INVESTIGATIONS ON PATHOLOGIES IN HIGHER SPIN FIELD THEORIES

M. SABIR, M. Sc.

THESIS SUBMITTED IN
PARTIAL FULFILMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

COCHIN UNIVERSITY

1978

CERTIFICATE

Certified that the work reported in the present thesis is based on the bona fide work done by M. Sabir, research scholar, under my guidance in the Department of Physics, Cochin University, and has not been included in any other thesis submitted previously for the award of any degree.

Cochin - 22
June 6, 1978

K. Babu Joseph,
Supervising Teacher.

DECLARATION

Certified that the work presented in this thesis is based on the original work done by me under the guidance of Dr. K. Babu Joseph in the Department of Physics, Cochin University, and has not been included in any other thesis submitted previously for the award of any degree.

Cochin - 22
June 6, 1978

Sabir

M. Sabir

PREFACE

The investigations presented in this thesis have been carried out by the author, as a full-time research scholar, during 1974-78 in the Physics Department of Cochin University.

The present thesis deal with studies on certain aspects of pathological higher spin field theories. It brings to light some new abnormalities and new examples of abnormal theories and also puts forward a novel approach towards the construction of trouble-free theories.

In Chapter I of this thesis, after a brief description of the different formulations of higher spin fields an extensive survey is made of the variety of pathologies afflicting such theories. The scope of some of the remedies suggested in the recent literature is also discussed. Chapter II contains a simple example of a pathological theory involving vector and scalar particles. It is shown that while certain coupling schemes lead to constraint breakdown and noninvariance of the S-matrix, there are other possible interactions which do not share these difficulties. In Chapter III, as a preliminary to an analysis of the question of relativistic covariance of a pathological theories at the classical level, the Dirac-Schwinger covariance condition is derived within a classical framework. The crucial role of the energy continuity equation is pointed out and the origin of higher order derivative terms is traced to the presence of higher derivatives of canonical coordinates and momenta in the energy density functional. Chapter IV contains a study of the problem of relativistic covariance at the classical level with regard to several interactions of a spin 1 field. It is found that while the acausal interactions lead to noncovariance, there are examples where the field propagation is causal but the theory is nevertheless noncovariant. Chapter V presents a new approach to higher spin field theory. It is suggested that the method of keeping the subsidiary conditions separate from the equations of motion in a consistent way by means of a Lagrange multiplier may be a solution to the various maladies of higher spin theories. The distinctive features of this approach are illustrated with the example of a spin 3/2 field. It is shown that the theory is causal and free of imaginary values in the energy spectrum when coupled to an external electromagnetic field. Quantization is carried out in an indefinite metric space and for minimal coupling a unitary S-matrix is constructed by introducing a fictitious particle and a new vertex. Chapter VI extends the Lagrange multiplier formalism to the description of a spin 2 field. The massless limit of the corresponding theory is also discussed.

The original contributions contained in this thesis are the following:

1. Observation of the possibility of constraint breakdown and noninvariance of the S-matrix for several couplings of a vector and scalar fields and the demonstration of the absence of such pathologies for other couplings.

2. A derivation of the Dirac-Schwinger covariance condition in classical field theory.

3. Detection of relativistic noncovariance at the classical level in the interactions of a spin 1 field and the observation of this possibility even in causal theories.

4. A new approach to the formulation of pathology-free theories and its application to spin 3/2 and spin 2 fields.

A part of these investigations has been published in the form of the following papers:

1. "Patholgies in interacting massive vector and scalar fields", Curr. Sci. **45**, 544 (1976)

2. "Relativistic noncovariance in interacting spin-1 field theories" J. Phys. A: Math. Gen. 9, 1951 (1976).

3. "The Dirac-Schwinger covariance condition in classical field theory" Pramana 9, 103 (1977).

4. "Lagrange multiplier formalism for a spin 3/2 field" J.Phys. A: Math. Gen. 10, 1225 (1977).

The investigations incorporated in this thesis have been conducted under the able and inspiring guidance of Dr. K. Babu Joseph, Department of Physics, Cochin University. The author is grateful to him for persistent interest, profound insight, and invaluable guidance.

Thanks are due to Professor Joy George who extended generous and sincere support to the progress of these investigations. The author wishes to record his hearty thanks to Professor Sathianandan, Head of the Department of Physics for his encouragement and valuable help.

The author is also grateful to Shri A.N.M. Shenoi, Shri M.N. Sreedharan Nair and Shri V.C. Kuriakose for their whole-hearted cooperation and for the immense help they have rendered over the years.

The author also wishes to thank Cochin University and University Grants Commission, New Delhi, for providing financial support in the form of a Junior Research Fellowship.

NOTATION AND CONVENTIONS

The metric used is $\delta_{\mu\nu}$, fourth component of a vector being assigned an imaginary value.

Greek indices denote 4-vectors and Latin indices 3-vectors. Summation over repeated indices will always be implied, unless stated otherwise.

The Dirac γ's (γ_μ , $\mu = 1, \ldots 4$) are all hermitian with square equal to one.

Complex conjugation and hermitian conjugation are both denoted by the same symbol (an asterisk, *) and it will be clear from the context which of these is meant. Conjugation is carried out on all imaginary units including the metric i. However, in dealing with complex 4-vectors, $A_\mu = (\vec{A}, i A_o)$ where $\vec{A} \neq \vec{A}^*$, $A_o \neq A_o^*$ it is convenient to work with the conjugate vector $A^*_\mu = (\vec{A}^*, i A_o^*)$ in which the metric is left unconjugated as distinct from $A_\mu^* = (\vec{A}^*, -i A_o^*)$.

Time derivatives will be frequently denoted by a dot i.e. $\partial_t A = \dot{A}$.

Natural units $\hbar = c = 1$ are used throughout.

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CHAPTER 1

INTRODUCTION

After a brief survey of various formulations of higher spin wave equations, an account is presented of the nature and variety of pathologies afflicting higher spin field theory when interactions are introduced. A discussion is also given of the scope and extent of the possible remedies suggested in the recent literature.

The interest in particles of higher spin (spin $s \geq 1$) dates back to the time Dirac[1] wrote his famous wave equation for the electron. While in the early days the study of higher spin particles was rather an academic pursuit, the discovery of higher spin resonant states in the fifties turned it into an important area of research. The ensuing activity has culminated in the realisation that the present day field theory, unless modified drastically, is incapable of giving a consistent account of higher spin particles and their interactions. Before going into the details of the various maladies of higher spin field theories a quick survey will be first made of the various formulations of higher spin field equations.

1.1 Relativistic wave equations for arbitrary spin

It is well known that a particle of spin s may be characterised by a definite irreducible representation of the three-dimensional rotation group which has both single and double valued representations. The principle of relativity, however, demands that physical laws be covariant under Lorentz transformations, and hence in setting up causal equations of motion it is the Lorentz group and its representations, and not the rotation group, that must be considered.

The homogeneous Lorentz group is isomorphic to the group SL(2,C), the group of unimodular transformations in a complex two dimensional space. The finite dimensional representations of this group are non-unitary, and the complex conjugate representations ("dotted spinors") are distinct fromthe "undotted spinor" representations. A finite dimensional irreducible representation $D^{jj'}$ is characterized by two independent sets of integral or half-integral values and has the dimensionality $(2j + 1)(2j' + 1)$. Elements of the corresponding representation space are spinors of $2j'$ dotted and $2j$ undotted indices. Spinor calculus developed in analogy with the usual tensor calculus allows one to find all possible equations invariant under SL(2,C)[2].

The simplest equation invariant under SL(2,C) is

$$\partial_{a\dot{b}}\varphi^{a} = im\,\varphi_{\dot{b}} \tag{1.1}$$

Here φ^{a} is a two-component spinor and $\partial_{a\dot{b}}$ is the spinorial derivative operator related to the four-derivative through the relation

$$\partial_{a\dot{b}} = \sigma_{\mu;a\dot{b}}\,\partial_{\mu} \tag{1.2}$$

where $\sigma_{\mu;a\dot{b}}$ is the spin-tensor

$$\sigma_{\mu;a\dot{b}} = (\vec{\sigma}, iI) \tag{1.3}$$

the σ_i being the Pauli spin matrices. The relation between a quantity and its complex conjugate being non-linear, eq.(1.1) must be rejected on account of its non-linearity or more particularly, it can hold only for m = 0 in which case one obtains the Weyl equation

$$\partial_{a\dot{b}}\varphi^{a} = 0 \tag{1.4}$$

The nonlinearity problem for $m \neq 0$ can be overcome in a different way by having on the rhs of eq.(1.1) not the complex conjugate of φ_a but of another spinor χ_a independent of φ_a :

$$\partial_{a\dot{b}}\varphi^{a} = im\chi_{\dot{b}} \tag{1.5}$$

There will now have to be another equation for $\chi_{\dot{b}}$

$$\partial^{a\dot{b}}\chi_{\dot{b}} = im\varphi^{a} \tag{1.6}$$

Equations (1.5) and (1.6) together constitute the spinorial form of the Dirac equation

$$(\gamma\cdot\partial + m)\psi(x) = 0 \tag{1.7}$$

where $\psi = \begin{pmatrix}\varphi_1\\ \varphi_2\\ \chi^{\dot{1}}\\ \chi^{\dot{2}}\end{pmatrix}$ is a four component spinor transforming according to the representation $\mathcal{D}^{(\frac{1}{2},0)} \oplus \mathcal{D}^{(0,\frac{1}{2})}$.

Under spatial rotations alone this degenerates into $D^{1/2} \oplus D^{1/2}$ and is a unique spin representation, invariant under parity transformation.

After the success of the spin ½ wave equation it was Dirac[3] himself who first suggested the way to formulate a wave equation to describe a particle of arbitrary spin s. Dirac, and Fierz and Pauli[4, 5] extended the way of arriving at eqs. (1.5) and (1.6) to the general case of spinor $\varphi^{a\, a_1 \ldots a_\ell}_{\dot{b}_1 \dot{b}_2 \ldots \dot{b}_k}$ transforming as $D^{(j_1, j_1')}$ with $j_1 = \frac{1}{2}(l+1)$, $j_1' = \frac{1}{2}k$. This spinor does not correspond to a unique spin since under the rotation sub-group

$$D^{(j_1, j_1')} \approx D^{(j_1+j_1')} \oplus D^{(j_1+j_1'-1)} \oplus \cdots \oplus D^{|j_1-j_1'|} \tag{1.8}$$

Introducing a second spinor $\chi^{a_1 \ldots a_\ell}_{\dot{b}\, \dot{b}_1 \ldots \dot{b}_k}$, as a natural generalization of the spin ½ case, the wave equation for arbitrary spin may be written in the form

$$\partial_{a\dot{b}}\, \varphi^{a\, a_1 \ldots a_\ell}_{\dot{b}_1 \ldots \dot{b}_k} = i m\, \chi^{a_1 \ldots a_\ell}_{\dot{b}\, \dot{b}_1 \ldots \dot{b}_k} \tag{1.9a}$$

$$\partial^{a\dot{b}}\, \chi^{a_1 \ldots a_\ell}_{\dot{b}\, \dot{b}_1 \ldots \dot{b}_k} = i m\, \varphi^{a\, a_1 \ldots a_\ell}_{\dot{b}\, \dot{b}_1 \ldots \dot{b}_k} \tag{1.9b}$$

The spinors φ and χ satisfy the Klein- Gordon equation. Supplementary conditions needed to pick the highest spin from (1.8) are provided by eqs. (1.9a, b). The spin of the particle is given by

$$s = \tfrac{1}{2}(k + l + 1) \qquad (1.10)$$

Fermions of spin $s = k + \frac{1}{2}$ may be described by the Rarita-Schwinger[6] theory. Define

$$\varphi^{a}_{\mu_1 \ldots \mu_k} = \prod_{j=1}^{k} \sigma_{\mu_j;}{}^{\dot b_j}{}_{a_j}\, \varphi^{a a_1 \ldots a_k}_{\dot b_1 \ldots \dot b_k} \qquad (1.11a)$$

$$\chi_{\dot b \mu_1 \ldots \mu_k} = \prod_{j=1}^{k} \sigma_{\mu_j;}{}^{\dot b_j}{}_{a_j}\, \chi^{a_1 \ldots a_k}_{\dot b \dot b_1 \ldots \dot b_k} \qquad (1.11b)$$

Then the tensor-spinor

$$\psi_{\mu_1 \ldots \mu_k} = \begin{pmatrix} \varphi^{1}_{\mu_1 \ldots \mu_k} \\ \varphi^{2}_{\mu_1 \ldots \mu_k} \\ \chi_{\dot 1 \mu_1 \ldots \mu_k} \\ \chi_{\dot 2 \mu_1 \ldots \mu_k} \end{pmatrix}$$

is symmetric in $\mu_1 \ldots \mu_k$ and satisfies the equations

$$(\gamma \cdot \partial + m)\, \psi_{\mu_1 \ldots \mu_k} = 0 \qquad (1.12)$$

$$\partial_{\mu_i} \psi_{\mu_1 \cdots \mu_i \cdots \mu_k} = 0 \qquad (1.13)$$

For any i, and as a consequence of these equations there also follows the further subsidiary conditions

$$\gamma_{\mu_i} \psi_{\mu_1 \cdots \mu_i \cdots \mu_k} = 0 \qquad (1.14)$$

for any i.

Similarly bosons of spin s are described by a symmetric tensor

$$\varphi_{\mu_1 \cdots \mu_i \cdots \mu_s} = \varphi_{\mu_i \cdots \mu_1 \cdots \mu_s} \qquad (1.15)$$

obeying the equations

$$(\Box - m^2) \varphi_{\mu_1 \cdots \mu_i \cdots \mu_s} = 0 \qquad (1.16)$$

$$\partial_{\mu_i} \varphi_{\mu_1 \cdots \mu_i \cdots \mu_s} = 0 \qquad (1.17)$$

for any i.

1.2 Matrix algebraic equations

A first-order wave equation

$$(\beta \cdot \partial + m) \psi_{DKP} = 0 \qquad (1.18)$$

similar in structure to the Dirac equation (1.7) but with the matrices obeying the algebra

$$\beta_\mu \beta_\nu \beta_\lambda + \beta_\lambda \beta_\mu \beta_\mu = \beta_\mu \delta_{\nu\lambda} + \beta_\lambda \delta_{\nu\mu} \qquad (1.19)$$

describing particles of both spin 0 and spin 1 was derived by Duffin[7], Kemmer[8] and Petiau[9] in the late 1930's. Bhabha[10,11], later showed that a general first-order system of relativistic wave equations can be set up which includes the Dirac and Duffin-Kemmer-Petiau equations as special cases. In Bhabha's theory it is assumed that all information pertaining to a physical system is contained in a first-order equation

$$(\beta_\mu \partial_\mu + \chi) \psi(x) = 0 \qquad (1.20)$$

where χ is a mass term, and no further subsidiary conditions, as for the Dirac-Fierz-Pauli equations, are required. It is the algebra of the β-matrices that is the principal object of study in this approach. Requiring invariance of (1.20) under a Lorentz transformation Λ leads to

$$\psi' = \Lambda \psi \, , \quad \Lambda^{-1} \beta_\mu \Lambda = \alpha_{\mu\nu} \beta_\nu \qquad (1.21)$$

For an infinitesimal Lorentz transformation $\alpha_{\mu\nu} =$

$\delta_{\mu\nu} + \epsilon_{\mu\nu} \, , \quad \Lambda = \frac{1}{2} \epsilon_{\mu\nu} I_{\mu\nu}$ with $I_{\mu\nu} = - I_{\nu\mu}$

one obtains the invariance condition in the form

$$[\, I_{\mu\nu} \,, \beta_\lambda \,] = \delta_{\nu\lambda} \beta_\mu - \delta_{\mu\lambda} \beta_\nu \qquad (1.22)$$

These relations, however, do not determine the algebra of the β-matrices completely, and in Bhabha's theory it is postulated further that

$$I_{\mu\nu} = -[\, \beta_\mu \,, \beta_\nu \,] \qquad (1.23)$$

It can be shown that the self-adjoint operators $I_{ab} = -I_{ba}$, a, b, = 1, 2, 3, 4, 5 with the components

$$I_{\mu 5} = -I_{5\mu} = \beta_\mu \,, \qquad I_{55} = 0 \qquad (1.24)$$

satisfy the commutation relations characterising the SO(5) group. Each inequivalent irreducible representation of the Lie algebra of SO(5) determines an inequivalent irreducible representation of the β-algebra. Each irreducible representation is specified by two numbers S and s both integers or half-integers $S \geq s \geq 0$. For spin $\frac{1}{2}$ and spin 1 the algebra generated are the Dirac and DKP algebras respectively. But for $S > 1$ the algebra is reducible and contains representations of all lower spins. Further for $S > 1$ the Bhabha fields do not satisfy the single mass Klein-Gordon equation but actually satisfies

$$\partial\cdot\beta\left[\Box-\varkappa^2\right]\left[\Box-\frac{\varkappa^2}{4}\right]\cdots\left[\Box-\frac{\varkappa^2}{(\mathcal{S}-1)^2}\right]$$

$$\times\left[\Box-\frac{\varkappa^2}{\mathcal{S}^2}\right]\psi \quad = 0 \tag{1.25}$$

for integer $\mathcal{S}$ and

$$\left[\Box-4\varkappa^2\right]\left[\Box-\frac{4}{9}\varkappa^2\right]\cdots\left[\Box-\frac{\varkappa^2}{(\mathcal{S}-1)^2}\right]$$

$$\times\left[\Box-\frac{\varkappa^2}{\mathcal{S}^2}\right]\psi \quad = 0 \tag{1.26}$$

for half-integer $\mathcal{S}$. Thus, in general, the Bhabha theory yields a multi-mass, multi-spin system. For a given spin itself there may be different mass states. This feature of the Bhabha equations was part of the reason for their being neglected until in recent times the need to formulate a pathology-free theory stimulated a revival of interest in them[12-18].

A modification of the Bhabha equations was suggested by Harish-Chandra[19-21] where, instead of the SO(5) algebra the β_μ's obey the condition

$$(\beta_\mu)^{2\mathcal{S}+1} \quad = \quad (\beta_\mu)^{2\mathcal{S}-1} \tag{1.27}$$

The resulting equations describe particles with unique masses, but Harish-Chandra found that for higher spins there are no finite algebras satisfying eq.(1.27). On this account, Harish-Chandra concluded that there may not exist fundamental particles of higher spin in nature.

1.3 Algebraic inconsistencies and Lagrangian formulation

The first instance of an inconsistency showing up in an interacting higher spin theory was noted by Fierz and Pauli[5] not long after Dirac suggested equations of the form (1.9) for describing particles of arbitrary spin. They observed that for higher spins eq. (1.9) leads to algebraically inconsistent results when the effect of an electromagnetic field is taken into account through the minimal substitution $\partial_\mu \to \pi_\mu = \partial_\mu - ie A_\mu$. This comes about because the π_μ considered as operators no longer commute with each other but satisfies

$$[\pi_\mu , \pi_\nu] = ie F_{\mu\nu} \tag{1.28}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor. For instance, eqs. (1.12) and (1.13) for spin 3/2 with minimal electromagnetic interaction introduced read

$$(\gamma \cdot \pi + m)\psi_\mu = 0 \tag{1.29}$$

$$\pi_\mu \psi_\mu = 0 \tag{1.30}$$

and the subsidiary condition is no longer consistent with eq. (1.29), as a consequence of eq. (1.28). The same thing can happen in integer spin fields with spin $s \geq 1$.

Fierz and Pauli suggested that the most expedient way of resolving this difficulty is to set up a Lagrangian formulation wherein the equations of motion as well as subsidiary condition follow from the variation of the same Lagrangian. If the interactions are then introduced via the Lagrangian the resulting equations of motion and the subsidiary conditions will retain their mutual compatability.

In general for higher spins the Lagrangian formulation requires the introduction of some auxiliary variables or fields. The Lagrangians for spin 3/2 and spin 2 fields given by Fierz and Pauli contain terms depending on such auxiliary fields and the vanishing of these functions as a result of the variation is ensured by a proper choice of certain numerical coefficients. Lagrangian formulations along these lines were developed for higher spins by Fronsdal[22] and Chang[23] for spins upto 4, and of late, these have been extended to fields of arbitrary spin by Singh and Hagen[24,25]. The Chang Lagrangian for a spin 3/2 field with an auxiliary spinor Φ is given by

$$\mathcal{L} = \bar{\psi}_\mu (\gamma\cdot\partial + m)\psi_\mu - m\,\bar{\psi}_\mu \partial_\mu \Phi - \bar{\Phi}\partial_\mu \psi_\mu - \frac{3}{2}\bar{\Phi}(\gamma\cdot\partial + 2m)\Phi \qquad (1.31)$$

This is equivalent to the Rarita-Schwinger Lagrangian if $\psi_\mu \rightarrow \psi_\mu + \gamma_\mu \Phi$ and coincides with the Lagrangian with an arbitrary parameter discovered by Moldauer and Case[26] if $\psi_\mu \rightarrow \psi_\mu + a\gamma_\mu \Phi$. Similarly the Fierz-Pauli Lagrangian for spin 2 field with an auxiliary scalar field is equivalent to the Lagrangian obtained by Nath[27]. Bhabha equations discussed previously may also be treated from a Lagrangian point of view.

1.4 Free Field quantization

Once a Lagrangian formulation is in hand canonical quantization of the field can be carried out. But for fields of spin s 1 this procedure becomes highly cumbersome due to the presence of subsidiary conditions which make not all components of the field canonically independent. Therefore, before attempting quantization, the canonically independent fields and momenta must be separated. Schwinger's[28] action principle approach wherein quantization follows simply by identifying the generator corresponding to the boundary variation with the infinitesimal generators of unitary transformation is highly rewarding in this context and has been applied by Moldauer and Case[26], Chang[23,29] and Singh and Hagen[24,25] to quantize higher spin fields.

An alternative, non-Lagrangian, approach was pioneered by Umezawa and Takahashi[30,31]. A generalization of the Yang-Feldman[32] method, this can be applied to the quantization of a field of arbitrary spin if the field equations are of the form

$$\Lambda_{\alpha\beta}(\partial)\,\varphi_\alpha(x) \quad = 0 \qquad (1.32)$$

provided the differential operator Λ satisfies certain specified conditions.

1.5 Pathologies in interacting higher spin theories

With the initial difficulties overcome without too much effort the field theory of higher spin particles remained a mere curiosity until experiments began to reveal the existence of resonances of progressively higher spins. Efforts to understand the dynamics of higher spin particles within the framework of field theory led to the shocking realisation that the theory was full of contradictions. Of these anomalies in higher spin field theory, the earliest one to be noted was, what has come to be known as the Johnson-Sudarshan[33] effect.

a) The Johnson-Sudarshan effect

While the quantisation of free fields of higher spin poses no problem, Johnson and Sudarshan in 1961 showed that a consistent quantisation of an interacting half-integer spin field of spin $s > \frac{1}{2}$ is imperiled by the presence of constraint relations. They proved a general theorem which asserts that for such fields the possibility of consistent quantisation requires that the equal-time anti-commutator must be functions of the other fields to which the field in question is coupled. This result may be illustrated with the example of the Rarita-Schwinger field ψ_μ in interaction with an external electromagnetic field with the Lagrangian density

$$\mathcal{L} = [\, \bar{\psi}_\mu (\pi_\sigma \gamma_\sigma + m)\psi_\mu - \bar{\psi}_\mu (\pi_\nu \gamma_\mu + \pi_\mu \gamma_\nu)\psi_\nu$$

$$+ \bar{\psi}_\mu \gamma_\mu (\pi_\rho \gamma_\rho - m)\, \gamma_\nu \psi_\nu \,] \qquad (1.33)$$

where $\pi_\mu = \partial_\mu - ie A_\mu$. The field equation obtained from eq. (1.33) is

$$(\pi_\rho \gamma_\sigma + m)\psi_\mu - (\pi_\nu \gamma_\mu + \pi_\mu \gamma_\nu)\,\psi_\nu$$

$$+ \gamma_\mu (\pi_\rho \gamma_\rho - m)\, \gamma_\nu \psi_\nu = 0 \qquad (1.34)$$

Eq.(1.34) does not prescribe the second time-derivative of all the field components and the fourth component of eq. (1.34) yields the primary constraint

$$-\pi_i \psi_i + \pi_i \gamma_i \gamma_j \psi_j - m \gamma_i \psi_i = 0 \qquad (1.35)$$

Defining that part of ψ_k which transforms as a spin 3/2 object under spatial rotations by

$$\psi_k^{3/2} = (\delta_{k\ell} - \tfrac{1}{3}\gamma_k \gamma_\ell)\,\psi_\ell \qquad (1.36)$$

the constraint relation (1.35) can be written as

$$(m - 2/3\, \pi_i \gamma_i)\, \gamma_k \psi_k + \pi_k \psi_k^{3/2} = 0 \qquad (1.37)$$

In Schwinger's action principle approach the equal-time commutation relations of the theory are determined by

$$[\, \chi(x)\, ,\, G\,] = i\, \delta \chi(x) \qquad (1.38)$$

where the generator G is obtained fromthe time-derivative terms in $\mathcal{L}$, and has the fgem

$$G = i \int d^3x\, [\, \dot{\psi}_k^{*} (\delta_{k\ell} - 1/3\, \gamma_k \gamma_\ell)\, \delta\psi_\ell + 2/3\, \psi_k^{*} \gamma_k \times \delta(\gamma_\ell \psi_\ell)\,] \qquad (1.39)$$

The equal-time commutators of $\psi_i^{3/2}$ following from eqs.(1.39), (1.38) and (1.37) are

$$\{ \psi_k^{3/2}(x)\, ,\, \psi_\ell^{3/2}(x') \}_{x_0 = x_0'} = 0 \qquad (1.40)$$

$$\{ \psi_k^{3/2}(x)\, ,\, \psi_k^{3/2 *}(x') \}_{x_0 = x_0'} = (\delta_{km} - 1/3\, \gamma_k \gamma_m)\, [\, \delta_{mn} - \pi_m \frac{2/3}{m^2 - 2/3\, e\vec{\sigma}\cdot\vec{H}}\, \pi_n\,]\, (\delta_{n\ell} - 1/3\, \gamma_n \gamma_\ell)\, \delta(\vec{x} - \vec{x}') \qquad (1.41)$$

where $\vec{H}$ is the magnetic field. The consistency of the quantization depends on the positive-definitêness of the expression on the rhs ofs eq.(1.41) which, obviously, is determined by the strength of the magnetic field. It will be positive only if $2/3\,|eH| < m^2$. If the quantization is carried out in a chosen frame of reference the anti-commutator (1.41) holds with the strength of the magnetic field H as measured in that frame. But for a non-vanishing external field a frame can always be found where $2/3\,|e\,H| > m^2$ and accordingly the commutation relations would develop an inconsistency.

