ (and for values of T' below $14 \pm 2^{\circ}C$), $d C > 0.005^{\circ}C$ and so should be taken as $0.01^{\circ}C$. In a tropical ocean $(T'-t)$ equal to $-13^{\circ}C$ (and T' equal to $14 \pm 2^{\circ}C$) is usually found between the depths of 150 m and 200 m. In the treatment that follows, we will assume a tropical ocean where t can be taken as $27 \pm 2^{\circ}C$, and a thermometer having V_0 equal to $105 \pm 10^{\circ}C$. In this case, the error introduced due to the addition of expansion correction may be taken as negligible upto 200 m and as $0.01^{\circ}C$ below 200 m. This means that the maximum error in the corrected thermometer reading is $\pm 0.01^{\circ}C$ upto 200 m and $\pm 0.02^{\circ}C$ below 200 m.

3.2. Pressure

The practical unit of measurement for pressure used in Physical Oceanography in 1 decibar which is equal to 10^5 dynes/cm^2 (Sverdrup *et al.* 1942). The pressure exerted per square centimetre by 1 metre of sea water is very nearly equal to 1 decibar and so the depth in metres and the pressure in decibars are expressed by nearly the same numerical values. Generally the former is slightly greater than 1 decibar but the difference $d P'$, given by

$$d P' = \rho g \times 100 \text{ dynes/cm}^2 - 1 \text{ decibar}$$

is usually less than 0.6% in a tropical sea. dP' is smaller and may even become negative in the surface layers of the ocean for latitudes towards the equator because both acceleration due to gravity and density are smaller under these conditions. Larger differences occur in deeper layers at higher latitudes because, both acceleration due to gravity and density become higher at these depths and latitudes.

A rough estimate of the depth is obtained by measuring the length of the wire rope payed out and the angle made by the wire with the vertical. This will give the true value of depth only when the depth and the wire angle are small. When both are large, the usual method of determining the depth is by comparing the corrected reading of the protected reversing thermometer with the corrected reading of an unprotected reversing thermometer. For the unprotected thermometer also we have to algebraically add the expansion correction and the index correction to get the corrected reading. The expression for the expansion correction for the unprotected reversing thermometer is given by

$$C_u = \frac{(T_w - t_u)\,(T'_u + Vo_u)}{K_u} \qquad (3.7)$$

where the symbols have the following meanings:

C_u - expansion correction to be added algebraically to the main thermometer reading of the unprotected reversing thermometer, in ^{o}C

T_w - corrected reading of the protected reversing thermometer, in ^{o}C

t_u - auxiliary thermometer reading of the unprotected reversing thermometer, in ^{o}C

T'_u - main thermometer reading of the unprotected reversing thermometer, in ^{o}C

Vo_u - volume of mercury below $0^{o}C$ mark of the main thermometer of the unprotected reversing thermometer, expressed in ^{o}C of the main thermometer scale

K_u - reciprocal coefficient of the relative expansion of mercury and thermometer glass. This value for most of the modern thermometers is $6100^{o}C$.

The index correction, I_u, is for the errors caused by the irregular cross section and for the irregularities of the scale etching. Hence the total correction

$$\Delta T_u = C_u + I_u \qquad (3.8)$$

and the corrected reading of the unprotected thermometer is

$$T_u = T'_u + \Delta T_u$$
$$= T'_u + C_u + I_u \qquad (3.9)$$

Since all the different quantities in the expression (3.8) are subject to measurement errors, addition of the expansion correction and index correction to the uncorrected main thermometer reading introduces errors into the corrected main thermometer reading and is given by

$$d\,T_u = d\,T'_u + d\,C_u + d\,I_u \qquad (3.10)$$

The error dI_u is neglected for the same reasons as was discussed in the case of the protected reversing thermometer. $d\,C_u$ is obtained by taking the total differential of the expression for C_u and is given by

$$d\,C_u = \left[\frac{d\,T_w}{T_w - t_u} - \frac{d\,t_u}{T_w - t_u} + \frac{d\,T'_u}{T'_u + Vo_u} \right.$$
$$\left. + \frac{d\,Vo_u}{T'_u + Vo_u} - \frac{dK_u}{K_u} \right] C_u \qquad (3.11)$$

Since each of the errors, $d\,T_w$, $d\,t_u$, dT'_u, $d\,Vo_u$ and dK_u are just as likely to be positive as negative, we should take all the terms with a positive sign to be sure of the maximum error in C_u. Hence the final expression for $d C_u$ is

$$d C_u = \left[\frac{d T_w + d t_u}{T_w + t_u} + \frac{d \Gamma'_u + d Vo_u}{T'_u + Vo_u} + \frac{d K_u}{K_u} \right] C_u \qquad (3.12)$$

The errors in Γ'_u, t_u, Vo_u and K_u are the same as the corresponding quantities in a protected thermometer. The errors in T_w have already been discussed in the preceding section. With these values for the different errors, $d C_u$ may be computed using the expression (3.12). $d C_u$ may be taken as correct to only two significant figures for the same reason as discussed in the preceding section.

Computation of $d C_u$ for different values of $(T_w - t_u)$ and $(\Gamma'_u + Vo_u)$, for both the cases when $d T_w$ is $\pm 0.01^\circ C$ and $\pm 0.02^\circ C$, results in the same numerical values as shown in Table II(A), the maximum difference being a unit in the third decimal place, for higher values of $(T_w - t_u)$ and $(\Gamma'_u + Vo_u)$. Hence the graph for $d C_u$ against $(T_w - t_u)$ for the two extreme values of $(\Gamma'_u + Vo_u)$, namely $70^\circ C$ and $200^\circ C$, will also be of the same shape and range as Fig. I(A), and so the generalisations arrived at from Fig. I(A) will hold good for an unprotected reversing thermometer also. Calculation of $d C_u$ for different values of $(T_w - t_u)$ (and T'_u) for an individual thermometer for a specified value of t is rather cumbersome, since T'_u increases roughly by 0.01°C for every 1 metre increase in depth. But it can easily be seen that since the difference between the protected and the unprotected thermometer readings is not large in the surface layers, a curve of δC_u drawn against $(T_w - t_u)$ (and T'_u) for a V_o value in the range from 95°C to 115°C and for t_u in the range from 25°C to 29°C will have the same shape and range as in Fig. I(B). For higher values of $(T_w - t_u)$ (and for lower values of T'_u), the difference in the readings of the unprotected and the protected thermometers increases and consequently the curve for δC_u shows, slightly higher values than the values shown by Fig. I(B). Hence we may reasonably assume that the error in the corrected reading of the unprotected thermometer is ±0.01°C upto a depth of 200 metres and ±0.02°C below 200 metres.

Depth is calculated using the formula (LaFond, 1951)

$$D = \frac{T_u - T_w}{Q \rho_m} \qquad (3.13)$$

where the symbols have the following meanings

D - depth in metres

T_u - corrected unprotected thermometer reading in °C

T_w - corrected protected thermometer reading in $^\circ C$

ρ_m - mean density of the water column above the depth of reversal of the thermometers

Q - pressure coefficient of the unprotected thermometer expressed in $^\circ C$ increase in the reading per 0.1 kg/cm^2 increase in pressure.

The error in the result using the above formula due to the errors in the different quantities may be obtained by taking the total differential of the expression.

$$d D = \left[\frac{dT_u}{(T_u - T_w)} - \frac{dT_w}{(T_u - T_w)} - \frac{dQ}{Q} - \frac{d\rho_m}{\rho_m}\right] D \qquad (3.14)$$

Since each of the errors dT_u, dT_w, dQ and $d\rho_m$ are just as likely to be positive as negative we should take all the terms with a positive sign in order to be sure of the maximum error in D. Hence

$$d D = \left[\frac{dT_u + dT_w}{T_u - T_w} + \frac{dQ}{Q} + \frac{d\rho_m}{\rho_m}\right] D \qquad (3.15)$$

We have already discussed dT_u and dT_w and found that their magnitude is $+0.01^\circ C$ upto a depth of 200 metres and $\pm 0.02^\circ C$ below 200 metres. Q and ρ_m are usually known correct to 4 significant figures and 6 significant figures respectively and so, the magnitudes of $\frac{dQ}{Q}$ and $\frac{d\rho_m}{\rho_m}$ will be small compared to the term,

$$\frac{\delta T_u + \delta T_w}{T_u - T_w}$$

Hence the former two terms may be neglected. The expression for δD may therefore be approximated as

$$\delta D = \left[\frac{\delta T_u + \delta T_w}{T_u - T_w} \right] D \tag{3.16}$$

The quantity $Q\rho_m$ is approximately equal to $0.01^\circ C/\text{metre}$ (LaFond, 1951) and D may be written as

$$D = \frac{T_u - T_w}{Q\rho_m} = \frac{T_u - T_w}{0.01^\circ C/m} \tag{3.17}$$

Hence, δD is given by

$$\delta D = \left[\frac{\delta T_u + \delta T_w}{(T_u - T_w)} \right] D$$

$$= \frac{\delta T_u + \delta T_w}{(T_u - T_w)} \times \frac{(T_u - T_w)}{0.01^\circ C/m}$$

$$= \frac{\delta T_u + \delta T_w}{0.01^\circ C/m}$$

$$= |(\delta T_u + \delta T_w)\, 100| \text{ metres} \tag{3.18}$$

The sum $(\delta T_u + \delta T_w)$ will be $0.02^\circ C$ upto 200 metres depth and $0.04^\circ C$ below 200 metres depth. So the error in δD may be taken as 2 metres in absolute magnitude upto a depth of 200 metres and 4 metres in absolute magnitude below 200 metres. Hence the total error in taking the pressure in decibars as numerically equal to depth in metres is the sum of $\delta P'$ and the pressure equivalent of the error in depth δD. Thus, for example, the maximum error in taking the pressure at 1000 metres depth equal to 1000 decibars is about 10 decibars in the tropical seas. The actual value is usually lesser since, as mentioned earlier, $\delta P'$ in the surface layers of the ocean, particularly at latitudes near the equator, is small.

3.3. Salinity

Water dissolves almost all known substances and in greater quantities compared to any other liquid. Determination of the total quantity of the dissolved substances is necessary since this affects the density of a water sample. Because of the complexity of sea water, direct determination of the total dissolved substances by chemical means is a near impossibility. Also it is not possible to obtain reproduceable results by evaporating sea water to dryness and weighing the residue since some of the materials present, chiefly chlorides, are lost in the last stages of drying. These difficulties were overcome by following a technique yielding reproduceable results which, although do not represent the total quantity of dissolved solids, do represent a quantity, closely related and called the salinity of water. This technique was established by an International Commission (Knudsen, 1901; Knudsen _et al_. 1902; Forch _et al_. 1902) and on the basis of its work salinity was defined as 'the total amount of solid material in grammes contained in one kilogramme of sea water when all carbonates have been converted to oxides, the bromine and iodine replaced by chlorine and all organic matter completely oxidised'. The salinity obtained according to this definition is slightly less than the total salt content. In fact, a water having a salinity of 34.32‰ will have a total salt content of 34.48‰ (Lyman and Fleming, 1940). Determination of salinity by the method of the International Commission is very rarely carried out because it is time consuming and difficult (Forch _et al_. 1902; Sorensen, 1902). Also it is subject to the following two sources of errors (Riley, 1975).

a) Hydrogen chloride is lost through hydrolysis of magnisium chloride

b) water of crystallisation is retained tenaciously

Morris and Riley (1964) describes a method for the gravimetric determination of salinity which eliminates the above sources of errors. This method, eventhough yields very accurate results with careful work, is not suitable for routine determinations as it is also time consuming.

Since Dittmar's analysis of 77 water samples collected during the Challenger Expedition (Dittmar, 1884), it became obvious that regardless of the absolute concentrations, the relative proportions of the different major constituents are virtually constant. This concept of constancy of composition of sea water is only slightly changed after more precise analyses of numerous sea water samples for the major constituents (Cox et al. 1967). Murray (1893) suggested that it is only necessary to determine the chlorine content in a definite weight of water to ascertain the respective quantities of other constituents. The total halogen content in a definite quantity of water is expressed in terms of chlorinity which is defined as the total amount of chlorine, bromine and iodine in grammes contained in one kilogramme of sea water assuming that the bromine and iodine have been replaced by chlorine (Forch et al. 1902; Sorensen, 1902). A relationship between salinity and chlorinity, namely,

$$S‰ = 0.030 + 1.8050\ Cl‰ \qquad (3.19)$$

was established by Knudsen (Knudsen et al. 1902) from the the salinity data of Sorensen (1902) and this equation became the working definition of salinity. One drawback of the above definition is that it changes every time the atomic weights were changed. Because of this problem, chlorinity was defined in the following way (Jacobsen and Knudsen, 1940): 'The number giving the chlorinity, in per mille, of a sea water sample is, by definition, identical with the number giving the mass with unit gramme of atomic weight silver just necessary to precipitate the halogens in 0.3285234 kilogramme of sea water sample'. This definition of chlorinity, being independent of any redetermination of atomic weights, has been used by all oceanographers and the chlorinity titration remained the preferred precision method for its determination until very recently.

Since mid-fifties, this working definition of salinity came to be questioned increasingly for three reasons.

1. Development of conductometric salinometers established that the chlorinity determination was no longer the only standard method for determining the salinity of sea water.

2. New investigations of the gravimetric salinity determination (Morris and Riley, 1964) threw doubt on the absolute accuracy of Sorensen's methods.

3. The small number and the non-representative distribution of sea water samples used by Sorensen cast doubt on the validity of Knudsen's relationship between salinity and chlorinity.

Because of these factors, a new investigation of the interrelationship between the measured parameters (chlorinity, conductivity ratio) and the derived parameters (salinity, sigma-t) was conducted and an International Joint Panel made the following recommendations (UNESCO, 1962,1965,1966,1968; Cox et al. 1967):

1. Salinity be defined in terms of conductivity ratio at 15^{o}C relative to the conductivity of 'standard sea water' having a salinity of 35‰ and chlorinity of 19.375‰. This means that the position of importance held by the chlorinity titration for the determination of salinity was removed and conductivity measurement took its place. Since the 'standard sea water' is prepared assuming constancy of composition of sea water, which is only nearly true (Cox et al. 1967), the new definition of salinity also is dependent on this concept (Culkin, 1965). This concept can be avoided only when salinity is defined in terms of absolute conductivity instead of relative conductivity.

2. The relationship between salinity and chlorinity be redefined by a truely proportional relationship, namely,

$$S‰ = 1.80655\ Cl‰ \quad \text{(Wooster } \underline{et\ al.}\text{ 1969)} \qquad (3.20)$$

When chlorinity is determined by titration, it be reported as chlorinity itself and use the above relationship to get an estimate of salinity.

The above relationship is compatible with the Knudsen relation and these two yield identical results at a salinity of 35‰. At 32‰ and 38‰ salinities, the difference is 0.0026‰ and at 6‰ salinity it is 0.025‰ (Lyman, 1969). This new definition of chlorinity is an arbitrary one and does not have a chemical meaning, but the salinity obtained this way is very close to the total salt content of sea water (Grasshoff, 1976).

Thus the two methods that are used for the routine determination of salinity of sea water are the following:

a) Chlorinity titration
b) Conductivity ratio measurement

a) <u>Chlorinity titration</u>

Salinity determination by means of chlorinity titration, even though old fashioned, is not completely outdated. One reason for this is that many small laboratories do not have the relatively expensive equipment needed for the conductometric determination of salinity. Another reason is that only small amount of sample is needed for the chlorinity titration compared to the conductivity measurement (Grasshoff, 1976).

The chlorinity of sea water is determined using Mohr-Knudsen method of titration in which the halogen ion in the sea water is titrated with silver nitrate using pottasium chromate as indicator. The silver nitrate solution used should be standardised against 'standard sea water'.

There are several sources of errors in this method.

1. The error caused in pipetting the sea water sample is ±0.01‰ in chlorinity on a chlorinity of 19‰ (Riley, 1965).

2. A temperature change by 2°C of the silver nitrate solution causes an error of ±0.01‰ in chlorinity (Riley, 1965).

3. The smallest division of a Knudsen burette is 0.01 milli litre. The reading accuracy of such a burette is only 0.005 milli litre, equivalent to 0.005‰ in

chlorinity, since the meniscus will interfere with the reading (Fomin, 1964).

4. A similar error of 0.005‰ in chlorinity is caused in the standardisation titration of the silver nitrate solution.

With correct calibration of the burette, proper care and experience, a standard deviation of 0.01‰ in chlorinity can be obtained (Grasshoff, 1976). Hence the error in the determination of chlorinity by the above method can be taken as ±0.02‰ at 95% confidence level. The corresponding error in salinity is obtained by taking the total differential of the equation (3.20),

as
$$\begin{aligned} d\,S‰ &= 1.80655\, d\,Cl‰ \\ &= \pm 0.036‰ \\ &\simeq 0.04‰ \end{aligned}$$

b) Conductivity ratio measurement

As already mentioned, the salinity determination by conductivity ratio measurement has almost completely superseded the chemical determination of chlorinity because of the increased speed, reliability and simplicity of modern salinometers and because of the fact that they can be used at sea by relatively unskilled personnel. Although Knudsen (Knudsen *et al.* 1902; Knudsen, 1903) had tried measurement of conductivity as a means to determine salinity more than 80 years ago, only recently, with the development of reliable electronic equipments (Brown and Hamon, 1961), could the method be effectively used for this purpose.

By international agreement (Wooster *et al.* 1969), the conductivity ratio measured by a salinometer is converted to a salinity value by the use of Tables (UNESCO, 1966) which have been compiled from calculation of salinities from conductivity ratios at 15°C using the fifth order polynomial obtained from the data by Cox *et al.* (1967) after adjusting the constant term by the addition of a small quantity (+0.00018) to make R_{15} = 1.00000 exactly corresponding to a salinity of 35.000‰.

$$\begin{aligned} S‰ = & -0.08996 + 28.29720\ R_{15} + 12.80832\ R_{15}^2 \\ & -10.67869\ R_{15}^3 + 5.98624\ R_{15}^4 \\ & -1.32311\ R_{15}^5 \qquad (3.22) \end{aligned}$$

If the conductivity ratio is measured at a temperature other than 15°C, the conductivity ratio R_t is converted to

R_{15} by the addition of a correction term Δ_{15} (t) given by the equation

$$\Delta_{15}(t) = R_{15} - R_t = 10^{-5} R_t(R_t-1)\ (t-15)$$
$$[96.7-72.0\ R_t + 37.3\ R_t^{\ 2}$$
$$-(0.63+0.21\ R_t^{\ 2}\ (t-15)] \qquad (3.23)$$

This correction term is also obtained from the UNESCO Tables.

Instrument manufactures of the bench type salinometers usually claim a reproduceability of 0.003‰ to 0.002‰ in salinity, if the measurement is carefully performed. But the accuracy of salinity determination by this method is much poorer under realistic field work conditions, even assuming no systematic errors, for the following reasons.

1) Investigation by Grasshoff and Hermann (1975) has shown that the comparability of salinity measurements performed with different instruments, even of the same type, is much worse than the reproduceability of an individual instrument, especially if the salinities are not close to the point of calibration. This source of error is partially overcome by introducing 'standard sea water' of different salinities.

