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Bidirectional communication using delay coupled chaotic directly modulated semiconductor lasers

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Abstract. Chaotic synchronization of two directly modulated semiconductor lasers with negative delayed optoelectronic feedback is investigated and this scheme is found to be useful for efficient bidirectional communication between the lasers. A symmetric bidirectional coupling is identified as a suitable method for isochronal synchronization of such lasers. The optimum values of coupling and feedback strength that can provide maximum quality of synchronization are identified. This method is successfully employed for encoding/decoding both analog and digital messages. The importance of a symmetric coupling is demonstrated by studying the variation of decoding efficiency with respect to asymmetric coupling.

Keywords. Isochronal synchronization; bidirectional communication; directly modulated semiconductor lasers; delayed optoelectronic feedback.

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1. Introduction

Chaotic synchronization is a widely investigated topic of research in the past few decades, due to its application in physical, chemical, biological as well as technological fields [1–10]. Synchronization of chaotic dynamics of lasers has attracted much attention in the recent years because of its potential application in secure communication [11–17]. Recently, secure high-speed long distance communication has been achieved using synchronization of chaotic lasers [18].

Long-wavelength directly modulated semiconductor lasers are the most preferred light source in the fibre-optic communication links because its output wavelength falls in the minimum loss and dispersion window of optical fibres. This has drawn attention to the study of its chaotic dynamics and synchronization. Providing GHz modulation to its input current is an efficient method for producing chaotic outputs from semiconductor lasers [19–26]. The dynamics of semiconductor lasers with direct current modulation is widely investigated from its application point of view in secure communication [27–38]. However, the mode gain reduction occurring in such systems due to nonlinear processes suppresses chaotic dynamics [27]. For InGaAsP lasers used in optical communication systems, the nonlinear gain reduction is very strong and its optimum value lies between 0.03 and 0.06. This system exhibits chaotic dynamics only for nonlinear gain reduction below 0.01 [27]. A positive-delayed optoelectronic feedback combined with strong current modulation is found to suppress chaotic dynamics and bistability in semiconductor lasers [28,29]. Chaotic synchronization of two such lasers with low values of nonlinear gain reduction factor is achieved and successfully applied for unidirectional optical secure communication [30,31]. A bidirectional coupling of two such lasers is also found to suppress chaotic dynamics [32]. It is recently reported that a negative-delayed optoelectronic feedback is capable of producing chaotic outputs from directly modulated semiconductor lasers with optimum value of nonlinear gain suppression factor [33].

Synchronization of two chaotic semiconductor lasers can be achieved using different coupling schemes under unidirectional [39,40] or bidirectional configuration [41,42] which can be either optical or optoelectronic, direct or delayed [43,44]. The majority of scientific investigation on the methods and properties of chaotic synchronization were concentrated on the unidirectional coupling between the oscillators in a master–slave configuration where the dynamics of the master system is reproduced by the slave system. Here a message is encoded onto the output of the chaotic transmitter and is decoded by the receiver. However, for effective communication between them, message transfer in both directions is very essential and this demands a bidirectional coupling. In addition to this, the significance of delays in the coupling channels enhances the importance of synchronization of two bidirectionally delay coupled chaotic oscillators.

Most of the methods proposed for synchronization of the delay coupled systems depend on the introduction of a third relay element between the two oscillators [45–47]. The use of a chaotic system which is different from the outer systems as the relay element is also equally effective in achieving isochronal (zero-lag) synchronization [45]. Isochronal synchronization between two semiconductor lasers reported recently uses a third mediator laser which is coupled bidirectionally to both the end lasers [46,48]. As there is no direct link between the oscillators which are isochronally synchronized, this method has limited applicability in the bidirectional secure communication. For this purpose, using a third element as a drive system [49] and adding a coupling signal along with the self-delayed feedback signal [50–52] are reported recently.

Here we investigate the possibility of bidirectional secure communication using two directly modulated semiconductor lasers with optimum value of nonlinear gain reduction and delayed negative optoelectronic feedback without using a third element. The optimum range of self-feedback and coupling fractions for achieving isochronal synchronization between the lasers is investigated. The isochronal synchronization achieved using the bidirectional coupling is further applied for bidirectional secure communication between them. Both analog and digital messages are successfully encoded and decoded simultaneously at the two lasers.