Johnson and Sudarshan further demonstrated that the Bhabha-Gupta[34,35] equation describing a particle with two mass states and spin 3/2 and spin ½ also suffers from the type of inconsistency described above. Later work brought to light further examples of similar pathological behaviour in higher spin theory. For a spin 2 field coupled to an external electromagnetic field this was demonstrated by Nath[27], for a spin 3/2 field interacting with a scalar field by Hagen[36], and for the interaction of a spin 3/2 particle with a pion and nucleon by Nath, Etemadi and Kimel[37].

A question as to the genuineness of the above type of pathology and whether it is a consequence of an improper method of quantization was raised first by Gupta and Repko[38]. They noted that the simplest choice of canonical variables corresponding to the commutation relations (1.41) failed to satisfy the Heisenberg equation of motion in the fully quantized theory. To save the situation they introduced a transformation in the canonical variables and on the assumption that this would result in a modification of the Johnson-Sudarshan anticommutator, they conjectured that the Johnson-Sudarshan inconsistency might be a spurious one. But, later, Kimel and Nath[39], using a generalized Yang-Feldman approach demonstrated the need for the canonical variable transformation of Gupta and Repko but they also showed that the Johnson-Sudarshan anticommutator is invariant under this transformation. The equivalence of the commutation relations obtained from the action principle approach was also established for the case of a spin 3/2 field interacting with a scalar and Dirac field[40,41]. These equivalence proofs were based on perturbative techniques and in the works already referred to, the assertion was proved only upto second order in the coupling constant. Soo[42] extended the Nath-Kimel method to fourth order, and proved the equivalence of the commutator obtained from the action principle and Yang-Feldman approach to this order. Following this Gluck, Hays and Kimel[43] demonstrated, with the simple example of a spin 0 - spin ½ system manifesting the pathologies of higher spin fields, the equivalence of Yang-Feldman and action principle quantisation without invoking perturbation theory. They conjectured that the same may be true of other pathological theories as well.

The suspicion that the quantization leading to the commutation relation (1.41) may be noninvariant was finally settled by Mainland and Sudarshan[44] by directly verifying that the Heisenberg equation is satisfied. Taking the explicit dependence of the external field on space and time coordinates they proved that the Heisenberg equation is satisfied in the general form

$$\dot{\upsilon} = d_0 \upsilon = -i [\upsilon, H] + \partial_0 \upsilon \qquad (1.42)$$

where d_0 denotes the total time derivative and ∂_0 the partial derivative. Subsequently, by a clarification of the meaning of Poincaré invariance when external fields are present and by an explicit calculation of the commutator of the spin 3/2 field with the generators of translations and homogeneous Lorentz transformation, Mainland and Sudarshan showed that the fields transform covariantly[45]. In the case of a spin 3/2 field interacting with a scalar field a similar conclusion was reached by Hagen[46]. Thus it becomes abundantly clear that the appearance of an indefiniteness in the anticommutator must be the result of a more fundamental flaw in the theory and not the result of non-covariant quantization.

(b) Acausal propagation

A landmark in the study of pathological higher spin field theory is the work of Velo and Zwanziger[46,47] who demonstrated that serious difficulties are present in the interacting higher spin fields when considered at the basic c-number level itself. Applying the method of characteristics[48] as the basic tool, they showed that interacting higher spin field equations interpreted as classical wave equations possess solutions that either do not propagate or propagate with a velocity exceeding that of light.

To appreciate the nature of the difficulties involved it may be profitable to start with a look at the mathematical description of wave propagation[48]. Non-dispersive wave propagation is associated with hyperbolic systems of partial differential equations. For such equations an initial value problem may be posed on a class of surfaces[5], called "space-like" with respect to the equation, and they possess solutions with wavefronts that travel along rays at finite velocities. The wave-fronts are, essentially, the characteristic surfaces of the system of equations. Characteristic surfaces are those space-like surfaces for which the initial value problem has no unique solution. Alternatively, they may be characterized as surfaces across which discontinuities can occur in the highest order derivatives. These characteristic surfaces are determined by the coefficients of the highest order derivatives present. The rays through any point form a ray-cone which again is determined by the coefficients of the higher order derivatives. In a hyperbolic system if coupling is introduced through lower order derivative the ray cone will remain unaffected, that is, the nature of the wave propagation will be the same in the free and in the interacting case. This is what happens in the examples of the Klein-Gordon and Dirac equations (both hyperbolic system) when they are coupled, as is usual, non-derivatively or through lower derivatives. In either case the ray-cone remains the light-cone.

On the other hand, in the description of higher spin fields with redundant components the Euler-Lagrangian equations will not form a hyperbolic system. Because of constraint conditions they will only constitute a degenerate system. However, in the free field case, they may be shown to be equivalent to a hyperbolic system describing wave propagation together with a supplementary set of constraint equations which are preserved in time. But there now arises the possibility that if in higher spin Lagrangians interactions are introduced through non-derivative or lower order derivative terms the resulting Euler-Lagrange equations might no longer be equivalent to a hyperbolic system with the light cone as the ray cone. Velo and Zwanziger[46,47] found several instances where the wave-front propagates across the light cone. Causality violation, in this sense, was first demonstrated with the example of a spin 3/2 particle in an external electromagnetic field and was immediately extended to several interactions of a spin 1 field.

The Velo-Zwanziger result may be illustrated by the example of a charged vector particle coupled to an external symmetric tensor potential through the Lagrangian

$$\mathcal{L} = -\frac{1}{2} G^*_{\mu\nu} G_{\mu\nu} - m^2 \varphi^*_\mu \varphi_\mu - \lambda \varphi^*_\mu T_{\mu\nu} \varphi_\nu \qquad (1.43)$$

where $G_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu$. The Euler-Lagrange equation that follows is

$$\partial_\mu (\partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu) - m^2 \varphi_\nu - \lambda T_{\nu\mu} \varphi_\mu = 0 \qquad (1.44)$$

and its fourth component constitutes the primary constraint

$$\nabla^2 \varphi_4 - \partial_i \partial_4 \varphi_i - m^2 \varphi_4 - \lambda T_{4\nu} \varphi_\nu = 0 \qquad (1.45)$$

The secondary constraint is obtained by taking the divergence of eq.(1.44)

$$m^2 \partial_\mu \varphi_\mu + \lambda \partial_\mu T_{\mu\nu} \varphi_\nu = 0 \qquad (1.46)$$

The true equation of motion that results when eq.(1.46) is substituted back in eq.(1.44) is

$$\begin{aligned} \partial_\mu \partial_\mu \varphi_\nu - \lambda m^{-2} \partial_\nu \partial_\mu (T_{\mu\lambda} \varphi_\lambda) - m^2 \varphi_\nu & \\ - \lambda T_{\mu\nu} \varphi_\mu = 0 & \qquad (1.47) \end{aligned}$$

The eq.(1.47) prescribes the second time derivatives of all the components and is not degenerate. From this it is apparent that every solution of eq.(1.44) satisfies the constraints (1.45) and (1.46) and the equation (1.47). Conversely it may be verified that every solution of eq.(1.47) satisfies eq.(1.44) and that if the constraints are satisfied at a particular time they will be satisfied always.

To analyse the nature of field propagation, Velo and Zwanziger made use of the fact[48] that the velocity of propagation is determined by the normals to the characteristic surfaces. These are found by replacing $\partial_\mu \to n_\mu$ in the highest order derivative terms and evaluating the determinant D(n) of the resulting coefficient matrix. In the present case

$$D(n) \qquad = \left| n^2 \delta_{\mu\nu} + \lambda m^{-2} n_\mu (n \cdot T)_\nu \right| \qquad (1.48)$$

The normals to the characteristic surfaces are determined by equating this determinant to zero

$$D(n) \qquad = (n^2)^3 (n^2 + \lambda m^{-2} n \cdot T \cdot n) = 0 \qquad (1.49)$$

Assuming for simplicity that only the T_{00} component of $T_{\mu\nu}$ is non-vanishing, one obtains

$$n_0^2 = \frac{1}{1 - \lambda m^{-2} T_{00}} \vec{n}^2 \qquad (1.50)$$

If $\lambda m^{-2} T_{00} > 1$, n_0 becomes imaginary and the system ceases to be hyperbolic. If $0 < \lambda m^{-2} T_{00} < 1$, the equations are hyperbolic, but the normals are time-like and hence the characteristic surfaces are space-like and the propagation becomes acausal. One can further show that the acausal ray is not eliminated by the constraints.

Following the surprising discovery of Velo and Zwanziger, extensive investigations along these lines by later workers brought to light many more instances of external couplings as well as mutual interactions of spin 1[49-54], spin 3/2[55-57] and spin 2[58] theories suffering from the affliction of acausal propagation. Madore and Tait[59] developed an alternate method to determine the nature of field propagation. This shock wave formalism, which is equivalent to the method of characteristics, is based on the fact that the characteristic surface is the surface of discontinuity in the highest order derivatives of field functions.

Acausality of propagation manifests itself in the quantised theory through the nonvanishing of the field commutator for space-like separations. That this was indeed the case was directly verified by Hortacsu[60] for a spin 3/2 particle in an electromagnetic field in a 1 + 1 dimensional model. Schroer, Seiler and Swieca[61] had earlier noted, following Capri[62], that the quantum problem in external potential with the source linear in the field is reducible to a discussion of the corresponding classical problem.

The question whether the method of characteristics is sufficient to detect the violation of causality was raised by Mathews and Seetharaman[63]. They observed that while for a charged spin 1 particle with an anomalous moment coupling to an external electromagnetic field the propagation is causal according to the method of characteristics, it has a tachyonic mode with the velocity of light as the minimum velocity. Hence they argued that the method of characteristics gives only a necessary condition but not a sufficient one for causal propagation. But Hagen[64] has pointed out that the tachyonic mode in the spin 1 field occurs only in the case of an external electromagnetic field and the mutually interacting theory of a spin 1 particle with an anomalous magnetic moment with an electromagnetic field at the quantized level is fully causal.

e) Constraint breakdown

Velo and Zwanziger[47] in studying causality violation in higher spin field theories came across another, and much worse form of pathology in the minimal electromagnetic coupling of a massive spin 2 field. They observed that when the minimal substitution $\partial_\mu \rightarrow \Pi_\mu = \partial_\mu - ieA_\mu$ is introduced into the Lagrangian

$$\mathcal{L} = -\partial_\lambda \psi^*_{\mu\nu}\, \partial_\lambda \psi_{\mu\nu} + 2\, \partial_\lambda \psi^*_{\mu\nu}\, \partial_\mu \psi_{\lambda\nu}$$
$$- \partial_\mu \psi^*_{\mu\nu}\, \partial_\nu \psi - \partial_\mu \psi^* \partial_\lambda \psi_{\mu\lambda} - \partial_\mu \psi^* \partial_\mu \psi$$
$$- m^2 (\psi^*_{\mu\nu} \psi_{\mu\nu} - \psi^* \psi) \tag{1.51}$$

where $\psi_{\mu\nu}$ is a symmetric tensor and $\psi = \psi_{\mu\mu}$, two of the constraints deduced from the Euler-Lagrange equation following from eq.(1.51) turn into equations of motion and the field acquires 6 independent degrees of freedom instead of the required 5. This is a strange phenomenon to which attention had been drawn by Federbush[65] a few years earlier. Federbush made use of a first-order formalism using a 50-component wave function (a symmetric tensor $\psi_{\mu\nu}$ and a third rank tensor $\Gamma_{\mu\nu\lambda} = \Gamma_{\nu\mu\lambda}$) to exhibit the inconsistency, and had also found that the introduction of an additional term (now called the "Federbush term") corresponding to a direct interaction with the electromagnetic field is the only way out of this impasse. Hagen[66], later showed that the Federbush equations when formulated in a second order form are identical to the Dirac-Fierz-Pauli equations for spin 2 field. That the Chang theory for a 30-component wave function with minimal coupling is also equivalent to the Federbush theory was demonstrated explicitly by Hagen. The one parameter family of Lagrangians obtained by Nath[27] and Tait[67] as generalizations of the Dirac-Fierz-Pauli Lagrangian preserve the proper number of constraints for minimal electromagnetic coupling. The conflict between minimality and the degrees of freedom has been traced to the ambiguity of derivative ordering when second derivatives are present[68].

Apart from spin-2 theory other instances for the occurrence of the above discussed pathology are provided by certain simple forms of interaction between a spin 3/2 particle and a nucleon and pion[36,37]. There are spin 1 theories also sharing this difficulty[69].

The relevance of the phenomenon of constraint breakdown in relation to the question of relativistic invariance will be discussed below.

(d) Noncovariance

While there was an initial doubt that the Lorentz covariance of a theory might be endangered by the orrurrence of the Johnson-Sudarshan inconsistency[33], Mainland and Sudarshan[44,45] later explicitly demonstrated the proper covariance of a spin 3/2 field with minimal electromagnetic coupling. However, another kind of covariance problem afflicting higher spin theories has been highlighted by Jenkins[70,71]. This has to do with the covariance property of the S-matrix in the interaction picture as given by

$$S = T \exp\left(-\int_{-\infty}^{+\infty} \mathcal{H}_I(x)\, d^4x\right) \tag{1.52}$$

where $\mathcal{H}_I(x)$ is the interaction Hamiltonian in the interaction picture. Now, the time-ordered product of two operators may, in general, contain noncovariant terms. In theories where the Mathews rule[72] holds the effect of such noncovariant terms gets cancelled with the effect of other noncovariant terms present in the interaction Hamiltonian in every order of perturbation theory. With the example of an interacting massive vector field Jenkins proved that the Mathews rule is violated and the S-matrix is noncovariant when the propagation of the field is acausal.

Another line of connection between the nature of field propagation and the covariance property was also observed by Jenkins[69] who considered simple theories of interacting massive spin 1 and spin 3/2 fields, and established that if there are acausal modes of propagation, it will be accompanied by a breakdown of Lorentz covariance. This stems from the impossibility of determining the independent field components in all frames simultaneously; constraints losses occur in a noncovariant fashion. That the same thing is true of a spin 2 particle in a homogeneous magnetic field was later noted by Mathews, Seetharaman and Prabhakaran[73].

(e) Appearance of complex energy eigenvalues

Another manifestation of the pathology-ridden nature of higher spin field theory was observed by Goldman, Tsai and Yildiz[74-77] who first realised that the spectrum of energy eigenvalues of a spin 1 particle with an anomalous moment coupling to an external homogeneous magnetic field may include complex values if the strength of the magnetic field exceeds a specified value. Exploiting the connection between the problem of a charged particle in a magnetic field and the harmonic oscillator, Mathews[78] developed a new method of obtaining the energy eigenvalues in the presence of a magnetic field which has the distinct advantage that it could easily be extended to spin 3/2 and spin 2 problems. Seetharaman, Prabhakaran and Mathews[79] studied the energy spectrum of a spin 3/2 particle in a homogeneous magnetic field and found that complex values appear when field strength exceeds a critical value $H = 3m^2/2e$ which is also the threshold for the onset of the Johnson-Sudarshan inconsistency. They further demonstrated that the norm of the Rarita-Schwinger wave function becomes indefinite in sign for magnetic field strengths beyond the critical value. Thus it becomes evident that the Johnson-Sudarshan effect has its roots in the troubles at the classical level itself, and that the emergence of complex eigenvalues in the energy spectrum is a symptom of these maladies. However, it appears that this type of difficulty is not directly related to the nature of field propagation and, as remarked in Sec. 1.5b there are examples where the field propagation is causal, in the sense of Velo and Zwanziger, but energy eigenvalues turn complex, depending on the external field. Occurrence of complex energy values in the case of a spin 2 particle in a homogeneous magnetic field was demonstrated by Prabhakaran, Seetharaman and Mathews[73].

(f) Instability

The instability phenomenon as described by Wightman[80] is another pitfall for higher spin theories. Wightman, in his endevour to understand whether the Johnson-Sudarshan inconsistency for a particle of higher spin in an external field is a general phenomenon or a consequence of some special assumptions, found that not all representations of SL(2, C) are equally suitable for describing higher spin fields. He demonstrated with examples that there are 'unstable' representations which cannot be embedded in a physically sensible theory where an external field is included. For such representations the ingoing and outgoing fields do not satisfy the same set of commutation relations and hence the physical interpretation of the theory fails.

In the examples so far studied by Wightman[81] it has strangely been the case that the stability of the representation and the causality of propagation in the sense of Velo and Zwanziger are mutually exclusive attributes. Thus the Dirac-Fierz-Pauli equations for a spin 3/2 particle is stable while the Hurley equations[82-84] for arbitrary spin which do not suffer from acausality are unstable.

1.6 Ways to avoid pathologies

While, over the years, more and more maladies afflicting higher spin theories have been discovered, and examples of these ailing theories have multiplied, there is, as yet, no definite understanding as to the failure of field theory to give a consistent description of higher spin particles and their interactions. However, in recent times attempts of varied sorts have been made to rid higher spin theory of its afflictions and to construct pathology-free formulations. These range from minor modifications in the interaction terms to alternate formulations without subsidiary conditions and the inclusion of gravitation and supersymmetry.

An example of the simple-minded approach of modifying the interaction term in an effort to get over a pathology was discussed in Sec.1.5e where it was mentioned that the inclusion of a Federbush term relieves the spin 2 theory of the difficulty of constraint loss. The work of

Shamaly and Capri[51] in regard to the causality of the Takahashi-Palmer[85] spin 1 field coupled to an electromagnetic field is another instance where this approach is successful. They showed that the acausal propagation of the minimally coupled Takahashi-Palmer field may be remedied by the inclusion of an anomalous moment coupling with a specified value of the anomalous moment. But this "counter term" approach, as Shamaly and Capri[51] observed, is of no avail in curing the acausal propagation of a spin 3/2 field in minimal interaction with an external electromagnetic field.

One of the earliest suggestions regarding a pathology-free formulation of higher spin theory was made by Velo[86] who by considering a multiplet of massive vector mesons in interaction established that the difficulties of constraint loss and acausal propagation may be avoided by choosing a Lagrangian of the Yang-Mills type and imposing a set of restrictions on the coupling constraints. Though the validity of Velo's conjecture remains yet to be investigated, it has become clear during recent times that a hopeful way of resolving the troubles may be through multi-mass, multi-spin theories.

A perusal of the inconsistencies of various kinds surveyed in sec. 1.5 reveals that their occurrence lies intimately tied with the existence of constraint relations.

A way out of the present confused situation may, hence, naturally be sought in theories that do not demand subsidiary conditions. Bhabha equations described in sec.1.2 belong to this category, and interest in Bhabha equations has been revived in the hope of obtaining a consistent theory of interactions of higher spin particles. The causality of propagation of higher spin Bhabha fields was studied by Baisya[87] and Nagpal[88,89] and by Krajcik and Nieto[15] in the course of their extensive investigations into the structure of Bhabha fields. The theory is causal at the c-number level, and causality at the q-number level can be assured if quantised with an indefinite metric. Apart from the multi-mass, multi-spin structure of the field, the introduction of negative probabilities is the real price that has to be paid to achieve consistency in the presence of interactions, and it appears that there is no way to eliminate the negative-normed states from the theory.

Another formulation of arbitrary spin theory that does not call for subsidiary conditions is due to Hurley[82-84]. This is based on the representation

$$[(s,0) \oplus (s-\tfrac{1}{2},\tfrac{1}{2})] \oplus [(0,s) \oplus (\tfrac{1}{2},s-\tfrac{1}{2})]$$

of SL(2,C) where s is any non-negative integer of half-odd integer. Equations of this type were considered a long time before[90-92], and it was realised that they describe parity doublets of particles of mass m and spin s. Of the members of the doublet one has all its states of positive norm and the other of negative norm. Hurley's method of eliminating the states of negative norm has the advantage that the introduction of minimal interaction with an external electromagnetic field does not destroy the causality of propagation of the field. However, as was noted in Sec.1.5f, Hurely's equations are unstable, in the sense of Wightman, and hence cannot be given a reasonable particle interpretation at the quantum level.

Another formulation of constraint-free equations for arbitrary spin fields which does not suffer from the trouble of acausality has been recently suggested by Hayward[93]. This theory, describing particles of unique mass and spin, starts from a definite representation of the inhomogeneous Lorentz group and the wave function for spin s has 8s components. The extraneous components get eliminated, not by means of subsidiary conditions but through the gauge freedom that is available in the theory. The introduction of an indefinite metric is essential to this formalism. Canonical quantization does not work for spin $s \geq 3/2$ and the absence of the Johnson-Sudarshan inconsistency and other troubles at the quantized level remain to be demonstrated.

For a first-order system of the Gelfand-Yaglom type[94], which include the Bhabha equations and the Hurley equations as special cases, Amar and Dozzio[95,96] derived a sufficient matrix condition that these equations retain causality under minimal coupling. They found that the origin of causality violation may be traced to the constraints inherent in the theory. Whether the causal theories thus identified can be successfully quantised and whether all other types of pathology are also absent in these theories are questions that remain to be investigated.

Another approach to a non-constraint theory for higher spin fields, discussed first by Munozek[97], was utilized by Fukuyama and Yamamoto[98] to formulate a consistent theory of interacting spin 3/2 field. Redundant components are eliminated, here, by the device of making the masses of the corresponding particles infinite. This is an extension of the ξ-limiting procedure proposed by Lee and Yang[128] for interacting spin 1 particles and the advantage offered by this formulation is that it avoids both the Johnson-Sudarshan problem and acausal propagation. However, an unpalatable feature of this approach is, apart from the infinite masses, that the causality of propagation depends critically on the way the limit is taken. Also, it may be noted that the introduction of a negative metric is crucial to this method.

There are a couple of examples of theories which have constraint conditions but are free from acausality. One of these is the Bhabha-Gupta equation[34,35] for a mixed spin 3/2 - spin ½ field. Prabhakaran, Seetharaman and Mathews,[99] found that while a proper choice of a parameter in the Bhabha-Gupta Lagrangian ensures causal field propagation for minimal coupling, the same choice renders the total charge of the free theory indefinite. That the total charge is indefinite in the causally propagating reducible theory proposed by Fisk and Tait[100] was also shown by these authors. They have further shown that the preservation of causality inthe Bhabha-Gupta field with minimal coupling is not a fortuitous result and that the propagation of this field remains causal when coupled to scalar fields or when a gravitational coupling is considered along with electromagnetic interaction[129,130]. It appears from these examples that only by starting with a reducible theory which will need an indefinite metric quantization, may causality of propagation be preserved in the presence of interactions. This in turn implies that if one is to allow the idea of fundamental higher spin fields, one will have to reconcile onself to an indefinite metric theory.

Recently, a clue as to the origin of higher spin pathologies was reported by Capri and Shamaly[101] who found that a free spin 3/2 field of the Dirac-Fierz-Pauli type as well as a spin 1 field with an electric quadrupole moment coupling to an external field exhibit a nonlocal structure. This is in contrast to the case of spin 0, ½ and 1 fields which do not have any non-locality even in the presence of minimal electromagnetic coupling and it led them to speculate that the trouble of acausal propagation in spin 3/2 and spin 1 fields may be due to their additional structure. However, Cox,[102] while demonstrating the acausality of the composite Harish-Chandra equation[103] for a spin 3/2 - spin ½ field has expressed doubt as to the conclusion of Capri and Shamaly. He points out that in the Harish-Chandra equation acausality can be traced to the constraint structure and not to the physical constituents of the composite field. Non-locality of the sort noted by Capri and Shamaly is attributable to the constraint relations themselves. Regarding higher spin particles as composites will in no way help to resolve the difficulties which plague the theories.

An entirely new possibility of arriving at a consistent theory of higher spin fields has been sought by the inclusion of gravitational fields along with other couplings. Madore[104] has shown that for a spin 3/2 field interacting with an external electromagnetic field causal propagation is possible if an additional coupling with a gravitational field is included, provided that only a "linear approximation" of the gravitational field is considered, and also that the charge e and mass m are related by the condition

$$m = \frac{e}{\sqrt{3G}}.$$

In view of these restrictions it is doubtful whether the above-mentioned result has any deep significance.