2) Samples used for the measurements by Cox et al. (1967), which form the basis for the International Oceanographic Tables, Vol.I, and the 'standard sea water' have been standardised with respect to the carbonate system. In fact, the composition of sea water is liable to changes in the carbonate system. Park (1965) and Grasshoff (1968) have demonstrated the effect of pH (as a measure of the state of the carbonate system) on the relative conductivity and have shown that the salinity, obtained by measurement of conductivity ratio, changes as pH changes, without changing the total salt content of the seawater. They have also demonstrated that this apparent change in salinity is largest in the pH range occurring in the natural sea water and for high salinities. The above studies have also shown the effect of other variable micro constituents on the apparent salinity of sea water, but their influence has less dominance compared to the changes in the carbonate system.

Considering the above sources of errors, it will be unrealistic to assume, in the salinity determination by the measurement of conductivity ratio, an accuracy better than 0.02‰ under realistic field work conditions (Grasshoff, 1976). The corresponding error in the chlorinity value may be taken as ±0.01‰.

3.4. On the accuracy of *in situ* salinometers

Accurate conductivity measurements *in situ* present difficulties additional to those experienced in the laboratory environment. Conductivity is a function, not only of salinity of sea water but also of temperature and pressure and these two parameters must be monitored for compensation purposes. The instrument which measures salinity (or conductivity), temperature and pressure *in situ* are known as STD and CTD probes. The former has an internal compensation circuit to produce a value for salinity directly from the three measured parameters.

There are several possible sources of errors in the data obtained using the STD and CTD probes.

1. When the time constants of the sensors differ, the compensation circuit of the STD probe produces characteristic transient spikes if a strong temperature gradient is present (Goulet and Culverhouse, 1972). In this situation, it is better to employ a CTD probe and to apply compensation for time constant differences during the calculation of salinity.

2. Another source of error in the STD probe data is the sudden jump in the salinity calibration which usually

occurs due either to a mechanical shock to the sensor head or to a dry solder joint in the probe electronics (Wilson, 1975). They are normally of the order of 0.1%. in salinity and may pass unnoticed unless a close watch is kept. Such calibration jumps can occur for the other parameters also. A usual remedy is to take at least one reversing bottle sample with each STD cast, the bottle being attached to the probe unit or just above it and triggered by messenger after equilibration of the reversing thermometers.

3. The conductivity obtained from the data of CTD probe is only a relative value, relative to the calibration of the instrument. Hence it is important to ascertain the conditions of salinity, temperature and pressure under which the unit was calibrated and also the value of absolute conductivity which was assumed for these conditions. Since the latter is not always explicitly stated, confusion on this point can lead to a significant error (Walker and Chapman, 1973).

4. A possible source of error in the computation of salinity from the conductivity - temperature - depth data from a CTD probe is from the equation used for the purpose. For example, equation for the computation of salinity from conductivity ratio and temperature given by Cox et al. (1967)

is true only in the range of temperature from 10°C to 30°C, although work is in progress to extend the range to 0°C (Wilson, 1975). The equation given by Perkin and Walker (1972) for the same purpose is true only in the range of temperature from -2°C to 20°C. These equations show a marked deterioration outside the stated ranges. The most widely used relationship for the pressure correction is that of Bradshaw and Schleicher (1965).

Hence it is evident that the present STD and CTD units, eventhough highly useful, are, by no means, ideal. Besides the problems of 'spiking' and calibration jump in the STD probes and the non-standardised calibration procedures and differences in calculation in the CTD probes, the available units are not very much reliable and tend to be too complicated and expensive. Some of the problems can be eliminated by taking a reversing water bottle sample during the cast which will allow a correction to be applied to the results from the _in situ_ probe to bring them into agreement with the International Oceanographic Tables. Hence the accuracy of an _in situ_ probe can never be better than that obtainable from a reversing water bottle sample. This situation will continue until instrumental stability is greatly improved, reliable absolute conductivity values are available and the validity of the relationship to calculate salinity in the International Tables is extended below 10°C. Only in such a situation can the emperical correction of the CTD and STD data using a reversing water bottle and reversing thermometers can be discarded.

CHAPTER IV

ERRORS IN THE COMPUTATION OF DEPENDENT OCEANOGRAPHIC VARIABLES

In the preceding chapter we have discussed the random errors that occur in the measurements of the independent oceanographic variables, namely, temperature, depth and salinity. The values of these variables are used to calculate the dependent oceanographic variables, namely, density *in situ*, sigma-t, specific volume and specific volume anomaly. Since the measurements of the independent variables are subject to random errors, the resulting dependent variables will also be in error, the magnitude of which may be obtained using the law of propagation of errors.

4.1. Density *in situ* and sigma-t

The density of any substance is defined as the mass per unit volume and in the cgs system it is stated in grammes per cubic centimetre. The specific gravity of any substance is defined as the ratio of its density to that of distilled water at $4^{o}C$ and under atmospheric pressure. Since in the cgs system the density of distilled water at $4^{o}C$ is unity, the specific gravities are numerically identical with densities. In oceanography, the term density is generally used eventhough specific gravity is the one that is meant.

The density of sea water depends upon the three independent variables, namely, temperature, salinity and pressure. The density of a sea water sample at the temperature and pressure at which it was collected, known as the density *in situ*, is denoted by the symbol $\rho_{s,t,p}$. Knudsen (1901) introduced, for the purpose of convenient handling of numerical values, a symbol $\sigma_{s,t,p}$ as a measure of density *in situ* defined as

$$\sigma_{s,t,p} = (\rho_{s,t,p}-1)\ 10^3 \qquad (4.1)$$

The corresponding quantity at atmospheric pressure is written as σ_t and that at atmospheric pressure and 0^oC as σ_o. σ_t may be obtained from the relation (Knudsen, 1901)

$$\sigma_t = -\Sigma_t + (\sigma_o+0.1324)\ [1-A_t+B_t(\sigma_o-0.1324)] \qquad (4.2)$$

where

$$\Sigma_t = \frac{(T-3.98)^2}{503.570} \times \frac{(T+283)}{(T+67.26)}, \qquad (4.3)$$

$$\sigma_o = -0.069+1.4708\ Cl-0.001570\ Cl^2$$
$$+0.0000398\ Cl^3, \qquad (4.4)$$

$$A_t = [4.7867\ T - 0.098185\ T^2 + 0.0010843\ T^3]\ 10^{-3}, \tag{4.5}$$

and

$$B_t = [18.030T - 0.8164\ T^2 + 0.01667\ T^3]\ 10^{-6} \tag{4.6}$$

$\sigma_{s,t,p}$ as a measure of density <u>in situ</u> may be obtained by the addition of the correction terms computed from formula established by Ekman (1908).

The error in the result of computation for σ_t due to the errors in the temperature and chlorinity measurements may be obtained by taking the total differential of the expression given above and the result may be written as follows.

$$d\sigma_t = \alpha\ dCl + \beta dT \tag{4.7}$$

where

$$\alpha = [1.4708 - 0.003140\ Cl + 0.0001194\ Cl^2] \times [1-A_t + 2\ \sigma_o\ B_t] \tag{4.8}$$

$$\beta = (\sigma_o+0.1324)\ [(\sigma_o-0.1324)\ (18.030-1.6328\ T + 0.05001\ T^2)\ 10^{-6} - (4.7867 - 0.196370\ T + 0.0032529\ T^2)\ 10^{-3}] - \Sigma_t\left(\frac{2}{T-3.98} + \frac{1}{T+283} - \frac{1}{T+67.26}\right)^* \qquad (4.9)$$

The coefficients α and β calculated for different values of temperature and chlorinity are shown as Table III(A) and Table III(B) respectively. Table III(B) shows that β Can be either positive or negative. But, since the error dT is just as likely to be positive as negative, we should take the term β dT with a positive sign to be sure of the maximum error in σ_t.

In chapter 3 we have discussed the magnitudes of the random errors in the measurements of temperature and

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*Fomin (1964) gives the expression for β as

$$\beta = (\sigma_o+0.1324)\left[(\sigma_o-0.1324)\ (18.030-1.6328\ T + 0.05001\ T^2)\ 10^{-6} - (4.7867 - 0.196370\ T + 0.0032529\ T^2)\ 10^{-3} - \Sigma_t\left(\frac{2}{T-3.98} + \frac{1}{T+283} - \frac{1}{T+67.26}\right)\right] \qquad (4.10)$$

Here the position of the double bracket is at the end of the expression which is wrong. This is not a printing mistake because Fomin actually used the wrong expression for the computation of values of β.

chlorinity. It was established that the error in the measurement of temperature in a tropical sea should be considered as $\pm 0.01^{\circ}C$ upto a depth of 200 m and as $\pm 0.02^{\circ}C$ below 200 m. The temperature observed at a depth of 200 m in a tropical sea is around $13^{\circ}C$ and so we can estimate the error committed in the measurement of temperature above $13^{\circ}C$ to be $\pm 0.01^{\circ}C$ and that below $13^{\circ}C$ to be $\pm 0.02^{\circ}C$. We have also established that the error in the chlorinity determinations by the method of chlorinity titration is ± 0.02‰ and that by the method of conductivity ratio measurement is ± 0.01‰. Assuming as above the errors in the determination of temperature and chlorinity, the errors in the computed values of σ_t for different values of temperature and chlorinity may be obtained using expression (4.7) and Tables III(A) and III(B). It can easily be observed that for almost all values of temperature and chlorinity, the error in σ_t is greater than 0.015 when chlorinity is determined by the method of conductivity ratio measurement and that the error is around ± 0.03 when the chlorinity is estimated by the method of chlorinity titration. Hence in the discussion that follows we will assume that the error in the computation of σ_t is ± 0.02 when the chlorinity is determined by the method of conductivity ratio measurement and is ± 0.03

Table III(A)

Variation of coefficient α with temperature and chlorinity

Cl ‰	T °C											
	-2	1	4	7	10	13	16	19	22	25	28	31
10	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.34
12	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.34
14	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.35	1.34
16	1.46	1.45	1.43	1.42	1.40	1.39	1.38	1.37	1.36	1.36	1.35	1.34
18	1.46	1.45	1.43	1.42	1.40	1.39	1.38	1.37	1.37	1.36	1.35	1.35
20	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.36	1.35
22	1.47	1.45	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36	1.36

Table III(β)

Variation of coefficient β with temperature and chlorinity

Cl ‰	T °C											
	-2	1	4	7	10	13	16	19	22	25	28	31
10	+0.03	-0.01	[illegible]	-0.10	-0.13	-0.17	-0.20	-0.23	-0.26	-0.28	-0.31	-0.33
12	+0.02	-0.03	-0.07	-0.11	-0.14	-0.17	-0.20	-0.23	-0.26	-0.29	-0.31	-0.34
14	+0.01	-0.04	-0.08	-0.12	-0.15	-0.18	-0.21	-0.24	-0.27	-0.29	-0.32	-0.34
16	0.00	-0.05	-0.09	-0.12	-0.16	-0.19	-0.22	-0.25	-0.27	-0.30	-0.32	-0.34
18	-0.02	-0.06	-0.10	-0.13	-0.17	-0.20	-0.22	-0.25	-0.28	-0.30	-0.32	-0.35
20	-0.03	-0.07	[illegible]	-0.14	-0.17	-0.20	-0.23	-0.26	-0.28	-0.31	-0.33	-0.35
22	-0.04	-0.08	-0.12	-0.15	-0.18	-0.21	-0.24	-0.26	-0.29	-0.31	-0.33	-0.35

when the chlorinity is determined by the method of chlorinity titration.

Cox <u>et al</u>. (1968) have pointed out that the absolute density of distilled water at 4°C and at atmospheric pressure is about 3 parts in 10^5 below unity and hence the absolute density of sea water is not quite equal to its specific gravity. They also pointed out that the distilled water used by Forch <u>et al</u>. (1902) in their specific gravity determinations was of unknown isotopic composition. Since the distillation procedure used by them is not given, it is now not possible to reproduce the standard used by Forch <u>et al</u>. (1902). This situation warranted a new investigation into the relationship between salinity, temperature and σ_t. Cox <u>et al</u>. (1970) have described an apparatus designed for this purpose. They found that Knudsen's tables gave slightly lower values of specific gravity over the salinity range of normal sea water and the deviation became larger for lower salinities. Investigations of Kremling (1971,1972) using a more accurate density comparison instrument, developed by Kratky <u>et al</u>. (1969), confirmed the above conclusions of Cox <u>et al</u>. (1970). Kremling's work showed that Knudsen's values were low, on an average, by 0.013 in σ_t in the salinity range from

9‰ to 39‰ and the deviation was about 0.025 in σ_t at a salinity of 5‰.

Using the data obtained from his experiments Cox *et al.* (1970) have developed an emperical relationship between temperature, salinity and σ_t, valid in the salinity range from 9‰ to 41‰, as:

$$\sigma_t = 8.00969062 \times 10^{-2} + 5.88194023 \times 10^{-2}\ T + 7.97018644 \times 10^{-1} S - 8.11465413 \times 10^{-3}\ T^2 - 3.25310441 \times 10^{-3}\ ST + 1.31710842 \times 10^{-4}\ S^2 + 4.76600414 \times 10^{-5}\ T^3 + 3.89187483 \times 10^{-5}\ ST^2 + 2.87971530 \times 10^{-6}\ S^2T - 6.11831499 \times 10^{-8}\ S^3 \tag{4.11}$$

As before, the error in the result of computation for σ_t due to the errors in the temperature and salinity measurements may be obtained by taking the total differential of the above expression and the result may be written as:

$$d\sigma_t = a\ ds + b\ dt \tag{4.12}$$

where

$$a = 7.9702 \times 10^{-1} - 3.2531 \times 10^{-3}\ T + 2.6342 \times 10^{-4}\ S + 3.8919 \times 10^{-5}\ T^2 + 5.7594 \times 10^{-6}\ ST - 1.8355 \times 10^{-7}\ S^2 \tag{4.13}$$

$$b = 5.8319 \times 10^{-2} - 1.6629 \times 10^{-2}\, T - 3.2531 \times 10^{-3}\, S + 1.4298 \times 10^{-4}\, T^2 + 7.7838 \times 10^{-5}\, ST + 2.8797 \times 10^{-6}\, S^2 \qquad (4.14)$$

The coefficients a and b are calculated for different values of temperature and salinity and are shown as Table IV(A) and Table IV(B) respectively. Table IV(B) shows that b can be either positive or negative. But, since the error dT is just as likely to be positive as negative, we should take the term bdT with a positive sign to be sure of the maximum error in σ_t.

Considering a tropical sea and following the same argument discussed in connection with the computational error in σ_t when Knudsen's relation is used for the purpose, we can estimate that the error committed in the measurement of temperature above 13°C is $\pm$0.01°C and that below 13°C is $\pm$0.02°C. As established in section 3.3, the error in the determination of salinity by the chlorinity titration method is $\pm$0.04‰ and that by the method of conductivity ratio measurement is $\pm$0.02‰. Quantifying as above the errors in the determination of temperature and salinity, the error in the computed value of σ_t for different values of temperature and salinity may be obtained using the relation (4.12) and Tables IV(A) and IV(B). Here also it may be observed

Table IV(A)

Variation of coefficient 'a' with temperature and salinity

S ‰	T °C											
	-2	1	4	7	10	13	16	19	22	25	28	31
15	0.81	0.80	0.79	0.78	0.77	0.77	0.76	0.75	0.75	0.75	0.74	0.74
20	0.81	0.80	0.79	0.78	0.77	0.77	0.76	0.76	0.75	0.75	0.74	0.74
25	0.81	0.80	0.79	0.78	0.78	0.77	0.76	0.76	0.75	0.75	0.75	0.74
30	0.81	0.80	0.79	0.79	0.78	0.77	0.77	0.76	0.76	0.75	0.75	0.75
35	0.81	0.80	0.79	0.79	0.78	0.77	0.77	0.76	0.76	0.75	0.75	0.75
40	0.81	0.80	0.80	0.79	0.78	0.77	0.77	0.76	0.76	0.76	0.75	0.75

Table IV(B)

Variation of coefficient 'b' with temperature and salinity

S ‰	T °C											
	-2	1	4	7	10	13	16	19	22	25	28	31
15	+0.04	0.00	-0.05	-0.09	-0.13	-0.17	-0.20	-0.23	-0.26	-0.29	-0.31	-0.33
20	+0.03	-0.02	-0.06	-0.10	-0.14	-0.18	-0.21	-0.24	-0.27	-0.29	-0.31	-0.33
25	+0.01	-0.04	-0.08	-0.12	-0.15	-0.19	-0.22	-0.25	-0.27	-0.30	-0.32	-0.34
30	-0.01	-0.05	-0.09	-0.13	-0.16	-0.20	-0.23	-0.26	-0.28	-0.30	-0.32	-0.34
35	-0.02	-0.07	-0.10	-0.14	-0.18	-0.21	-0.24	-0.26	-0.29	-0.31	-0.33	-0.34
40	-0.04	-0.08	-0.12	-0.15	-0.19	-0.22	-0.25	-0.27	-0.29	-0.31	-0.33	-0.35

that the magnitude of the error in the computed value of σ_t is such that d σ_t may be taken as ± 0.02 when the salinity is determined by the method of conductivity ratio measurement and as ± 0.03 when the salinity is determined by the method of chlorinity titration.

This shows that the result can be in error by ± 0.02 in σ_t even when using the best available methods for the measurement of temperature and salinity. For more accurate results we may have to go for instrumental methods. One such instrument described by Cox et al. (1970), eventhough gives accurate results (the accuracy claimed is ± 0.008 in σ_t), is not suitable for routine work because of its complexity. Kratky et al. (1969) has developed a more accurate density comparison instrument suitable for routine work. Although this instrument appears to be rather slower in operation compared to modern salinometers, it gives better results compared to computational methods. The accuracy claimed for the instrument is $\pm 1.5 \times 10^{-6}$ in density for a variation of temperature by $\pm 0.01^{\circ}C$. Assuming the temperature variation in the instrument to be within $\pm 0.01^{\circ}C$, since the maximum error in the determination of temperature is $\pm 0.02^{\circ}C$, the maximum error in the determination of density *in situ* will be 4.5×10^{-6} or less than half a unit in the second decimal place of the σ_t value. This means that, if the accuracy claimed for the instrument can really be achieved in the routine work, the result will be at least four times more accurate than the result obtained from computational methods.

4.2. Specific volume and specific volume anomaly

Instead of density *in situ*, its reciprocal, $\alpha_{s,t,p}$, called the specific volume *in situ*, is generally used in Dynamic Oceanography. In order to avoid writing a large number of decimals, the specific volume is commonly expressed as an anomaly, δ defined as

$$\delta = \alpha_{s,t,p} - \alpha_{35,0,p} \tag{4.15}$$

where $\alpha_{35,0,p}$ is the specific volume of a 'standard ocean' of salinity 35‰ at $0^{\circ}C$ and at pressure p expressed in decibars. The values of $\alpha_{35,0,p}$ for the 'standard ocean' were first tabulated by Bjerknes and Sandstrom (1910). The value of the anomaly δ, which contains the departures of the real ocean from the 'standard ocean' depends on the temperature, salinity and pressure and hence can be expressed as

$$\delta = \delta_s + \delta_t + \delta_{s,t} + \delta_{s,p} + \delta_{t,p} + \delta_{s,t,p} \tag{4.16}$$

The last term of the above expression, $\delta_{s,t,p}$, is so small that it can always be neglected. The first three terms, which are independent of pressure, are usually grouped together and the sum is known as the thermo-steric anomaly, δ_T. The value of δ_T may be obtained from the value of σ_t using the relation

$$\delta_T = 0.02736 - \frac{10^{-3}\sigma_t}{1+10^{-3}\sigma_t} \qquad (4.17)$$

Hence

$$\delta = \delta_T + \delta_{s,p} + \delta_{t,p} \qquad (4.18)$$

The values of the three terms on the right hand side of the above expression may be obtained from Oceanographic Tables and hence the specific volume anomaly in situ may be computed. Sverdrup (1933) was the first to compile these tables.