2. Laser model

The dynamics of semiconductor lasers with direct current modulation and negative-delayed optoelectronic feedback can be represented by rate equations for the photon density (P), carrier density (N) and driving current (I)

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{1}{\tau_{\mathrm{e}}}\left\{\left(\frac{I}{I_{\mathrm{th}}}\right) - N - \left[\frac{N-\delta}{1-\delta}\right]P\right\} \tag{1}$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{\tau_{\mathrm{p}}}\left\{\left[\frac{N-\delta}{1-\delta}\right](1-\varepsilon P)P - P + \beta N\right\} \tag{2}$$

$$I(t) = I_{\mathrm{b}} + I_{\mathrm{m}}\sin(2\pi f_{\mathrm{m}}t) - r_{\mathrm{s}} \times (P(t-\tau)). \tag{3}$$

Here τ_{e} and τ_{p} are the electron and photon lifetimes, $\delta = n_0/n_{\mathrm{th}}, \varepsilon = \varepsilon_{\mathrm{NL}}S_0$ are the dimensionless parameters where n_0 is the carrier density required for transparency, $n_{\mathrm{th}} = (\tau_{\mathrm{e}}I_{\mathrm{th}}/eV)$ is the threshold carrier density, $\varepsilon_{\mathrm{NL}}$ is the factor governing the nonlinear gain reduction occurring with an increase in S, $S_0 = \Gamma(\tau_{\mathrm{p}}/\tau_{\mathrm{e}})n_{\mathrm{th}}$, I_{th} is the threshold current, e is the electron charge, V is the active volume, Γ is the confinement factor and β is the spontaneous emission factor. $I_{\mathrm{b}} = b \times I_{\mathrm{th}}$, is the bias current where b is the bias strength. $I_{\mathrm{m}} = m \times I_{\mathrm{th}}$ is the modulation current where m is the modulation depth and f_{m} is the modulation frequency [27]. The self-feedback strength is denoted as r_{s} and the delay time is denoted as τ_{s}. Each laser receives a total delayed feedback equivalent to that required for producing chaotic outputs [33]. The parameter values for which this system produces chaotic outputs are given in table 1.

Two such lasers L_1 and L_2 are coupled to each other through the coupling signal generated using each other's output. The schematic diagram of the bidirectionally coupled directly modulated semiconductor lasers with delayed optoelectronic feedback is shown in figure 1. Each of the laser diodes L_1 and L_2 is driven with

Table 1. Parameter values used for numerical simulation.

Parameter	Value
τ_{c}	3 ns
τ_{p}	6 ps
ε	0.05
δ	$692 \times 10^{(-3)}$
β	$5 \times 10^{(-5)}$
f_{m}	0.8 GHz
I_{th}	26 mA
m	0.55
b	1.5
r	0.02
τ	3.78 ns

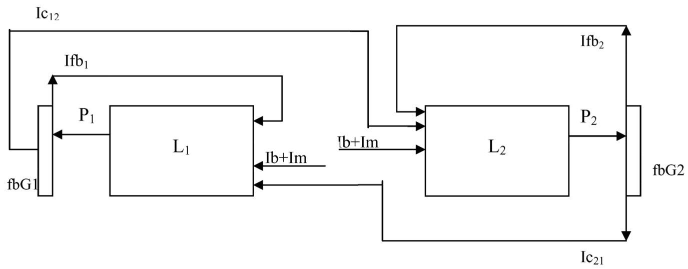

Figure 1. Schematic of bidirectionally coupled directly modulated semiconductor lasers with negative optoelectronic feedback. The laser diodes L_1 and L_2 are driven by bias current I_b and a GHz modulation current I_m. fbG_1 and fbG_2 are feedback generators which split the light outputs into required fractions for self-feedback and coupling signals delayed by appropriate times and generate proportional signals Ifb_1, Ifb_2 and coupling signals Ic_{12} and Ic_{21}.

a bias current and is modulated with a sinusoidal GHz current. In addition, each laser receives a coupling current proportional to the output of the other laser. A fraction of the light output from the laser diode L_1 is converted into electronic signal after providing the required delay and is fed back to it's input as the feedback current Ifb_1 in addition to its injection current. Another fraction is converted into a proportional current signal after providing the appropriate delay and is fed to the input of the laser diode L_2 as the coupling signal Ic_{12}. Similarly, a fraction of the output of laser diode L_2 is converted to a proportional current signal after providing the required delay and is fed to its own input as the feedback current Ifb_2. Another fraction is converted into a proportional current after providing the required delay and is fed to the input of L_1 as its coupling signal Ic_{21}. The input current equations of the two lasers are represented as follows:

$$I_1(t) = I_\mathrm{b} + I_\mathrm{m}\sin(2\pi f_\mathrm{m} t) - r_\mathrm{s} \times (P_1(t-\tau_\mathrm{s})) - r_\mathrm{c} \times (P_2(t-\tau_\mathrm{c})) \qquad (4)$$

$$I_2(t) = I_\mathrm{b} + I_\mathrm{m}\sin(2\pi f_\mathrm{m} t) - r_\mathrm{s} \times (P_2(t-\tau_\mathrm{s})) - r_\mathrm{c} \times (P_1(t-\tau_\mathrm{c})). \qquad (5)$$

Here, r_s and r_c are the self-feedback and coupling strengths and τ_s and τ_c are the feedback and coupling delay times. The total strength of the signal comprising of the feedback and coupling strengths is kept within the optimum value of delayed feedback for producing chaotic outputs. The delay times of coupling and self-feedback signals are also fixed at their optimum values. The above equations are numerically simulated using fourth-order Runge–Kutta method with parameter values given in table 1.