However, the recent development of supergravity[105] theories has raised a fresh hope of yielding a consistent, causal spin 3/2 description[106, 107]. The supersymmetry algebra introduced by Akulov and Volkov[108] and by Wess and Zumino[109] is a graded Lie algebra which has representations that act in the space of helicity states of two massless particles of adjacent spins s and $s-\frac{1}{2}$. The (2, 3/2) representation is identified with the supergravity multiplet. In the supergravity theory of Freedman and van Niewenhuizen[106] a massless spin 2 and a massless spin 3/2 field are coupled in a locally supersymmetric Lagrangian. The supersymmetric invariance of this theory is essentially a fermionic gauge invariance of the Rarita-Schwinger field noted by Rarita and Schwinger themselves. It is hoped that there exists a super-Higgs mechanism by which the spin 3/2 particle acquires mass.

Because the higher spin fields involved in supergravity theory are massless the acausality problem does not arise in this theory[110]. That this theory is also ghost-free, at least in the true-approximation, has been proved by Das and Freedman[111]. These authors have emphasised that some of these results are formal, in a sense, and it is not clear because of the curved nature of the space-time involved whether the fully interacting theory would remain causal. Freedman[112], recently, coupled the (2, 3/2) multiplet with the (1, ½) multiplet, and it is said to be first causal, ghost-free theory of an electromagnetic-like interaction of spin 3/2 field. However, a cosmological term occurs in this which leads to some difficulties, and no way is yet known how to handle this term.

Even if supersymmetry theories manage to overcome the present obstacles, and the inclusion of gravity becomes accepted as an essential ingredient in formulating higher spin theories one perplexing question, a question also raised by Krajcik and Nieto[18], will remain to be answered. What is wrong with conventional field theory and why does the simple Dirac equation work?

This thesis has been concerned with the pathologies that afflict the conventional formulations of higher spin fields. Some new abnormalities of a spin 1 field interacting with a scalar field or a Dirac field are reported here. The possibility of the breakdown of relativistic invariance even in theories that are free of acausal propagation is a significant result brought out by this study. A new approach towards the construction of pathology-free theory by means of a consistent Lagrange multiplier scheme is developed in this thesis and is applied to spin 3/2 and spin 2 problems.

CHAPTER II

PATHOLOGIES IN INTERACTING MASSIVE VECTOR AND SCALAR FIELDS

The interaction between a real (complex) vector field and a real (complex) scalar field is studied with reference to the questions of causality violation and loss of constraints. For certain couplings, at a particular value of the scalar field, the secondary constraint breaks down in a Lorentz-covariant fashion and the propagation can either be causal or acausal at this value. The corresponding quantised theories are, however, not Lorentz-covariant. Other couplings are constructed which are free of these pathologies.

2.1 Introduction

Pathologies abound in higher spin field theories and the inconsistencies of a massive spin 1 field have been discussed by a number of authors starting from Velo and Zwanziger[47, 69, 71]. A study is here made of the mutual interactions between a spin 1 field and a spin 0 field under several coupling schemes with reference to the possibility of pathologies of various sorts making their appearance. The method of characteristics is applied to analyze the nature of the field propagation and the possibility of constraint losses is studied. The Lorentz covariance of these theories both at the classical and at the fully quantised level is also examined. It is found that for the mutual interaction between a massive spin 1 field and a charged scalar field with the simplest form of derivative coupling the secondary constraint equation is lost at a particular value of the scalar field. The field propagation may or may not be causal at this distinguished value. Such a situation may be referred to as one of "uncertain causality". But the loss of constraints occurs in a Lorentz-covariant manner and hence does not lead to any noncovariance of the sort observed by Jenkins[71] at the classical level. The quantised theory, on the other hand, is shown to be not Lorentz-covariant in that the S-matrix in the interaction picture contains surface-dependent terms in violation of the Mathews rule. However, when the interaction Lagrangian is modified by the addition of a suitable term (which for the case of a complex scalar field, is also necessary to ensure current conservation) the resulting theory is found to be non-pathological and the corresponding quantized theory is seen to possess Lorentz-covariance. Similar results are also established for the interactions between neutral spin 1 and neutral scalar fields, complex spin 1 and neutral scalar fields and complex spin 1 and complex scalar fields.

2.2 Interaction between neutral spin 1 and complex scalar field

A neutral spin 1 field may be described by the vector formalism with the Lagrangian

$$\mathcal{L}_V = -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right) - \frac{m^2}{2} A_\mu A_\mu \tag{2.1}$$

The Lagrangian for the charged scalar field is

$$\mathcal{L}_S = -\partial_\mu \varphi^* \partial_\mu \varphi - \mu^2 \varphi^* \varphi \tag{2.2}$$

The simplest parity-invariant coupling of the neutral vector field to a complex scalar field is of the form

$$\mathcal{L}_I = ig\,[\,\partial_\mu\varphi^*\cdot\varphi - \varphi^*\partial_\mu\varphi\,]\,A_\mu \tag{2.3}$$

The equations of motion that follow from the Lagrangian

$$\mathcal{L} = \mathcal{L}_V + \mathcal{L}_S + \mathcal{L}_I \tag{2.4}$$

are

$$(\Box - m^2)A_\mu - \partial_\mu\partial_\lambda A_\lambda = -ig\,(\partial_\mu\varphi^*\cdot\varphi - \varphi^*\partial_\mu\varphi) \tag{2.5a}$$

$$(\Box - \mu^2)\,\varphi = ig\,(2\,\partial_\lambda\varphi\cdot A_\lambda + \varphi\,\partial_\lambda A_\lambda) \tag{2.5b}$$

$$(\Box - \mu^2)\,\varphi^* = -ig\,(2\,\partial_\lambda\varphi^*\cdot A_\lambda + \varphi^*\,\partial_\lambda A_\lambda) \tag{2.5c}$$

Eq.(2.5a) is not a true equation of motion since it does not contain the second time derivative of A_4. The fourth component of eq.(2.5a)

$$\begin{aligned} L_4 \equiv\; & \nabla^2 A_4 - \partial_4\nabla\cdot\vec{A} - m^2 A_4 \\ & + ig\,(\partial_4\varphi^*\cdot\varphi - \varphi^*\partial_4\varphi) = 0 \end{aligned} \tag{2.6}$$

constitutes a primary constraint. A secondary constraint relation is obtained by taking the divergence of eq.(2.5a) and making use of eqs.(2.5b) and (2.5c).

$$\begin{aligned} \partial_\lambda A_\lambda = \; & 2g^2/m^2\,(\varphi^*\varphi\,\partial_\lambda A_\lambda + \partial_\lambda\varphi^*\cdot\varphi\,A_\lambda \\ & + \varphi^*\,\partial_\lambda\varphi\cdot A_\lambda) \end{aligned} \tag{2.7}$$

In principle eqs. (2.6) and (2.7) should be solvable for A_4, $\partial_4 A_4$ in terms of A_i and $\partial_\mu A_i$ in order that the vector field should have three components required for a spin 1 particle. However, in the present case, when the scalar field assumes a value for which $\varphi^*\varphi = m^2/2g^2$ the secondary constraint relation (2.7) breaks down, and the vector field acquires an additional degree of freedom.

When $2g^2/m^2\varphi^*\varphi \neq 1$, eq.(2.7) may be substituted in eq.(2.5a) to yield the constraint-free equation

$$(\Box - m^2) A_\nu - 2g^2/m^2 \left[\varphi^*\varphi \, \partial_\nu \partial_\lambda A_\lambda + \partial_\lambda A_\lambda \partial_\nu (\varphi^*\varphi) + \partial_\nu \{ A_\lambda \partial_\lambda (\varphi^*\varphi) \} \right] + ig\,(\partial_\nu \varphi^* \cdot \varphi - \varphi^* \partial_\nu \varphi) = 0 \qquad (2.8)$$

The new equation of motion (2.8) supplemented by the constraints (2.6) and (2.7), assumed valid only on an initial space-like surface x_0 = constant, is equivalent to the original equation (2.5a), it being possible to show that the constraint relations are preserved in time (For proof see Appendix I).

A Cauchy problem can now be set up for the system comprising eqs.(2.5a), (2.5c) and (2.8) with the initial data specified on a space-like surface x_0 = constant and satisfying the constraints (2.6) and (2.7). The characteristic determinant of this system is found by replacing $\partial_\mu \rightarrow n_\mu$ in the second order derivative terms and evaluating the determinant of the coefficients of A_μ, φ and φ^*. Choosing the special frame of reference in which $n_\mu = (0, 0, 0, in_0)$ and evaluating the 6 x 6 determinant and generalising to an arbitrary frame, one obtains

$$D(n) = (n^2)^6 \left[1 - \frac{2g^2}{m^2} \varphi^* \varphi \right] \qquad (2.9)$$

Normals to the characteristic surface are determined by setting $D(n) = 0$. The field propagation is light-like and hence causal unless the factor $(1 - \frac{2g^2}{m^2} \varphi^* \varphi)$ vanishes and it has been assumed in this derivation that this is not the case. If, however, this is permitted eq.(2.5) becomes a degenerate system and at the value $\varphi^* \varphi = \frac{m^2}{2g^2}$ field propagation becomes undetermined in the sense that normals can be space-like, light-like or time-like. It cannot be immediately concluded whether causality is violated or not at this value of $\varphi^* \varphi$. This has happened because it is precisely at this value of the scalar field that the secondary constraint breaks down.

Since the procedure adopted in setting up the Cauchy problem is not manifestly covariant, to check whether any Lorentz noncovariance has crept in, the Cauchy problem may be reformulated by specifying the initial data not on a constant-time hyperplane, but on an arbitrary space-like surface σ with normal n_μ. On such a surface the constraint relations (2.6) and (2.7) become

$$n_\mu L_\mu = 0 \tag{2.10}$$

$$(1 - \frac{2g^2}{m^2} \varphi^* \varphi)\, n_\lambda \partial_\lambda n_\rho A_\rho = f(n_\lambda A_\lambda, A_{t\lambda}, \partial_\rho A_{t\lambda}, \varphi, \varphi^*) \tag{2.11}$$

where

$$A_{t\mu} = A_\mu - n_\mu n_\lambda A_\lambda \tag{2.12}$$

[The detailed form of the rhs of eq.(2.11) is not important for the present discussion]. Eqs.(2.10) and (2.11) are solvable for $n_\mu A_\mu$ and $n_\lambda \partial_\lambda n_\rho A_\rho$ in terms of $A_{t\mu}, \partial_\mu A_{t\nu}$, φ and φ^* and their derivatives unless the factor $(1 - 2g^2/m^2 \varphi^* \varphi)$ vanishes. Since this factor is independent of the normals, the constraint breaks down when it occurs, is a frame-independent phenomenon and hence the present theory does not exhibit the type of Lorentz noncovariance at the classical level observed by Jenkins[69].

Despite this fact it will now be shown that the corresponding quantized theory is not Lorentz-covariant.

To deal with the second quantized version the interaction Hamiltonian corresponding to the Lagrangian (2.3) is calculated first. In the Heisenberg picture it has the form

$$\mathcal{H}_I = -\mathcal{L}_I - g^2/2m^2 (\dot{\varphi}^* \varphi - \varphi^* \dot{\varphi})^2 - g^2 \varphi^* \varphi A_0^2 \qquad (2.13)$$

To develop a perturbation expansion for the S-matrix it is advantageous to go from the Heisenberg picture over to the interaction picture. However, the appearance of time-derivatives of field operators in the interaction Hamiltonian renders the transformation non-trivial. Making use of the general formula given, for example, by Nishijima[113]

$$\mathcal{H}_I^{int} = -\mathcal{L}_I^{int} + \frac{1}{2} \sum_\alpha \left(\frac{\partial \mathcal{L}_I}{\partial \dot{\varphi}_\alpha} \right)^2 \qquad (2.14)$$

the interaction Hamiltonian in the interaction picture is found to be

$$\mathcal{H}_I = -\mathcal{L}_I - g^2/2m^2 (\dot{\varphi}^* \varphi - \varphi^* \dot{\varphi} + 2g \varphi^* \varphi A_0)^2 + g^2 \varphi^* \varphi A_0^2 \qquad (2.15)$$

where all the operators are understood to be in the interaction picture. The Hamiltonian contains normal-dependent terms and is not covariant. When the S-matrix elements are evaluated using the Dyson formula (1.52), Wick decomposition of the time-ordered products gives rise to additional normal-dependent terms arising from the contractions

$$\underbrace{\partial_\mu \varphi^*(x_1)\, \partial_\nu \varphi(x_2)}_{} = \partial_\mu^{(1)} \partial_\nu^{(2)} \Delta_F(x_1 - x_2) + i\,\delta_{\mu 4}\,\delta_{\nu 4}\,\delta^{(4)}(x_1 - x_2) \qquad (2.16)$$

$$\underbrace{A_\mu(x_1)\, A_\nu(x_2)}_{} = \left(\delta_{\mu\nu} - m^{-2}\partial_\mu \partial_\nu\right)\Delta_F(x_1 - x_2) + i m^{-2}\,\delta_{\mu 4}\,\delta_{\nu 4}\,\delta^{(4)}(x_1 - x_2) \qquad (2.17)$$

where Δ_F is the Feynman propagator of the scalar field.

In the examples where the Mathews rule[72] holds, the normal-dependent terms in the Hamiltonian get cancelled with the normal-dependent terms from the contractions, and the S-matrix will be free of normal-dependent terms. Direct computation now shows that the present theory is an exception to the generalised Mathew's rule; normal-dependent terms drop off to second order in coupling constant but some such terms survive cancellation in the fourth order (Details are given in Appendix II).

A modification of the above discussed theory with the interaction Lagrangian (2.3) may now be envisaged with the addition of a term depending on the square of the coupling constant; thus

$$\mathcal{L}_I = ig\,(\partial_\mu\varphi^*\cdot\varphi - \varphi^*\partial_\mu\varphi)\,A_\mu$$
$$- g^2\varphi^*\varphi\, A_\mu A_\mu \qquad (2.18)$$

The resulting equations of motion are

$$(\Box - m^2)A_\mu - \partial_\mu\partial_\lambda A_\lambda = ig\,(\partial_\mu\varphi^*\cdot\varphi - \varphi^*\partial_\mu\varphi) + 2g^2\varphi^*\varphi\, A_\mu \qquad (2.19a)$$

$$(\Box - \mu^2)\varphi = ig\,(2\partial_\lambda\varphi\cdot A_\lambda + \varphi\,\partial_\lambda A_\lambda) + g^2\varphi\, A_\lambda A_\lambda \qquad (2.19b)$$

$$(\Box - \mu^2)\varphi^* = -ig\,(2\partial_\lambda\varphi^*\cdot A_\lambda + \varphi^*\partial_\lambda A_\lambda) + g^2\varphi^*A_\lambda A_\lambda \qquad (2.19c)$$

The fourth component of eq.(2.19a) constitutes a primary constraint. However, the secondary constraint obtained in this case by taking the divergence of eq.(2.19a) and making use of eqs.(2.19b) and (2.19c) is

$$\partial_\lambda A_\lambda = 0 \qquad (2.20)$$

which is the same as the usual constraint condition for a free vector field, and this has come about because the current to which the vector field is coupled is conserved.

As a consequence, in the present case, there arises no question of loss of constraints at any value of the scalar field.

The true equation of motion resulting when eq.(2.20) is substituted in eq. (2.19a) is

$$(\Box - m^2) A_\mu = ig\,(\partial_\mu \varphi^* \cdot \varphi - \varphi^* \partial_\mu \varphi) + 2g^2 \varphi^* \varphi A_\mu \tag{2.21}$$

The characteristic determinant of the system of eqs.(2.21) (2.19b) and (2.19c) is

$$D(n) = (n^2)^6 \tag{2.22}$$

which implies that the ray-cone coincides with the light-cone and hence the propagation is causal.

The interaction Hamiltonian in the interaction picture for the coupling (2.18) is

$$\mathcal{H}_I = -\mathcal{L}_I - g^2/2m^2\,(\dot{\varphi}^*\varphi - \varphi^*\dot{\varphi})^2 + g^2\varphi^*\varphi A_0^2 \tag{2.23}$$

When the S-matrix elements are evaluated with the above Hamiltonian using the formula (1.52), explicit calculation verifies that normal-dependent terms from the Hamiltonian (2.22) and from the contractions (2.16) and (2.17) cancel each other to fourth order in coupling constant (see Appendix II)

A combinatorial argument can now be invoked to prove that such cancellation will occur in all higher orders, and hence the theory is Lorentz-covariant in the sense that S-matrix does not contain normal-dependent terms.

Studying in the fashion outlined above a non-derivative interaction between vector and scalar fields bilinear in both fields given by the interaction Lagrangian

$$\mathcal{L}_I = - g^2 \varphi^* \varphi A_\mu A_\mu \tag{2.24}$$

it is found that the secondary constraint breaks down only if $\varphi^*\varphi = -m^2/2g^2$ which is not possible because $\varphi^*\varphi$ is a positive-definite quantity. The characteristic determinant is given by

$$D(n) = (n^2)^6 (1 + 2g^2/m^2 \varphi^* \varphi) \tag{2.25}$$

and since the second factor cannot vanish, the nature of field propagation is always determined and is evidently causal. The corresponding quantized theory, on examination, is found to be Lorentz-covariant.

2.3 Coupled neutral vector and scalar fields

The simplest interaction between a neutral vector field and a neutral scalar field is introduced by means of the Lagrangian

$$\mathcal{L}_I = g(\partial_\mu \varphi) \varphi A_\mu \tag{2.26}$$

The above interaction is forbidden by the requirement of SU(2) invariance if both the scalar and vector field belong to an isotriplet. However, taking this to be a model interaction, the causality and related problems of the interacting fields are investigated. Analysing the field equations implied by (2.26) it is seen that the secondary constraint is lost when $\varphi^2 = m^2/g^2$ and

$$D(n) = (n^2)^5 (1 - g^2/m^2 \varphi^2) \qquad (2.27)$$

Consequently, when the field assumes a value for which $\varphi^2 = m^2/g^2$ the causality question cannot be settled unambiguously. Though the loss of constraints in the present case occurs in a frame-independent manner, leaving intact the relativistic covariance at the classical level it turns out that for the second-quantised version of this theory a Lorentz-covariant S-matrix cannot be constructed.

When the interaction (2.26) is modified by the addition of a new term $-g^2/2\ \varphi^2 A_\mu A_\mu$ the pathology mentioned above disappears and the quantised theory is covariant.

2.4 Complex vector fields

When a complex vector field is coupled to a neutral scalar field or complex scalar field with the interaction Lagrangian

$$\mathcal{L}_I = g\, \partial_\mu \varphi \cdot \varphi \, (A_\mu + A_\mu^*) \tag{2.28}$$

or

$$\mathcal{L}_I = ig\, (\partial_\mu \varphi^* \cdot \varphi - \partial_\mu \varphi \cdot \varphi^*) (A_\mu + A_\mu^*) \tag{2.29}$$

proceeding as in the examples in Sec. 2.2, results similar to those found there, are obtained.

2.5 Conclusion

The above analysis of the interacting field equations for various couplings between spin 1 and scalar fields brings out certain interesting results. It is found that some couplings lead to causal and trouble-free theories while others yield theories that suffer from several pathologies such as loss of constraints and noncovariance. In these cases the secondary constraint of the vector field equation is lost at a particular value of the scalar field. At this value the characteristic determinant gives no information about the field propagation. The causality of field propagation is "undetermined" in a sense. Though the corresponding quantised theories do not possess covariant S-matrices, no noncovariance arises in the manner suggested by Jenkins[69]. The loss of constraints appears as a covariant phenomenon and is present in all frames of reference. Nevertheless, if, in classical field theory, there is a breakdown of constraints and even if the causality problem does not possess a unique solution, Lorentz noncovariance will always make its appearance in the quantized version. This may be taken as an elaboration of the Jenkins conjecture that a classical field theory manifesting causality violation is not Lorentz-covariant at the quantized level. The subtle connection between causality and Lorentz-covariance will again be discussed in Chapter IV.

APPENDIX I

It will be shown that eq.(28) assumed valid for all times, together with the constraint relations (2.6) and (2.7) assumed valid at the initial time, is equivalent to the equation (2.5a). Taking the divergence of eq.(2.8) one obtains

$$(\Box - m^2)\left[\partial_\lambda A_\lambda - 2g^2/m^2\,(\varphi^*\varphi\,\partial_\lambda A_\lambda + A_\lambda\,\partial_\lambda\varphi^*\cdot\varphi + A_\lambda\,\varphi^*\,\partial_\lambda\varphi)\right] = 0 \tag{1A.1}$$

i.e. the constraint (2.7) obeys the Kelin-Gordon equation and so if it is satisfied at one instant, it will be satisfied at all future times. Making use of eq.(2.7) now valid for all times, in eq. (2.8), eq. (2.5a) is recovered and hence eq.(2.6) which is the fourth component of eq.(2.8) is also valid for all times.

APPENDIX II

Cancellation of normal-dependent terms

Non-covariant terms due to the normal-dependent terms in eq.(2.14) occur, to the lowest order, in g^2. Normal-dependent terms arising from contractions (2.16) and (2.17) also appear in this order. Using the formula (1.52), after the Wick decomposition is performed, the S-matrix element to order g^2 is obtained as

$$
\begin{aligned}
S^{(2)} &= i g^2/2m^2 \int : \left(\dot{\varphi}^*(x)\, \varphi(x) - \varphi^*(x)\, \dot{\varphi}(x) \right) : \; d^4x \\
&\quad - i g^2 \int : \varphi^*(x)\, \varphi(x)\, A_0^2(x) : \; d^4x \\
&\quad + g^2/2! \iint \Big[\Big(-\partial_\mu \underbrace{\varphi^*(x_1)\, \varphi(x_1)\, \varphi^*(x_2)\, \partial_\nu \varphi(x_2)}_{} \\
&\qquad - \varphi^*(x_1)\, \partial_\mu \underbrace{\varphi(x_1) \cdot \partial_\nu \varphi^*(x_2)}_{} \cdot \varphi(x_2) \Big) A_\mu(x_1)\, A_\nu(x_2) \\
&\quad + \left(\partial_\mu \varphi^*(x_1) \cdot \varphi(x_1) - \varphi^*(x_1)\, \partial_\mu \varphi(x_1) \right) \Big(\partial_\nu \varphi^*(x_2) \cdot \varphi(x_2) \\
&\quad - \varphi^*(x_2)\, \partial_\nu \varphi(x_2) \Big) \underbrace{A_\mu(x_1)\, A_\nu(x_2)}_{} \Big]
\end{aligned}
\tag{1A.2}
$$

where only those terms giving rise to noncovariance have been included. Making use of equations (2.16) and (2.17) it is easily seen that the normal-dependent terms arising from the contractions in eq.(1A.2) get cancelled with the first two terms of eq. (1A.2).

To the fourth order in g the S-matrix element is

$$
\begin{aligned}
S^{(4)} &= 2ig^4/m^2 \int :(\varphi^*(x)\,\varphi(x)\,A_0(x))^2:\ d^4x \\
&- ig^4/m^2 \iint T\,[\,(\dot\varphi^*\varphi - \varphi^*\dot\varphi)(x_1)\ \varphi^*\varphi A_0(x_1) \\
&\times (\partial_\mu\varphi^*\cdot\varphi - \varphi^*\partial_\mu\varphi)(x_2)\ A_\mu(x_2)\,]\ d^4x_1\ d^4x_2\,. \\
&+ (-i)^2 g^4/2! \iint T\,[\,{}^1\!/_{4m^4}\ (\dot\varphi^*\varphi - \varphi^*\dot\varphi)^2(x_1)\,(\dot\varphi^*\varphi - \varphi^*\dot\varphi)^2(x_2) \\
&+ (\varphi^*\varphi A_0^2)(x_1)(\varphi^*\varphi A_0^2)(x_2) - {}^1\!/_{2m^2}\ (\dot\varphi^*\varphi - \varphi^*\dot\varphi)^2(x_1) \\
&\times (\varphi^*\varphi A_0^2)(x_2)\,]\ d^4x_1\ d^4x_2 \\
&+ (-i)^3 g^4/3! \iiint T\,[\,{}^1\!/_{2m^2}\ (\partial_\mu\varphi^*\cdot\varphi - \varphi^*\partial_\mu\varphi)(x_1)\ (\partial_\nu\varphi^*\cdot\varphi \\
&- \varphi^*\partial_\nu\varphi)(x_2)\ (\dot\varphi^*\varphi - \varphi^*\dot\varphi)(x_3)\ A_\mu(x_1)\,A_\nu(x_2) - (\partial_\mu\varphi^*\cdot\varphi \\
&- \varphi^*\partial_\mu\varphi)(x_1)(\partial_\nu\varphi^*\cdot\varphi - \varphi^*\partial_\nu\varphi)(x_2)(\varphi^*\varphi A_0^2)(x_3) \\
&\times A_\mu(x_1)\,A_\nu(x_2)\,]\ d^4x_1\ d^4x_2\ d^4x_3 \\
&+ (-i)^4 g^4/4! \iiiint T\,[\,(\partial_\mu\varphi^*\cdot\varphi - \varphi^*\partial_\mu\varphi)(x_1)\ (\partial_\nu\varphi^*\cdot\varphi - \varphi^*\partial_\nu\varphi)(x_2) \\
&\times (\partial_\varrho\varphi^*\cdot\varphi - \varphi^*\partial_\varrho\varphi)(x_3)(\partial_\lambda\varphi^*\cdot\varphi - \varphi^*\partial_\lambda\varphi)(x_4)\ A_\mu(x_1)\,A_\nu(x_2) \\
&\times A_\varrho(x_3)\,A_\lambda(x_4)\,]\ d^4x_1\ d^4x_2\ d^4x_3\ d^4x_4 \qquad (1A.3)
\end{aligned}
$$

Further normal-dependent terms appear in eq.(1A.3) when the time-ordered products are replaced by normal products and contractions. It is found that of these non-covariant terms all except those arising from the first two terms renders the matrix element normal-dependent and hence noncovariant. These non-covriant terms can be traced to the g^3 and g^4 dependent terms in the Hamiltonian (2.14). The absence of these terms in the modified Hamiltonian (2.23) ensures the covariance of the corresponding S-matrix.