The error in the result of computation for δ due to the errors in the measurements of temperature, salinity and pressure may be obtained by taking the total differential of the above expression.

$$d\delta = d\delta_T + d\delta_{s,p} + d\delta_{t,p} \qquad (4.19)$$

The magnitudes of the errors $d\delta_{s,p}$ and $d\delta_{t,p}$ are small compared to the magnitude of error $d\delta_T$ so that the former two are neglected in the following discussions.

Hence

$$d\delta \simeq d\delta_T \tag{4.20}$$

$d\delta_T$ may easily be obtained from the relation (4.17) as

$$d\delta_T = -\frac{10^{-3} d\,\sigma_t}{(1+10^{-3}\sigma_t)^2} \tag{4.21}$$

Hence

$$d\delta \simeq d\delta_T = -\frac{10^{-3}\,\sigma_t}{(1+10^{-3}\sigma_t)^2} \tag{4.22}$$

The negative sign in the above expression is immaterial since $d\sigma_t$ is just as likely to be positive as negative.

We have already discussed the magnitude of $d\sigma_t$ and found that it is ± 0.02 in σ_t when salinity is obtained using the method of conductivity ratio measurement and is ± 0.03 in σ_t, when the salinity is measured using the method of chlorinity titration. Using these values for the error in σ_t it can easily be estimated, using the expression (4.22), that the error in the specific volume anomaly is ± 2 c l/ton when the former method is used for the determination of salinity and is ± 3 c l/ton when the latter method is used for the purpose*. Since the error in the determination of σ_t by

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*There is a widely used graphical method described by Montgomery (1954) and Montgomery and Wooster (1954) for instrumental method, more specifically, when the instrument developed by Kratky *et al.* (1969) is used for the purpose, is less than half a unit in its second decimal place, the corresponding error in the value of

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the determination of the values of thermosteric anomaly at different depths from known values of temperature and salinity. The method involves, as a first step, drawing of the temperature-depth curve for a station on a graph sheet which has the temperature marked along the abscissa, depth and salinity marked along the ordinate and super imposed with a family of curves representing thermosteric anomaly. Salinity is then plotted against temperature (This is essentially a T-S curve). Using these two curves, depths of chosen values of thermosteric anomalies are obtained.

The smallest division on the above mentioned graph sheet is equivalent to 0.05^{o}C in temperature and 0.05‰ in salinity and so plotting of a point on the T-S curve introduces an additional error of $\pm$0.025^{o}C in temperature and $\pm$0.025‰ in salinity. Consequently the total error in the values of temperature and salinity, to be used for the purpose of computing the error in the value of thermosteric anomaly obtained from the station curves, specific volume anomaly will be less than half cl/ton.

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should be taken as $\pm 0.045^{\circ}C$ and $\pm 0.045‰$ respectively, assuming that salinity is obtained using the method of conductivity ratio measurement. This results in an error of approximately ± 4 cl/ton in the value of thermosteric anomaly obtained graphically.

CHAPTER V

ERRORS IN THE COMPUTATION OF DERIVED QUANTITIES

In chapter IV the magnitudes of errors that are caused in the computation of dependent oceanographic variables due to the random errors in the measurements of the independent oceanographic variables were discussed. These dependent oceanographic variables are used in the computations of the derived quantities, namely, the dynamic depth anomalies of isobaric surfaces and the relative currents. Since the dependent oceanographic variables are subject to errors, the derived quantities computed from them are also subject to errors, the magnitudes of which are discussed in this chapter.

5.1. Dynamic depth anomaly of isobaric surfaces

Level surfaces are surfaces that are everywhere normal to the force of gravity. Since potential energy of a mass remains constant on such a surface, they are surfaces of constant gravitational potential or constant geopotential. The ideal sea surface is a level surface with zero geopotential value assigned to it. The geopotential values of other equipotential surfaces are obtained by calculating the amount of work, W, required to move a unit mass from the ideal sea surface a geometrical distance h along a plumb line

$$\text{i.e.,} \qquad W = gh \tag{5.1}$$

where g is the acceleration due to gravity.

The numerical value of the geopotential depends on the units used in gh. Bjerknes (1910) introduced, as a unit of the geopotential, the dynamic decimetre which represents the work that must be done to lift a unit mass along the plumb line about one geometric decimetre. The practical unit is the dynamic metre, D, and is defined by

$$D(\text{dyn m.}) = \frac{gh}{10} \left(\frac{m}{sec}\right)^2 \tag{5.2}$$

where h is expressed in metres and g in metre/sec^2. Since the numerical value of g, when expressed in m/sec^2, is less than 10, the dynamic metre is slightly ~~greater~~ smaller numerically than the geometric depth of one metre. A level surface in the ocean can then be defined as a surface of equal dynamic depth below the ideal sea surface.

The static pressure P at any depth h metres below the sea surface is given by the weight of the water column of unit cross-section between the sea surface and depth h.

If $\bar{\rho}$ is the mean density of the water column, then

$$P = \bar{\rho}\, gh \tag{5.3}$$

Substituting for gh from equation (5.2),

$$\begin{aligned} P &= 10\, \bar{\rho}\, D \text{ bar} \\ &= \bar{\rho}\, D \text{ decibar} \end{aligned} \tag{5.4}$$

Hence dynamic depth expressed in dynamic metres is given by

$$\begin{aligned} D &= \frac{P}{\bar{\rho}} \\ &= P\bar{\alpha} \end{aligned} \tag{5.5}$$

where P is expressed in decibars and $\bar{\alpha}$ is the mean specific volume of the water column. Since the density and the specific volume of the waters in the ocean vary in both the horizontal and vertical directions, accurate computation of the dynamic depth requires the infenitesimal form of the equation (5.5).

$$dD = \bar{\alpha}\, dp \tag{5.6}$$

The dynamic distance between two isobaric surfaces with pressures P_o and P is obtained from equation (5.6) as

$$D = \int_{P_o}^{P} \bar{\alpha}\, dP \tag{5.7}$$

If P_o refers to the pressure at the sea surface, then $P_o = 0$ and D represents dynamic depth of an isobaric surface with pressure P.

Using equation (4.15), equation (5.7) may be written as

$$D = \int_{P_o}^{P} \alpha_{35,0,P}\, dp + \int_{P_o}^{P} \bar{\delta}\, dp$$

$$= D_{35,0,P} + \Delta D. \tag{5.8}$$

The first term on the right hand side of the equation (5.8), namely,

$$D_{35,0,P} = \int_{P_o}^{P} \alpha_{35,0,P}\, dp \tag{5.9}$$

contains the contribution of a standard ocean which is invariable. The second term, namely,

$$\Delta D = \int_{P_o}^{P} \bar{\delta}\, dp \tag{5.10}$$

contains the departures of the real ocean from the standard ocean and is called the dynamic depth anomaly between the isobaric surfaces P_o and P. Since only ΔD is variable between two isobaric surfaces with pressures P_o and P, the relative geopotential intervals between the isobaric surfaces can be obtained from an evaluation of ΔD, if $\bar{\delta}$, is known as a function of depth.

Equation (5.10) may be used in the construction of dynamic topographies of given isobaric surfaces to provide maps showing lines of equal dynamic depth anomaly of an isobaric surface below the sea surface. The dynamic topographies thus obtained are known as relative topographies since they are obtained with reference to the sea surface. Since the actual topography of the sea surface more or less deviates from a level surface, to get the absolute topography of the different isobaric surfaces we should select, as a reference surface, an isobaric surface which coincides with a level surface. A reference surface of this kind is called a reference level and can usually be found only deep below the ocean surface. Since the isobaric surface and the level surface coincide at a reference level, no water motion is possible at that level, and hence, may be called a level of no motion. When such a reference level is selected, the absolute topographies of the different isobaric surfaces are obtained by computing the dynamic height anomalies of the different isobaric surfaces taking this reference level as the zero reference surface. For this computation, equation (5.10) may be modified by changing the origin of the co-ordinate system from the sea surface to the reference level and assuming the Z axis to be directed vertically upwards, as

$$\Delta D = -\int_{P_o}^{P} \bar{\delta}\, dp = -\int_{0}^{P-P_o} \bar{\delta}\, dp$$

$$= \int_{0}^{P_o-P} \bar{\delta}\, dp \qquad (5.11)$$

where P_o is the pressure at the reference level and P is the pressure at any other isobaric surface.

In practice, metres of depth are substituted for decibars of pressure in equation (5.11). This is permissible because the error introduced due to this substitution will be negligibly small, particularly in the lower latitudes and in the upper layers of the ocean, as we have already seen in section 3.2.

$$\therefore \Delta D = \int_{0}^{H_o-h} \bar{\delta}\, dz$$

$$= \int_{0}^{Z} \bar{\delta}\, dz \qquad (5.12)$$

where H_o and h are depths from the sea surface to the reference level and any other isobaric surface respectively and Z is the height of any isobaric surface above the reference level. Equation (5.12), in finite difference form, useful for computations with discrete data, is

$$\Delta D = \sum_{1}^{n} \bar{\delta}_i \, \Delta z_i \qquad (5.13)$$

where n is the number of intervals into which the distance between the level surface and the given isobaric surface is divided.

The error in the computation of dynamic height anomaly due to the errors inherent in the determination of specific volume anomaly and depth may be obtained by taking the total differential of equation (5.13).

$$d\Delta D = \sum_{1}^{n} \Delta z_i \; d\bar{\delta}_i + \sum_{1}^{n} \bar{\delta}_i \; d\Delta z_i \qquad (5.14)$$

The contribution of the second term on the right hand side of the expression (5.14) to the total error in the determination of dynamic height anomaly may be estimated as follows. If we assume that Δz_i are error free depth increments, then an error in the determination of depth will appear as a change in the value of $\bar{\delta}_i$. We have earlier, in section 3.2 estimated the magnitude of error in the determination of depth as ± 2 m upto a depth of 200 m and ± 4 m below 200 m. The change in value of $\bar{\delta}_i$, for such depth differences, is negligible except where the variation of specific volume anomaly with depth is large. Sufficiently large variation which can introduce an appreciable change in $\bar{\delta}_i$ for depth differences equal to the magnitude of error in the determination of depth is found only in the thermocline region, that is roughly in the depth range from 50 m to 350 m. If the selected reference level is sufficiently deep, the error contributed by this term to the total error in the computation of dynamic height anomaly will be small compared to that contributed by the first term and so, this may be neglected. Hence expression (5.14) becomes

$$d\Delta D = \sum_{1}^{n} \Delta z_i \; d\bar{\delta}_i$$

$$= \sum_{1}^{n} \Delta z_i \; d\delta_i \qquad (5.15)$$

$\bar{\delta}_i = \delta_i$ because $\bar{\delta}_i$ is obtained, in practical computations, from just two values of specific volume anomalies, found at the two extremities of the depth interval Δz_i.

Expression (5.15) shows that the error in the determination of dynamic height anomaly is the result of the sum of a large number of individual errors. Since the errors are just as likely to be positive as negative, their sum will never be large and the result should be obtained using the normal law of errors. Since we know only the maximum value of each individual error, we should use the formula (2.24) derived in section 2.3, which is given as

$$\sigma = \frac{\sqrt{s}\,\lambda\epsilon}{\sqrt{3}}$$

When the depth intervals Δz_i are constant and since we have assumed that $d\delta_i$ is constant throughout the water column

$$\ell\epsilon = \Delta z \; d\delta \qquad (5.16)$$

Hence expression (~~5.15~~ 2.24) may be written as

$$\sigma = \frac{\sqrt{S}\,\Delta z \; d\delta}{\sqrt{3}} \qquad (5.17)$$

Normally the total depth is divided into several depth ranges, each of which is divided into equal depth intervals. In such situations we should calculate σ for each depth range using formula (5.17) and then get the standard deviation of the combined error distribution which represents the distribution of error in the determination of dynamic height anomaly. This can be obtained using formula (2.22) of section 2.3, given as

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots\cdots}$$

where σ_1, σ_2, etc. are standard deviations for the different depth ranges. Twice the value of the standard deviation of the combined distribution may be taken as the maximum error in the determination of dynamic height anomaly, since more than 95%. of the time the error will be within the range of $\pm 2\,\sigma$ (Section 2.3).

Formula (5.17) shows that σ increasea as S increases keeping Δz constant. This means that deeper we select the reference level, the more will be the error in the computed dynamic height anomaly. Also the formula shows that the larger the number of depth intervals into which a specific range of depth is divided, the smaller will be the computational error.

To illustrate computation of error in the dynamic height anomaly of an isobaric surface, let us compute the same for the zero decibar surface with reference to 1500 decibar surface. If we assume that data are available for the standard depths accepted by the National Oceanographic Data Centre, then there are three depth intervals of 10 m each in the depth range from 0 m to 30 m, one depth interval of 20 m in the depth range from from 30 m to 50 m, four depth intervals of 25 m each in the depth range from 50 m to 150 m, three depth intervals of 50 m each in the depth range from 150 m to 300 m and twelve depth intervals of 100 m each in the depth range from 300 to 1500 m. The value of the maximum error, $\ell\epsilon$, for the 10 m depth interval is obtained using formula (5.16) as

$$\ell\epsilon = \Delta z \; d\delta$$

$$= 10 \text{ decibar} \times 2 \text{ cl/ton (Section 4.2)}$$

$$= 10 \times 10^5 \; \frac{gm\ cm}{sec^2 \times cm^2} \times \frac{2 \times 10 \times cm^3}{10^5\ gm}$$

$$= 20 \; \frac{cm^2}{sec^2} = \frac{20}{10^4} \; \frac{m^2}{sec^2}$$

$$= \frac{20}{10^5} \text{ dyn. m} = 0.2 \text{ dyn. mm.}$$

Now the standard deviation σ for the depth range from 0 m to 30 m is obtained using formula (5.17) as

$$\sigma = \frac{\sqrt{S}\,\Delta z\, d\delta}{\sqrt{3}} = \frac{\sqrt{3} \times 0.2}{\sqrt{3}}$$

$$= 0.2 \text{ dyn. mm.}$$

Similarly standard deviation for the other depth ranges are computed after calculating the corresponding maximum errors and the results are tabulated in Table V. Using the values of the standard deviation for different depth ranges given in Table V, the standard deviation of the combined error distribution is obtained using formula (2.22) of section 2.3 as

$$\sigma = \sqrt{\sigma_1^2 + \sigma_1^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2}$$

$$= \sqrt{(0.2)^2 + (0.2)^2 + (0.6)^2 + (1.0)^2 + (4.0)^2}$$

$$= \sqrt{0.04 + 0.04 + 0.36 + 1 + 16}$$

$$= \sqrt{17.44} = 4.2 \text{ dyn. mm.}$$

Table V

Standard deviation of maximum error for different depth ranges

Depth range (m)	Depth interval (m)	Maximum error (dyn. mm)	σ for the depth range (dyn. mm)
0- 30	10	0.2	0.2
30- 50	20	0.4	0.2
50-150	25	0.5	0.6
150-300	50	1.0	1.0
300-1500	100	2.0	4.0

Table VI

Values of dΔD for different isobaric surfaces with reference to 1500 d bar surface

Depth (m)	0	10	20	30	50	75	100	125	150	200	250	300
dΔD (dyn.mm)	8	8	8	8	8	8	8	8	8	8	8	8

Depth (m)	400	500	600	700	800	900	1000	1100	1200	1300	1400	15
dΔD (dyn.mm)	8	7	7	7	6	6	5	5	4	3	2	

The maximum error in the computation of dynamic height anomaly of the zero isobaric surface with reference to 1500 d bar surface is twice the above value of σ and so

$$d\Delta D = 2 \times 4.2 = 8.4 \simeq 8 \text{ dyn. mm},$$

corrected to the nearest whole number. In a similar manner the maximum errors in the computation of dynamic height anomalies of the other isobaric surfaces with reference to 1500 d bar surface are computed and are presented in Table VI.

5.2. Relative current

Relative current perpendicular to the vertical plane between two stations A and B is computed using Helland-Hansen formula (Sandstrom and Helland-Hansen, 1903), derived from Bjerkne's circulation theorem (Bjerknes 1898, 1900) with the assumptions that motion is non-accelerated and frictionless and that the observations are taken simultaneously. Though not absolutely justified, these assumptions are used for the computation of currents in the ocean, since the oceanic condition is generally quasi-permanent.

Helland-Hansen's formula is derived by equating the pressure gradient force acting down slope of a sloping isobaric surface with the Coriolis force acting upslope. Solving for V, we get

$$V = \frac{10\ (\Delta D_A - \Delta D_B)}{L.\ 2\ \omega\ \sin\ \phi} \qquad (5.18)$$

where

$\Delta D_A - \Delta D_B$ = difference in anomalies of dynamic height at stations A and B in dynamic metres.

L = distance between stations in metres

ω = angular velocity of the earth equal to 0.729×10^{-4} radians/sec

ϕ = mean latitude between the stations

V = relative current velocity normal to the line joining the two stations, in metres per second.

The current is called relative current because the current computed using the formula is relative to any unknown current at the reference level. To get the absolute current, the reference level selected should be a level of no motion.

The error in the computation of relative current is due to the errors in the computed value of difference in dynamic height anomalies, in the measurements of distance between stations and in the latitude angle. This may be obtained by taking the total differential of the expression (5.18).

$$dV = \frac{10d\ (\Delta D_A - \Delta D_B)}{L.\ 2\ \omega \sin \phi} - \frac{10\ (\Delta D_A - \Delta D_B)}{L^2.\ 2\ \omega \sin \phi}\ dL$$

$$- \frac{10(\Delta D_A - \Delta D_B)}{L.2\ \omega \sin^2 \phi} \cos \phi\ d\phi \qquad (5.19)$$

The second and third terms on the right hand side of the equation (5.19) may be discarded being negligible compared to the first term assuming that the measurements of distance between stations, L and latitude angle, ϕ are done accurately. This is generally true, particularly with the use of modern navigational aids. Hence the error in the relative current may be written as

$$dV = \frac{10d\ (\Delta D_A - \Delta D_B)}{L.\ 2\ \omega \sin \phi} \qquad (5.20)$$

Expression (5.20) shows that the error in the computed current is directly proportional to the error in the difference in dynamic height anomaly and is inversely proportional to the distance between stations and the sine of the latitude angle*.