3. Results and discussion

To study the dynamics of synchronization, the coupling strength is increased slowly from zero to the optimum value and the corresponding variation in the correlation coefficient between the photon densities P_1 and P_2 of lasers L_1 and L_2 is estimated.

The correlation coefficient is calculated using the equation defined in [53]. The synchronization is also studied using a newly introduced synchronization index, viz. synchronization error decay coefficient (SEDC) [54]. This measure is defined based on the evolution of synchronization error between the coupled systems. It is a good measure of synchronization with zero and 1 as the two bounds indicating no synchronization and complete synchronization and also an indicator of the nature of stability of synchronization with its negative and positive values indicating unstable and stable synchronization regimes.

By varying the coupling strength from zero to the optimum value of delayed feedback required as given in table 1, the system is slowly shifted from uncoupled state to closed loop coupled state and finally to an open loop coupled state. When the coupling strength is zero, each laser runs independent of each other and is driven to chaotic state totally by its own self-feedback signal. As the coupling strength is increased, the self-feedback strength is decreased by an equal amount thus keeping the total feedback signal equal to the optimum value of feedback required for producing chaotic outputs. Finally, as the coupling strength becomes equal to the required amount of total feedback, the self-feedback is cut off, thus making the coupling an open loop one or equivalently a bidirectional lag coupling where each laser is driven to chaos by the delayed output of the other laser.

Figures 2a and 2b and 3a and 3b show the variation of instantaneous correlation coefficient and SEDC respectively between the lasers L_1 and L_2 with respect to the increase in coupling strength and the corresponding decrease in feedback strength. When the coupling is zero, the correlation coefficient and SEDC between the outputs of the two lasers is zero indicating no synchronization between them. As the coupling strength increases, the coupling becomes closed loop, where each laser receives a feedback signal from its own output as well as a coupling signal from the other laser. One can find that the synchronization quality increases when the coupling strength increases and attains a maximum of 0.99 for both SEDC and correlation coefficient at a coupling strength of 0.01. Here, the self-feedback strength also reaches 0.01 thus providing maximum symmetry in total feedback signals. With further increase in coupling strength, the quality of synchronization decreases as indicated by the decrease in correlation coefficient and SEDC. When the coupling strength reaches its maximum value, the self-feedback of individual lasers is cut off thus making the coupling an open loop one which is equivalent to the bidirectional lag coupling. It can be inferred that highest correlation occurs at a point when both self-feedback and coupling strength are equal and also equal to half of the required feedback. From these results, it is clear that the quality of synchronization is highly dependent on the symmetry of feedback signals. The role of symmetry of the feedback signals in providing high quality synchronization can be attributed to their roles in driving the systems to chaotic state. This shows that a symmetric closed loop scheme is effective for synchronization of two such lasers in bidirectional coupling configuration when compared to open loop or bidirectional lagged coupling.

The possibility of using the above scheme for secure communication is investigated using both analog and digital messages. The messages are encoded onto the chaotic outputs of each laser using chaotic modulation scheme. The messages are added to the outputs of each laser and a fraction of the total signal is fed back

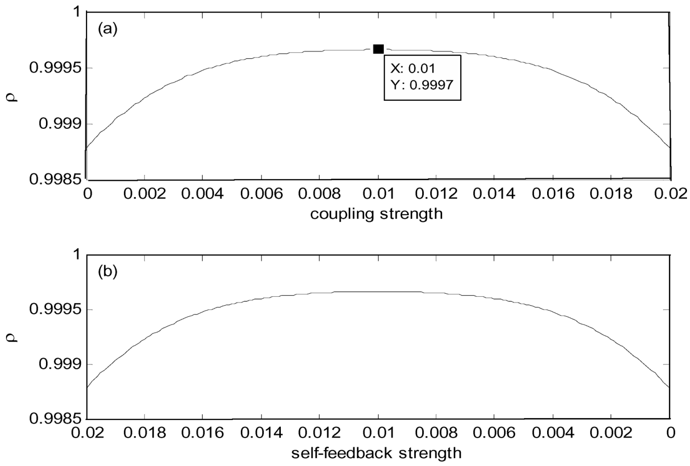

Figure 2. Variation of correlation coefficient (ρ) with (**a**) increase in coupling strength, (**b**) decrease in feedback strength, for bidirectional coupling scheme.