CHAPTER III

THE DIRAC-SCHWINGER COVARIANCE CONDITION IN CLASSICAL FIELD THEORY

As a preliminary to the study of the relativistic covariance of higher spin theories at the classical level, a straight-forward derivation of the Dirac-Schwinger condition is given within the framework of classical field theory. The crucial role of the energy continuity equation is pointed out. The origin of higher order derivatives of delta function is traced to the presence of higher order derivatives of canonical coordinates and momenta in the energy-density functional.

3.1 Introduction

It is well known that the proper relativistic covariance of a quantum field theory set up via an action principle is not ensured by the formal invariance of the Lagrangian. Dirac[114] and Schwinger[115-119] have stated that a sufficient condition for the relativistic covariance of a quantum field theory is the energy-density commutator condition

$$
\begin{aligned}
1/i\,[T_{00}(x), T_{00}(x')] &= -(T_{0k}(x) + T_{0k}(x'))\,\partial_k\delta(\vec{x}-\vec{x}') \\
&\quad + (b_k(x) + b_k(x'))\,\partial_k\delta(\vec{x}-\vec{x}') + (c_{klm}(x) + c_{klm}(x')) \\
&\quad \times \partial_k\partial_l\partial_m\delta(\vec{x}-\vec{x}') + \cdots
\end{aligned}
\tag{3.1}
$$

where $b_k = \partial_l \beta_{lk}$ and $\beta_{kl} - \beta_{lk} = \partial_m \gamma_{mkl}$ and the energy-density is that obtained from a symmetrical energy-momentum tensor constructed by the Belinfante[120] prescription. For a class of theories that Schwinger calls 'local' the Dirac-Schwinger (DS) condition is satisfied in the simplest form with $b_k = c_{klm} = \cdots = 0$. Spin s ($s \leq 1$) theories belong to this class. For canonical theories in which simple canonical commutation relations hold Brown[121] has given two new proofs of the DS condition.

In Chapters I and II of this thesis several instances of higher spin theories have been noted where relativistic covariance seems to be upset at the quantised or at the classical level. As a preliminary to a detailed study of the covariance problem at the c-number level in such cases, an investigation is carried out in the present chapter of the significance of the DS condition for a classical field theory in a general setting. Considering a Poisson bracket (PB) realisation of the Poincaré group Lie algebra, a derivation of the DS condition is presented starting from the basic definition of PB of two functions of canonical field coordinates and momenta. In the simple case where the energy-density is a functional of canonical coordinates and momenta and their first derivatives, only the first derivative of the delta function appears on the rhs of eq.(3.1), and the general form of this coefficient can be identified by appealing to the energy continuity equation. The Poincaré group structure relations serve only to determine the quantities b_k in eq. (3.1).

The present study reveals that at the classical level the absence of higher derivatives of delta function on the rhs of eq.(3.1) is not in any way directly related to the spin of the field and that such terms appear inevitably when the energy density is a functional of higher derivatives of fields and momenta. A derivation of the DS condition in the case of a generalized mechanics with higher derivatives of canonical coordinates and momenta is also given.

3.2 DS condition for simple systems

For a simple system described by a set of N independent dynamical variables ψ_α, π_α $(\alpha = 1 \cdots N)$ it is assumed that the energy density derived from a symmetric energy momentum tensor is a functional of only ψ_α, π_α and their first derivatives. The PB of the energy densities at two distinct points at equal times is

$$\{ T_{00}(x), T_{00}(x') \} = \sum_\alpha \int d^3y \left(\frac{\delta T_{00}(x)}{\delta \psi_\alpha(y)} \frac{\delta T_{00}(x')}{\delta \pi_\alpha(y)} - \frac{\delta T_{00}(x)}{\delta \pi_\alpha(y)} \frac{\delta T_{00}(x')}{\delta \psi_\alpha(y)} \right) \tag{3.2}$$

Under the above-stated assumptions

$T_{00} = T_{00}[\psi_\alpha, \pi_\alpha, \partial_k \psi_\alpha, \partial_k \pi_\alpha]$, and the functional derivatives are

$$\frac{\delta T_{00}(x)}{\delta \psi_\alpha(y)} = \frac{\partial T_{00}(x)}{\partial \psi_\alpha(x)} \delta(\vec{x}-\vec{y}) - \frac{\partial T_{00}(x)}{\partial \partial_k \psi_\alpha(x)} \frac{\partial}{\partial y_k} \delta(\vec{x}-\vec{y}) \tag{3.3a}$$

$$\frac{\delta T_{00}(x)}{\delta \pi_\alpha(y)} = \frac{\partial T_{00}(x)}{\partial \pi_\alpha(x)} \delta(\vec{x}-\vec{y}) - \frac{\partial T_{00}(x)}{\partial \partial_k \pi_\alpha(x)} \frac{\partial}{\partial y_k} \delta(\vec{x}-\vec{y}) \tag{3.3b}$$

Making use of eq.(3.3) it follows that

$$\{ T_{oo}(x), T_{oo}(x') \} = \sum_{\alpha}' \Big[\Big(\frac{\partial T_{oo}(x)}{\partial \pi_\alpha(x)} \frac{\partial T_{oo}(x')}{\partial \partial_{k'} \psi_\alpha(x')}$$

$$- \frac{\partial T_{oo}(x)}{\partial \psi_\alpha(x)} \frac{\partial T_{oo}(x')}{\partial \partial_{k'} \pi_\alpha(x')} + \frac{\partial T_{oo}(x')}{\partial \pi_\alpha(x')} \frac{\partial T_{oo}(x)}{\partial \partial_k \psi_\alpha(x)}$$

$$- \frac{\partial T_{oo}(x')}{\partial \psi_\alpha(x')} \frac{\partial T_{oo}(x)}{\partial \partial_k \pi_\alpha(x)} \Big) \partial_k \delta(\vec{x}-\vec{x}')$$

$$+ \Big(\frac{\partial T_{oo}(x)}{\partial \partial_k \pi_\alpha(x)} \frac{\partial T_{oo}(x')}{\partial \partial_{\ell'} \psi_\alpha(x')} - \frac{\partial T_{oo}(x)}{\partial \partial_k \psi_\alpha(x)} \frac{\partial T_{oo}}{\partial \partial_{\ell'} \pi_\alpha(x')} \Big)$$

$$\times \partial_k \partial_\ell \delta(\vec{x}-\vec{x}') \tag{3.4}$$

where $\partial_{k'}$ denotes $\frac{\partial}{\partial x'_k}$. Invoking the Schwinger identity[119]

$$[f(x) g(x') + f(x') g(x)] \partial_k \delta(\vec{x}-\vec{x}')$$

$$= [f(x) g(x) + f(x') g(x')] \partial_k \delta(\vec{x}-\vec{x}') \tag{3.5}$$

and an easily verified analogous identity involving the second derivative of the delta function (See Appendix)

$$[f_k(x) g_\ell(x') - g_k(x) f_\ell(x')] \partial_k \partial_\ell \delta(\vec{x}-\vec{x}')$$

$$= - [\partial_k f_k(x) \cdot g_\ell(x) - \partial_k g_k(x) \cdot f_\ell(x)$$

$$+ \partial_{k'} f_k(x') \cdot g_\ell(x') - \partial_{k'} g_k(x') \cdot f_\ell(x')] \partial_\ell \delta(\vec{x}-\vec{x}') \tag{3.6}$$

the expression on the rhs of eq.(3.4) simplifies to

$$\{ T_{00}(x) \, , \, T_{00}(x') \} = (f_{0k}(x) + f_{0k}(x')) \times \partial_k \delta(\vec{x} - \vec{x}') \tag{3.7}$$

where

$$f_{0k} = \sum_{\alpha} \Big(\frac{\partial T_{00}}{\partial \pi_\alpha} \frac{\partial T_{00}}{\partial \partial_k \psi_\alpha} - \frac{\partial T_{00}}{\partial \psi_\alpha} \frac{\partial T_{00}}{\partial \partial_k \pi_\alpha}$$

$$+ \frac{\partial T_{00}}{\partial \partial_k \pi_\alpha} \partial_\ell \frac{\partial T_{00}}{\partial \partial_\ell \psi_\alpha} - \frac{\partial T_{00}}{\partial \partial_k \psi_\alpha} \partial_\ell \frac{\partial T_{00}}{\partial \partial_\ell \psi_\alpha} \Big) \tag{3.8}$$

Integration of (3.7) over x' yields

$$\{ T_{00}(x) \, , \, P_0 \} = \partial_k f_{0k}(x) \tag{3.9}$$

where $P_0 = \int d^3x \; T_{00}(x)$ is the total energy.

If now it is assumed that the system possesses space-time translational invariance, P_0 being the generator of time-translation, the equation (3.9) for $T_{00}(x)$ can be cast into the form

$$d_0 T_{00}(x) - \partial_k f_{0k}(x) = 0 \tag{3.10}$$

where d_0 represents the total time derivative. From (3.10) and the energy continuity equation

$$d_0 T_{00}(x) + \partial_k T_{0k}(x) = 0 \tag{3.11}$$

it may be inferred that

$$f_{0k} = - T_{0k} + b_k \qquad (3.12)$$

where b_k is an arbitrary function with $\partial_k b_k = 0$. Hence the energy density PB may be written in the form

$$\{ T_{00}(x) , T_{00}(x') \} = - (T_{0k}(x) + T_{0k}(x')) \times \partial_k \delta(\vec{x}-\vec{x}') + (b_k(x) + b_k(x')) \partial_k \delta(\vec{x}-\vec{x}') \qquad (3.13)$$

Upto this point the concept of relativistic covariance has not been used in any manifest way. In order that a field theory be relativistically covariant the ten generators P_μ , $J_{\mu\nu}$ constructed in terms of the fundamental field variables must obey the structure relations characterising the geometric nature of the Poincaré group. Since the present discussion is wholly confined to the classical regime, it is the PB realization of the Poincaré algebra that is of interest

$$\{ P_\mu , P_\nu \} = 0 \qquad (3.14a)$$

$$\{ J_{\mu\nu} , P_\rho \} = \delta_{\mu\rho} P_\nu - \delta_{\nu\rho} P_\mu \qquad (3.14b)$$

$$\{ J_{\mu\nu} , J_{\rho\lambda} \} = \delta_{\nu\lambda} J_{\mu\rho} - \delta_{\mu\lambda} J_{\nu\rho} - \delta_{\nu\rho} J_{\mu\lambda} + \delta_{\mu\rho} J_{\nu\lambda} \tag{3.14c}$$

Of these relations, only the following PB's

$$\{ J_{ok} , P_o \} = - P_k \tag{3.15a}$$

$$\{ J_{ok} , J_{ol} \} = - J_{kl} \tag{3.15b}$$

pertain to the Lorentz covariance of the theory in its proper sense; the rest of the structure relations (3.14) follow from the three dimensional invariance and the vector property of the boost-generators J_{ok}.

The generators in eq.(3.15) may be expressed in terms of the energy-momentum tensor as follows:

$$P_k = \int T_{ok}(x) \, d^3x \tag{3.16a}$$

$$J_{ok} = x_o P_k - \int x_k T_{oo}(x) \, d^3x \tag{3.16b}$$

$$J_{kl} = \int \left(x_k T_{ol}(x) - x_l T_{ok}(x) \right) d^3x \tag{3.16c}$$

Relation (3.15a) is already allowed by the general form of eq. (3.13) and in order that eq.(3.15b) be satisfied b_k must obey the condition

$$\int \left(x_k b_\ell(x) - x_\ell b_k(x) \right) d^3x = 0 \qquad (3.17)$$

For this volume integral to vanish the integrand must be of the form of a three-divergence i.e.

$$x_k b_\ell - x_\ell b_k = \partial_m a_{mk\ell} \qquad (3.18)$$

where a_{mkl} is an arbitrary function antisymmetric in the indices k and l. From eq.(3.18), with the aid of the fact that b_k is divergenceless, it is easily deducible that b_k itself must be a divergence

$$b_k = \partial_\ell \beta_{\ell k} \qquad (3.19)$$

where $\beta_{\ell k} = x_\ell b_k + \partial_m a_{mkl}$. It also follows that the antisymmetric part of $\beta_{k\ell}$ is a divergence.

3.3 DS condition for non-conservative systems

In the case where the Lagrangian of a system has an explicit space-time dependence translational invariance can no longer be invoked and eq.(3.10) will be replaced in this instance by

$$d_0 T_{00}(x) - \partial_k f_{0k}(x) = \partial_0 T_{00}(x) \qquad (3.20)$$

where a slight change of notation has been introduced in that ∂_0 now denotes explicit differentiation with respect to time whereas ∂_k still retains the usual meaning. Energy momentum conservation does not hold in this case and eq. (3.11) is replaced by

$$d_0 T_{00}(x) + \partial_k T_{0k}(x) = \partial_0 \mathcal{L} \tag{3.21}$$

From eqs. (3.20) and (3.21) it may readily be inferred that f_{ok} must be of the form

$$f_{ok} = -T_{ok} + b_k \tag{3.22}$$

with $\partial_k b_k = \partial_0 (T_{00} - \mathcal{L})$. Since b_k is not divergenceless eq. (3.15a) is not satisfied automatically. Making use of eqs. (3.16a) and (3.16b) to evaluate the lhs of eq. (3.15a) it is found that

$$\{ J_{0k}, P_0 \} = -\int (T_{0k}(x) - b_k(x)) \, d^3x \tag{3.23}$$

and hence in order that the structure relation be satisfied b_k must be restricted by requiring

$$\int b_k(x) \, d^3x = 0 \tag{3.24}$$

This implies $b_k = \partial_\ell \beta_{\ell k}$ where $\beta_{\ell k}$ is arbitrary. As in sec.3.2 the structure relation (3.15b) imposes on $\beta_{k\ell}$ the restriction

$$\beta_{k\ell} - \beta_{\ell k} = \partial_m \gamma_{mk\ell} \tag{3.25}$$

It is worth noting that in contrast to the case of conservative systems, the coefficient b_k in the present case need not be divergenceless but other conditions on b_k are the same as before.

3.4 DS condition in higher derivative field theories

A Hamiltonian formulation of a generalized mechanics with higher order derivatives of field variables has been considered by Coelho de Souza and Rodrigues[121]. They have shown that the PB of two functions A and B is to be defined as

$$\{A, B\} = \sum_{\alpha=1}^{N} \sum_{m=0}^{s-1} \int \Big(\frac{\delta A}{\delta \psi_\alpha^{(m)}} \frac{\delta B}{\delta \pi_{\alpha/m+1}} - \frac{\delta A}{\delta \pi_{\alpha/m+1}} \frac{\delta B}{\delta \psi_\alpha^{(m)}} \Big) \, d^3x \tag{3.26}$$

where $\psi_\alpha^{(m)} = \frac{d^m}{dx_0^m} \psi_\alpha$ and $\pi_{\alpha/m+1}$ is the momentum conjugate to $\psi_\alpha^{(m)}$, s is the order of the highest derivative involved. If $F = \int \mathcal{F} \, d^3x$, by definition

$$\frac{\delta F}{\delta \psi_\alpha} = \frac{\partial \mathcal{F}}{\partial \psi_\alpha} - \partial_{i_1} \frac{\partial \mathcal{F}}{\partial \partial_{i_1} \psi_\alpha} + \cdots$$

$$+ (-1)^s \partial_{i_1} \cdots \partial_{i_s} \frac{\partial \mathcal{F}}{\partial \partial_{i_1} \cdots \partial_{i_s} \psi_\alpha} \tag{3.27}$$

Using the definition (3.26) to evaluate the PB $\{ T_{00} , T_{00} \}$ where T_{oo} is a functional of $\psi_\alpha^{(m)}$, $\Pi_{\alpha / m+1}$ and their derivatives upto order s yields the general result

$$\{ T_{00}(x) , T_{00}(x') \} = (a_k(x) + a_k(x')) \partial_k \delta(\vec{x} - \vec{x}')$$

$$+ (c_{klm}(x) + c_{klm}(x')) \partial_k \partial_l \partial_m \delta(\vec{x} - \vec{x}') + \cdots \tag{3.28}$$

The coefficients a_k, c_{klm}, . . . appearing in eq.(3.28) may be evaluated directly in any particular case. When the highest order derivative in T_{oo} is the second, it is found

$$a_k = \sum_\alpha \sum_{m=0}^{1} \Big[\Big(\frac{\partial T_{00}}{\partial \Pi_{\alpha/m+1}} \frac{\partial T_{00}}{\partial \partial_k \psi_\alpha^{(m)}}$$

$$- \frac{\partial T_{00}}{\partial \psi_\alpha^{(m)}} \frac{\partial T_{00}}{\partial \partial_k \Pi_{\alpha/m+1}} + \frac{\partial T_{00}}{\partial \partial_k \Pi_{\alpha/m+1}} \partial_l \frac{\partial T_{00}}{\partial \partial_l \psi_\alpha^{(m)}}$$

$$- \frac{\partial T_{00}}{\partial \partial_k \psi_\alpha^{(m)}} \partial_l \frac{\partial T_{00}}{\partial \partial_l \Pi_{\alpha/m+1}} \Big)$$

$$- \left(\partial_j \frac{\partial T_{00}}{\partial \partial_k \psi_\alpha^{(m)}} \; \partial_i \frac{\partial T_{00}}{\partial \partial_i \partial_j \pi_{\alpha/m+1}} + \partial_j \frac{\partial T_{00}}{\partial \partial_j \psi_\alpha^{(m)}} \; \partial_i \frac{\partial T_{00}}{\partial \partial_i \partial_k \pi_{\alpha/m+1}} \right.$$

$$\left. + \partial_j \frac{\partial T_{00}}{\partial \partial_i \psi_\alpha^{(m)}} \; \partial_i \frac{\partial T_{00}}{\partial \partial_j \partial_k \pi_{\alpha/m+1}} \right) + \left(\partial_j \frac{\partial T_{00}}{\partial \partial_k \pi_{\alpha/m+1}} \; \partial_i \frac{\partial T_{00}}{\partial \partial_i \partial_j \psi_\alpha^{(m)}} \right.$$

$$\left. \left. + \partial_j \frac{\partial T_{00}}{\partial \partial_j \pi_{\alpha/m+1}} \; \partial_i \frac{\partial T_{00}}{\partial \partial_i \partial_k \psi_\alpha^{(m)}} + \partial_j \frac{\partial T_{00}}{\partial \partial_i \pi_{\alpha/m+1}} \; \partial_i \frac{\partial T_{00}}{\partial \partial_j \partial_k \psi_\alpha^{(m)}} \right) \right] \tag{3.29}$$

$$c_{ijk} = \sum_{m=0}^{1} \sum_{\alpha} \left(\frac{\partial T_{00}}{\partial \partial_i \psi_\alpha^{(m)}} \frac{\partial T_{00}}{\partial \partial_j \partial_k \pi_{\alpha/m+1}} - \frac{\partial T_{00}}{\partial \partial_i \pi_{\alpha/m+1}} \frac{\partial T_{00}}{\partial \partial_j \partial_k \psi_\alpha^{(m)}} \right) \tag{3.30}$$

Here use has been made of the following identity (See Appendix)

$$(f_i(x)\, g_{jk}(x') + g_{jk}(x)\, f_i(x'))\, \partial_i \partial_j \partial_k \delta(\vec{x}-\vec{x}')$$

$$= (f_i(x)\, g_{jk}(x) + f_i(x')\, g_{jk}(x'))\, \partial_i \partial_j \partial_k \delta(\vec{x}-\vec{x}')$$

$$- (\partial_j f_i(x)\, \partial_k g_{jk}(x) + \partial_j f_j(x)\, \partial_k g_{ik}(x)$$

$$+ \partial_j f_k(x)\, \partial_k g_{ij}(x) + \partial_{j'} f_i(x')\, \partial_{k'} g_{jk}(x')$$

$$+ \partial_{j'} f_j(x') \, \partial_{k'} g_{ik}(x') \quad + \partial_{j'} f_k(x') \, \partial_{k'} g_{ij}(x') \Big)$$

$$\times \partial_i \delta(\vec{x} - \vec{x}') \tag{3.31}$$

When still higher derivatives are involved a_k, c_{klm}, . . . will have additional contributions from these higher derivatives and they may be evaluated using identities similar to (3.31) for terms with higher derivatives of delta function.

Integrating eq.(3.26) over x'

$$\{ T_{00}(x) \, , P_0 \} \quad = \partial_k a_k + \partial_k \partial_\ell \partial_m c_{k\ell m} + \cdots \tag{3.32}$$

Assuming space-time translational invariance this becomes

$$\dot{d_0} T_{00}(x) - \partial_k a_k - \partial_k \partial_\ell \partial_m c_{k\ell m} - \cdots = 0 \tag{3.33}$$

From this and the energy continuity equation(3.11) it may be inferred that

$$a_k + \partial_\ell \partial_m c_{k\ell m} + \cdots = -T_{0k} + B_k \tag{3.34}$$

with $\partial_k B_k = 0$.

Hence the energy density PB assumes the form

$$\{T_{00}(x), T_{00}(x')\} = -(T_{0k}(x) + T_{0k}(x'))$$

$$\times\, \partial_k \delta(\vec{x}-\vec{x}') + (b_k(x) + b_k(x'))\, \partial_k \delta(\vec{x}-\vec{x}')$$

$$+ (c_{k\ell m}(x) + c_{k\ell m}(x'))\, \partial_k \partial_\ell \partial_m \delta(\vec{x}-\vec{x}') + \cdots \tag{3.35}$$

where

$$b_k(x) = B_k(x) - \partial_\ell \partial_m c_{k\ell m}(x) \tag{3.36}$$

Using the form of the energy density PB as given by eq.(3.35) to evaluate the rhs of the structure relation (3.15a)

$$\{J_{0k}, P_0\} = -\int d^3x\, [\, T_{0k}(x) - b_k(x)$$

$$- \partial_\ell \partial_m c_{k\ell m}(x) - \cdots]$$

$$= -\int d^3x\, [\, T_{0k}(x) - B_k(x)\,] \tag{3.37}$$

Hence the structure relation is satisfied only if

$$\int d^3x\ B_k(x) = 0 \tag{3.38}$$

which means that $B_k = \partial_\ell \alpha_{k\ell}$ where $\alpha_{k\ell}$ is arbitrary. Eq. (3.36) implies that b_k must itself be a divergence i.e.

$$b_k = \partial_\ell\, [\, \alpha_{k\ell} - \partial_m c_{k\ell m} - \cdots] \tag{3.39}$$
$$= \partial_\ell \beta_{k\ell}$$

In order that the structure relation (3.15b) be obeyed the coefficients b_k must be further restricted by

$$\int d^3x \;\left(x_k\, b_l(x) - x_l\, b_k(x) \right) \qquad = 0 \tag{3.40}$$

Substituting eq. (3.39) in eq.(3.40) it follows

$$\int d^3x \;\left(\beta_{kl} - \beta_{lk} \right) \qquad = 0 \tag{3.41}$$

or, the antisymmetric part of β_{kl} is a divergence. Thus the conditions that b_k has to satisfy are in the present case the same as those for b_k in eq. (3.1).

3.5 Conclusion

The DS condition has been derived within the framework of classical field theory. The general form of this condition is determined by the energy continuity equation. For simple cases this takes on a form involving only the first derivative of the delta function while the presence of higher derivatives of fields and canonical momenta in the energy density invites the appearance of higher derivatives of delta functions. This makes clear the fact that for these higher order terms to appear it is not necessary that a higher spin be associated with the field system. On the other hand, the appearance of higher derivatives of delta function must be taken to mean that the energy density functional in such theories contains higher derivatives of canonical coordinates and momenta.

While it is the translational invariance of the theory that determines the general form of the DS condition, Lorentz covariance of the theory finds expression through the restrictions imposed on the coefficients b_k. A point of difference between the classical DS condition and the quantum version may be noted here. In the classical case for simple systems the coefficient b_k is divergenceless whereas there is no such restriction in the quantum case. However, when the energy density has an explicit space-time dependence or when higher derivatives are present b_k obeys the same condition as in the quantum case. Despite these differences which may not be too significant the quantum-classical analogy would presumably allow an extension of the result herein obtained regarding the correspondence between higher order Lagrangians and higher order derivatives of delta function to the quantum domain.