To illustrate the computation of error in the relative current relative to 1500 d bar surface, let us assume that the distance between the two stations A and B is 100 kilometres and the average latitude angle is 5^{o}. The maximum error in the difference between the dynamic height anomalies at the two stations should be computed as twice the standard deviation of the combined distribution resulting from the combination of the

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equation (5.18) will lead to computation of infinite current at the equator. But the fact that halving the distance between stations will also lead to unrealistic results, by doubling the error in the computed current, is not recognised by many oceanographers. Montgomery and Stroup (1962) found that decrease of station spacing will not always result in more details of the current distribution and stated: 'In effect, the stations (9,11,......., 27) midway between whole degrees of latitude are neglected. It was thought that the details gained by halving the spacing of verticals would be largely unreal'. Again while discussing the representation of distribution of geostrophic flow through a vertical section they stated: 'It might be thought that a continuous distribution of geostrophic velocity component could be attained by sufficiently two error distributions of the dynamic height anomalies at the two stations: Hence

$$d\,(\Delta D_A - \Delta D_B) = 2\sqrt{4^2 + 4^2} = 2\sqrt{2} \times 4 = 11.3$$

$$\triangleq \quad 11 \text{ dyn. mm}$$

The error in the computed current, dV, is computed from

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reducing the station spacing. The result, however, would be an increasingly irregular pattern, ultimately bearing no resemblence to the actual current (because with decreasing spacing even small fluctuations of the observed specific volume would lead to increasingly swift and narrow current bands of alternating direction)'. One of the reasons for the above results obtained by Montgomery and Stroup (1962), even though they have taken a comparatively shallow reference surface at 300 m which should have resulted in less computational error, may be the increased error inherent in the method of obtaining values of thermosteric anomaly described by Montgomery (1954) and Montgomery and Wooster (1954) (Section 4.2). Ofcourse, smaller spacing of verticals will result in gained details for the vertical sections of the dependent and independent parameters, but not of the distribution of currents.

the formula (5.20) and using the LaFond's tables (LaFond, 1951) as

$$dV = 9 \text{ cm/sec}$$

Figure 2 shows the variation of the error in the computed current (relative to 1500 d bar surface) with latitude angle for an error of 10 dyn. mm in the difference in dynamic height anomaly and for a pair of stations separated by 100 km. The curve shows that the error is less than 1 cm/sec above 40^{o} latitude angle and increases towards the equator. Very near the equator the rate of increase is very large.

Any computed current which is less than or equal to the magnitude of the error itself should be considered as unreal. Since the error is given in absolute magnitude, its effect is serious where the current velocities are small, particularly very near the equator where the magnitude of error increases due to latitude effect.

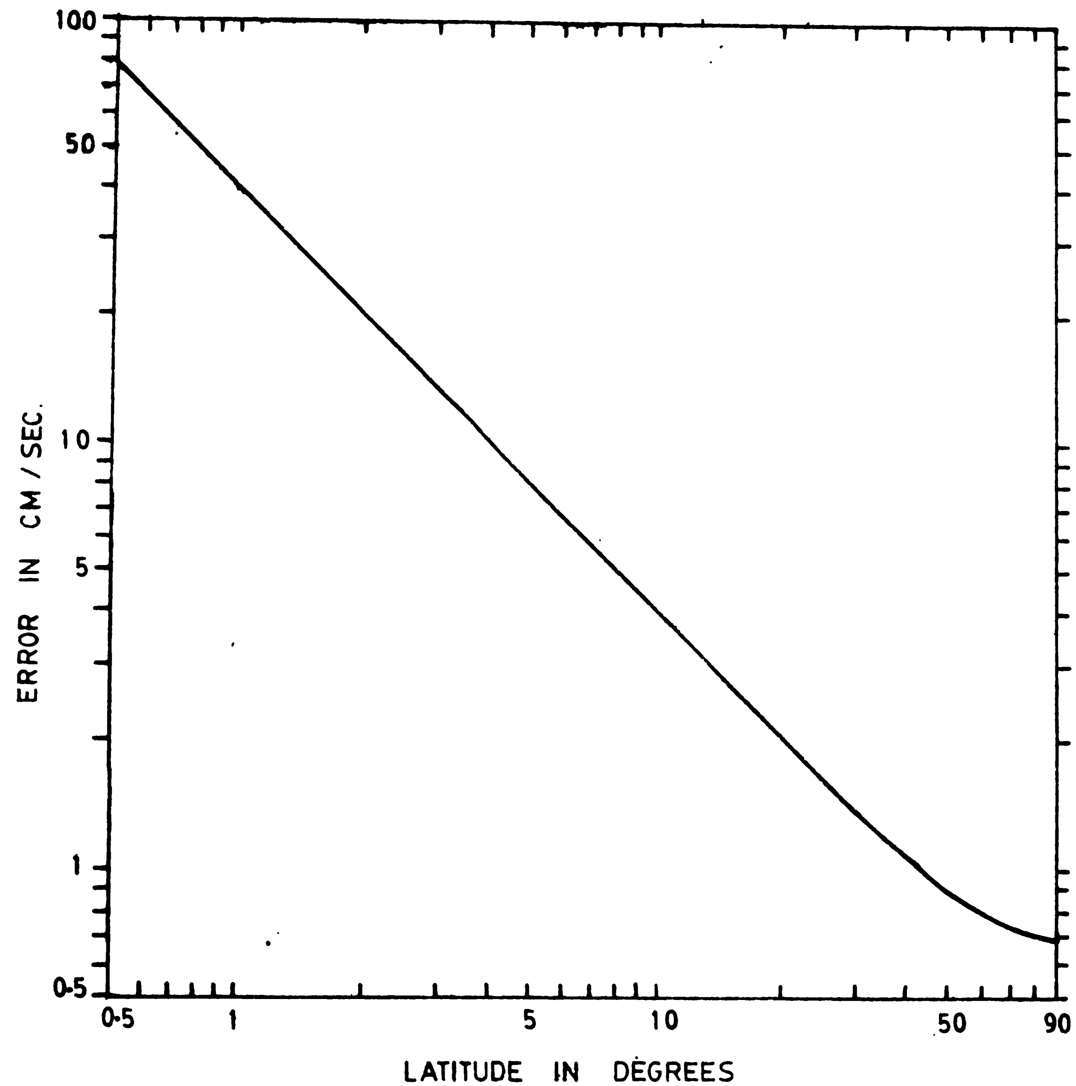

Fig. 2: Absolute error in the computation of geotrophic current for 10 dyn.mm. error in the difference in dynamic height anomaly and for 100 Km station spacing (relative to 1500 db)

CHAPTER VI

IDENTIFICATION OF A ZERO REFERENCE LEVEL

To get the absolute topography of the different isobaric surfaces, an isobaric surface which coincides with a level surface should be selected as a zero reference level. Since no water motion is possible at this level, it is called a level of no motion. Proper selection of a zero reference level is important for the presentation of an oceanic circulation pattern. This chapter presents a method for the identification of a zero reference level.

6.1. Determination of an appropriate reference surface

For the determination of the absolute magnitudes of computed currents, the reference surface used for the computation of the relative currents should either be a level of no motion or the distribution of the current velocity along the reference surface should be known. The latter method is not widely used for the following reasons:

1) Current measurement at sea is very difficult and subject to uncertainties.

2) The measured current is the total current which may include the effects of winds, tides, internal waves, etc. in addition to the geostrophic component of the current. Hence, for the computation of absolute currents, identification of a 'level of no motion' is usually attempted.

Literature cites several methods for the identification of a level of no motion. Some earlier investigators assumed a level of no motion as deep as possible reasoning that the velocity decreases with depth and that at great depths the isobaric surfaces are nearly horizontal. This method has the disadvantage that the error in the computed current may make the picture of the circulation pattern completely unreliable since the error in the computed currents increases with increase in the depth of reference surface. Other methods developed by different authors were critically examined by Fomin (1964) and may be summarised as follows:

a) Dietrich's method

The idea that the intermediate oxygen minimum layer in the ocean corresponds to a layer with minimum horizontal water motion was first advanced by Jacobsen (1916) and was further developed and applied in practice by Wust (1935) and Dietrich (1936). Several authors have refuted this idea (Rossby, 1936; Iselin, 1936; Wattenburg, 1938; Seiwell, 1937) and have argued that an oxygen minimum layer need not necessarily be a layer of minimum current velocity. Also Sverdrup (1938) showed that an oxygen minimum layer cannot have any general dynamic significance.

b) Parr's method

Parr (1938) advanced the idea that ~~since~~ the thickness of a layer bounded by two isopycnic surfaces (surfaces of equal density) cannot remain constant in the region of a current and should vary perpendicular to the direction of current and where the thickness of such a layer is constant, the horizontal water motion is zero or it is a layer of no motion. He also suggested a graphical method to identify such a layer. Fomin (1964) showed that the above is a necessary but not a sufficient condition for the existance of a layer of no motion because the vertical variation of current velocity depends not only on the slope of the isopycnic surfaces but also on the vertical density gradient. A layer bounded by two isopycnic surfaces in a region where there is a strong vertical density gradient will be least distorted compared to the overlaying and underlaying layers in the presence of a strong geostrophic current.

c) Hidaka's method I

The current velocity field in the sea is in constant interaction with the field of any physical or chemical property of sea water and there is mutual adjustments between these two fields. This fact was used by Hidaka (1949) to develop a method for the determination of the level of no motion in the sea from the salinity distribution. He argued that the surface where the second derivative of the vertical salinity distribution, $\frac{\partial^2 S}{\partial z^2}$, is equal to zero should be a level of no motion. Fomin (1964) showed that Hidaka's method provides the identification of a surface that has very definite structural features of the salinity field, but that such features are not uniquely related to the current velocity field.

d) Hidaka's method II

Hidaka (1940a,1940b,1950) suggested another method, for the identification of the level of no motion, based on continuity considerations for the stationary distribution of certain physical and chemical properties of sea water. He considered a volume of water in the form of a tetrahedral prism that extends from sea surface to the bottom. Assuming continuity of water volume, Hidaka obtained a set of equations, the solution of which lead to the identification of the level of no motion. Defant (1941a), however, has raised objection on the practical applicability of this method on two grounds:

i) The assumption on the continuity of water volume is not strictly true theoretically because, continuity condition requires constant mass and not constant volume.

ii) The set of equations obtained by Hidaka is practically inconsistent and cannot be solved with the existing accuracy of measurements at sea.

e) <u>Sverdrup's method</u>

Sverdrup *et al.* (1942) suggested a method for the identification of the level of no motion which uses the known fact that, in the steady state, the total water transport through any cross section of an oceanic area, which extends from one shore to the other, should be zero. He considered a horizontal reference surface and argued that the reference surface will be in the layer of no motion when the water transport through the section above the reference surface is equal to the water transport below the reference surface. Fomin (1964)

- G 3427 -

points out that this method is unsuitable on three counts:

i) The currents that compensate each other need not necessarily be in the vertical plane. They can be in the horizontal plane as well.

ii) The accuracy of water density determination is insufficient for a successful computation of water transport, particularly in the deep layers.

iii) For a successful identification of the level of no motion, the pure drift component of the current should also be considered which will not only complicate the computation but also introduce additional errors caused by the uncertainty of the wind field that corresponds to the stationary case.

f) Defant's method

Defant (1941b), while comparing the differences in the relative dynamic depth anomalies of given isobaric surfaces between adjacent oceanographic stations in the Atlantic ocean, found that at certain levels these differences were practically constant over a large depth interval. Such constant relative pressure differences between adjacent stations can be interpreted in two ways:

i) The whole layer of deep water has a constant velocity.

ii) The whole layer is uniformly at rest.

The first interpretation should be considered as unreasonable since the constant velocity in most cases proves to be considerably larger than the surface velocity, a result which is against the present oceanographic experience. Hence, Defant concluded that the second interpretation is valid and the layer may be considered as a layer of no motion.

Eventhough Defant's method is the most justified and widely used one for the identification of the layer of no motion, it is not without objections. Wust (1951) put forward two arguments against the method:

i) It is doubtful whether we can find a level of no motion in a layer of constant relative pressure differences when the current computed has the same direction above and below the layer.

ii) If current is computed in the immediate vicinity of the bottom relative to a level of no motion identified in a layer of constant relative pressure differences, it is frequently found that the magnitude of the computed current exceeds the magnitude of the current at the surface and in a direction opposite to the direction of the surface current. This is an unreasonable result according to the present knowledge of the oceans.

Fomin (1964) pointed out that in a vertical section of the differences in the relative dynamic depth anomalies of isobaric surfaces between adjacent stations, selection of the depth of level of no motion is not always unique because, in some cases, there may be several layers with constant relative pressure differences and selection of any one of them is arbitrary. Also, for some pairs of stations, there may not be a layer with constant relative pressure difference at all. He also pointed out that in a region where the current velocity is low, Defant's method is liable to fail because the relative dynamic depth differences between adjacent stations in such regions will be of the same order of magnitude as the computational error itself. The method is liable to fail in a weakly stratified water body for the same reason. It will give good results only in strongly stratified bodies of water and in regions where the current velocity is high.

Sastry and D'Souza (1971) have reported that application of Defant's method in the Arabian sea region did not yield a zero level which could be accepted with any reasonable degree of certainty.

g) Mamaev's method

Mamaev (1955) suggested a method which is very similar to Defant's method. His method is based on the fact that in a layer where the differences in relative dynamic depth anomaly between adjacent stations is constant, the specific volume anomalies should be equal. Hence a vertical distribution of the differences in specific volume anomalies at two neighbouring stations will show a zero value whenever the differences in the relative dynamic depth anomalies between these two stations are constant. This method has the advantage that the minimum or zero value of the differences in specific volume anomaly is easily found compared to a layer where the differences in dynamic depth anomalies are constant. Since this method is similar to Defant's method in principle, this method also is subject to the objections raised in the context of the latter.

6.2. Identification of a level where the current velocity is negligibly small.

The foregoing discussion on the methods used for the identification of the level of no motion in the sea brings out the fact that there is no fool-proof method for the same. In a complicated mass field, as in the real ocean, it is unreasonable to think either that the level of no motion coincides with an isobaric surface over large areas or that it coincides with real surfaces in the ocean such as isothermal surface, isohaline surface, etc. In oceanic areas where the current continues from the surface right to the bottom without changing direction, a level of no motion will not exist at all. A surface where there is no water motion should be considered as having a very complex topography, which sometimes crop out at the surface where there are two opposing currents in the horizontal plane. The identification of such a surface in an oceanic area is beyond the means of present day dynamic methods in oceanography. Hence it is advisable to try to identify a level where the water motion is negligibly small and use this level for the computation of currents so that the computed currents will not be very much different from the absolute currents. A method for the identification of such a level has been described by Fomin (1964) by the application of the density model of Shtokman (1950,1951) to the oceanic mass field.

The component of the geostrophic current velocity, V_x, in the northern hemisphere, perpendicular to a vertical cross section is obtained by equating the pressure gradient force with the Coriolis force, assuming that the motion is non-accelerated and frictionless.

i.e., $$f\, V_x \rho = \frac{\partial p}{\partial y} \qquad (6.1)$$

where $f = 2\,\omega \sin\phi$ is the Coriolis parameter, ρ is the density of water and p is the sea pressure. The right hand side of equation (6.1) may be written in terms of density ρ by the use of hydrostatic equation, namely,

$$dp = g\,\rho\, dz \qquad (6.2)$$

where g is the acceleration due to gravity and z is the depth from the sea surface. Integrating Equation (6.2) we get

$$\int_0^z dp = p = g \int_0^z \rho\, dz \qquad (6.3)$$

Differentiating equation (6.3) with respect to y

$$\frac{\partial p}{\partial y} = g \int_0^z \frac{\partial \rho}{\partial y}\, dz \qquad (6.4)$$

Substituting equation (6.4) in equation (6.1)

$$f\, V_x \bar{\rho} = g \int_0^z \frac{\partial \rho}{\partial y}\, dz$$

$$\therefore \quad V_x = \frac{g}{f\bar{\rho}} \int_0^z \frac{\partial \rho}{\partial y}\, dz \qquad (6.5)$$

where $\bar{\rho}$ is the average density of the water column.

Shtokman's density model, when applied to the oceanic mass field, assumes that the density at the reference surface, which is the lower boundary of the baroclinic layer, is constant and has a value equal to $\rho(o)$ and that the deviation of density, $\rho(x,y,z)$, at any point from this constant value is given as the product of two functions, one of which depends only on the vertical coordinate, and the other only on the horizontal coordinate.

i.e., $\rho(o) - \rho(x,y,z) = \delta(z)\, f(x,y)$ (6.6)

where $f(x,y)$ is known as the function of influence. Hence, along a vertical, $f(x,y)$ should be a constant.

i.e., $\rho(o) - \rho(z) = K\delta(z)$ (6.7)

The assumption that the function of influence is constant implies that, in an oceanic region where the vertical distribution curves of density are similar in appearance, the function $\delta(z)$ will be different only by a constant multiplier. This will be true in oceanic areas where the T-S curves are similar and similarity of T-S curves is retained over very large oceanic areas.

Now let us locate the origin of the coordinate system on the assumed reference surface with the z axis directed vertically upwards. Substituting equation (6.6) in equation (6.5) we get

$$V_x(z) = - \frac{g}{f\bar{\rho}} \frac{\partial f(x,y)}{\partial y} K \int_0^z \delta(z)\, dz \qquad (6.8)$$

Assuming z = H at the sea surface, the geostrophic current at the sea surface is

$$V_x(H) = \frac{-g}{f\bar{\rho}} \frac{\partial f(x,y)}{\partial y} K \int_0^H \delta(z)\, dz \qquad (6.9)$$

Solving for $\frac{\partial f(x,y)}{\partial y}$ from equation (6.9) and substituting in equation (6.8) we obtain

$$V_x(z) = V_x(H) \frac{\int_0^z \delta(z)\, dz}{\int_0^H \delta(z)\, dz} = V_x(H)\, \phi(z) \qquad (6.10)$$

where $\phi(z)$ is known as the stratification function. Equation (6.10) describes the vertical distribution of geostrophic current velocity and may be used to compute any subsurface current, once the absolute value of the geostrophic component of the sea surface current is known by measurement. Equation (6.10) shows that the strarification function will be the same where the function $\delta(z)$ at different points is different only by a constant multiplier.

Integration of equation (6.10) between the limits $z = 0$ to $z = H$ yields

$$\int_0^H V_x(z)\,dz = \int_0^H V_x(H)\,\phi(z)\,dz$$

$$= V_x(H) \int_0^H \phi(z)\,dz$$

$$= \frac{V_x(H)}{F(H)} \qquad (6.11)$$

where the function $F(H)$ is defined by

$$F(H) = \frac{1}{\int_0^H \phi(z)\,dz}$$

$$= \frac{\int_0^H \delta(z)\,dz}{\int_0^H \int_0^z \delta(z)\,(dz)^2} \qquad (6.12)$$

If the geostrophic current velocity component in equation (6.11) is considered as average current between two stations at a distance L, then the time rate of water transport between the verticals at the two stations between the sea surface and the assumed reference surface is given by

$$L \int_0^H V_x(z)\,dz = \frac{L\,V_x(H)}{F(H)} \qquad (6.13)$$

It is easy to see that since $L\ V_x(H)$ is a constant, the time rate of water transport varies inversely with the function $F(H)$. It will be illustrated later in this section that function $F(H)$ decreases in magnitude as the value of H increases, and becomes constant below a particular value of depth of the assumed reference surface. Hence the time rate of water transport remains constant irrespective of the reference surface selected, once it is below the particular reference surface where the value of the function $F(H)$ becomes constant. This means that the layer of water below this reference surface is a layer of no motion. Hence the reference surface where the value of the function $F(H)$ becomes constant is a level of no motion. The function $F(H)$ will be the same in a region where the function $\delta(z)$ is different only by a constant multiplier.

In fact, Shtokman's density model, when applied to the mass field of the ocean, does not ensure that the reference level thus identified is actually a level where the geostrophic current velocity is zero. The model only tells that if a current exists at the selected reference level, it will continue unabated to the bottom, because the constancy of the function $F(H)$ in a layer indicates zero vertical density gradient, a condition which cannot change the magnitude of current in the vertical. The result that reference surface identified by the application of Shtokman's model to the oceanic mass field is a level of no motion is due to a tactical assumption that the velocity of geostrophic current at the sea bottom is zero. Since we generally expect only negligibly small current at the bottom of the sea where depths are large, the reference surface selected by the application of Shtokman's density model may be considered as a surface where the geostrophic current velocity is negligibly small and hence may be considered as a zero surface for the purpose of computing absolute currents.