APPENDIX

The identities (3.6) and (3.31) are readily verified by integrating either side over x or x'. The proof of eq.(3.6) is essentially based on the identity

$$f_k(x)\,\partial_k\partial_\ell\, g_\ell(x) - g_k(x)\,\partial_k\partial_\ell\, f_\ell(x)$$
$$= \partial_k\,\left[\, f_k(x)\,\partial_\ell\, g_\ell(x) - g_k(x)\,\partial_\ell\, f_\ell(x)\,\right] \qquad (3A.1)$$

In a similar way eq. (3.31) is established by noting that

$$f_i(x)\,\partial_i\partial_j\partial_k\, g_{jk}(x) + \partial_i\partial_j\partial_k f_i(x)\cdot g_{jk}(x)$$
$$= \partial_i\partial_j\partial_k\,\left[\, f_i(x)\, g_{jk}(x)\,\right]$$
$$- \partial_i\,\left[\,\partial_j f_i(x)\,\partial_k\, g_{ik}(x) + \partial_j f_j(x) \times \partial_k\, g_{ik}(x) + \partial_j f_k(x)\,\partial_k\, g_{ij}(x)\,\right] \qquad (3A.2)$$

CHAPTER IV

RELATIVISTIC NONCOVARIANCE IN INTERACTING SPIN 1 FIELD THEORY

The covariance problem in certain pathological theories at the classical level is studied within a Poincaré group framework making use of the DS condition. It is shown that while the acausal interactions of a spin 1 field lead to noncovariance there are examples where the propagation is causal but the theory is nevertheless non-covariant.

4.1 Introduction

It has been mentioned in Chapters I and II that in higher spin theories causality violation seems to be accompanied by a breakdown of Lorentz covariance. Two types of noncovariance have been observed so far. Of these the noncovariance at the quantized level pertains to be noninvariance of the S-matrix calculated perturbatively. The other type of noncovariance, manifesting itself at the classical level, arises out of the impossibility of determining all dependent components simultaneously in all frames of reference. These results do not establish Lorentz noncovariance in a rigorous manner because (a) the validity of the perturbation expansion in acausal theories is somewhat dubious[122]; (b) the attributed noncovariance at the classical level is somewhat conjectural.

The present work is aimed towards an investigation of the covariance question of interacting higher spin systems within a Poincaré group framework at the basic c-number level. The DS condition is the main tool of this study. Several interactions of a spin 1 field are chosen as examples. The acausal symmetric tensor and quadrupole couplings of a charged vector field are shown not to possess Lorentz covariance. At the same time it is found that the Pauli moment coupling of a spin 1 particle to an external magnetic field which, though causal, has the difficulty of imaginary values in the energy eigenvalue spectrum is also non-covariant. A similar analysis of mutually interacting vector fields and Dirac fields under various coupling schemes reveals that while the acausal interactions are all non-covariant, there are two cases of derivative couplings which are causal but not covariant. The implications of these results are discussed in some detail.

4.2 Noncovariance

In the DS covariance condition which was established in the context of classical field theory in Chapter III, it was found that while the general form of the PB given by (3.1) arises solely from the energy continuity equation, the covariance requirements are reflected through the restrictions on the coefficient b_k. The primary condition on b_k is that it must be a three-divergence. If in a specific instance this condition is violated, the Poincare group structure relations (3.15a) and (3.15b) will not be satisfied, and the theory will fail to be Lorentz-covariant. In the examples considered below it is shown that the PB $\{T_{00}(x), T_{00}(x')\}$ contains in addition to $-[T_{0k}(x) + T_{0k}(x')] \times \partial_k \delta(\vec{x}-\vec{x}')$ a term of the form $-[f_{0k}(x) + f_{0k}(x')] \times \partial_k \delta(\vec{x}-\vec{x}')$ where f_{0k} is not a three-divergence.

4.3 Examples

a) Symmetric tensor coupling

Consider the interaction of a massive charged vector field $\varphi_\mu(x)$ with an external symmetric tensor field $W_{\mu\nu}(x)$. The Lagrangian is taken to be

$$\mathcal{L} = -\frac{1}{2} G_{\mu\nu} (\partial_\mu \varphi_\nu^* - \partial_\nu \varphi_\mu^*) - \frac{1}{2} G_{\mu\nu}^* (\partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu) + \frac{1}{2} G_{\mu\nu}^* G_{\mu\nu} - \mu^2 \varphi_\mu^* \varphi_\mu - \lambda \varphi_\mu^* W_{\mu\nu} \varphi_\nu \tag{4.1}$$

It has been proved that the field propagation is acausal for this interaction (see Chapter I) and constraint breakdown can occur in this theory in a frame-dependent manner[69]. To study the covariance of this theory by applying the DS condition the energy and momentum densities of the system are first computed:

$$\begin{aligned} T_{oo} = {} & G_{oi}^* G_{oi} + \frac{1}{2} (\partial_i \varphi_j^* - \partial_j \varphi_i^*)(\partial_i \varphi_j - \partial_j \varphi_i) \\ & + \mu^2 \varphi_i^* \varphi_i + \lambda W_{ij} \varphi_i \varphi_j^* + \frac{1}{\mu^2 - \lambda W_{oo}} \\ & \times \{ \partial_i G_{oi}^* \partial_j G_{oj} - \lambda W_{io} (\varphi_i^* \partial_j G_{oj} + \varphi_i \partial_j G_{oj}^*) \\ & + \lambda^2 W_{io} W_{jo} \varphi_i \varphi_j^* \} \end{aligned} \tag{4.2}$$

$$T_{0k} = G_{0i}^{*} \partial_k \varphi_i + G_{0i} \partial_k \varphi_i^{*} \tag{4.3}$$

If, for simplicity, it is now assumed that only the W_{00} component of the symmetric tensor field is non-vanishing eq.(4.2) simplifies to

$$T_{00} = [\, G_{0i}^{*} G_{0i} + \frac{1}{2} (\partial_i \varphi_j^{*} - \partial_j \varphi_i^{*}) (\partial_i \varphi_j - \partial_j \varphi_i)$$

$$+ \mu^2 \varphi_i^{*} \varphi_i + \frac{1}{\mu^2 - \lambda W_{00}} \partial_i G_{0i}^{*} \partial_j G_{0j} \,] \tag{4.4}$$

It may easily be verified that the above assumption does not invalidate any of the contentions of the present wokk. Making use of the basic PB relations

$$\{ \varphi_i(x) , \pi_j(x') \} = \delta_{ij} \, \delta(\vec{x} - \vec{x}') \tag{4.5a}$$

$$\{ \varphi_i^{*}(x) , \pi_j^{*}(x') \} = \delta_{ij} \, \delta(\vec{x} - \vec{x}') \tag{4.5b}$$

$$\{ \varphi_i(x) , \varphi_j(x) \} = \{ \varphi_i(x) , \varphi_j^{*}(x) \} = 0 \tag{4.5c}$$

$$\{ \pi_i(x) , \pi_j(x') \} = \{ \pi_i(x) , \pi_j^{*}(x') \} = 0 \tag{4.5d}$$

which in the present case become

$$\{ \varphi_i(x) , G_{0j}^{*} \} = - \delta_{ij} \, \delta(\vec{x} - \vec{x}') \tag{4.6a}$$

$$\{ \varphi_i^*(x) \ , \ G_{0j}(x) \} \qquad = \ - \delta_{ij} \ \delta(\vec{x} - \vec{x}') \tag{4.6b}$$

with all other PBs vanishing, the PB $\{ T_{00}(x) \ , \ T_{00}(x') \}$ is evaluated:

$$\begin{aligned}
\{ T_{00}(x) \ , \ T_{00}(x') \} \quad = \ - \Big[& \{ \partial_K \varphi_\ell(x) \cdot G^*_{0\ell}(x) \\
& + \partial_K \varphi^*_\ell(x) G_{0\ell}(x) \} - \{ \partial_K \ (\varphi^*_\ell G_{0\ell}(x) \ + \varphi_\ell(x) \ G^*_{0\ell}(x)) \} \\
& - \frac{\lambda W_{00}(x)}{\mu^2 - \lambda W_{00}} \ \{ \varphi_K(x) \ \partial_\ell G^*_{0\ell}(x) \ + \varphi^*_K \ \partial_\ell G_{0\ell}(x) \} \\
& + \ \{ \partial_{K'} \varphi_\ell(x') \cdot G^*_{0\ell}(x') \ + \partial_{K'} \varphi^*_\ell(x) \cdot G_{0\ell}(x') \} \\
& - \ \{ \partial_{K'} \ (\varphi^*_\ell(x') G_{0\ell}(x') \ + \ \varphi_\ell(x') G^*_{0\ell}(x')) \} \ - \ \frac{\lambda W_{00}(x')}{\mu^2 - \lambda W_{00}(x')} \\
& \times \ \{ \varphi_K(x') \ \partial_{\ell'} G_{0\ell}(x') \ + \ \varphi^*_K(x') \ \partial_{\ell'} G_{0\ell}(x') \} \Big] \\
& \times \ \partial_K \ \delta(\vec{x} - \vec{x}')
\end{aligned} \tag{4.7}$$

where $\partial_{K'}$ denotes differentiation with respect to x_k' . The term within the first curly bracket on the rhs of eq. (4.7) may easily be identified as the momentum density T_{ok}. The second curly bracket contains a term of the form of a three divergence. However, the term $f_{ok} = \dfrac{\lambda W_{00}}{\mu^2 - \lambda W_{00}}$ $\times (\varphi_K \partial_\ell G^*_{0\ell} \ + \varphi^*_K \partial_\ell G_{0\ell})$ which appears in addition to these, cannot be written as a three-divergence and, as argued before, breaks the covariance of the theory.

b) Quadrupole coupling

The second example is provided by the quadrupole moment coupling of the charged vector field to an external electromagnetic field with an interaction Lagrangian

$$\mathcal{L}_I = -e\,(\varphi_\lambda^* Q_{\lambda\mu\nu}\partial_\mu\varphi_\nu + \varphi_\lambda Q_{\lambda\mu\nu}\partial_\mu\varphi_\nu^*) \qquad (4.8)$$

where $Q_{\lambda\mu\nu} = \partial_\lambda F_{\mu\nu}$ and $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$ is the field tensor of the electromagnetic field. If the only non-vanishing components of the electromagnetic field tensor are $F_{i4} = -iE_i$, so that the field is a simple electrostatic field, the energy and momentum densities are

$$\begin{aligned} T_{00} = \Big[& G_{0i}^* G_{0i} + \tfrac{1}{2}(\partial_i\varphi_j^* - \partial_j\varphi_i^*)(\partial_i\varphi_j - \partial_j\varphi_i) \\ & + \mu^2\varphi_i^*\varphi_i + 1/\mu^2\, \partial_i G_{0i}^*\, \partial_j G_{0j} + e/\mu^2 \\ & \times \{ \partial_k G_{0k}\, \partial_i(\varphi_j^*\partial_j E_i) + \partial_k G_{0k}^*\, \partial_i(\varphi_j\partial_j E_i) \} \\ & - 3e^2/\mu^2\, \partial_i(\varphi_j^*\partial_j E_i)\, \partial_k(\varphi_\ell\partial_\ell E_k) \Big] \end{aligned} \qquad (4.9)$$

$$T_{0k} = (G^{*}_{0i} + e\,\varphi^{*}_{j}\,\partial_j E_i)\,\partial_k \varphi_i + (G_{0i} + e\,\varphi_j\,\partial_j E_i)\,\partial_k \varphi^{*}_{i} \qquad (4.10)$$

Though the canonical momenta contain additional terms arising from the derivative coupling in eq.(4.8), because of the specific features of the electrostatic field the basic PBs are found to be same as those given by eq.(4.6). Making use of these relations to evaluate the energy density PB it is seen that on the rhs there is an additional term given by

$$f_{0k} = 3e^2/\mu^2 \Big[\{ \partial_i E_k \cdot G_{0i}\ \partial_j (\varphi^{*}_{\ell}\,\partial_\ell E_j) + \partial_i E_k \cdot G^{*}_{0i}\ \partial_j (\varphi_\ell\,\partial_\ell E_j) \} - 1/\mu^2 \{ \partial_i G^{*}_{0i} \times (\partial_j \partial_k E_j)\ \partial_\ell (\varphi_m\,\partial_m E_\ell) + \partial_i G_{0i}\ (\partial_j \partial_k E_j) \times \partial_\ell (\varphi^{*}_{m}\,\partial_m E_\ell) \} \Big] \qquad (4.11)$$

Since this term cannot be written as a three-divergence it follows that the theory which is known to be acausal[47] is also non-covariant.

c) Anomalous moment coupling

The coupling of a spin 1 particle to an external electromagnetic field via the anomalous magnetic moment with the interaction Lagrangian

$$\mathcal{L}_I = -iek\, \varphi_\mu^* F_{\mu\nu} \varphi_\nu \tag{4.12}$$

has a special interest in that while the propagation of the field is causal at the classical level[47] it is known that the energy eigenvalue spectrum can develop imaginary values depending on the field strength (See chapter I). In the particular case where the external field is a homogeneous magnetic field the energy and momentum densities corresponding to eq. (4.12) is

$$T_{oo} = [\, G_{oi}^* G_{oi} + \tfrac{1}{2} (\partial_i \varphi_j^* - \partial_j \varphi_i^*)(\partial_i \varphi_j - \partial_j \varphi_i)$$

$$+ \mu^2 \varphi_i^* \varphi_i + 1/\mu^2\ \partial_i G_{oi}^*\ \partial_j G_{oj} + iek\, F_{ij} \varphi_i \varphi_j^*] \tag{4.13}$$

$$T_{ok} = G_{oi}\, \partial_k \varphi_i^* + G_{oi}^*\, \partial_k \varphi_i \tag{4.14}$$

Evaluating the energy density PB with the help of the basic PB's (4.6) an additional term

$$f_{ok} = iek/\mu^2\, [\, \partial_i G_{oi} \cdot F_{jk} \varphi_j^* + \partial_i G_{oi}^* \cdot F_{jk} \varphi_j \,] \tag{4.15}$$

is encountered on the rhs which is not in the form of a divergence and this again proves that the theory is not covariant at the classical level.

(d) Coupling with the Dirac field

In the case of mutual interaction between a massive neutral vector field and Dirac field, Shamaly and Capri[49] have shown that direct couplings of the scalar, vector, pseudovector and tensor type all lead to causal theories. On the other hand, for derivative couplings, only the vector and tensor couplings with the interaction Lagrangians

$$\mathcal{L}_{DV} = g_{DV}(\bar{\Psi}\gamma_\mu\psi)\ \partial_\mu\ \varphi_\nu^2 \qquad (4.16)$$

and

$$\mathcal{L}_{DT} = g_{DT}(\bar{\Psi}\sigma_{\mu\nu}\psi)\ \partial_\mu\varphi_\nu \qquad (4.17)$$

are causal, while the scalar, pseudoscalar and pseudovector couplings exhibit acausal behaviour. Analyzing the Lorentz covariance of these derivatively coupled theories at the classical level exactly as in the preceding examples, the surprising result emerges that the theories that follow from the Lagrangian (4.16 and (4.17) wherein no pathologies were previously reported do not possess Lorentz covariance. The energy density PB for these couplings contain the extra terms

$$f_{ok}^{DV} = g_{DV}/m^2(\bar{\Psi}\gamma_\ell\psi)\ \partial_\ell(\partial_j G_{oj})\ \varphi_k \qquad (4.18)$$

$$f_{0k}^{DT} = g_{DT}/m^2 \, (\bar{\Psi}\gamma_0\psi \cdot G_{0k} + \bar{\Psi}\sigma_{kl}\Psi \cdot \varphi_l) \tag{4.19}$$

which are not three dimensional divergences. That the acausal theories are also noncovariant is proved exactly in the same fashion.

4.4 Conclusion

The present work sheds new light on some aspects of pathological behaviour in higher spin theories. It has been shown that Lorentz covariance at the classical level does not exist in theories where acausality is present. But the most important fact emerging from the present study is that the onset of Lorentz noncovariance when interactions are introduced is a more general sort of phenomenon than acausal field propagation. Examples have been provided of theories where the causality of propagation is impeccable but are, nevertheless, non-covariant.

Observation of a new kind of anomaly in interacting spin-1 field theories by Aimsaar and Kõiv[123] which appears to be in definite correlation with noncovariance but not in immediate relation with acausality lends further corroboration to some of the results of the present work. These authors note that for a coupling bilinear in the vector field the interaction Hamiltonian $\mathcal{H}_I$ in the interaction picture is in general an infinite series in the fields to which the vector field is coupled. The vertex corresponding to such an interaction will include an infinite number of lines of other fields in addition to one passing vector boson line. For the interactions considered in the present work they conclude that

(i) for the symmetric tensor and quadrupole couplings both of which are acausal and non-covariant the expression for $\mathcal{H}_I$ is an infinite series;

(ii) every derivative-type coupling to a Dirac field, bilinear in the vector field, leads to an infinite series for $\mathcal{H}_I$. All of these are non-covariant but only the derivative vector type is causal;

(iii) the derivative tensor coupling to the Dirac field which is causal but non-covariant contains a self-interaction of the Dirac field.

The only apparent disagreement between the results of the present work and the conclusions of Ainsaar and Koiv concerns the anomalous moment coupling which is causal but non-covariant. It rurns out that this interaction has a finite expression for $\mathcal{H}_I$. This may be attributed to the fact that the electromagnetic field in the present work has been treated as external while Ainsaar and Kõiv consider mutual interactions. This becomes understandable when one recalls Hagen's conclusion, mentioned in Chapter I, that the fully quantized theory of anomalous moment coupling of a spin-1 field to an electromagnetic field is trouble-free[64].

As a final remark it is noted that the inference regarding the non-covariance of theories in Sec.4.3 may hold in the quantized version as well if PB relations are replaced by commutation(anti-commutation) relations. Thus non-covariance may be present at the quantized level in the Heisenberg picture itself.

CHAPTER V

LAGRANGE MULTIPLIER FORMALISM FOR A SPIN 3/2 FIELD

As a possible solution to the various pathologies haunting higher spin field theories a new method is suggested where the subsidiary conditions are kept separate from the equations of motion in a consistent way by means of a Lagrange multiplier. The distinct features of this approach are illustrated with reference to a massive spin 3/2 field. It is shown that in this formulation the field propagation is causal when coupled to an external field and that the energy spectrum in a magnetic field is real. Quantization is carried out in an indefinite metric space. For minimal coupling a unitary S-matrix is constructed by introducing a fictitious particle and an additional vertex.

5.1 Introduction

The problem of formulating a consistent field theory of higher spin particle interactions is as yet an unsolved one. The variety of maladies afflicting higher spin theory ranging from acausal propagation and inconsistency in quantization to break-down of Lorentz covariance and appearance of imaginary energy eigenvalues were surveyed in Chapter I. Some of the suggested remedies were also described there. This Chapter explores a new possibility of constructing a pathology-free higher spin theory by introducing a Lagrange multiplier formalism.

In the conventional formulation, as originally laid by Fierz and Pauli[5], the subsidary conditions required for elimination of redundant components are derived along with equations of motion by the variation of the Lagrangian. These constraint relations, as a rule, get modified when interaction terms are introduced into the Lagrangian. It is proposed that the alternative of keeping the subsidiary conditions separate from the equations of motion in a consistent way by means of a Lagrange multiplier may be a solution to the above mentioned pathologies of higher spin field theories. Nakanishi[125], Hsu and Sudarshan[126-127] have made use of a Lagrange multiplier formalism to construct manifestly renormalizable theories of massive vector fields and massive and massless Yang-Mills fields. But the motivation of the present work has a different origin. It tries to take into account the existence of constraint relations from the very outset, and these constraint relations are kept away from the influence of interactions which otherwise lead to troubles of various sorts.

In this chapter the method outlined above is applied to a spin 3/2 field. A Lagrange multiplier formalism is set up for a massive spin 3/2 field and the field is quantised in an indefinite metric space. It is demonstrated that the present theory is free of the usual pathologies when interaction with an electromagnetic field is introduced. However, it happens that the S-matrix is not automatically unitary in the present approach and a procedure is outlined for the restoration of the unitarity of the scattering amplitude.

A discussion of a spin 1 field in the Lagrange multiplier formalism is presented in Appendix I.

5.2 Formulation

A spin 3/2 field may be described by means of a 16-component vector-spinor ψ_μ (spinor index suppressed). The most general Lagrangian leading to an equation of motion that has at most first derivatives in it has the form

$$\mathcal{L} = \bar{\psi}_\mu(x) \Lambda_{\mu\nu}(\partial) \psi_\nu(x) \tag{5.1}$$

with

$$\Lambda_{\mu\nu}(\partial) = -(\gamma\cdot\partial + m)\,\delta_{\mu\nu} - A\,(\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu) - B\,\gamma_\mu\,\gamma\cdot\partial\,\gamma_\nu - C\,m\,\gamma_\mu\gamma_\nu \qquad (5.2)$$

where A, B, C are three parameters. If these parameters are restricted by the conditions

$$A \neq -\tfrac{1}{2} \qquad (5.3a)$$

$$B = 3/2\,A^2 + A + \tfrac{1}{2} \qquad (5.3b)$$

$$C = -(3\,A^2 + A + 1) \qquad (5.3c)$$

the Lagrangian (5.1) yields the Rarita-Schwinger theory for the irreducible spin 3/2 field which is known to be afflicted by troubles of different sorts when minimal electromagnetic coupling is introduced.

In the present formulation (5.1) is taken as the Lagrangian to start with. It is now assumed that there is a constraint relation between the 16 components of the field of the form

$$\gamma_\mu \psi_\mu(x) = 0 \qquad (5.4)$$

Unlike in the usual treatment, the Lagrangian (5.1) is varied under the assumption that the constraint relation exists independently, and it is incorporated into the Lagrangian by means of a Lagrange multiplier. The Lagrange multiplier thus introduced must be a function of space-time and hence may be treated as an additional field variable. The variation of the multiplier field in the Lagrangian yields the constraint relation a posteriori. The effective Lagrangian with the constraint incorporated is

$$\mathcal{L} = \bar{\Psi}_\mu \Lambda_{\mu\nu} \Psi_\nu + \eta \bar{\xi} \gamma_\mu \Psi_\mu + \eta \bar{\Psi}_\mu \gamma_\mu \xi \tag{5.5}$$

where ξ is the multiplier field introduced and η is a real number. It is evident that if the Lagrangian (5.5) is to be Lorentz-invariant the multiplier field must have the transformation property of a four-spinor.

The equation of motion obtained by varying $\bar{\Psi}_\mu$ is

$$\begin{aligned}[(\gamma\cdot\partial + m)\,\delta_{\mu\lambda} + A(\gamma_\mu \partial_\lambda + \gamma_\lambda \partial_\mu) + B\,\gamma_\mu\, \gamma\cdot\partial\, \gamma_\lambda \\ + C m\, \gamma_\mu \gamma_\lambda]\, \Psi_\lambda = \eta\, \gamma_\mu \xi \end{aligned} \tag{5.6}$$

and the equation resulting from the variation of $\bar{\xi}$ is the constraint condition in eq.(5.4). Because of eq.(5.9) eq.(5.6) reduces to

$$(\gamma\cdot\partial + m)\,\Psi_\mu + A\,\gamma_\mu\, \partial_\lambda \Psi_\lambda = \eta\, \gamma_\mu \xi \tag{5.7}$$

Multiplying eq.(5.7) successively with γ_μ and ∂_μ and making use of eq.(5.4), the following relations result:

$$(1/2 + A)\, \partial_\lambda \Psi_\lambda = \eta\, \xi \tag{5.8}$$

$$m\,\partial_\lambda\psi_\lambda + (1+A)\,\gamma\cdot\partial\,\partial_\lambda\psi_\lambda = \eta\,\gamma\cdot\partial\,\xi \tag{5.9}$$

Eq.(5.8) expresses the multiplier field in terms of and may be utilized to eliminate ξ from eq.(5.7). If the arbitrary parameter A is assigned a value $-\frac{1}{2}$ then $\xi = 0$, from eq.(5.8), and hence the multiplier field will disappear altogether from the formalism. But it will be seen below that to achieve consistent quantisation a different choice of A will have to be made. When A $\neq -\frac{1}{2}$ eq. (5.8) may be substituted into eq.(5.9) to derive

$$(\gamma\cdot\partial + 2m)\,\xi = (\gamma\cdot\partial + 2m)\,\partial_\lambda\psi_\lambda = 0 \tag{5.10}$$

When A $= -\frac{1}{2}$, though $\xi = 0$, from eq.(5.9) it is still possible to derive the relation

$$(\gamma\cdot\partial + 2m)\,\partial_\lambda\psi_\lambda = 0 \tag{5.11}$$

It appears, therefore, that the present theory contains in addition to the spin 3/2 particle of mass m, a spin $\frac{1}{2}$ particle of mass 2m. This becomes further evident from an inspection of the equations of motion in the rest frame as is done below.

From eqs. (5.7), (5.8) and (5.9) it follows

$$(\gamma\cdot\partial + m)\,\psi_\mu = \frac{1}{2}\gamma_\mu\,\partial_\lambda\psi_\lambda = \frac{1}{(1+2A)}\gamma_\mu\,\xi \tag{5.12}$$

where the last equality holds if $A \neq -\frac{1}{2}$. The components of ψ_μ do not satisfy the Klein-Gordon equation, but eqs. (5.10) and (5.12) together yield

$$(\Box - 4m^2)(\gamma\cdot\partial + m)\psi_\mu = 0 \tag{5.13}$$

and

$$(\Box - 4m^2)(\Box - m^2)\psi_\mu = 0 \tag{5.14}$$

From eq.(5.12) and its conjugate, with the aid of eq.(5.9) one can derive the conserved current

$$j_\mu(x) = i\,\bar{\psi}_\nu\gamma_\mu\psi_\nu(x) \tag{5.15}$$

The total charge is given by

$$Q = \int d^3x\,(\psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_3^*\psi_3 - \psi_0^*\psi_0) \tag{5.16}$$

In the present the total charge is not positive - definite. This fact may be demonstrated by examining eq.(5.12) in the rest frame. Let

$$\psi_\lambda(x) = w_\lambda(\vec{k},E)\,e^{i(\vec{k}\cdot\vec{x} - Et)} \tag{5.17}$$

be a plane-wave solution of eq.(5.12) with positive energy. In the rest frame with $\vec{k} = 0$, eq. (5.12) becomes, when eq.(5.17) is substituted,

$$(-\gamma_4 E + m) w_\mu = -\tfrac{1}{2}\gamma_\mu E w_4 \tag{5.18}$$

For $\mu = 4$ this becomes

$$(-\tfrac{1}{2}\gamma_4 E + m) w_4 = 0 \tag{5.19}$$

If $E = m$, w_4 will vanish identically as is the case in Rarita-Schwinger theory. However the present formulation also admits the value $E = 2m$ so that w_4 has non-vanishing components corresponding to this. These non-vanishing terms contribute negative terms in the expression (5.16) and the total charge becomes indefinite.

The different spin components of the field corresponding to the two possible mass states m and $2m$ may be identified by taking a closer look at equation (5.18). With $\mu = 1, 2, 3$ and $E = m$ eq.(5.18) becomes

$$(-\gamma_4 + 1) w_i = 0 \tag{5.20}$$

This implies that only the upper components of w_i survive, and these constitute six nonvanishing components. That w_4 vanishes in this case has been noted before. The constraint relation (5.9) becomes in this instance

$$\sigma_i v_i = 0 \tag{5.21}$$

where v_i is the two-component spinor forming the non-vanishing part of w_i and the σ_i are Pauli matrices. The repre-

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sentation of Dirac matrices given by

$\gamma_i = \begin{pmatrix} 0 & -i\sigma_i \\ i\sigma_i & 0 \end{pmatrix}$ has been used in obtaining eq.(5.21). Because of eq.(5.21) the six components are reduced to four independent components which constitute the spin 3/2 part of the field.

When E = 2m, eq. (5.18) becomes, for μ = 1, 2, 3

$$(-2\gamma_4 + 1)\, w_i = -\gamma_i w_4 \tag{5.22}$$

It is evident from eq.(5.19) that in this case only the upper component u_4 of w_4 can be non-vanishing. Then it follows from eq.(5.22) that the upper components of w_i vanish, and the lower components are related to u_4 by the relation

$$-i\sigma_i u_4 = 3v_i \tag{5.23}$$

Thus the six components of v_i are all expressible in terms of u_4, and there remains only two independent components for the field. Since the subsidiary condition (5.4) now becomes

$$i\sigma_i v_i = u_4 \tag{5.24}$$

which is compatible with eq.(5.23), it does not bring in any further reduction in the number of components. The two independent solutions corresponding to the value $E = 2m$ constitute the spin $\frac{1}{2}$ part of the field.

Before turning to the problem of quantisation a few remarks may be in order concerning the choice of the constraint condition (5.4). In the irreducible description of a spin 3/2 particle the vector-spinor satisfies, in addition to eq.(5.4), another constraint relation

$$\partial_\mu \psi_\mu(x) = 0 \tag{5.25}$$

If the Lagrange multiplier formalism developed above it is attempted to take into account both sets of constraint conditions by introducing one more multiplier field by means of the Lagrangian

$$\mathcal{L} = \bar{\psi}_\mu \Lambda_{\mu\nu} \psi_\nu + \eta(\bar{\xi}\gamma_\mu\psi_\mu + \bar{\psi}_\mu\gamma_\mu\xi) - \lambda(\bar{\chi}\partial_\mu\psi_\mu + \partial_\mu\bar{\psi}_\mu\cdot\chi) \tag{5.26}$$

the equation of motion becomes

$$(\gamma\cdot\partial + m)\psi_\mu = \eta\gamma_\mu\xi + \lambda\partial_\mu\chi \tag{5.27}$$

By operating with γ_μ and ∂_μ on eq.(5.27) successively, the following relations are obtained.

$$4\eta\xi + \lambda\,\gamma\cdot\partial\chi = 0 \tag{5.28}$$

$$\eta\,\gamma\cdot\partial\xi + \lambda\,\Box\chi = 0 \tag{5.29}$$

Eq.(5.28) may be used for eliminating ξ in favour of the field variable χ. But χ will survive in the equations and will constitute a spin ½ field. This comes about because of the presence of derivatives in the constraint (5.25). Since the introduction of two multiplier fields offers no definite advantage, the rest of the work follows the Lagrange multiplier formalism with a single multiplier field.

5.3 Quantization

Since in the Lagrangian (5.5) all the components are varied independently the canonical quantisation procedure may be employed to obtain the commutation relations of the field components. Canonically conjugate momenta Π_μ of ψ_μ are

$$\Pi_\mu = i\,\bar{\psi}_\nu (L_4)_{\nu\mu} \tag{5.30}$$

where

$$(L_4)_{\nu\mu} = \gamma_4 \delta_{\nu\mu} + A(\gamma_\mu \delta_{\nu 4} + \gamma_\nu \delta_{\mu 4}) + B\,\gamma_\nu \gamma_4 \gamma_\mu$$

Standard canonical anticommutation relations are given by

$$\{\psi_\mu(x), \Pi_\nu(x')\}_{x_0 = x_0'} = i\,\delta_{\mu\nu}\,\delta(\vec{x}-\vec{x}') \tag{5.31a}$$

$$\{\psi_\mu(x), \psi_\nu(x')\}_{x_0 = x_0'} = \{\Pi_\mu(x), \Pi_\nu(x')\}_{x_0 = x_0'} = 0 \tag{5.31b}$$

from which it follows that

$$\{ \psi_\mu(x) , \bar{\psi}_\nu(x') \}_{x_0 = x_0'} = (L_4)^{-1}_{\mu\nu} \, \delta(\vec{x}-\vec{x}') \tag{5.32}$$

In the Rarita-Schwinger theory with the restrictions (5.3a), b, c) L_4 is singular and its inverse does not exist. When the conditions (5.3) are not satisfied $(L_4)^{-1}$ is given by

$$\begin{aligned} (L_4)^{-1}_{\mu\nu} = \; & [(1+2A+3A^2-2B) \, \gamma_4 \, \delta_{\mu\nu} \\ & - (A^2-A-2B)(\gamma_\mu \delta_{\nu 4} + \gamma_\nu \delta_{\mu 4}) - 2(2A \\ & + A^2+2B) \, \gamma_4 \, \delta_{\mu 4} \, \delta_{\nu 4} + (A^2-B) \, \gamma_\mu \gamma_4 \gamma_\nu] \\ & \times 1/(1+2A+3A^2-2B) \, . \end{aligned} \tag{5.33}$$

Since the parameters A and B are arbitrary, they may be chosen in such a way that the commutation relation (5.32) is consistent with the subsidiary condition (5.4). It is easily verified that consistency is achieved by the choice A = 1 and B = 3/4. With A and B so determined the commutation relations may be written as

$$\begin{aligned} \{ \psi_\mu(x) , \bar{\psi}_\nu(x') \}_{x_0 = x_0'} = \; & [\delta_{\mu\nu} \gamma_4 + \tfrac{1}{2} \gamma_\mu \gamma_4 \gamma_\nu \\ & - (\delta_{\mu 4} \gamma_\nu + \delta_{\nu 4} \gamma_\mu) + 2 \gamma_4 \delta_{\mu 4} \delta_{\nu 4}] \, \delta(\vec{x}-\vec{x}') \end{aligned} \tag{5.34}$$

The requirements of relativistic covariance, local commutativity and the equations (5.4), (5.13) and (5.14) determine the general form of the commutator for arbitrary separations to be

$$\{ \psi_\mu(x), \bar{\psi}_\nu(x') \} = -ia\,(\gamma\cdot\partial - m)\,[\,\delta_{\mu\nu} - \tfrac{1}{3}\gamma_\mu\gamma_\nu$$

$$+ \tfrac{1}{3}(\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu) - 2/3m^2\,\partial_\mu\partial_\nu\,]\,\Delta(x-x';m^2)$$

$$+ ib\,(\partial_\mu + \tfrac{1}{2}m\gamma_\mu)(\gamma\cdot\partial - 2m)(\partial_\nu + \tfrac{1}{2}m\gamma_\nu)$$

$$\times\,\Delta(x-x';4m^2) \tag{5.35}$$

where

$$\Delta(x-x';m^2) = -i/(2\pi)^3 \int d^4k\ \epsilon(k_0)\,\delta(k^2+m^2)\,e^{ik\cdot x} \tag{5.36}$$

In order that the above expression be consistent with the equal time commutation relations given by eq.(5.34) the constants a and b must be chosen as a = 1 and b = 2/3m^2. Eqs. (5.34) and (5.8) with A = -1 may be employed in deriving the further commutation relations

$$\{ \xi(x), \bar{\psi}_\nu(x') \} = -3i/\eta\,(\gamma\cdot\partial - 2m)(\partial_\nu + \tfrac{1}{2}m\gamma_\nu)$$

$$\times\,\Delta(x-x';4m^2) \tag{5.37}$$

$$\{ \xi(x) , \xi(x') \} = \frac{27im^2}{2\eta^2} (\gamma\cdot\partial - 2m) \Delta(x-x'; 4m^2) \quad (5.38)$$

The last of the commutation relations shows that the spin ½ field has a negative metric in Hilbert space.

The Feynman propagator of the field, defined as the vacuum expectation value of the time-ordered product of field operators is easily evaluated in the usual manner. Decomposing the field operators into positive and negative frequency parts and making use of the general commutation relations (5.35) it is found that

$$\begin{aligned} S_{F\mu\nu}(x_1-x_2) &= \langle 0 | T \psi_\mu(x_1) \bar{\psi}_\nu(x_2) | 0 \rangle \\ &= \theta(t_1-t_2) [d_{\mu\nu}(\partial) \, i \, \Delta^+ (x_1-x_2; m^2) \\ &\quad + d'_{\mu\nu}(\partial) \, \Delta^+ (x_1-x_2; 4m^2)] \\ &\quad - \theta(t_2-t_1) [d_{\mu\nu}(\partial) \, i \, \Delta^- (x_1-x_2; m^2) \\ &\quad + d'_{\mu\nu}(\partial) \, i \, \Delta^- (x_1-x_2; 4m^2)] \end{aligned} \quad (5.39)$$

where

$$\begin{aligned} d_{\mu\nu}(\partial) &= -(\gamma\cdot\partial - m) [\delta_{\mu\nu} - \tfrac{1}{3} \gamma_\mu \gamma_\nu \\ &\quad + \tfrac{1}{3} (\gamma_\mu \partial_\nu - \gamma_\nu \partial_\mu) - {}^2\!/_{3m^2} \, \partial_\mu \partial_\nu] \end{aligned} \quad (5.40)$$

$$d'_{\mu\nu}(\partial) = 2/3m^2 \,(\partial_\mu + \tfrac{m}{2}\gamma_\mu)(\gamma\cdot\partial - 2m)(\partial_\nu + \tfrac{m}{2}\gamma_\nu) \qquad (5.41)$$

and $\Delta^{\pm}(x)$ are the positive and negative frequency parts of $\Delta(x)$. In the present formulation the normal-dependent terms that arise when θ-functions are commuted past the derivatives cancel each other and the propagator is rigorously given by

$$\begin{aligned} S_{F\mu\nu}(x_1-x_2) &= -(\gamma\cdot\partial - m)\,[\,\delta_{\mu\nu} - \tfrac{1}{3}\gamma_\mu\gamma_\nu \\ &\quad + \tfrac{1}{3}(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu) - 2/3m^2\,\partial_\mu\partial_\nu\,]\,\Delta_F(x_1-x_2;m^2) \\ &\quad - 2/3m^2\,(\partial_\mu + \tfrac{m}{2}\gamma_\mu)(\gamma\cdot\partial - 2m)(\partial_\nu + \tfrac{m}{2}\gamma_\nu) \\ &\quad \times \Delta_F(x_1-x_2;4m^2) \end{aligned} \qquad (5.42)$$

where

$$\Delta_F(x_1-x_2;m^2) = -i/(2\pi)^4 \int d^4k \, \frac{e^{ik\cdot x}}{k^2+m^2-i\epsilon} \qquad (5.43)$$

5.4 Interactions and pathologies

To discuss the question of pathologies in the presence of interaction the coupling of the above described mixed spin 3/2 - spin ½ field to an electromagnetic field may be considered. To begin with, it is assumed that the electromagnetic field is an external one.

a) Causality of propagation

Introducing minimal coupling into the Lagrangian (5.5) with the subsidiary condition kept unchanged the gauge-invariant field equation in the presence of an external electromagnetic field is obtained as

$$(\gamma \cdot D + m)\psi_\mu + A\,\gamma_\mu D_\lambda \psi_\lambda = \gamma_\mu \xi \tag{5.44}$$

where $D_\mu = \partial_\mu - ie A_\mu$. Taking into account the subsidiary condition (5.4) it is easily verified that ξ satisfies the equation

$$(\gamma \cdot D + m)\xi = 4ie/(1+2A)\, F_{\mu\lambda}\gamma_\lambda \psi_\mu \tag{5.45}$$

where $F_{\mu\lambda}$ is the electromagnetic field tensor $F_{\mu\lambda} = \partial_\mu A_\lambda - \partial_\lambda A_\mu$. Expressing ξ in terms of $D_\lambda \psi_\lambda$, the equation (5.44) may be rewritten in the form

$$(\gamma \cdot D + m)\psi_\mu - \tfrac{1}{2}\gamma_\mu D_\lambda \psi_\lambda = 0 \tag{5.46}$$

It is evident from this equation that the propagation character is unaffected by the coupling and that the characteristic determinant D(n), where n denotes a unit normal to the wave-front, has the same value as in the free field case and is given by

$$D(n) = \tfrac{1}{2}(n^2)^8 \tag{5.47}$$

so that the propagation is light-like and hence causal.

(b) Energy eigenvalues in a homogeneous magnetic field

The method developed by Mathews[78] is applied to determine the energy eigenvalue spectrum in the presence of a homogeneous magnetic field.

Define

$$D_\pm = D_1 \pm i D_2$$

$$\psi_\pm = \psi_1 \pm i \psi_2$$

$$\gamma_\pm = \frac{1}{2} (\gamma_1 \pm i \gamma_2)$$

Eq.(5.46) with $\mu = 1, 2$ can be manipulated to give two equations that involve ψ_+ and ψ_- only decoupled from each other

$$E \gamma_4 \gamma_+ \psi_+ + 2 (\gamma_+ D_+ + \gamma_- D_-) \gamma_- \psi_+ - (\gamma_3 D_3 - m) \gamma_+ \psi_+ = 0 \tag{5.48}$$

$$E \gamma_4 \gamma_- \psi_- + 2 (\gamma_+ D_+ + \gamma_- D_-) \gamma_+ \psi_- - (\gamma_3 D_3 - m) \gamma_- \psi_- = 0 \tag{5.49}$$

for stationary state solutions with a time-dependence e^{-iEt}. With ψ_+ partioned in the form $\psi_+ = \begin{pmatrix} \varphi_+ \\ \chi_+ \end{pmatrix}$ and ψ_- in the form $\psi_- = \begin{pmatrix} \varphi_- \\ \chi_- \end{pmatrix}$

eq.(5.48) becomes a pair of coupled equations

$$(E+m)\,\sigma_+\chi_+ \;+\; (2eH)^{1/2}\,(2a\sigma_+\sigma_- \;+\; a_3\sigma_3\sigma_+)\,\varphi_+ \;=\; 0 \qquad (5.50)$$

$$(E-m)\,\sigma_-\chi_- \;+\; (2eH)^{1/2}\,(2a\sigma_+\sigma_- \;+\; a_3\sigma_3\sigma_+)\,\chi_+ \;=\; 0 \qquad (5.51)$$

where

$$a = (2eH)^{-1/2}D_+ \quad , \quad a^+ = (2eH)^{-1/2}D_-$$

The fact that a and a^+ satisfy an algebra equivalent to that of a harmonic oscillator may now be exploited to reduce eqs.(5.50) and (5.51) into an ordinary matrix eigenvalue equation. Since a_3 commutes with all other operators it may be replaced by its eigenvalues in eqs.(5.50) and (5.51). Defining the states $|n,\alpha\rangle$, $|n,\beta\rangle$ by

$$N\,|n,\alpha\rangle = n\,|n,\alpha\rangle \quad , \quad N\,|n,\beta\rangle = n\,|n,\beta\rangle$$

$$\sigma_3\,|n,\alpha\rangle = |n,\alpha\rangle \quad , \quad \sigma_3\,|n,\beta\rangle = -\,|n,\beta\rangle \qquad (5.52)$$

where N = a^+ a, it is seen that the solutions of eq.(5.50) are of the general form

$$\begin{aligned} \varphi_+ &= c_1\,|n_1,\beta\rangle \\ \chi_+ &= c_2\,|n_2,\beta\rangle \end{aligned} \qquad (5.53)$$

Substituting in eq.(5.50) and equating the coefficients of $|n,\alpha\rangle$ on both sides a set of relations connecting c_1 and c_2 are obtained which can be written completely as

$$Hc = Ec \qquad (5.54)$$

where $c = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

and

$$H = \begin{pmatrix} m & -(2eH)^{1/2} a_3 \\ -(2eH)^{1/2} a_3 & -m \end{pmatrix}$$

The eigenvalues of H are easily found to be given by

$$E^2 = m^2 \left[1 + a_3^2 \cdot 2eH/m^2 \right] \qquad (5.55)$$

and hence are all real. The same procedure can be applied to eq.(5.51). The components of ψ_3 and ψ_4 on the other hand can be expressed in terms of ψ_+ , ψ_- and their space-derivatives.

5.5 S-matrix and unitarity

Consider the interaction of the vector-spinor field with a quantised electromagnetic field having the Lagrangian

$$\mathcal{L} = \bar{\psi}_\mu \Lambda_{\mu\nu}(D) \psi_\nu + \bar{\xi} \gamma_\mu \psi_\mu + \bar{\psi}_\mu \gamma_\mu \xi + 1/4\, F_{\mu\nu} F_{\mu\nu} \tag{5.56}$$

where $\Lambda_{\mu\nu}(D)$ denotes the expression (5.2) with $\partial_\mu \to D_\mu$, and the conjugation is now taken to be that in an indefinite metric space. Physical states in the theory must not contain the negative-metric carrying spin ½ particle and the physical states are defined by the condition

$$\xi^{(+)}(x) \,|\,phys\rangle = 0 \;, \quad \bar{\xi}^{(+)}(x) \,|\,phys\rangle = 0 \tag{5.57}$$

When the interaction is switched on the spin ½ particle does not remain free but gets coupled to the electromagnetic field, as is shown by eq.(5.45). Because of the interaction of the negative metric particle the S-matrix defined in the physical subspace of the indefinite metric space will not be automatically unitary. However, by adopting the method followed by Hsu[125, 126] to deal with a similar contingency in a different context, the unitarity of the S-matrix may be restored by introducing a fictitious particle and a new vertex. That this is possible may be demonstrated by examining the scattering amplitude for the self-energy process of the spin 3/2 - spin ½ particle $\psi(p) \to \psi(p-k)\,\gamma(k) \to \psi(p)$ which is obtained, in the second order, to be

$$S^{(2)} = e^2/V \int d^4k \ \bar{u}^{\alpha}_{\mu}(p) \gamma_\lambda \Big[d_{\mu\nu}(ik) \frac{-i}{(p-k)^2+m^2}$$

$$+ 2/3m^2 \{ i(p-k)_\mu + \tfrac{1}{2} m\gamma_\mu \} \frac{-1}{\gamma\cdot(p-k)-2im}$$

$$\times \{ i(p-k)_\nu + \tfrac{1}{2} m\gamma_\nu \} \Big] \gamma_\rho \ u^{\alpha}_{\nu}(p) \ \delta_{\lambda\rho} \ \frac{(-i)}{k^2} \tag{5.58}$$

where u^{α}_{μ} (α = 1, 2, .. . 6) are the wave functions the explicit forms of which are not needed for the present discussion.

This contains an extra amplitude coming from

$$e^2/V \int d^4k \ 2/3m^2 \ \bar{u}^{\alpha}_{\mu}(p)\gamma_\lambda \ \{ i(p-k)_\mu + \tfrac{1}{2} m\gamma_\mu \}$$

$$\times \frac{-1}{\gamma\cdot(p-k)-2im} \{ i(p-k)_\nu + \tfrac{1}{2} m\gamma_\nu \} \gamma_\rho \ u^{\alpha}_{\nu}(p)$$

$$\times \delta_{\lambda\rho} \ \frac{-i}{k^2} \tag{5.59}$$

This extra contribution may be removed by introducing a fictitious spin ½ particle F intothe theory. Let the corresponding Dirac field be ζ. If this interaction of the F particle is chosen to have the form $\bar{\psi}_\mu \gamma_\nu (\partial_\nu + \frac{m}{2}\gamma_\nu)\zeta A_\mu$ it is straightforward to verify that the extra amplitude to the self-energy is cancelled (in the second order) by the contribution from the process $\psi(p) \to F(p-k)\,\gamma(k) \to \psi(p)$.

It is conjectured that the cancellation is valid in all orders of perturbation theory.

5.6 Conclusion

The Lagrange multiplier formalism, though successful in eliminating some of the pathologies such as causality violation and existence of imaginary energy eigen-values in a homogeneous magnetic field and difficulties of quantization, leads to a multi-mass, multi-spin theory with indefinite metric. This result seems to be in accord with conjecture of Prabhakaran, Seetharaman and Mathews[99] that for half-integer spins greater than ½ causality in the presence of an electromagnetic interaction may be retained only if one starts with a free theory which is reducible and wherein the total charge is indefinite. The notable fact is that the requirement that the subsidiary conditions remain separate from the equations motion leads naturally to such a formalism. Unlike in the Rárita-Schwinger case, the Feynman propagator turns out to be covariant in the present theory of spin 3/2 field. The results of this study lend ample sup ort to the currently popular view that if there exist fundamental higher spin fields, they must have a multi-mass, multi-spin structure. Though an indefinite metric is inevitable in the present approach the unitarity problem may be handled by the introduction of further ghost fields. The somewhat ad hoc nature of this procedure may not be a serious drawback when viewed in the context of the totally unsatisfactory state of the present day higher spin field theory.

APPENDIX I

Lagrange multiplier formalism for a spin 1 field

The Lagrange multiplier formalism may easily be adapted for the description of a spin 1 field. In this case the present formalism becomes almost indistinguishable from Nakanishi's method[124] of constructing a renormalisable massive vector field with the introduction of a scalar ghost field.

The starting Lagrangian in the present treatment is the same as in conventional formulation

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu)(\partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu) - \tfrac{1}{2} m^2 \varphi_\mu \varphi_\mu \qquad (5A.1)$$

The constraint restricting the field components is

$$\partial_\mu \varphi_\mu = 0 \qquad (5A.2)$$

Introducing a scalar multiplier field to incorporate the constraint (5A.2) in (5A.1), the Lagrangian may be reformulated as

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu)(\partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu) - \tfrac{1}{2} m^2 \varphi_\mu \varphi_\mu - \eta \chi \partial_\mu \varphi_\mu \qquad (5A.3)$$

with the resulting equation of motion

$$(\Box - m^2)\varphi_\mu = \eta \partial_\mu \chi \qquad (5A.4)$$

From (5A.4) and (5A.2) it is seen that χ satisfies the equation

$$\Box \chi = 0 \tag{5A.5}$$

and hence corresponds to a massless scalar field. From (5A.5) and (5A.4) it follows that the components of satisfy the multi-Klein-Gordon equation

$$\Box(\Box - m^2)\varphi_\mu = 0 \tag{5A.6}$$

Canonical quantization of the field is easily carried out by defining the conjugate momenta

$$\pi_k = \dot{\varphi}_k + \partial_k \varphi_0 \tag{5A.7a}$$

$$\pi_0 = \eta \chi \tag{5A.7b}$$

and the equal-time commutation relations are

$$[\varphi_\mu(x), \varphi_\nu(x')]_{x_0 = x_0'} = 0 \tag{5A.8a}$$

$$[\varphi_\mu(x), \dot{\varphi}_\ell(x')]_{x_0 = x_0'} = i\,\delta_{\mu\ell}\,\delta(\vec{x}-\vec{x}') \tag{5A.8b}$$

$$[\dot{\varphi}_k(x), \dot{\varphi}_\ell(x')]_{x_0 = x_0'} = 0 \tag{5A.8c}$$

$$[\varphi_\mu(x), \chi(x')]_{x_0 = x_0'} = i/\eta\,\delta_{\mu 0}\,\delta(\vec{x}-\vec{x}') \tag{5A.8d}$$

$$[\dot{\varphi}_k(x), \chi(x')]_{x_0 = x_0'} = i/\eta\,\partial_k\,\delta(\vec{x}-\vec{x}') \tag{5A.8e}$$

$$[\chi(x), \chi(x')]_{x_0=x_0'} = 0 \tag{5A.8f}$$

The general commutation relation that satisfies the requirements of relativistic invariance, locality, eq.(5A.6) and that reproduces (5A.8) at equal time is

$$[\varphi_\mu(x), \varphi_\nu(x')] = i\,(\delta_{\mu\nu} - 1/m^2\,\partial_\mu\partial_\nu)\,\Delta(x-x'; m^2)$$

$$+\; i/m^2\,\partial_\mu\partial_\nu\,\Delta(x-x'; 0) \tag{5A.9}$$

The Feynman propagator for the field is

$$\Delta_{F\mu\nu}(x) = (\delta_{\mu\nu} - 1/m^2\,\partial_\mu\partial_\nu)\,\Delta_F(x; m^2)$$

$$+\; 1/m^2\,\partial_\mu\partial_\nu\,\Delta_F(x; 0) \tag{5A.10}$$

To demonstrate the absence of acausal propagation in the present formalism when interactions are introduced consider the equation

$$(\Box - m^2)\,\varphi_\mu - \eta\,\partial_\mu\chi + j_\mu = 0 \tag{5A.11}$$

which results when the vector field is coupled to a current j_μ. Since the constraint relation would remain unmodified it follows from(5A.11) that

$$\Box\chi = 1/\eta\,\partial_\mu j_\mu \tag{5A.12}$$

and by combining (5A.11) and (5A.12) one obtains

$$\Box(\Box - m^2)\varphi_\mu + \Box j_\mu - \partial_\mu \partial_\nu j_\nu = 0 \tag{5A.13}$$

It is evident from (5A.13) that if j_μ does not involve derivatives higher than the first of φ_μ, the propagation will remain light-like and hence causal. Thus the symmetric tensor coupling and the self-interaction of the neutral vector field which are acausal in the conventional formulation are causal in the present theory.