Shtokman's density model cannot be applied in an oceanic region where the current changes in direction with depth and where it first increases and then decreases with depth. This is because of the assumption that the function of influence is constant along the vertical and so the velocity of geostrophic current decreases with depth without change in direction. Hence the boundary between two oppositely directed currents can only be vertical.

This method is applied to the Arabian sea region for the identification of a zero reference level where the geostrophic current velocity is negligibly small. The data used are obtained from the Cruise No.31-0197 of the U.S. Research Vessel 'Atlantis' in the Arabian sea during August-September 1963. Fig.3 gives the location of the stations used in the present study.

We have already seen that Shtokman's density model can be applied only in an oceanic region where the magnitude of current decreases with depth and where the direction of current does not change with depth. This cannot be generally expected in the surface layers, particularly in a tropical oceanic region like the Arabian sea. So, for the purpose of computing function F(H) using Shtokman's density model, an oceanic region below about 1500 m is selected where generally the current velocity is expected only to decrease with depth with no change in direction. A region below 1400 m is taken for the purpose so that z = H now represents the 1400 d bar surface.

The method of computing function F(H) for Station No.82 is shown in Table VII assuming that the lower boundary of the current is 4000 m. For the purpose of this computation it is convenient to write the equation for function F(H) in finite difference form as

Fig. 3: Map showing location of stations.

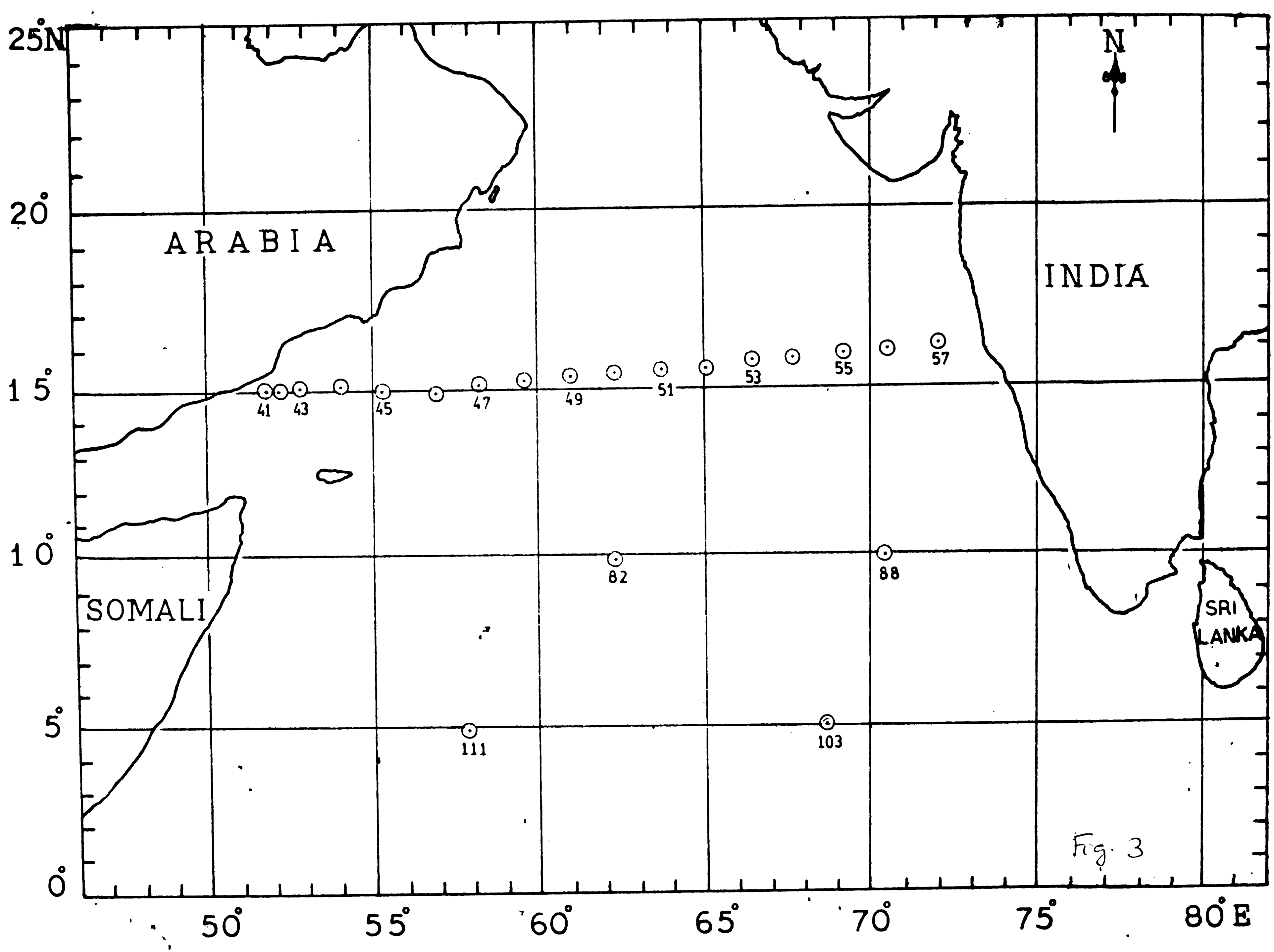
25°N
20°
15°
10°
5°
0°
50°
55°
60°
65°
70°
75°
80°E
N
ARABIA
INDIA
SOMALI
SRI LANKA
41
43
45
47
49
51
53
55
57
82
88
111
103

Fig. 3

Table VII

Computation of function F(H)

H-z (m)	σ_t	$k\delta(z)$	$K\overline{\delta(z)}$	$K\overline{\delta(z)}\Delta z$	$I = K\sum_{o}^{z}\overline{\delta(z)}\Delta z$	$\bar{I}$	$\bar{I}\Delta z$
1	2	3	4	5	6	7	8
1400	27.65	0.16			64.0		
			0.14	14.0		57.0	5700
1500	27.69	0.12			50.0		
			0.09	22.5		38.8	9700
1750	27.75	0.06			27.5		
			0.05	12.5		21.3	5325
2000	27.78	0.03			15.0		
			0.02	10.0		10.0	5000
2500	27.80	0.01			5.0		
			0.01	5.0		2.5	1250
3000	27.81	0.00			0.0		
			0.00	0.0		0.0	0.0
4000	27.81	0.00			0.0		

$$\sum_{o}^{H} \bar{I}\Delta z = 26975$$

$$F(H) = 2.37 \times 10^{-3}$$

$$= 2.37 \times 10^{-5}\ \text{cm}^{-1}$$

$$F(H) = \frac{\int_0^H \delta(z)\,dz}{\int_0^H \int_0^z \delta(z)\,(dz)^2}$$

$$= \frac{\sum_0^H \overline{\delta(z)}\,\Delta z}{\sum_0^H \left(\sum_0^z \overline{\delta(z)}\,\Delta z\right)\Delta z} \qquad (6.14)$$

In Table VII, the first column gives the depth of the different isobaric surfaces from the sea surface. The vertical distribution of σ_t is given in the second column. The third column shows the values of $[K\delta(z)]$, that is, values of $[\sigma_t(o) - \sigma_t(z)]$. The $[K\delta(z)]$ values averaged by layers are given in the fourth column. In the fifth column are given the products of the average values of $[K\delta(z)]$ and the corresponding depth intervals. The sixth column gives the summation $\sum_0^z \overline{\delta(z)}\Delta z$ where z can have values in the interval $0 \leq z \leq H$. Column 7 contains values of the sixth column averaged by layers. The last column shows the average values of the seventh column multiplied by the corresponding depth intervals. Now the function F(H) is obtained by dividing the number in the first row of the sixth column, which represents $K \sum_0^H \overline{\delta(z)}\Delta z$, by the sum of the values of the last column, which represents $K\sum_{0}^{H}(\sum_{0}^{z}\overline{\delta(z)}\Delta z)\Delta z$. To convert the result to the cgs system, we must multiply the same by 10^{-2}. The same computations are repeated for the same station assuming the lower boundary of the current at 3000 m, 2500 m, 2000 m and 1750 m. Similarly function F(H) for different depths of the lower boundary of current are computed for the stations 48,88,103 and 111. The stations selected are well-spread over the whole of the Arabian sea region so that a zero surface identified with this method may be applied to the whole of this oceanic region. The computed values of function F(H) are tabulated in Table VIII. The first column shows the assumed depths of the lower boundary of the current from the sea surface and the other columns show the computed values of function F(H) for the different depths and for the selected stations. Fig.4 is a graphical representation of function F(H) for the stations 48,88 and 103. Both Table VIII and Fig.4 show that the function F(H) depends little on position and agree rather well in magnitude. This means that in the region selected the water is comparatively uniform in the horizontal plane. The T-S characteristics of the water masses of the Indian ocean published by Sastry (1971) and Sastry and D'Souza (1972) confirm this fact by showing that the T-S curves of the water masses of the Arabian sea region coincide

Table VIII

Values of $F(H) \times 10^{-5}\ cm^{-1}$ for different depths of assumed reference surfaces

H-z (m)	Station Numbers				
	48	82	88	103	111
1750	7.07	7.38	7.72	7.38	7.72
2000	4.80	4.92	4.69	4.92	4.69
2500	2.87	3.39	2.83	2.92	2.83
3000	2.25	2.37	2.24	2.27	2.18
4000	2.25	2.37	2.24	2.27	1.53

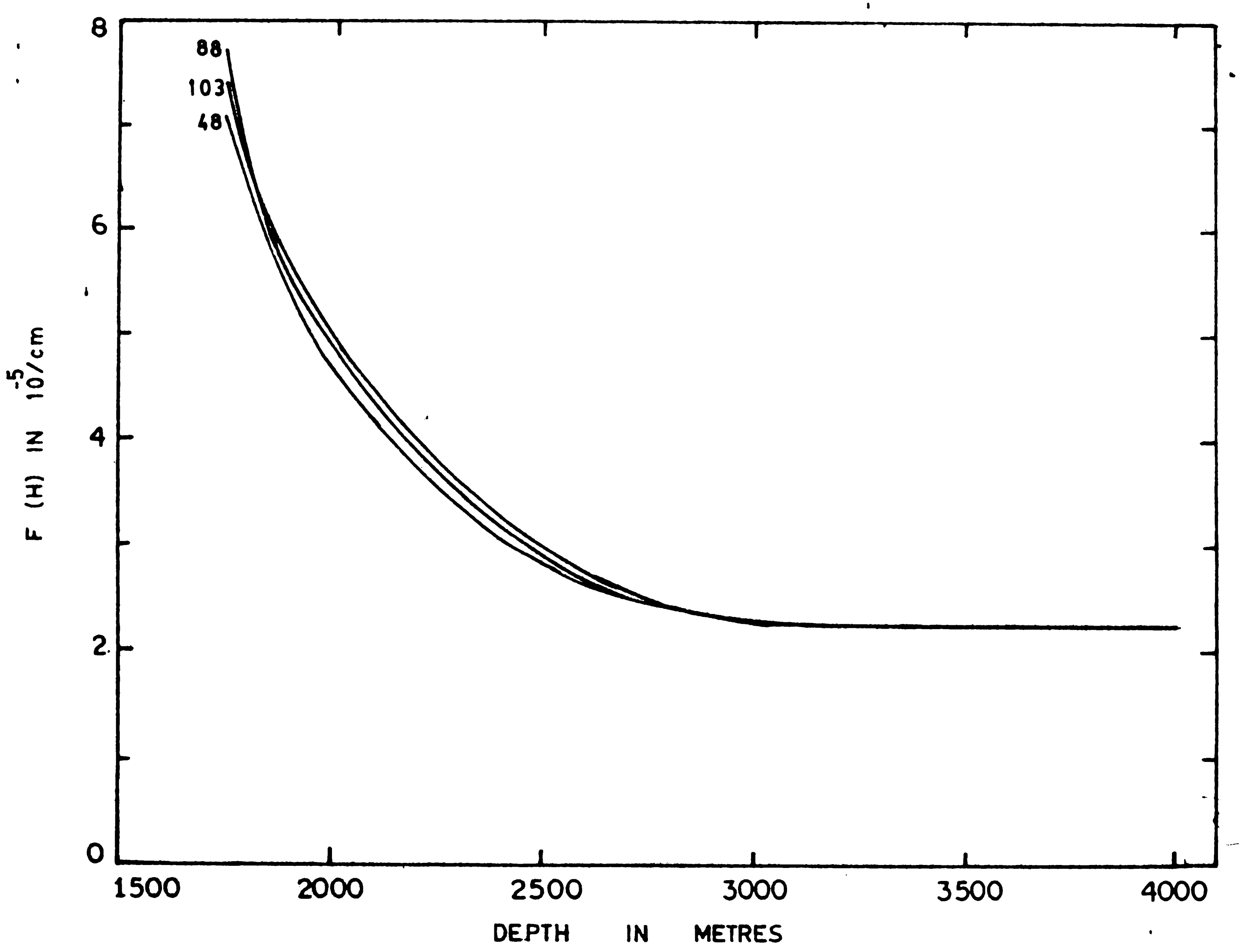

Fig. 4: Curves of function F(H) at three stations.

below 40 cl/ton isosteric surface which represents an oceanic region below 1500 m. The table and the figure also show that the function F(H) changes rapidly in the comparatively shallow regions and except for station 111 (Table VIII), the function F(H) is independent of depth when the depth is greater than 3000 m. At station 111 the vertical density gradient is not zero below 3000 m but is very weak compared to the upper layers. Hence, we may take the 3000 d bar surface as a zero surface in the Arabian sea with negligibly small geostrophic current.

CHAPTER VII

COMPUTATION OF DYNAMIC HEIGHT ANOMALY

In the preceding chapter we have found that 3000 d bar surface is an ideal surface to be used as a zero surface in the Arabian sea for the purpose of computation of the derived quantities, namely, anomalies of dynamic height of isobaric surfaces and geostrophic currents. Equation (5.17) has shown that the error in the dynamic height anomaly of an isobaric surface increases with increase in depth of the reference surface. It can further be shown, that the maximum error in the dynamic height anomaly of the zero isobaric surface with reference to 3000 d bar surface is 20 dyn. mm., two and a half times more than the maximum error calculated with 1500 d bar surface as a reference surface. This increased error in the computation of dynamic height anomaly of the sea surface will make the results on the surface circulation pattern more undependable. To reduce the error caused by the selection of a deep isobaric surface as a zero surface, a method is used in which the dynamic height anomaly of the different isobaric surfaces are computed using 1500 d bar surface as a reference surface and then the computed geostrophic current velocities are reduced to the value relative to the 3000 d bar surface using Shtokman's density model. The 1500 d bar surface is selected for the following reasons:

i) 1500 d bar surface is a surface where the density variation in the horizontal is very small. The dynamic topography of the different isobaric surfaces presented with the 1500 d bar surface as a reference surface will not be very different, for practical purpose, from that presented with 3000 d bar surface as the reference surface.

ii) According to the NODC accepted standard depths, there are frequent sampling points upto 1500 m depth which will help to reduce the error in the computed dynamic height anomaly.

iii) The mass field in the oceanic region below 1500 d bar surface is suitable for the application of Shtokman's density model so that the geostrophic currents computed with reference to 1500 d bar surface can be reduced to the value relative to 3000 d bar surface using this model.

7.1. Extending the selected zero reference surface into shallow regions.

Once the reference surface for the purpose of computation is selected, we must extend the computations into shallow regions of the ocean where the depth is less than the depth of selected reference surface. Different methods available for this purpose were critically examined by Fomin (1964) which may be summarised as follows:

7.1.1. Helland-Hansen's method

In this method proposed by Helland-Hansen (1934), the block of solid earth in the form of a triangle formed by the reference line, the bottom line and the vertical at the shallow station is replaced by an imaginary water mass. If it is assumed that the gradient current velocity and the horizontal pressure gradient are zero along the bottom line, then the isobaric and isosteric surfaces in the imaginary water mass must be horizontal. This implies a motionless water mass. If the points of intersection of the isosteric lines with the bottom line are projected horizontally into the vertical at the shallow station, we get the vertical distribution of specific volume anomaly at the shallow station upto the reference surface and these values may be used for the computation of currents with reference to the selected zero surface.

The above method is based on the assumption that gradient current velocity and hence the horizontal pressure gradient are zero near the bottom. According to Fomin (1954) this assumption is not justified because the velocity of total current at a solid boundary becomes zero due to friction and the consequent appearance of a compensating current. This compensating current cannot be determined by dynamic methods. Another disadvantage of the method is that its application requires graphical construction.

7.1.2. Jacobsen and Jensen method

Jacobsen and Jensen (1926) proposed a method which is also based on the assumption that the current velocity and the horizontal pressure gradient along the bottom line are zero. Here also the solid earth formed by the reference line, the bottom line and the vertical at the shallow station is replaced by an imaginary motionless water mass. But this method has two more additional assumptions.

a) The bottom line between the two stations, one of which is shallow, is rectilinear.

b) The isosteres are equidistant near the bottom. These two assumptions result in the isosteres in the imaginary water mass being equidistant and parallel.

The practical computation is done in the following manner. Let A and B are two stations of which A is a shallow station and B a deep station through the bottom of which passes the zero reference surface which is horizontal. Now assume a horizontal reference surface passing through the bottom of the shallow station A and compute the differences in dynamic height anomalies between the two stations with reference to this assumed reference level. Then add to this the correction

$$\Delta = \frac{1}{2} \Delta p \; (\delta_{B_1} - \delta_{A_1}) \qquad (7.1)$$

where Δp is the pressure difference between the assumed reference surface and the zero reference surface and δ_{B_1} and δ_{A_1} are the specific volume anomalies at the assumed reference level for the two stations B and A respectively. The sum will then be the difference in dynamic height anomalies between the two stations with reference to the zero reference surface.

We have already seen in section 7.1.1. that the assumption of zero gradient current velocity and zero pressure gradient near the bottom is not justified. Eventhough this method has an advantage over the Helland-Hansen method in that its application does not require any graphical constructions, the additional assumptions concerning the structure of the density field near the bottom is not justified because under actual conditions water density field rarely has such a structure (Fomin, 1964).

7.1.3. Goren's method

Goren (1948) proposed a method in which he suggested that the density field be extrapolated in the imaginary water mass in such a manner that all the isosteres on each horizontal level in the imaginary water mass have a constant slope equal to the slope of the isostere at that level on the bottom line. This construction will give the required vertical distribution of the specific volume anomaly at the shallow station with reference to the zero reference surface.

The advantage of this method over the two methods already discussed is that here the solid block of earth along the profile located above the zero reference surface is replaced with an imaginary moving baroclinic water mass. This eliminates the unjustified assumptions made in the former two methods that the geostrophic current velocity and the pressure gradient are zero near the bottom. But this method has also some short comings, the most objectionable one being the requirement that the slope of all the isosteres should be constant at each horizontal in the imaginary water mass, a requirement very difficult to justify (Fomin, 1964). Further, the application of this method requires graphical construction.

Another method suggested by Fomin (1964) is th simple extrapolation of the specific volume anomaly of water along the profile in the imaginary water mass which will provide the vertical distribution of the specific volume anomaly at the shallow station upto the zero reference level. In this case also, solid block of earth along the profile located above the zero reference surface is replaced with a moving water mass. The subjective errors inherent in the method, however, make it most unsuitable, particularly if the shallow station is located at the boundary of the ocean under study.

7.2. A new method suggested for the extension of computation into shallow regions

The foregoing discussion on the different methods used for the extension of computation into the shallow regions of the ocean has brought out the fact that none of them are without short comings. Hence a new method is suggested for this purpose in which most of the objection raised in the preceding pages are avoided.