The rest of the development of the theory may be carried out along the lines followed for the spin 3/2 case.

CHAPTER VI

A SPIN 2 FIELD IN LAGRANGE MULTIPLIER FORMALISM

The Lagrange multiplier method is extended to the description of a spin-2 field. The method bypasses the problem of constraint break-down in the presence of interactions and the theory is shown to remain causal when coupled minimally to an electromagnetic field. Canonical quantization of the field is carried out and a covariant propagator obtained. The massless limit of the theory is also discussed.

6.1 Introduction

In the preceding chapter a Lagrange multiplier formalism was developed and applied to a spin 3/2 field. Although the resulting theory was one of multi-mass, multi-spin type with an indefinite metric, it was found to possess definite advantages over conventional formulations especially in eliminating pathologies that usually arise in presence of interactions. In this chapter a Lagrange multiplier formalism is set up for a spin 2 field described by a symmetric tensor. Quantization of the field is carried out and the absence of acausal propagation is demonstrated. A discussion is also given of the physically interesting case of the massless limit of the theory.

6.2 Formulation

An irreducible spin-2 field of mass m is described by the set of equations

$$(\Box - m^2)\,\psi_{\mu\nu} = 0 \qquad (6.1)$$

$$\partial_\mu \psi_{\mu\nu} = 0 \qquad (6.2)$$

$$\psi_{\mu\mu} = 0 \qquad (6.3)$$

where $\psi_{\mu\nu}$ is symmetric tensor of rank 2. A general Lagrangian density $\mathcal{L}_0$ given by

$$\mathcal{L}_0 = -\partial_\lambda \psi^*_{\mu\nu}\, \partial_\lambda \psi_{\mu\nu} - m^2 \psi^*_{\mu\nu}\psi_{\mu\nu} + \partial_\lambda \psi^*_{\mu\nu}\, \partial_\mu \psi_{\lambda\nu}$$
$$+ \partial_\mu \psi^*_{\mu\nu}\, \partial_\lambda \psi_{\lambda\nu} + A\,(\partial_\mu \psi^*_{\mu\nu}\, \partial_\nu \psi + \partial_\nu \psi^* \partial_\mu \psi_{\mu\nu})$$
$$+ B\, \partial_\nu \psi^* \partial_\nu \psi + m^2 C\, \psi^* \psi \qquad (6.4)$$

where $\psi = \psi_{\mu\mu}$ and A, B, C are arbitrary parameters, will yield the equations (6.1) - (6.3) when A, B, C are restricted by the conditions

$$A \neq -\tfrac{1}{2} \qquad (6.5a)$$

$$B = 3/2\, A^2 + A + \tfrac{1}{2} \qquad (6.5b)$$

$$C = 3\, A^2 + A + 1 \qquad (6.5c)$$

This is the formulation as given by Nath[27] and under the stipulated conditions the Lagrangian (6.4) is equivalent to the Fierz-Pauli[5] Lagrangian for the spin 2 field. In presence of minimal electromagnetic coupling these formulations suffer from acausality of propagation and inconsistency of quantization.

In the Lagrange multiplier method $\mathcal{L}_0$ is the Lagrangian to start with, and the constraint relation (6.2) is assumed to exist independently and is incorporated into the Lagrangian by means of a multiplier field χ_μ which for the sake of relativistic invariance must be a vector.

$$\mathcal{L} = \mathcal{L}_0 + \eta\,(\chi_\nu^* \partial_\mu \psi_{\mu\nu} + \chi_\nu \partial_\mu \psi^*_{\mu\nu}) \qquad (6.6)$$

Here it may be noted that only one of the set of constraint relations (6.2) and (6.3) is taken into account; a discussion of the formulation with both sets of constraints incorporated is given in Appendix I.

Variation of $\psi^*_{\mu\nu}$ in eq.(6.6) gives the equation

$$(\Box - m^2)\psi_{\mu\nu} - (\partial_\lambda \partial_\mu \psi_{\lambda\nu} + \partial_\lambda \partial_\nu \psi_{\lambda\mu}) - A\,\partial_\mu\partial_\nu\psi$$

$$- \delta_{\mu\nu}\,[\,A\,\partial_\rho\partial_\lambda\psi_{\rho\lambda} + B\,\Box\psi\,] + \delta_{\mu\nu} m^2 C\psi$$

$$= \tfrac{1}{2}\,(\partial_\mu \chi_\nu + \partial_\nu \chi_\mu) \qquad (6.7)$$

and the variation of χ^*_μ reproduces the constraint (6.2). Making use of eq.(6.2) in eq.(6.7) it follows

$$(\Box - m^2)\psi_{\mu\nu} - A\,\partial_\mu\partial_\nu\psi - \delta_{\mu\nu}\,B\,\Box\psi$$

$$+ \delta_{\mu\nu} m^2 C\,\psi = \tfrac{1}{2}\,(\partial_\mu\chi_\nu + \partial_\nu\chi_\mu) \qquad (6.8)$$

Putting $\mu=\nu$ and summing over μ in eq.(6.8) one obtains

$$(1 - A - 4B)\,\Box\psi + (4C-1)\,m^2\psi = \partial_\mu\chi_\mu \qquad (6.9)$$

Taking the divergence of eq.(6.8) there results, when account is taken of the subsidiary condition (6.2)

$$-(A+B)\Box\partial_\nu\psi + Cm^2\partial_\nu\psi = \tfrac{1}{2}(\Box\chi_\nu + \partial_\nu\partial_\mu\chi_\mu) \quad (6.10)$$

Equations (6.9) and (6.10) together yield the relation

$$\Box\chi_\nu = (2B-A-1)\Box\partial_\nu\psi - (2C-1)m^2\partial_\nu\psi \quad (6.11)$$

In contrast to the situation of the spin 3/2 field with the constraint (5.4) it appears that in the present case, because of the derivative nature of the constraint, the multiplier field χ_μ cannot be expressed locally in terms of the components of $\psi_{\mu\nu}$. However, the multiplier field can be eliminated from eq.(6.8) by operating with $\Box$ on either side of the equation and making use of eq. (6.11):

$$\Box(\Box-m^2)\psi_{\mu\nu} + (1-2B)\Box\partial_\mu\partial_\nu\psi - \delta_{\mu\nu}B(\Box)^2\psi$$
$$+ \delta_{\mu\nu}Cm^2\Box\psi + (2C-1)m^2\partial_\mu\partial_\nu\psi = 0 \quad (6.12)$$

If it is now assumed that $B = C \neq 1/3$ it is immediately seen from eq. (6.12) that

$$\Box(\Box-m^2)\psi = 0 \quad (6.13)$$

and as a consequence

$$(\Box)^2(\Box-m^2)\psi_{\mu\nu} = 0 \quad (6.14)$$

The symmetric tensor in the present theory possesses six independent components. In addition a vector field is present, the divergence of which is related to ψ through eq.(6.9). Hence it may be expected that the present formulation would contain besides the spin 2 particle, a particle of spin 1 and another of spin 0. This inference is further borne out by a study of the quantized version of the above formalism.

6.3 Quantization

Since in the Lagrangian (6.6) the variations of the field components are all taken to be independent of one another, the canonical quantization procedure may be adopted for deriving the field commutation relations. The canonical momenta conjugate to $\psi_{\mu\nu}$ and $\psi^*_{\mu\nu}$ are defined by

$$\begin{aligned}
\Pi_{\mu\nu} &= \dot{\psi}^*_{\mu\nu} - i/2\,(\partial_\mu \psi^*_{4\nu} + \partial_\nu \psi^*_{4\mu}) \\
&\quad - i/2\,(\delta_{\mu 4}\,\partial_\rho \psi^*_{\rho\nu} + \delta_{\nu 4}\,\partial_\rho \psi^*_{\rho\mu}) \\
&\quad - iA/2\,(\delta_{\mu 4}\,\partial_\nu \psi^* + \delta_{\nu 4}\,\partial_\mu \psi^*) - B\,\dot{\psi}^*\,\delta_{\mu\nu} \\
&\quad - i/2\,(\delta_{\mu 4}\,\chi^*_\nu + \delta_{\nu 4}\,\chi^*_\mu) - i A\,\delta_{\mu\nu}\,\partial_\rho \psi^*_{\rho 4}
\end{aligned} \tag{6.15a}$$

$$\begin{aligned}
\Pi^*_{\mu\nu} &= \dot{\psi}_{\mu\nu} - i/2\,(\partial_\mu \psi_{4\nu} + \partial_\nu \psi_{4\mu}) \\
&\quad - i/2\,(\delta_{\mu 4}\,\partial_\rho \psi_{\rho\nu} + \delta_{\nu 4}\,\partial_\rho \psi_{\rho\mu}) \\
&\quad - iA/2\,(\delta_{\mu 4}\,\partial_\nu \psi + \delta_{\nu 4}\,\partial_\mu \psi) - B\,\dot{\psi}\,\delta_{\mu\nu} \\
&\quad - i/2\,(\delta_{\mu 4}\,\chi_\nu + \delta_{\nu 4}\,\chi_\mu) - i A\,\delta_{\mu\nu}\,\partial_\rho \psi_{\rho 4}
\end{aligned} \tag{6.15b}$$

The Hamiltonian density of the field is easily evaluated with the aid of eqs. (6.15), (6.6) and eqs. (6.7) and (6.2).

$$
\begin{aligned}
\mathcal{H} = \Big[& (\pi^*_{ij} + i\,\partial_i \psi_{4j})(\pi_{ij} + i\,\partial_i \psi^*_{4j}) \\
& + B/(1-3B)\, \{ \pi^*_{kk}\pi_{jj} + i/2\,(\pi^*_{kk}\,\partial_j \psi^*_{4j} + \pi_{kk}\,\partial_j \psi_{4j}) \} \\
& + \partial_j \psi_{ij}\,(\partial_k \psi^*_{ik} + \partial_i \psi^*_{44} - 2i\,\pi_{i4}) \\
& + \partial_j \psi^*_{ij}\,(\partial_k \psi_{ik} + \partial_i \psi_{44} - 2i\,\pi^*_{i4}) \\
& - i\,\partial_j \psi_{4j}\, \{ \pi_{44} + B/2(1-3B)\, \pi_{kk} \} \\
& - i\,\partial_j \psi^*_{4j}\, \{ \pi^*_{44} + B/2(1-3B)\, \pi^*_{kk} \} \\
& - \partial_i \psi^*_{j\nu}\,\partial_j \psi_{i\nu} + \partial_i \psi^*_{\mu\nu}\,\partial_i \psi_{\mu\nu} + m^2 \psi_{\mu\nu}\psi_{\mu\nu} \\
& - B\,\partial_i \psi^*\,\partial_i \psi^* - m^2 C\,\psi^*\psi \Big]
\end{aligned}
\qquad (6.16)
$$

As might have been expected the Hamiltonian density is not positive-definite due to the presence of additional spin constituents, and hence quantization will have to be carried out in an indefinite metric space. Taking the symmetry of the field components into account canonical commutation relations may be written in the form

$$[\psi_{\mu\nu}(x)\,,\,\Pi_{\rho\lambda}(x')]_{x_0=x_0'} = i/2\,(\delta_{\mu\rho}\delta_{\nu\lambda} + \delta_{\mu\lambda}\delta_{\nu\rho}) \times \delta(\vec{x}-\vec{x}') \tag{6.17a}$$

$$[\psi^*_{\mu\nu}(x)\,,\,\Pi^*_{\rho\lambda}(x')]_{x_0=x_0'} = i/2\,(\delta_{\mu\rho}\delta_{\nu\lambda} + \delta_{\mu\lambda}\delta_{\nu\rho}) \times \delta(\vec{x}-\vec{x}') \tag{6.17b}$$

$$[\psi_{\mu\nu}(x)\,,\,\psi_{\rho\lambda}(x')]_{x_0=x_0'} = [\psi_{\mu\nu}(x)\,,\,\psi^*_{\rho\lambda}(x')] = 0 \tag{6.17c}$$

$$[\Pi_{\mu\nu}(x)\,,\,\Pi_{\rho\lambda}(x')]_{x_0=x_0'} = [\Pi_{\mu\nu}(x)\,,\,\Pi^*_{\rho\lambda}(x')]_{x_0=x_0'} = 0 \tag{6.17d}$$

These commutation relations are not automatically consistent with the constraint relation (6.2). By requiring that eq.(6.17) be compatible with the constraint relations the above commutation relations can be reexpressed in terms of $\psi_{\mu\nu}$, $\dot{\psi}_{\mu\nu}$ and χ_μ (Appendix II).

The consistency of the quantization procedure can be demonstrated by showing that the Heisenberg equations

$$\dot{\psi}_{\mu\nu}(x) = 1/i\,[\psi_{\mu\nu}(x)\,,\,H] \tag{6.18a}$$

$$\dot{\Pi}_{\mu\nu}(x) = 1/i\,[\Pi_{\mu\nu}(x)\,,\,H] \tag{6.18b}$$

where $H = \int d^3x\, \mathcal{H}$ and $\mathcal{H}$ is as given by eq. (6.16), reproduces equations (6.8) and (6.2) when the canonical commutators (6.17) are employed in eq. (6.18). To see this consider first the equations for ψ_{i4} and ψ_{44} which by eq. (6.17) are

$$\dot{\psi}_{i4} = 1/i \, [\psi_{i4}, H] = -i\,\partial_j \psi_{ij} \tag{6.19a}$$

$$\dot{\psi}_{44} = 1/i \, [\psi_{44}, H] = -i\,\partial_j \psi_{4j} \tag{6.19b}$$

These two equations together are identical with the constraint condition (6.2). Consider now the equations for the components ψ_{ij} and π^{*}_{ij}

$$\begin{aligned} \dot{\psi}_{ij} = 1/i \, [\psi_{ij}, H] &= \pi^{*}_{ij} + 1/2\,(\partial_i \psi_{4j} + \partial_j \psi_{4i}) \\ &\quad + B/(1-3B)\; \delta_{ij}\, \pi^{*}_{kk} \end{aligned} \tag{6.20}$$

$$\begin{aligned} \dot{\pi}^{*}_{ij} = 1/i \, [\pi^{*}_{ij}, H] &= -i\,(\partial_i \pi^{*}_{j4} + \partial_j \pi^{*}_{i4}) + \partial_i \partial_j \psi_{44} \\ &\quad + \partial_k^2 \psi_{ij} - m^2 \psi_{ij} - B\,\delta_{ij}\,\partial_k^2 \psi + m^2 C\,\delta_{ij}\,\psi \end{aligned} \tag{6.21}$$

Eq.(6.20) is an identity corresponding to one of the relations in the definitions of the canonical momenta (6.15a). Taking the time-derivative of eq.(6.20) and substituting from eq.(6.21) and making use of the already derived equations (6.19) and the defining relation (6.15a), the following second order equation emerges

$$\begin{aligned} &(\Box - m^2)\,\psi_{ij} - B\,\delta_{ij}\,\Box\psi - A\,\partial_i \partial_j \psi \\ &\quad + \delta_{ij}\, m^2 C\,\psi - 1/2\,(\partial_i \chi_j + \partial_j \chi_i) = 0 \end{aligned} \tag{6.22}$$

This equation is identical with the ij components of eq.(6.8) The rest of the equations of motion are recovered in a similar fashion by considering the Heisenberg equations for π^*_{i4} and π^*_{44} .

The general form of the commutator $[\psi_{\mu\nu}(x), \psi^*_{\rho\lambda}(x')]$ for arbitrary separations can be inferred by invoking the principles of relativistic covariance and local commutativity and the eqs. (6.14) and (6.2).

$$
\begin{aligned}
[\psi_{\mu\nu}(x), \psi^*_{\rho\lambda}(x')] = a\ \Big[& \tfrac{1}{2}(\delta_{\mu\rho}\delta_{\nu\lambda} + \delta_{\mu\lambda}\delta_{\nu\rho}) \\
& - \delta_{\mu\nu}\delta_{\rho\lambda} + m^{-2}(\delta_{\mu\nu}\partial_\rho\partial_\lambda + \delta_{\rho\lambda}\partial_\mu\partial_\nu) \\
& - \tfrac{1}{2} m^{-2}(\delta_{\mu\rho}\partial_\nu\partial_\lambda + \delta_{\mu\lambda}\partial_\nu\partial_\rho + \delta_{\nu\rho}\partial_\mu\partial_\lambda \\
& + \delta_{\nu\lambda}\partial_\mu\partial_\rho)\Big]\ \Delta(x-x'; m^2) + b\,\Big[m^{-2}(\delta_{\mu\nu}\partial_\rho\partial_\lambda \\
& + \delta_{\rho\lambda}\partial_\mu\partial_\nu) - \tfrac{1}{2} m^{-2}(\delta_{\mu\rho}\partial_\nu\partial_\lambda + \delta_{\mu\lambda}\partial_\nu\partial_\rho \\
& + \delta_{\nu\rho}\partial_\mu\partial_\lambda + \delta_{\nu\lambda}\partial_\mu\partial_\rho)\Big]\ \Delta(x-x'; 0)
\end{aligned} \tag{6.23}
$$

Apart from the constants a and b the coefficients of the various terms in eq.(6.23) have been so chosen that the constraint relation (6.2) is consistent with the general commutator. If the coefficients a and b are assigned the values $a = i$, $b = -i$ the equal-time commutation relations (Appendix II) can be recovered from eq.(6.24) for the choice of the parameter $B = \frac{1}{2}$. In a similar manner the following general commutation relations involving χ_μ are written down:

$$[\psi_{\mu\nu}(x), \chi_\rho^*(x')] = -i(\delta_{\mu\rho}\partial_\nu + \delta_{\nu\rho}\partial_\mu - \delta_{\mu\nu}\partial_\rho)\Delta(x-x';0) \qquad (6.24)$$

$$[\chi_\mu(x), \chi_\nu^*(x')] = -i(\delta_{\mu\nu} - \partial_\mu\partial_\nu)\Delta(x-x') \qquad (6.25)$$

These commutation relations will be consistent with the equal-time commutators given in Appendix II provided the arbitrary parameter A is set equal to zero.

The Feynman propagator of the field is evaluated in the customary way. In this case the normal-dependent terms drop off and the propagator is rigorously given by

$$\begin{aligned}\Delta_{F\,\mu\nu,\rho\lambda}(x-x') &= \langle 0|T\psi_{\mu\nu}(x)\psi^*_{\rho\lambda}(x')|0\rangle \\ &= [\,1/2\,(\delta_{\mu\rho}\delta_{\nu\lambda} + \delta_{\mu\lambda}\delta_{\nu\rho}) \\ &\quad + \delta_{\mu\nu}\delta_{\rho\lambda} + m^{-2}(\delta_{\mu\nu}\partial_\rho\partial_\lambda + \delta_{\rho\lambda}\partial_\mu\partial_\nu) \\ &\quad - 1/2\,m^{-2}(\delta_{\mu\rho}\partial_\nu\partial_\lambda + \delta_{\mu\lambda}\partial_\nu\partial_\rho + \delta_{\nu\lambda}\partial_\mu\partial_\rho\end{aligned}$$

$$+ \delta_{\nu\rho}\, \partial_\mu \partial_\lambda)]\ \Delta_F (x-x'; m^2)$$

$$- [m^{-2} (\delta_{\mu\nu}\, \partial_\rho \partial_\lambda + \delta_{\rho\lambda}\, \partial_\mu \partial_\nu) - \tfrac{1}{2} m^{-2} (\delta_{\mu\rho}\, \partial_\nu \partial_\lambda$$

$$+ \delta_{\mu\lambda}\, \partial_\nu \partial_\rho + \delta_{\nu\lambda}\, \partial_\mu \partial_\rho + \delta_{\nu\rho}\, \partial_\mu \partial_\lambda)]$$

$$\times\ \Delta_F (x-x'; 0) \tag{6.26}$$

This may be written in the form

$$\begin{aligned}
&\Delta_{F\,\mu\nu,\rho\lambda} (x-x') \\
&\qquad = \Delta^{(2)}_{F\,\mu\nu,\rho\lambda} (x-x') \\
&\qquad - 2/3\ (\delta_{\mu\nu} - m^{-2} \partial_\mu \partial_\nu) (\delta_{\rho\lambda} - m^{-2} \partial_\rho \partial_\lambda) \\
&\qquad \times\ \Delta (x-x'; m^2) - [m^{-2} (\delta_{\mu\nu}\, \partial_\rho \partial_\lambda + \delta_{\rho\lambda}\, \partial_\mu \partial_\nu) \\
&\qquad - \tfrac{1}{2} m^{-2} (\delta_{\mu\rho}\, \partial_\nu \partial_\lambda + \delta_{\mu\lambda}\, \partial_\nu \partial_\rho + \delta_{\nu\rho}\, \partial_\mu \partial_\lambda \\
&\qquad + \delta_{\nu\lambda}\, \partial_\mu \partial_\rho)]\ \Delta_F (x-x'; 0)
\end{aligned} \tag{6.27}$$

where $\Delta^{(2)}_F$ is the usual spin 2 propagator[31].

$$\Delta^{(2)}_{F\,\mu\nu,\rho\lambda}(x-x')$$

$$= [\ \tfrac{1}{2}(\delta_{\mu\rho}\delta_{\nu\lambda} + \delta_{\mu\lambda}\delta_{\nu\rho}) - \tfrac{1}{3}\delta_{\mu\nu}\delta_{\rho\lambda}$$

$$+ \tfrac{1}{3}m^{-2}(\delta_{\mu\nu}\partial_\rho\partial_\lambda + \delta_{\rho\lambda}\partial_\mu\partial_\nu) - \tfrac{1}{2}m^{-2}$$

$$\times(\delta_{\mu\rho}\partial_\nu\partial_\lambda + \delta_{\mu\lambda}\partial_\nu\partial_\rho + \delta_{\nu\rho}\partial_\mu\partial_\lambda$$

$$+ \delta_{\nu\lambda}\partial_\mu\partial_\rho) + \tfrac{2}{3}m^{-4}\partial_\mu\partial_\nu\partial_\rho\partial_\lambda\ ]\ \Delta_F(x-x';m^2)$$

(6.28)

The other propagators are given by

$$\langle 0|\,T\,\psi_{\mu\nu}(x)\,\chi^*_\rho(x')\,|0\rangle = -(\delta_{\mu\rho}\partial_\nu + \delta_{\nu\rho}\partial_\mu - \delta_{\mu\nu}\partial_\rho)\Delta_F(x-x';0) \qquad (6.29)$$

$$\langle 0|\,T\,\chi_\mu(x)\,\chi^*_\nu(x')\,|0\rangle = -(\delta_{\mu\nu} - \partial_\mu\partial_\nu)\,\Delta_F(x_1-x_2) - i\,\delta_{\mu4}\delta_{\nu4}\,\delta^{(4)}(x_1-x_2) \qquad (6.30)$$

Since χ_μ represents a negative-metric-carrying ghost particle which will not be present in physical states the above propagators will not be of further use, and the non-covariant form of (6.30) is of no consequence.

6.4 Electromagnetic interaction

The interaction of the above described symmetric tensor field to an external electromagnetic field will now be considered. The questions of constraint loss and the Johnson-Sudarshan type of inconsistency on quantization do not at all arise in the present framework. It will now be shown that the present formulation is free of acausality of propagation also.

Introducing minimal coupling into the Lagrangian (6.6) by the replacement $\partial_\mu \to D_\mu = \partial_\mu - ie A_\mu$ the equations of motion resulting fromthe variations of $\psi^*_{\mu\nu}$, $\chi^*_{\mu\nu}$ become

$$(D^2 - m^2)\psi_{\mu\nu} - (D_\lambda D_\mu \psi_{\lambda\nu} + D_\lambda D_\nu \psi_{\lambda\mu}) - A\, D_\mu D_\nu \psi$$

$$- \delta_{\mu\nu}\left[D_\rho D_\lambda \psi_{\rho\lambda} + B D^2 \psi \right] + \delta_{\mu\nu} C m^2 \psi$$

$$= D_\mu \chi_\nu \qquad (6.31)$$

$$D_\mu \psi_{\mu\nu} = 0 \qquad (6.32)$$

In contrast to the situation of the spin 3/2 theory developed in the previous chapter where the constraint condition was free of derivatives, the requirement of gauge invariance, in the present case, leads to a minimal modification of the constraint condition (6.2). Substituting eq.(6.32) in eq. (6.31), with the aid of the relation

$$[\, D_\mu , D_\nu \,] = ie\, F_{\mu\nu} \qquad (6.33)$$

where $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$ one obtains,

$$\begin{aligned} (D^2 - m^2)\,\psi_{\mu\nu} &- A\, D_\mu D_\nu \psi - \delta_{\mu\nu}\, B\, D^2\psi \\ &+ \delta_{\mu\nu}\, C m^2 \psi - ie\,(F_{\lambda\mu}\,\psi_{\lambda\nu} + F_{\lambda\nu}\,\psi_{\lambda\mu}) \\ &= D_\mu \chi_\nu \end{aligned} \qquad (6.34)$$

Contracting the indices μ and ν in eq. (6.34) and taking the divergence of eq. (6.34) and combining the two relations it is found that the equation obeyed by χ_ν is

$$\begin{aligned} D^2\chi_\nu &= -A\, D^2 D_\nu \psi - B\, D_\nu D^2\psi + C m^2 D_\nu \psi \\ &\quad - ie\,(F_{\mu\lambda}\, D_\lambda \psi_{\mu\nu} - F_{\lambda\nu}\, D_\mu \psi_{\lambda\mu}) \end{aligned} \qquad (6.35)$$

This equation may be used for eliminating χ_ν from eq. (6.31) and resulting equation is obtained as

$$D^2 (D^2 - m^2)\,\psi_{\mu\nu} + B\, D_\mu D_\nu D^2\psi - \delta_{\mu\nu}\, B\, (D^2)^2\psi$$
$$+ \text{ lower derivative terms } = 0 \qquad (6.36)$$

The lower derivative terms have not been shown explicitly because these are not important in determining the nature of propagation which depends solely on the highest order derivatives. The characteristic determinant of the system of equations (6.36) is evaluated with the result

$$D(n) \quad = (1-B)^3 (n^4)^{10} \tag{6.37}$$

Since $B \neq 1$ in the present theory (the choce $B = \frac{1}{2}$ having been made for consistent quantization of free field) the characteristic determinant will not vanish identically, and setting $D(n) = 0$ it is evident that the propagation of the interacting field is light-like and hence causal.

6.5 Massless limit

Another merit of the present formulation is that the limit $m \to 0$ can be taken smoothly without encountering any difficulty. The equation of motion in this case is

$$\Box \psi_{\mu\nu} - \delta_{\mu\nu} B \Box \psi - A \partial_\mu \partial_\nu \psi$$

$$= \tfrac{1}{2} (\partial_\mu \chi_\nu + \partial_\nu \chi_\mu) \tag{6.38}$$

but the constraint equation remains the same as eq.(6.2). The formalism can be developed in a way exactly analogous to the massive case. It suffices to note here that in the massless case eq.(6.39) is invariant under the gauge transformation

$$\psi_{\mu\nu}(x) \longrightarrow \psi_{\mu\nu}(x) + \tfrac{1}{2}\left[\partial_\mu \xi_\nu(x) + \partial_\nu \xi_\mu(x)\right] \qquad (6.39)$$

$$\chi_\mu(x) \longrightarrow \chi_\mu(x) \qquad (6.40)$$

provided the gauge functions $\xi_\mu(x)$ obey the conditions

$$\Box\, \xi_\mu(x) = 0 \qquad (6.41)$$

$$\partial_\mu \xi_\mu(x) = 0 \qquad (6.42)$$

This gauge-freedom will reduce the number of independent components in the massless case.

The Feynman propagator of the massless field is obtained by letting $m \longrightarrow 0$ in eq. (6.26)

$$\begin{aligned} D_{F\mu\nu,\rho\lambda}(x-x') = {} & \left[\tfrac{1}{2} (\delta_{\mu\rho}\delta_{\nu\lambda} + \delta_{\mu\lambda}\delta_{\nu\rho}) - \delta_{\mu\nu}\delta_{\rho\lambda} \right] \Delta_F(x-x';0) + \left[\delta_{\mu\nu}\partial_\rho\partial_\lambda + \delta_{\rho\lambda}\partial_\mu\partial_\nu \right. \\ & - \tfrac{1}{2}(\delta_{\mu\rho}\partial_\nu\partial_\lambda + \delta_{\mu\lambda}\partial_\nu\partial_\rho + \delta_{\nu\rho}\partial_\mu\partial_\lambda \\ & \left. + \delta_{\nu\lambda}\partial_\mu\partial_\rho) \right] E(x-x') \end{aligned} \qquad (6.43)$$

where $E(x - x') = \lim_{m \to 0} \frac{\partial}{\partial m^2} \Delta_F (x-x'; m^2)$

The expansion

$$\Delta_F (x-x'; m^2) = \Delta_F (x-x'; 0) + m^2 \frac{\partial}{\partial m^2} \Delta_F (x-x'; m^2)\Big|_{m^2=0} + \cdots \quad (6.44)$$

was used in deriving (6.43). In the expression (6.43) the term $[\frac{1}{2} (\delta_{\mu\rho} \delta_{\nu\lambda} + \delta_{\mu\lambda} \delta_{\nu\rho}) - \delta_{\mu\nu} \partial_{\rho\lambda}] \Delta_F(x-x'; 0)$ corresponds to the actual propagator of a massless spin-2 field while the other term represents the ghose particle. One noteworthy feature of the present approach is that the true massless spin 2 propagator is obtained simply by taking the limit $m \to 0$ in eq. (6.27) whereas this does not happen in the conventional formulations even if it is assumed that the derivative terms can be left out.

6.6 Conclusion

The Lagrange multiplier formalism for the spin 2 field has the attractive feature that pathologies like constraint loss, inconsistency in quantization and acausal propagation in presence of interaction do not appear here. However, this is achieved at the cost of introducing an indefinite metric theory and the problem of non-unitarity of scattering amplitude encountered in the spin 3/2 case is bound to arise in this context as well. Though, on account of the difficulty of isolating the negative-metric components, the procedure followed in the spin 3/2 case for restoring unitarity is not easy to carry out, it is hoped that the procedure of introducing additional vertices and fictitious particles will resolve the difficulty in the spin 2 case also.

The possibility of discussing the massless case in a natural way is another advantage of the present approach, and this may be used as a spring-broad for the formulation of a new gravitational theory involving tensor and additional ghost particles.

APPENDIX I

LAGRANGE MULTIPLIER FORMALISM WITH TWO SETS OF MULTIPLIER FIELDS

A Lagrange multiplier formalism can be developed by incorporating both sets of constraints (6.2) and (6.3). The Lagrangian in this case may be taken as

$$\mathcal{L} = \mathcal{L}_0 + \eta (\chi_\nu^* \partial_\mu \psi_{\mu\nu} + \chi_\nu \psi^*_{\mu\nu}) + \lambda (\xi^* \psi + \xi \psi^*) \tag{6A.1}$$

where ξ is a scalar multiplier field. The equation of motion is

$$(\Box - m^2) \psi_{\mu\nu} = \eta/2 \ (\partial_\mu \chi_\nu + \partial_\nu \chi_\mu) - \lambda \delta_{\mu\nu} \xi \tag{6A.2}$$

Taking into account the constraints (6.2) and (6.3) it follows from eq.(6A.2) that

$$\eta \partial_\mu \chi_\mu - 4\lambda \xi = 0 \tag{6A.3}$$

$$\eta/2 (\Box \chi_\nu + \partial_\nu \partial_\mu \chi_\mu) = \lambda \partial_\nu \xi \tag{6A.4}$$

Eq.(6A.3) expresses the scalar multiplier field ξ in terms of $\partial_\mu \chi_\mu$. By combining eqs.(6A.3) and (6A.4) the equation obeyed by χ_μ can be derived.

$$(\Box)^2 \chi_\nu = 0 \tag{6A.5}$$

From eqs.(6A.5) and (6A.2) it follows further that

$$(\Box)^2 (\Box - m^2)\psi_{\mu\nu} = 0 \tag{6A.6}$$

which is the same as eq.(6.14).

The quantization procedure can be developed along the lines in sec.6.3. However, a difficulty that appears in this case is that the Heisenberg equations reproduce the field equation (6A.2) only if $\partial_\mu \chi_\mu = 0$. Another obstacle is that a general commutator satisfying the constraints (6.2) and (6.3) and other requirements, while reproducing the equal-time commutation relations, cannot be written down. It is on account of these complexities that a single multiplier formalism has been adopted in the text.

APPENDIX II

The equal-time commutation relations involving the components of $\psi_{\mu\nu}$, $\psi^*_{\mu\nu}$, $\dot{\psi}_{\mu\nu}$, $\dot{\psi}^*_{\mu\nu}$, χ_μ and χ^*_μ are as follows:

$$[\psi_{ij}(x), \dot{\psi}^*_{kl}(x')] = i\left[\tfrac{1}{2}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}) + \frac{B}{1-3B}\delta_{ij}\delta_{kl}\right]\delta(\vec{x}-\vec{x}') \qquad (6A.7)$$

$$[\psi_{ij}(x), \dot{\psi}^*_{4k}(x')] = [\psi_{ij}(x), \dot{\psi}^*_{44}(x')] = 0 \qquad (6A.8)$$

$$[\psi_{i4}(x), \dot{\psi}^*_{jk}(x')] = [\psi_{i4}(x), \dot{\psi}^*_{44}(x')] = 0 \qquad (6A.9)$$

$$[\psi_{44}(x), \dot{\psi}^*_{ij}(x')] = [\psi_{44}(x), \dot{\psi}^*_{i4}(x')] = 0 \qquad (6A.10)$$

$$[\dot{\psi}_{ij}(x), \dot{\psi}^*_{kl}(x')] = 0 \qquad (6A.11)$$

$$[\dot{\psi}_{i4}(x), \dot{\psi}^*_{kl}(x')] = \left[\tfrac{1}{2}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}) + \frac{B}{1-3B}\delta_{ij}\delta_{kl}\right] \times \partial_j\delta(\vec{x}-\vec{x}') \qquad (6A.12)$$

$$[\dot{\psi}_{44}(x), \dot{\psi}^*_{ij}(x)] = 0 \qquad (6A.13)$$

$$[\dot{\psi}_{i4}(x), \dot{\psi}^*_{j4}(x')] = [\dot{\psi}_{44}(x), \dot{\psi}^*_{44}(x')] = 0 \qquad (6A.14)$$

$$[\dot{\psi}_{i4}(x), \dot{\psi}^*_{44}(x')] = 0 \qquad (6A.15)$$

$$[\psi_{i4}(x), \chi^*_j(x')] = -\delta_{ij}\delta(\vec{x}-\vec{x}') \qquad (6A.16)$$

$$[\psi_{ij}(x), \chi_k^*(x')] = 0 \tag{6A.17}$$

$$[\psi_{44}(x), \chi_j^*(x')] = 0 \tag{6A.18}$$

$$[\psi_{ij}(x), \chi_4^*(x')] = -\frac{(A+B)}{1-3B}\delta_{ij}\,\delta(\vec{x}-\vec{x}') \tag{6A.19}$$

$$[\psi_{i4}(x), \chi_4^*(x')] = 0 \tag{6A.20}$$

$$[\psi_{44}(x), \chi_4^*(x')] = -\delta(\vec{x}-\vec{x}') \tag{6A.21}$$

$$[\dot{\psi}_{ij}(x), \chi_k^*] = -i\Big[\delta_{ik}\,\partial_j\,\delta(\vec{x}-\vec{x}') + \delta_{jk}\,\partial_i\,\delta(\vec{x}-\vec{x}') + \frac{A+B}{1-3B}\delta_{ij}\,\partial_k\,\delta(\vec{x}-\vec{x}')\Big] \tag{6A.22}$$

$$[\dot{\psi}_{i4}(x), \chi_j^*(x')] = 0 \tag{6A.23}$$

$$[\dot{\psi}_{44}(x), \chi_i^*(x')] = i\,\partial_i\,\delta(\vec{x}-\vec{x}') \tag{6A.24}$$

$$[\dot{\psi}_{ij}(x), \chi_4^*(x')] = 0 \tag{6A.25}$$

$$[\dot{\psi}_{i4}(x), \chi_4^*(x')] = i\,\frac{A+B}{1-3B}\,\delta(\vec{x}-\vec{x}') \tag{6A.26}$$

$$[\dot{\psi}_{44}(x), \chi_4^*(x')] = 0 \tag{6A.27}$$

$$[\chi_k(x), \chi_l^*(x')] = 0 \tag{6A.28}$$

$$[\chi_4(x)\ ,\ \overset{*}{\chi}_4(x')] = 0 \tag{6A.29}$$

$$[\chi_i(x)\ ,\ \overset{*}{\chi}_4(x')] = -\frac{(1+2A-B)}{1-3B}\ \partial_i\,\delta(\vec{x}-\vec{x}') \tag{6A.30}$$

References

1. P.A.M. Dirac, Proc.Roy.117, 610 (1928)

2. E.M. Corson, Introduction to Tensors, Spinors and Relativistic Wave Equations (Blackie, 1953)

3. P.A.M. Dirac, Proc. Roy.Soc. A 155, 447 (1936)

4. M. Fierz, Helv.Phys. Acta 12, 3 (1939)

5. W. Pauli and M. Fierz, Proc. Roy.Soc. A 173, 211 (1939)

6. W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941)

7. R.J. Duffin, Phys. Rev. 54, 1114 (1938)

8. N. Kemmer, Proc. Roy.Soc. A 173, 91 (1939)

9. G. Petiau, Acad. R.Belg. Cl.Sci. Mem. 8 16 No.2 (1936)

10. H.J. Bhabha, Rev. Mod. Phys. 17, 200 (1945)

11. H.J. Bhabha, Rev. Mod. Phys. 21,451 (1949)

12. R.A. Krajcik and M.M. Nieto, Phys. Rev. D10, 4049 (1974)

13. R.A. Krajcik and M.M. Nieto, Phys. Rev. D11, 1442 (1975)

14. R.A. Krajcik and M.M. Nieto, Phys. Rev. D11, 1459 (1975)

15. R.A. Krajcik and M.M. Nieto, Phys. Rev. D13, 924 (1976)

16. R.A. Krajcik and M.M. Nieto, Phys. Rev. D14, 418 (1976)

17. R.A. Krajcik and M.M. Nieto, Phys. Rev. D15, 433 (1977)

18. R.A. Krajcik and M.M. Nieto, Phys. Rev. D15, 445 (1977)

19. Harish-Chandra, Proc. Roy.Soc. A186, 502 (1946)

20. Harish-Chandra, Proc. Camb. Philos. Soc. 43, 414 (1947)

21. Harish-Chandra, Phys. Rev. 71, 793 (1947)

22. C. Fronsdal, Nuovo Cimento Suppl. 9, 416 (1958)

23. S.J. Chang, Phys. Rev. 161, 1308 (1967)

24. L.P.S. Singh and C.R. Hagen, Phys. Rev. D9, 898 (1974)

25. L.P.S. Singh and C.R. Hagen, Phys. Rev. D9, 910 (1974)

26. P.A. Moldauer and K.M. Case, Phys. Rev. 102, 279 (1956)

27. L.M. Nath, Nucl. Phys. 68, 660 (1965)

28. J. Schwinger, Phys. Rev. 91, 713 (1953)

29. S.J. Chang, Phys. Rev. 148, 1259 (1966)

30. Y. Takahashi and H. Umezawa, Progr. Theor. Phys. 9, 14 (1953)

31. Y. Takahashi, An Introduction to Field Quantization (Pergamon Press, 1969)

32. C.N. Yang and D. Feldman, Phys. Rev. 79, 972 (1950)

33. K. Johnson and E.C.G. Sudarshan, Ann. Phys. 13, 126 (1961)

34. H.J. Bhabha, Phil. Mag. 43, 33 (1952)

35. K.K. Gupta, Proc. Roy. Soc. A222, 118 (1954)

36. C.R. Hagen, Phys. Rev. D4, 2204 (1971)

37. L.M. Nath, B. Etemadi and J.D. Kimel, Phys. Rev. D3, 2153 (1971)

38. S.N. Gupta and W.W. Repko, Phys. Rev. 177, 1921 (1969)

39. J.D. Kimel and L.M. Nath, Phys. Rev. D6, 2132 (1972)

40. B. Etemadi, Ph.D. thesis, Florida State University 1971 (unpublished)

41. L.M. Nath, B. Etemadi and J.D. Kimel, Florida State University Report No. HEP-72-8-4 1972 (Unpublished)

42. W.F. Soo, Phys. Rev. D8, 667 (1973)

43. D. Gluch, P. Hays and J.D. Kimel, Phys. Rev. D9, 1964 (1974)

44. G.B. Mainland and E.C.G. Sudarshan, Phys. Rev. D8, 1088 (1973)

45. G.B. Mainland and E.C.G. Sudarshan, Phys. Rev. D10, 3343 (1974)

46. G. Velo and D. Zwanziger, Phys. Rev. 186, 1337 (1969)

47. G. Velo and D. Zwanziger, Phys. Rev. 188, 2218 (1969)

48. R. Courant and D. Hilbert, Methods of Mathematical Physics, Vol.2, Chap. VII (Wiley-Interscience, N.Y., 1969)

49. J.D. Jenkins, Lett. Nuovo Cimento 6, 103 (1973)

50. J.D. Jenkins, Lett. Nuovo Cimento 7, 559 (1973)

51. A. Shamaly and A.Z. Capri, Lett. Nuovo Cimento 3, 467 (1972)

52. A. Shamaly and A.Z. Capri, Ann. Phys. 74, 503 (1972)

53. A. Shamaly and A.Z. Capri, Can. J. Phys. 52, 919 (1974)

54. A.S. Wightman, in Partial Differential Equations, edited by Lipman Bers et.al. (American Mathematical Society, Providence 1973) p.441

55. L.P.S. Singh, Phys. Rev. D7, 1256 (1973)

56. A. Shamaly and A.Z. Capri, Can. J. Phys. 52, 919 (1974)

57. G.W. Gibbons, J. Phys. A: Math. Gen. 9, 145 (1976)

58. G. Velo, Nucl. Phys. B43, 389 (1972)

59. J. Madore and W. Tait, Comm. Math. Phys. 30, 201 (1973)

60. M. Hortacsu, Phys. Rev. D9, 1928 (1974)

61. B. Schroer, R. Seiler and J.A. Swieca, Phys. Rev. D2, 2927 (1970)

62. A.Z. Capri, J. Math. Phys. 10, 575 (1969)

63. P.M. Mathews and M. Seetharaman, Phys. Rev. D8, 1815 (1973)

64. C.R. Hagen, Phys. Rev. D9, 498 (1974)

65. P. Federbush, Nuovo Cimento 19, 572 (1961)

66. C.R. Hagen, Phys. Rev. D6, 984 (1972)

67. W. Tait, Phys. Rev. D5, 3272 (1972)

68. S. Rai Choudhury, Lett. Nuovo Cimento 5, 698 (1972)

69. J.D. Jenkins, J. Phys. A: Math. Nucl. Gen. 7, 1129 (1974)

70. J.D. Jenkins, J. Phys. A: Math. Nucl. Gen. 6, 1211 (1973)

71. J.D. Jenkins, J. Phys. A: Math. Nucl.Gen. 6, 1935 (1973)

72. P.T. Mathews, Phys. Rev. 76, 684 (1949)

73. P.M. Mathews, M. Seetharaman and J. Prabhakaran, Phys. Rev. D14, 1021 (1976)

74. T. Goldman and W.-y. Tsai, Phys. Lett. 36B, 467 (1971)

75. W.-y. Tsai and A. Yildiz, Phys. Rev. D4, 3643 (1971)

76. T. Goldman and W.-y. Tsai, Phys. Rev. D4, 3648 (1971)

77. T. Goldman and W.-y. Tsai and A. Yildiz, Phys. Rev. D5, 1926 (1972)

78. P.M. Mathews, Phys. Rev. D9, 365 (1974)

79. M. Seetharaman, J. Prabhakaran and P.M. Mathews, Phys. Rev. D12, 458 (1975)

80. A.S. Wightman, in Proceedings of the Fifth Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1968 p.290

81. A.S. Wightman, in Studies in Mathematical Physics edited by E.H. Lieb, B. Simon and A.S. Wightman (Princeton University Press, Princeton, N.J., 1976) p.423.

82. W.J. Hurley, Phys. Rev. D4, 3605 (1971)

83. W.J. Hurley, Phys. Rev. Lett. 29, 1475 (1972)

84. W.J. Hurley, Phys. Rev. D10, 1185 (1974)

85. Y. Takahashi and R. Palmer, Phys. Rev. D1, 2974 (1970)

86. G. Velo, Nucl. Phys. B65, 427 (1973)

87. H.L. Baisya, Nucl. Phys. B23, 633 (1970)

88. A.K. Nagpal, Nucl. Phys. B53, 634 (1971)

89. A.K. Nagpal, Nucl. Phys. B80, 208 (1974)

90. J.D. Harris, Ph.D. thesis, Purdue University, 1955 (unpublished)

91. S.J. Chang, Phys. Rev.Lett 17, 1024, (1966)

92. W.-K. Tung, Phys. Rev. 156, 1385

93. R.W. Haymaker, The Dynamics of Fields of Higher Spin NBS Monograph No.154 (U.S. Dept of Commerce, Washington, D.C., 1976)

94. I.M. Gelfand, R.A. Minlos, and Z. Ya-Shapiro, Representations of the Rotation and Lorentz Group and Their Applications (Pergamon, 1963)

95. V. Amar and U. Dozzio, Lett. Nuovo Cimento 5, 355 (1972)

96. V. Amar and U. Dozzio, Lett. Nuovo Cimento 12, 659 (1975)

97. M. Munczek, Phys. Rev. 164, 1794 (1967)

98. T. Fukuyama and K. Yamamoto, Progr. Theor. Phys. 49, 304, (1973)

99. J. Prabhakaran, M. Seetharaman and P.M. Mathews, J. Phys. A. Math. Gen. 8, 560 (1975)

100. C. Fisk and W. Tait, J.Phys. A: Math. Nucl. Gen. 6, 383 (1973)

101. A.Z. Capri and A. Shamaly, Can. J. Phys. 54, 1089 (1976)

102. W. Cox, J.Phys. A: Math. Gen. 10, 1409 (1977)

103. Harish-Chandra, Proc. Roy. Soc. A192, 195 (1947)

104. J.Madore, Phys. Lett. 55B, 217 (1975)

105. B. Zumino and P. Nath, in Proceedings of the Conference on Gauge Theories and Modern Field Theory, Northeastern University 1975, edited by A. Arnowitt and P. Nath (Cambridge,Mass., MIT Press)

106. D.Z. Freedman and P. van Nieuwenhuizen, Phys. Rev. D13, 3214 (1976)

107. S. Ferrara and P. van Nieuwenhuizen, Phys. Rev. Lett. 37, 1669 (1976)

108. D.V. Volkov and V.P. Akulov, JETP Lett. 16, 438 (1972)

109. J. Wess and B. Zumino, Nucl. Phys. B70, 39 (1974)

110. S. Deser and B. Zumino, Phys. Lett. 62B, 335 (1976)

111. A. Das and D.Z. Freedman, Nucl.Phys. B114, 271 (1976)

G 1879

112. D.Z. Freedman, Phys. Rev. D15, 1173 (1977)

113. K. Nishijima, Fields and Particles, Benjamin, 1969)

114. P.A.M. Dirac, Rev. Mod. Phys. 34, 592 (1962)

115. J. Schwinger, Phys. Rev. 127, 324 (1962)

116. J. Schwinger, Phys. Rev. 130, 406 (1963)

117. J. Schwinger in Theoretical Physics (IAEA, Vienna, 1963)

118. J. Schwinger, Phys. Rev. 130, 800 (1963)

119. J. Schwinger, in Lectures in Particles and Field Theory Brandeis Summer Institute in Theoretical Physics (Englewood Cliffs: Prentice Hall Inc. 1964)

120. J. Belinfante, Physica 6, 1887 (1939)

121. S.G. Brown, Phys. Rev. 158, 1608 (1967)

122. G. Velo and D. Zwanziger, Lett. Nuovo Cimento 15, 39 (1976)

123. A. Ainsaar and M. Kõiv, Izvestia Akademee Nauk Eston-skoy SSR, Fizika Matimatica 26, 469 (1977)

124. N.Nakanishi, Phys. Rev. D5, 1324 (1972)

125. J.P. Hsu, Phys. Rev. D8, 2069 (1974)

126. J.P. Hsu, Phys. Rev. D9, 1113 (1974)

127. J.P. Hsu and E.C.G. Sudarshan, Phys Rev. D9, 1678 (1974)

128. T.D. Lee and C.N. Yang, Phys. Rev. 128, 885 (1962)

129. J. Prabhakaran, M. Seetharaman, and P.M. Mathews, Phys. Rev. D12, 3191 (1975)

130. J. Prabhakaran, T.R. Govindarajan and M. Seetharaman, Nucl. Phys. B127, 537 (1977)

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