In this method, the dynamic relief along a profile of the isobaric surface corresponding to the deepest sampling depth at the shallow station is linearly extrapolated along the profile in the shallow region. This gives the dynamic height anomaly of the isobaric surface at the shallow station with reference to the zero surface. If A, B and C are three stations in a line, of which C is a shallow station, then the required dynamic height anomaly at the shallow station is given by

$$\Delta D_{C'} = \left[\frac{\Delta D_{A'} - \Delta D_{B'}}{AB}\right] BC + \Delta D_{B'} \tag{7.2}$$

where $\Delta D_{A'}$, $\Delta D_{B'}$ and $\Delta D_{C'}$ are dynamic height anomalies of the isobaric surface passing through the deepest sampling depth at station C, and AB and BC are distances between stations A and B and stations B and C respectively.

The following are some of the advantages of this method:

a) In this method the section of the solid earth along the profile located above the zero reference surface is replaced with a moving baroclinic water mass.

b) Since it is a linear extrapolation, its application does not require graphical construction.

c) The linear extrapolation done only at one depth avoids accumulation of subjective errors.

If the values of the independent oceanographic parameters are missing at one depth at a particular station and if their values are known for the neighbouring stations on either side, the dynamic height anomaly at that depth may be obtained by linear interpolation. If A_1, B_1 and C_1 are three stations in a line in that order and if the values of the independent oceanographic parameters are missing at some particular depth at station B_1, then the dynamic depth anomaly at the depth where the values are missing is obtained as

$$\Delta D_{B_1'} = \left[\frac{\Delta D_{A_1'} - \Delta D_{C_1'}}{A_1C_1}\right] B_1C_1 + \Delta D_{C_1'} \qquad (7.3)$$

where $\Delta D_{A_1'}$, $\Delta D_{B_1'}$ and $\Delta D_{C_1'}$ are dynamic height anomalies of the isobaric surface passing through the depth at station B_1 where the independent oceanographic parameters are missing and A_1C_1 and B_1C_1 are distances between station A_1 and C_1 and stations B_1 and C_1 respectively.

Assuming 1500 d bar surface as the reference surface and then extending the same into the shallow regions using the above method for the purpose of computation, the dynamic height anomalies of the different isobaric surfaces, corresponding to the standard depths above 1500 d bar surface, were computed for stations along the profile approximately 15° latitude in the Arabian sea. The station positions are shown in Fig.3.

7.3. Smoothening the dynamic relief in a profile of an isobaric surface

In section 5.1 we have seen that the computed dynamic height anomaly of an isobaric surface will be in error by an amount which can be calculated using the method described in that section. Table VI of the same section gives the magnitudes of errors in the computed dynamic height anomalies of the different isobaric surfaces with reference to the 1500 d bar surface. The computed dynamic relief of an isobaric surface in a profile, so, will be in error by an amount depending on the isobaric surface selected. The actual dynamic relief of the isobaric surface will be one among the infinite number of possible reliefs that can be drawn within the error limits. In this section, the selection of a dynamic relief is done by a method of smoothening suggested by Fomin (1964) which will provide a more dependable picture of the dynamic topography of the isobaric surface as well as the vertical distribution of geostrophic current.

The dynamic height anomalies of the different isobaric surfaces are computed, as explained in section 7.2, for the stations which lie approximately along 15°N latitude in the Arabian sea. The computed dynamic reliefs of the different isobaric surfaces are then drawn and shown with dashed lines in Figs. 5-8. The lines on both sides of the computed dynamic profile are the limits of the error interval. The error limits taken were $1/\sqrt{2}$ times the maximum error for the concerned isobaric surface because the dynamic relief of an isobaric surface in a profile highlights the differences in dynamic height anomalies between adjacent stations and, as we have seen earlier (section 5.2), the total error in the difference in dynamic height anomalies is not the sum of the errors but the sum divided by $\sqrt{2}$*. The actual dynamic relief of an isobaric surface will be one among the many that are possible between the error limits. Those that are selected for the different isobaric surfaces are shown by thick lines in the figures and are the smoothest possible curves between the error limits. The smoothest

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*Fomin (1964) took the error limits as equal to the maximum error itself which, in the light of the above, should be considered as incorrect.

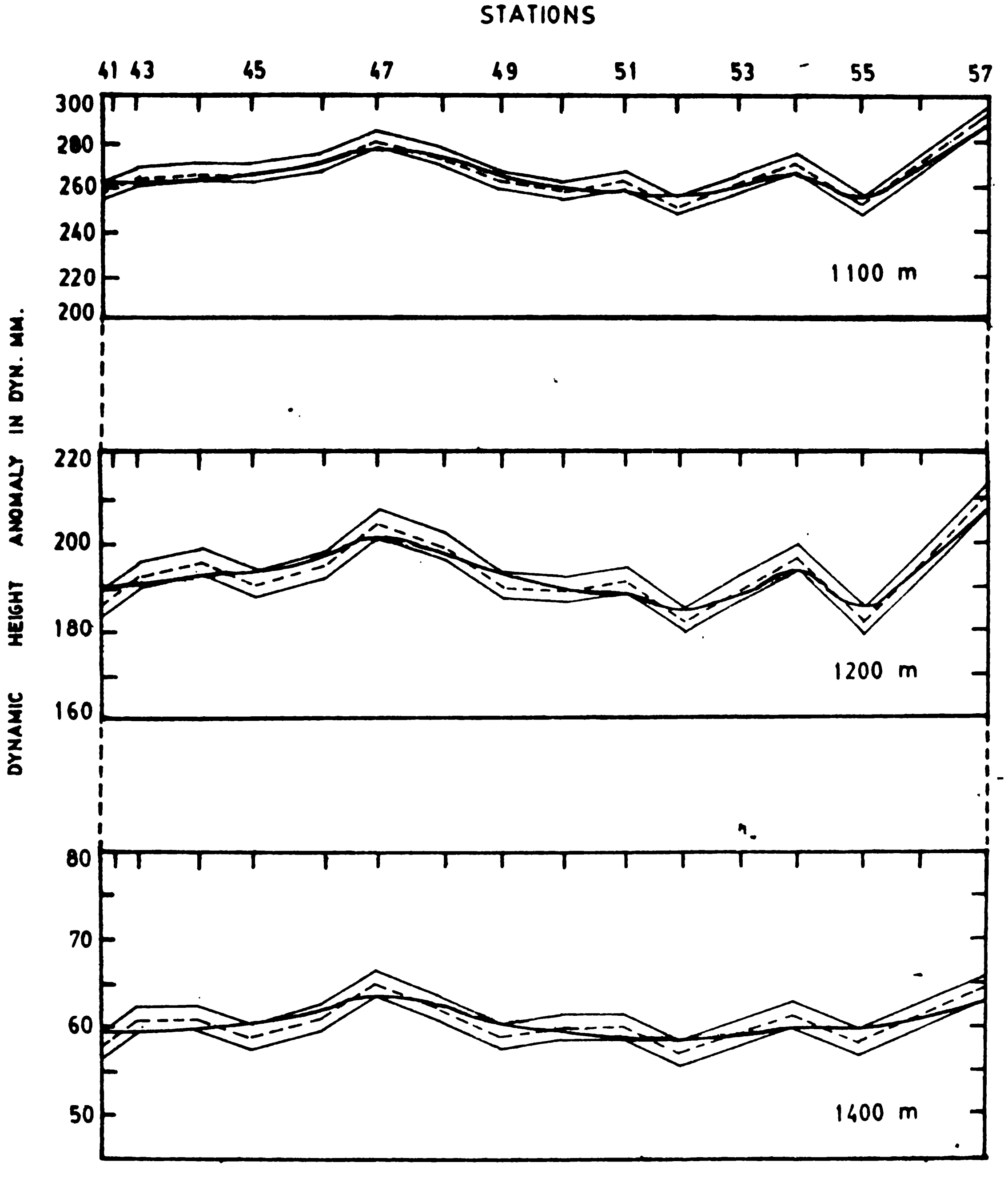

Fig. 5: Dynamic relief of different isobaric surfaces along approximately 15°N Latitude in the Arabian Sea.

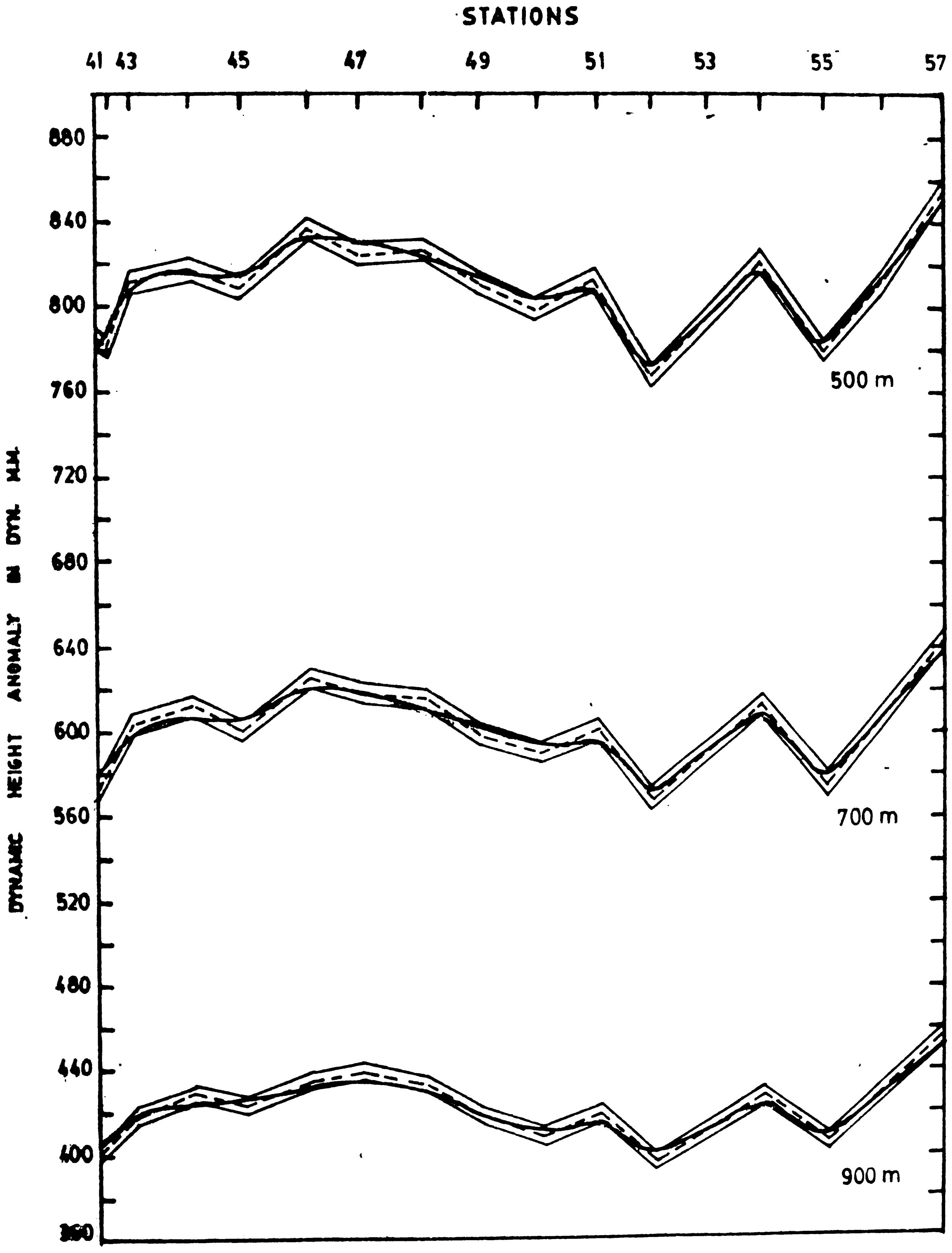

Fig. 6: Dynamic relief of different isobaric surfaces along approximately 15°N Latitude in the Arabian Sea.

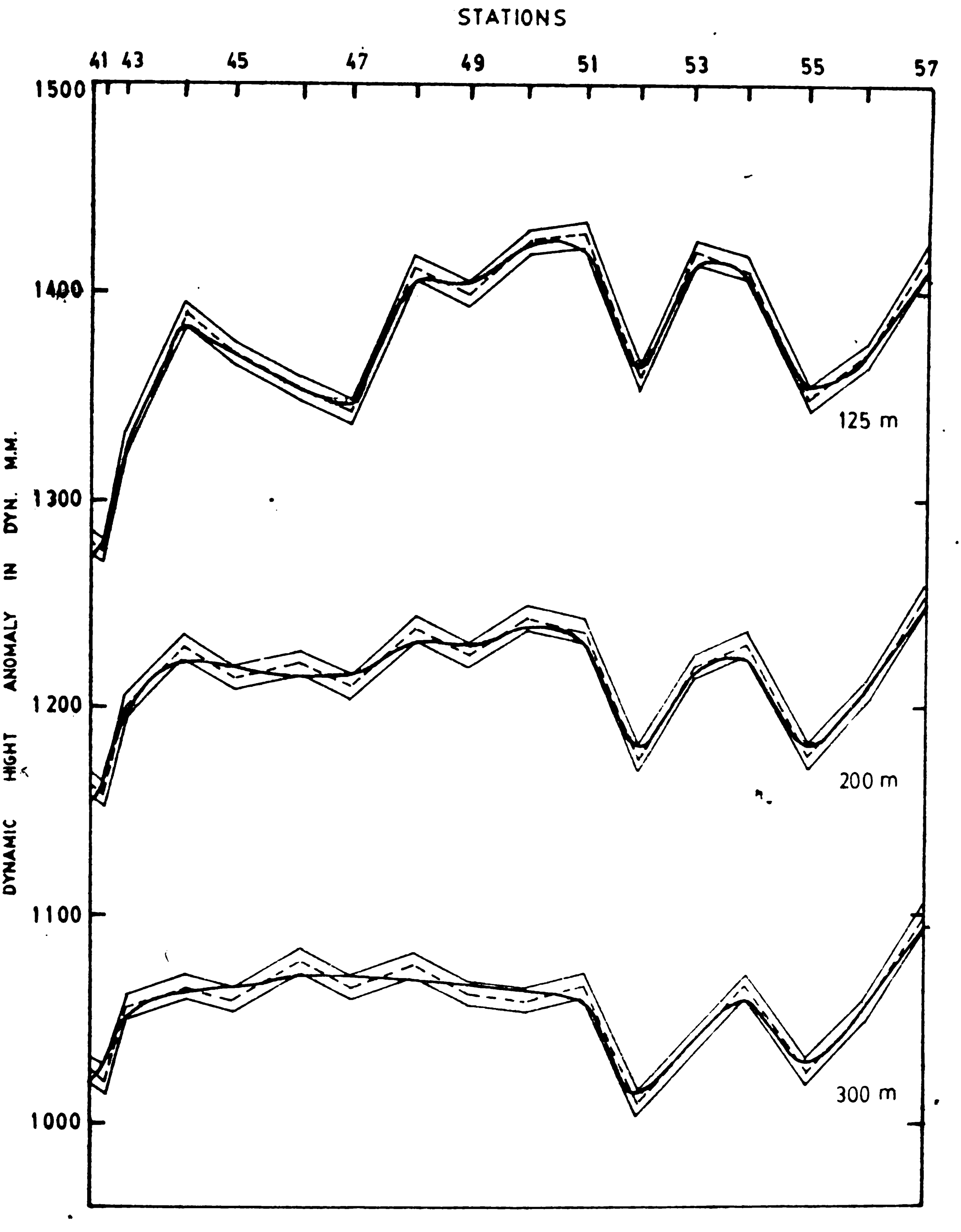

Fig. 7: Dynamic relief of different isobaric surfaces along approximately 15° N Latitude in the Arabian Sea.

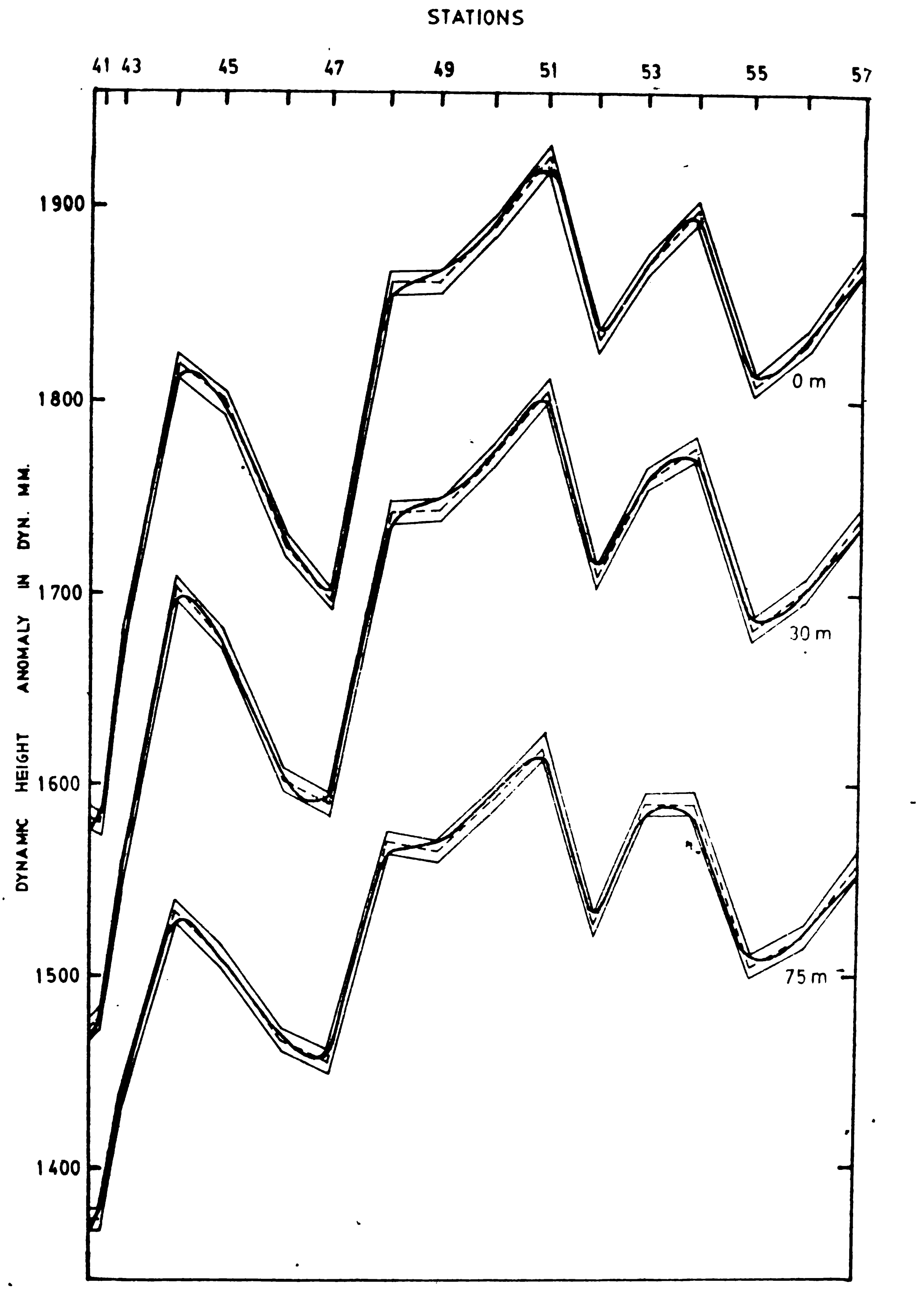

Fig. 8: Dynamic relief of different isobaric surfaces along approximately 15° Latitude in the Arabian Sea.

possible curve is selected because under oceanic condition, in general, large horizontal gradient of any parameter should be considered as unnatural and because any distortion to the curve between the error limits should be considered as unjustified*.

From the smoothened curves, obtained as above, the new values of the dynamic height anomalies of different isobaric surfaces for all the stations in the profile are obtained.

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*Smoothening is a process very commonly employed in Physical Oceanography in the processing and analysis of oceanographic data. For example, Montgomery (1954) and Stroup (1954) recommended a method of analysis of serial oceanographic data in which the shape of the station curves for any particular station is influenced by the data from the nearby stations and this procedure, they claimed, ensure meridional continuity of features of distribution. This is nothing but smoothening of data in-between stations. Again, oceanographers generally 'follow the trend' when they draw isolines of any parameter in sections and are not very particular to draw the lines through the exact plotted points.

CHAPTER VIII

COMPUTATION OF GEOSTROPHIC CURRENTS

We have obtained the smoothened values of the dynamic height anomalies along the profile with reference to 1500 d bar surface for the stations located approximately along $15^{\circ}N$ latitude in the Arabian sea (Fig.3) in section 7.3. These smoothened values are used for the computation of relative currents using the Helland-Hansen formula. The values thus obtained are to be reduced to 3000 d bar surface to obtain geostrophic currents with reference to the selected zero reference surface (section 6.2). This chapter deals with the procedure for the reduction of the computed geostrophic currents to the required zero reference surface.

8.1. Reduction of the computed geostrophic current with reference to 1500 d bar surface to 3000 d bar surface

We have seen in chapter VI that the equation (6.10), namely:

$$V_x(z) = V_x(H)\ \psi\ (z)$$

describes the vertical distribution of the absolute value of the geostrophic current velocity in an oceanic region where Shtokman's density model is applicable and so may be used to compute any subsurface current. It is not essential that the absolute value of the sea surface current should be known for the computation of the velocities of subsurface currents. The computation can be done if we know the difference between the geostrophic current velocities at any two depths. If we know the difference in geostrophic current velocities at the sea surface and at some depth z_1, by measurement, then the absolute value of the geostrophic current velocity at the sea surface, $V_x(H)$, can be obtained by sub.tracting $V_x(H)$ from both sides of equation (6.10) and then solving for $V_x(H)$.

$$V_x(z_1) - V_x(H) = V_x(H)\,\phi\,(z_1) - V_x(H)$$

$$= V_x(H)\,[\phi\,(z_1) - 1]$$

$$\therefore V_x(H) = \frac{V_x(z_1) - V_x(H)}{\phi\,(z_1) - 1}$$

$$= \frac{V_x(H) - V_x(z_1)}{1 - \phi\,(z_1)} \qquad (8.1)$$

Substituting equation (8.1) in equation (6.10), a formula for computing the vertical distribution of geostrophic current velocity from a knowledge of the relative value of geostrophic current between two depths is obtained as

$$V_x(z) = \frac{[V_x(H) - V_x(z_1)]\ \phi\ (z)}{1 - \phi\ (z)} \qquad (8.2)$$

We have already seen in section (6.2) that Shtokman's density model may be applied to the oceanic mass field below 1400 m. Take the term $[V_x(H) - V_x(z_1)]$ of equation (8.2) as the relative current at 1400 d bar surface relative to 1500 d bar surface, It can be computed using the Helland-Hansen method. Also, the stratification function ϕ (z) relative to the assumed zero surface at 3000 m depth can be computed for the same region. Once the quantities $[(V_x(H) - V_x(z_1)]$ and $\phi(z)$ are computed, the geostrophic current relative to 3000 d bar surface at any depth below 1400 d bar surface can be obtained using equation (8.2). The computed currents, relative to 1500 d bar surface, at different isobaric surfaces above 1400 d bar surface, can be reduced to the values relative to 3000 d bar surface by adding algibraically to these computed values the currents computed at 1500 d bar surface. The current at 1400 d bar surface may be obtained either by adding the current computed at 1500 d bar surface or by the use of equation (8.1).

The method of computation of stratification function ϕ (z) for station No.82 is shown in Table IX, assuming that the lower boundary of the current is 3000 m. For the purpose of this computation, it is convenient to write the equation for the stratification function ϕ (z) in the finite difference form as

$$\phi(z) = \frac{\int_0^z \delta(z)\, dz}{\int_0^H \delta(z)\, dz} = \frac{\sum_0^z \widetilde{\delta(z)}\, \Delta z}{\sum_0^H \widetilde{\delta(z)}\, \Delta z} \qquad (8.3)$$

The first six steps of the computation of function ϕ (z) have already been explained in section 6.2. The seventh column of Table IX gives the value of ϕ (z) as obtained by the division of the different values in the sixth column by the value at the top of the column. Similarly the function ϕ (z) was computed for the stations 48,83,103 and 111 and are tabulated in Table X. The stations used are the same as those used for the computation of the function F(H) and are well spread over the whole of the Arabian sea (Fig.3). Table X shows that the function ϕ (z) is not very different for these stations which means that in the region selected water is very uniform in the horizontal plane as already seen from Table VIII. An average value of the function ϕ (z) is shown as the last column of Table X. The deviation of the function ϕ (z) at any station from this average value will not be greater than 4 units in its second decimal place. This

Table IX

Computation of stratification function ϕ (z)

H-z (m)	σ_t	$K\delta(z)$	$K\,\overline{\delta(z)}$	$K\overline{\delta(z)}\Delta z$	$K\sum_{0}^{z}\overline{\delta(z)}\Delta z$	ϕ (z)
1	2	3	4	5	6	7
1400	27.65	0.16			64.0	1.00
			0.14	14.0		
1500	27.69	0.12			50.0	0.78
			0.09	22.5		
1750	27.75	0.06			27.5	0.43
			0.05	12.5		
2000	27.78	0.03			15.0	0.23
			0.02	10.0		
2500	27.80	0.01			5.0	0.08
			0.01	5.0		
3000	27.81	0.00			0.0	0.00

Table X

Values of ϕ (z) for different stations for an assumed reference surface at 3000 m

H-z (m)	Station numbers					ϕ (z)
	48	82	88	103	111	
1400	1.00	1.00	1.00	1.00	1.00	1.00
1500	0.81	0.78	0.82	0.80	0.80	0.81
1750	0.47	0.43	0.49	0.47	0.47	0.47
2000	0.27	0.23	0.26	0.27	0.27	0.26
2500	0.07	0.08	0.07	0.07	0.07	0.07
3000	0.00	0.00	0.00	0.00	0.00	0.00

means that, in view of equation (6.10), the error in the computed current using the average value of the function ϕ (z) will not be greater than 4% of its value relative to 3000 d bar surface.

The velocity at z = H given by equation (8.1), namely,

$$V_x(z) = \frac{[V_x(H) - V_x(z_1)]}{1 - \phi\ (z_1)}$$

will be correct only if the absolute value of the relative current $[V_x(H) - V_x(z_1)]$ is known by measurement. If the relative current is obtained by computations using classical Helland-Hansen method, an error will be introduced into the computed value of the geostrophic velocity at z = H due to the error committed in the computation of the density of sea water. The magnitude of this error may be estimated as follows: Using formula (5.18) the relative current between two depths, z = H and z = z_1, is obtained in the classical Helland-Hansen method as:

$$V_x(H) - V_x(z_1) = \frac{10(\Delta D_A - \Delta D_B)}{L.\ 2\ \omega \sin\phi} \tag{3.4}$$

If this value of the relative current is substituted in equation (8.1), we get

$$V_x(H) = \frac{10(\Delta D_A - \Delta D_B)}{L.2\,\omega \sin\phi\,[1 - \Phi(z_1)]}$$

$$= \frac{10(\Delta D_A - \Delta D_B)\int_0^H \delta(z)\,dz}{L.2\omega \sin\phi\,[\int_0^H \delta(z)\,dz - \int_0^{z_1} \delta(z)\,dz]} \qquad (8.5)$$

By taking the total differential of equation (8.5) and dividing by the equation, we get the relative error in the computation of the absolute current velocity at z = H as:

$$\frac{dV_x(H)}{V_x(H)} = \frac{d(\Delta D_A - \Delta D_B)}{\Delta D_A - \Delta D_B} + \frac{2d\sigma_t[z_1 \int_0^H \delta(z)dz - H \int_0^{z_1} \delta(z)dz]}{\int_0^H \delta(z)dz . \int_{z_1}^H \delta(z)dz} \qquad (8.6)$$

The magnitude of the relative error in the determination of the geostrophic current at 1400 m relative to 3000 m by the application of Shtokman's density model if the relative current at 1400 m relative to 1500 m is obtained by Helland-Hansen method may be computed using the expression (8.6). The first term on the right hand side of this expression is approximately equal to unity because the error in the difference in dynamic height anomaly at 1400 d bar surface relative to 1500 d bar surface will be of the same order of magnitude as the difference in dynamic height anomalies itself. The contribution of the second term on the right hand side of expression (8.6) may be estimated using Table IX to get the values of the different integrals. This value is about 0.7. Hence the total relative error is 1.7 or 170%. But if the classical method is used for the computation of current at 1400 d bar surface relative to 3000 d bar surface, without the application of Shtokman's model, the relative error may be 500% or more. It may be expected that the error from this source becomes negligibly small when the dynamic relief of the 1400 d bar surface is smoothened as explained earlier in section 7.3.

Using the smoothened values of the dynamic height anomalies of the different isobaric surfaces for the stations in the profile approximately along 15^{O}N latitude in the Arabian sea (Fig.3), obtained using the methods explained earlier, the relative currents relative to 1500 d bar surface were computed at all the different isobaric surfaces using classical Helland-Hansen method. Then using the average value of the function ϕ (z) given in Table X, the geostrophic currents at the different depths below 1400 d bar surface relative to 3000 d bar surface were computed. Using these computed currents at 1500 d bar surface, the computed currents above this level were adjusted to reduce them to values relative to

3000 d bar surface as explained earlier. The results are used to draw the vertical distribution of geostrophic current velocity along the profile relative to 3000 d bar surface and are shown as Figs. 9 and 10. Figs. 11 and 12 are the corresponding vertical distribution of geostrophic current velocities when the computations were done using classical methods.

Comparison of Figs. 9 and 10 with the corresponding Figs. 11 and 12, brings out certain important observations. Results of the computation of geostrophic current using smoothened values of the dynamic height anomalies of the different isobaric surfaces show a substantial northerly current between stations 41 and 42. The classical method yields only an erratic current of small magnitude in this region and this current changes direction as the depth increases. The fact that a substantial northerly current does exist in the region is evident from the raising trend of the isanosteres between the stations towards the coast shown in the diagram depicting the vertical distribution of thermosteric anomaly along the same profile produced by Sastry and D'Souza (1971) and reproduced as Fig.13 in this work. The failure of the classical method here may be due to the high computational errors arising out of the small distance between the stations.

Figs. 9 and 10:

Geostrophic current (cm/sec) relative to 3000 decibars along approximately $15^{\circ}N$ Latitude in the Arabian Sea, computed using the smoothened data (flow is towards south in the shaded area).

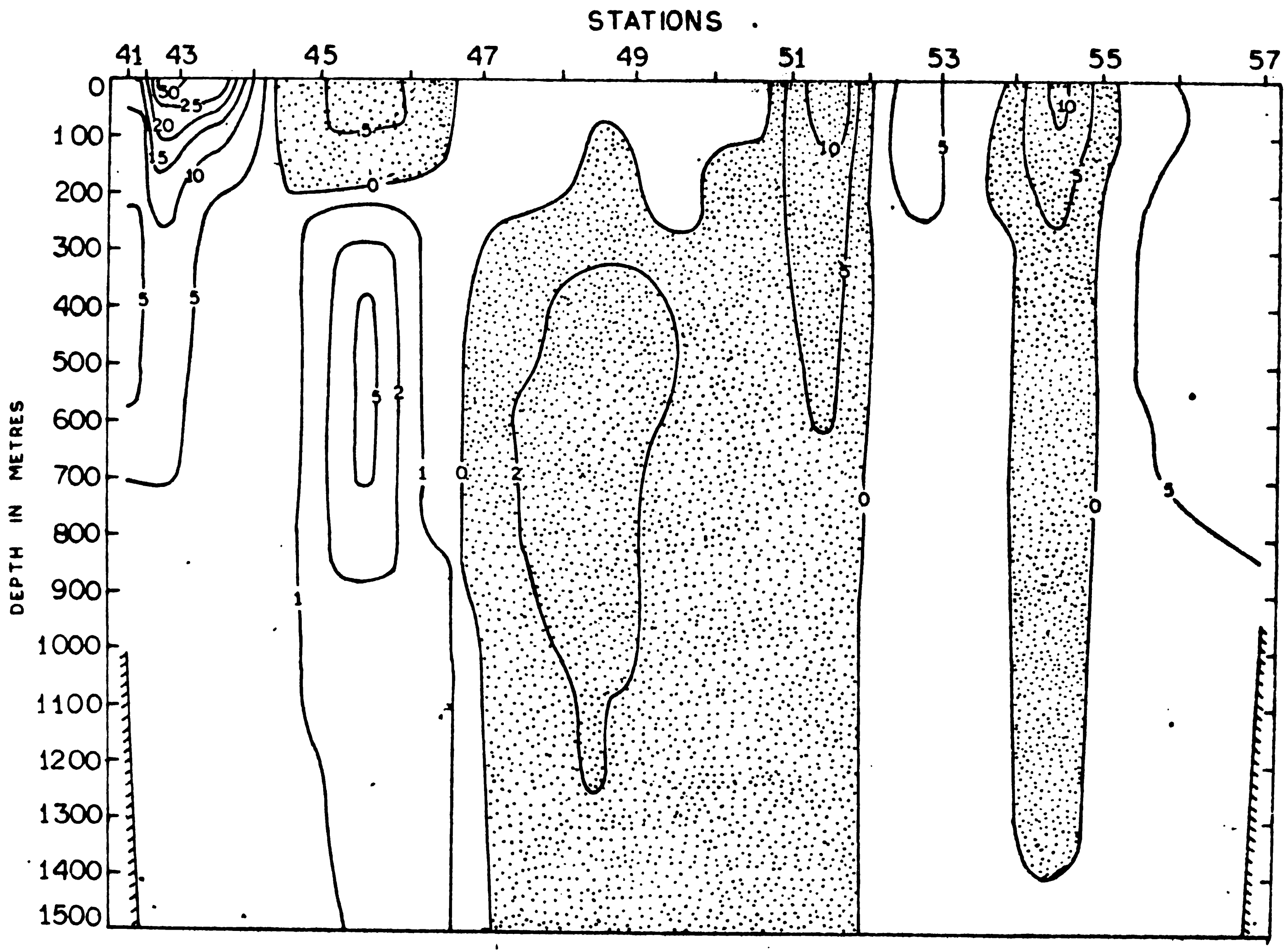
STATIONS
41
43
45
47
49
51
53
55
57
DEPTH IN METRES
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500

Fig. 9

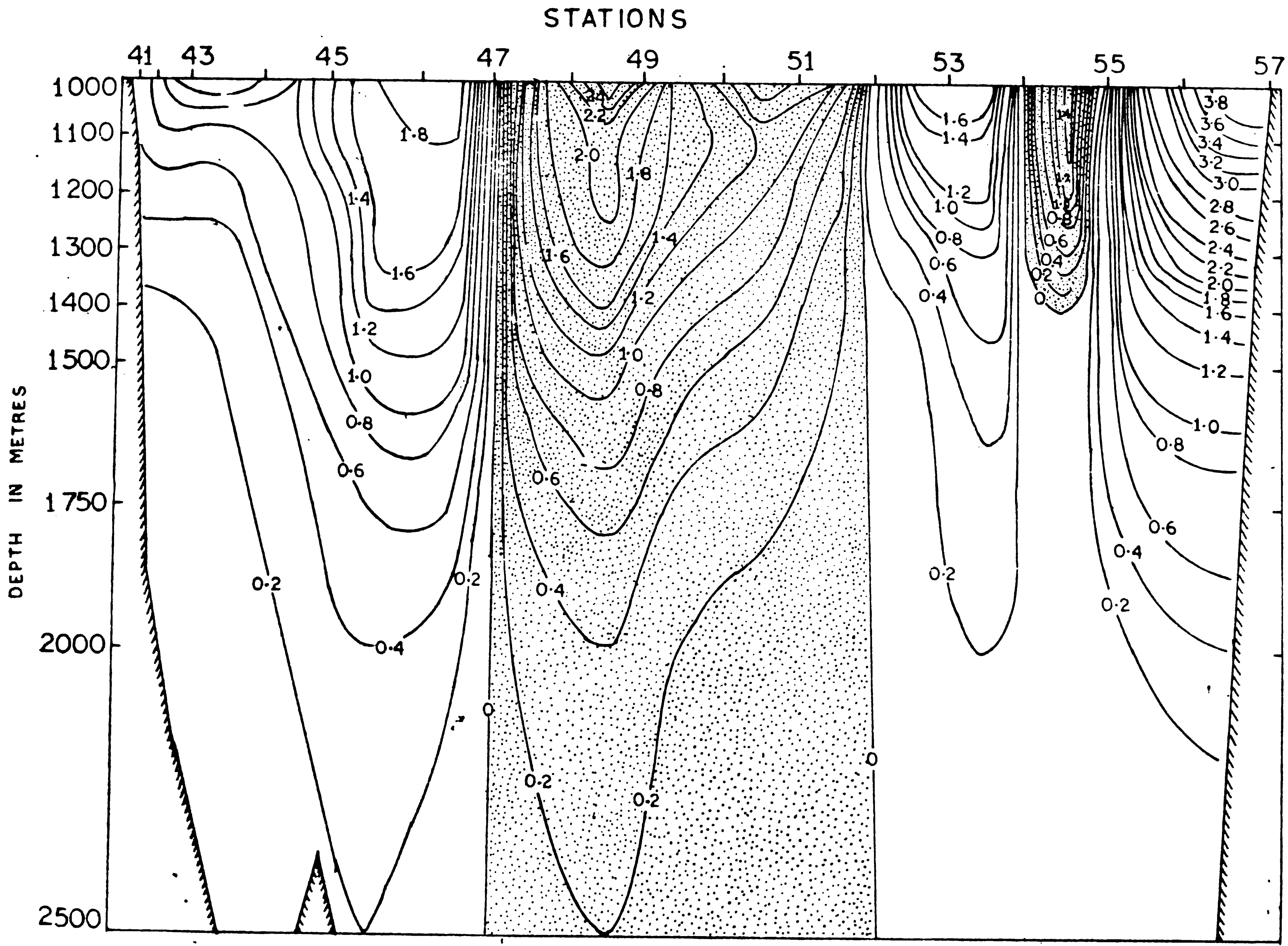
STATIONS
41
43
45
47
49
51
53
55
57
DEPTH IN METRES
1000
1100
1200
1300
1400
1500
1750
2000
2500

Fig. 10

Figs. 11 and 12:

Geostrophic current (cm/sec) relative to 3000 decibars along approximately 15°N Latitude in the Arabian Sea, computed using classical methods (flow is towards south in the shaded area).

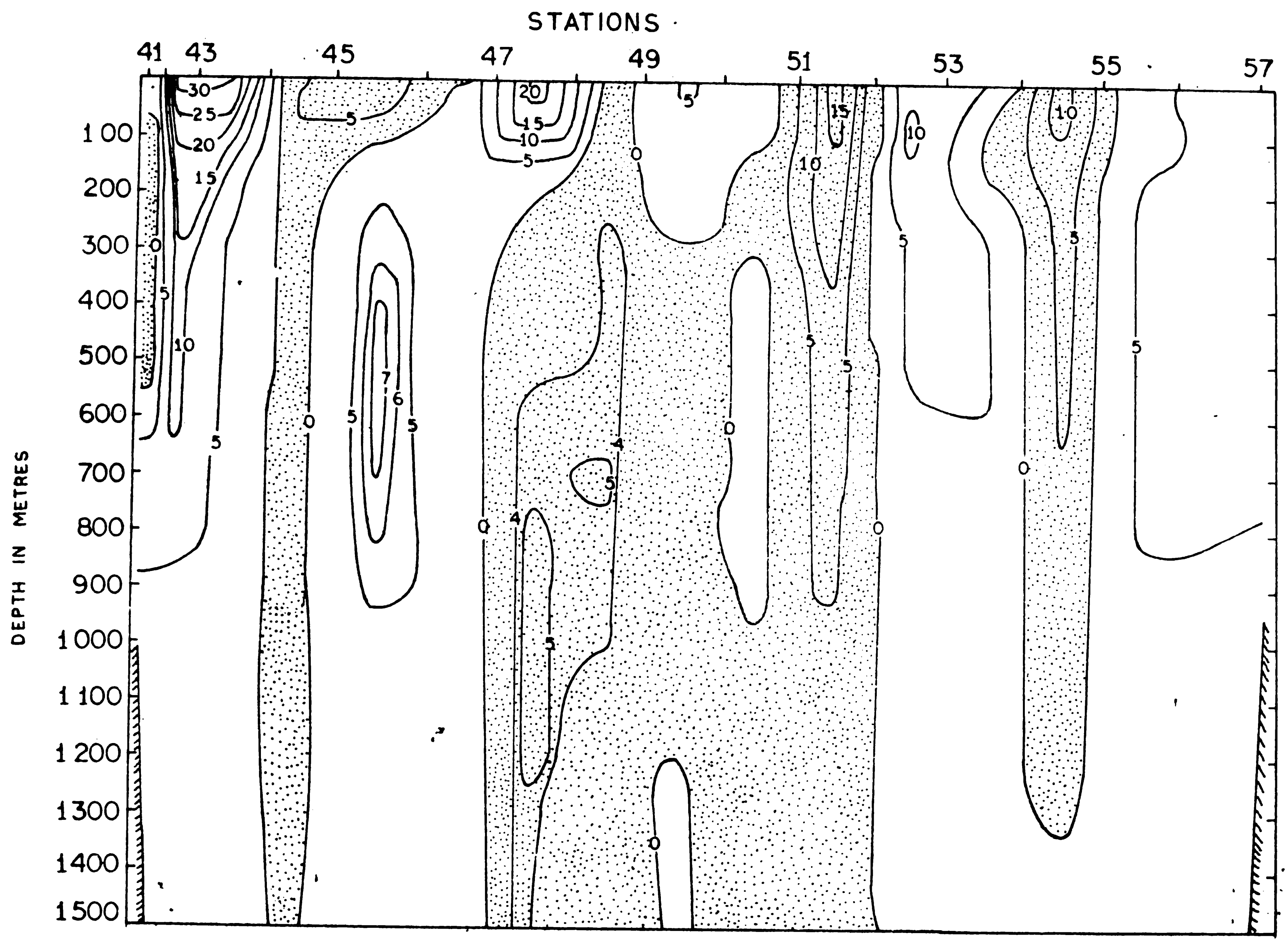
STATIONS
41
43
45
47
49
51
53
55
57
DEPTH IN METRES
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500

Fig. 11

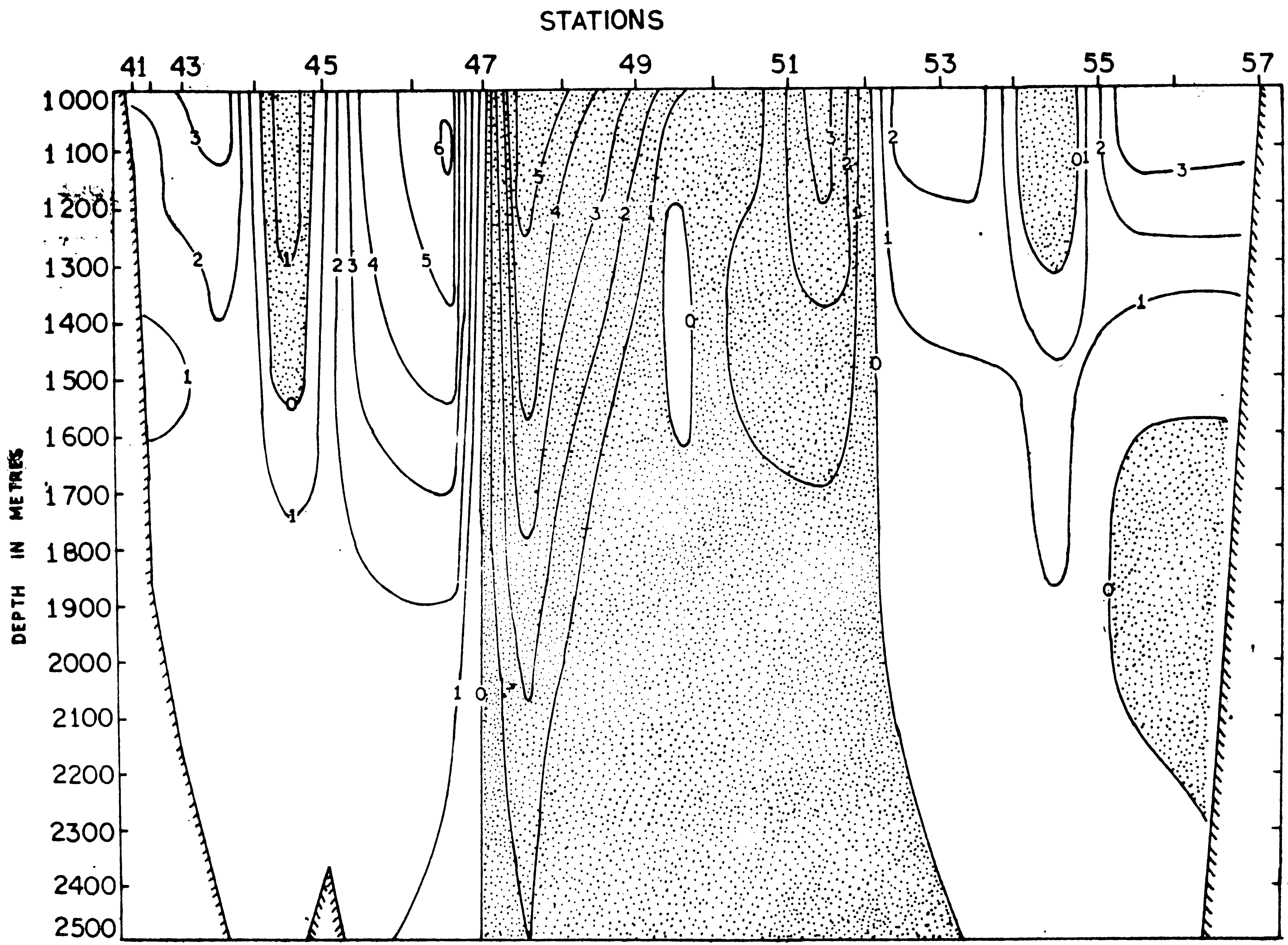
STATIONS
41
43
45
47
49
51
53
55
57
DEPTH IN METRES
1000
1100
1200
1300
1400
1500
1600
1700
1800
1900
2000
2100
2200
2300
2400
2500

Fig ·12

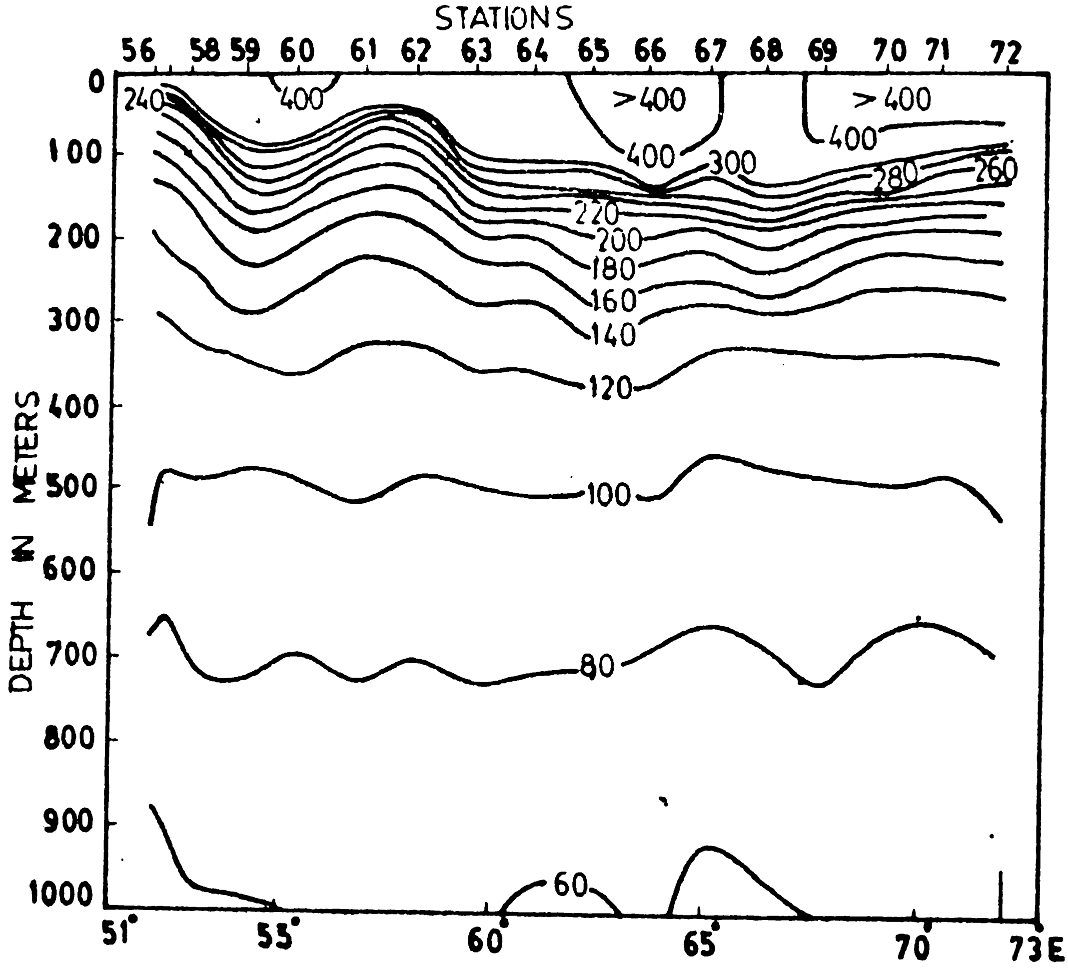

Fig. 13: Thermosteric anomaly (cl/T) along approximately 15°N Lat. in the Arabian Sea (after Sastry and D'Souza,1971)

Another observation is that the computed currents which are within the limits of accuracy of computation, gets smoothened out in the smoothening process. Under certain situations the smoothening procedure causes a change in the direction of such currents. Thus the southerly currents below 200 m between stations 44 and 45, below 1600 m between stations 56 and 57 and above 100 m between stations 48 and 49 seen in Figs.11 and 12 are seen to be northerly in Figs.9 and 10. Similarly the northerly currents below 300 m between stations 50 and 51 and below 1200 m between stations 49 and 50 in Figs.11 and 12 are seen to be southerly in Figs.9 and 10. The magnitudes of computed currents is generally decreased by the smoothening but under certain situations an increase can also occur as we have already seen between stations 41 and 42 where the computational error is very large.

8.2. Summary and conclusions

The measurement of the independent variables at sea is subject to random errors. These errors get propagated, through the series of computational procedures, into the final derived results. The magnitude of errors in the final results may, at times, be of such magnitude as to vitiate the results themselves. The present study is an attempt to examine the limits of errors contained in the basic data and the magnitude of the error component in the derived results of Dynamical Oceanography.

The general theory of errors relevant in the context of the studies on the propogation of errors in physical oceanographic computations have been discussed in chapter II. After discussing the type of errors in section 2.1, the general formula for errors is derived in section 2.2. The applications of the general formula for errors to the fundamental operations of arithmetic have been dealt with in section 2.3. In section 2.4 after discussing the normal law of errors, an expression for the computation of the standard deviation, σ of a normal error distribution representing the distribution of errors in the summation of S values, when only the maximum individual error is known, is derived.

The errors in the measurements of the independent oceanographic variables, namely, temperature, pressure and salinity are discussed in chapter III. In section 3.1 is discussed the magnitude of error committed in the measurement of temperature using the protected reversing thermometer. There it is shown that addition of expansion correction increases the magnitude of error in the temperature measurement generally and that in a tropical sea the magnitude of the total error in the temperature measurement should be taken as $\pm 0.01^{\circ}C$ upto a depth of 200 m and as $\pm 0.02^{\circ}C$ below the depth of 200 m. The error in the determination of sea pressure is dealt with in section 3.2. It is shown that the error committed in the replacement of pressure expressed in decibars by depth expressed in metres is negligibly small in the tropical seas. The error committed in the reading of an unprotected reversing thermometer is also discussed in this section. It is seen that addition of expansion correction increases the magnitude of error in the reading. The magnitude of the total error in this case also is equal to $\pm 0.01^{\circ}C$ upto a depth of 200 m and $\pm 0.02^{\circ}C$ below the depth of 200 m. The corresponding errors in the depth measurements are equal to ± 2m upto a depth of 200 m and ± 4 m below the depth of 200 m. The errors committed in the determination of salinity are discussed in section 3.3. The error committed in the determination of salinity by the method of titration is ± 0.04‰ and by the method of conductivity ratio measurement is ± 0.02‰. The accuracy of measurements using in situ instruments is discussed in section 3.4. It is shown that the accuracy obtained is much poorer compared to the classical method of obtaining the values of the independent variables.

Errors in the computation of dependent oceanograph variables due to the propogation of errors are discussed in chapter IV. In section 4.1 it is seen that irrespecti of the formula employed for the computation of σ_t, the error in the density determination shou be taken as ± 0.03 in σ_t when salinity is determined using the method of titration and as ± 0.02 in σ_t when salinity is determined using the method of conductivity ratio measurement. The corresponding errors in the computation of specific volume anomaly is shown to be equal to ± 3 cl/ and ± 2 cl/ton respectively in section 4.2.

In chapter V the errors committed in the computations of derived quantities, namely, dynamic heigh anomaly of isobaric surfaces and relative currents are discussed. In section 5.1 the magnitudes of errors in th computation of dynamic height anomaly of different isobaric surfaces relative to a reference isobaric surface at 1500 m are obtained using the normal law of errors. It is found that the deeper the reference surfac selected, the larger is the error in the determination of the dynamic height anomaly of an isobaric surface. The magnitudes of errors in the computation of relative currents is discussed in section 5.2. It is seen that the magnitude of error in the computation of relative current is directly proportional to the error in the determination of the difference in dynamic height anomalies between two stations and is inversely proportional both to the distance between the two stations and the sine of the latitude angle. It is also seen that if the distance between stations is 100 km, the absolute magnitude of error in the determination of relative current with reference to 1500 d bar surface is less than 1 cm/sec above 40^{0} latitude angle and increases towards the equator. Very near the equator the rate of increase is very large.

After critically reviewing the existing methods for the identification of a level of no motion in section 6.1, in section 6.2 a method for the identification of a level where the geostrophic current is negligibly small, by the application of Shtokman's density model to the oceanic massfield is discussed. It is shown that Shtokman's density model, may be applied to the mass field of the ocean below roughly 1500 m. A function F(H) is computed in this section by the application of the model to the oceanic mass field below 1400 m. This function decreases in magnitude as the depth of the selected reference surface increases and becomes constant at a depth equal to 3000 m. This depth, below which the function F(H) assumes a constant value, is shown, to be a level where the magnitude of the geostrophic current is negligibly small.

In section 7.1, the existing methods for the extension of the selected zero reference surface into shallow regions for the purpose of computation are critically examined. In section 7.2 a new method is proposed for the same purpose which essentially consists of the extrapolation of the dynamic height anomaly in a profile of the isobaric surface which represents the deepest sampling depth at the shallow station. It is shown that this method is free of the objections raised in connection with the existing methods. A method of smoothening of the dynamic relief of an isobaric surface in a profile within certain error limits, is described in section 7.3. It is shown that the error limit for the above purpose should be $1/\sqrt{2}$ times the maximum error computed for the dynamic height anomaly of the isobaric surface concerned. It is also shown that the smoothened values of the dynamic height anomalies of isobaric surfaces will give a more dependable picture of the oceanic circulation pattern.

In sec. 8.1 of chapter VIII, the reduction of the computed current relative to 1500 d bar surface to the zero reference surface at 3000 m depth is dealt with. It is shown that the relative current computed at 1400 d bar surface relative to 1500 d bar surface may be used for obtaining the currents at different isobaric surfaces below 1400 m, if the stratification function, $\phi(z)$, resulting from the application of Shtokman's density model to the region, is known. The current computed at different depths above 1500 d bar surface may be obtained by the algebraic addition of the current at 1500 d bar surface to the different computed currents. The results thus obtained for the stations lying approximately along 15°N latitude in the Arabian sea are compared with results obtained using the classical method.

The studies showed that the graphical method suggested by Montgomery (1954) and Montgomery and Wooster (1954) for the determination of thermosteric anomaly, δ_T, introduces, in the computed current, an error, that is equal to 2 times the error committed when the values are obtained using classical methods. Consequently, when the values of δ_T are obtained graphically, the computed circulation pattern may become undependable, particularly when the circulation is weak and is in the equatorial latitudes.

Proper selection of a zero reference surface is important for the presentation of an oceanic circulation pattern. If a reference level is selected arbitrarily and if a current does exist at that level, the computed currents in the layers above will be different from the actual currents. Also the wrong selection of a reference level will result in the computation of an oppositely directed current below this level which increases in magnitude towards the bottom, a result which is against the present day oceanographic experience.

Since the error in the computed current increases very rapidly towards the equator, computed current using the classical method is very undependable in these regions, particularly when the current is weak. Smoothening of the dynamic relief, which may be successful in the higher latitudes for the presentation of a dependable circulation pattern, may prove unsuccessful very near the equator since small variations in the values of dynamic height anomalies that produce large currents in these latitudes will be smoothened out, if they are within the error limits.

It is seen that decrease in station spacing will increase the error in the computed current and may lead to undependable results, particularly, when the values of thermosteric anomalies are obtained using the graphical method suggested by Montgomery (1954) and Montgomery and Wooster (1954). The error in the computed current committed when this method is used, is again doubled if the station spacing is halved. This means that the error in the computed current will be 4 times that given in Fig.2 if we take the station spacing as equal to 50 km. Hence the error in the computed current when the graphical method is used to obtain the required data and when the station spacing is 50 km will be ±4 cm/sec above 40° latitude, and ±36 cm/sec at 5° latitude. Such large errors may induce a band structure in the computed current pattern, particularly in the lower latitudes, when station spacing is reduced, a phenomenon observed by Montgomery and Stroup (1962). Hence it is advisable not to go for decrease in station spacing, in the hope of getting a detailed current structure. The station spacing, preferably, should not be less than 1° latitude or longitude angle.

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