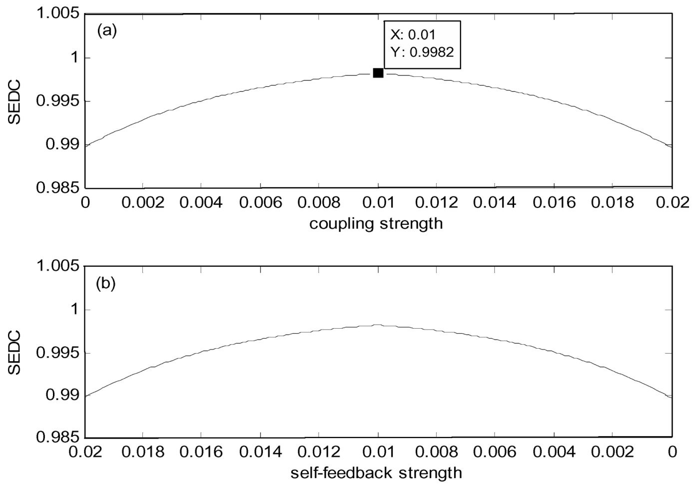

Figure 3. Variation of synchronization decay coefficient (SEDC) with (**a**) increase in coupling strength, (**b**) decrease in feedback strength, for bidirectional coupling scheme.

to their own inputs as self-feedback signal and another fraction is fed to the other laser's input as coupling signal. The amplitude and frequency of the message signal are chosen appropriately so as to ensure that the chaotic carrier properly masks them. For ensuring proper masking of the message in the chaos of the carrier signal, the message amplitudes should be restricted to <12% of the maximum value of the transmitter output amplitude. For the same reason the modulation index which is the ratio of frequency of message to the modulation frequency of the input current of the transmitter should be restricted to <1 [31].

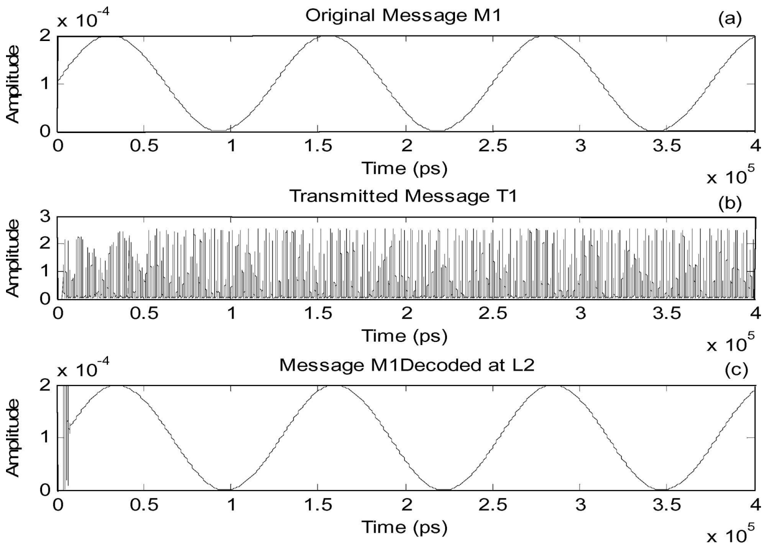

Figure 4. Encoding of analog message M_1 using bidirectional coupling scheme with chaotic modulation: (**a**) original message M_1 of amplitude 1*e*-4, modulation index of 0.01, (**b**) total transmitted signal T_1 and (**c**) message M_1 decoded at L_2.

Analog messages M_1 and M_2 of amplitude 1*e*-4 and modulation indices 0.01 and 0.02 are encoded on the outputs P_1 and P_2 of lasers L_1 and L_2, using chaotic modulation scheme. The total signal transmitted from laser L_1 and laser L_2 can be represented as follows:

$$T_1(t) = P_1(t) + M_1(t) \tag{6}$$

$$T_2(t) = P_2(t) + M_2(t). \tag{7}$$

A fraction of the total signal of each laser is transmitted to each other and another fraction is fed back to its own input as its delayed feedback signal. As the message signals enter the dynamics of the transmitter lasers, this becomes equivalent to chaotic modulation scheme. The self-feedback delay time and the travelling time of the coupling signal are kept equal to the optimum value as given in table 1. The feedback fraction and coupling fraction are kept equal at 0.01 so as to implement symmetric closed loop coupling scheme which gives maximum synchronization as indicated by the synchronization studies. The messages are decoded at L_1 and L_2 by subtracting the delayed output of L_1 and L_2 from the received signals T_2 and T_1 respectively. The decoding is represented as follows:

$$M_1'(t) = T_1(t-\tau) - P_2(t-\tau) = [P_1(t-\tau) + M_1(t-\tau) - P_2(t-\tau)] \tag{8}$$

$$M_2'(t) = T_2(t-\tau) - P_1(t-\tau) = [P_2(t-\tau) + M_2(t-\tau) - P_1(t-\tau)]. \tag{9}$$

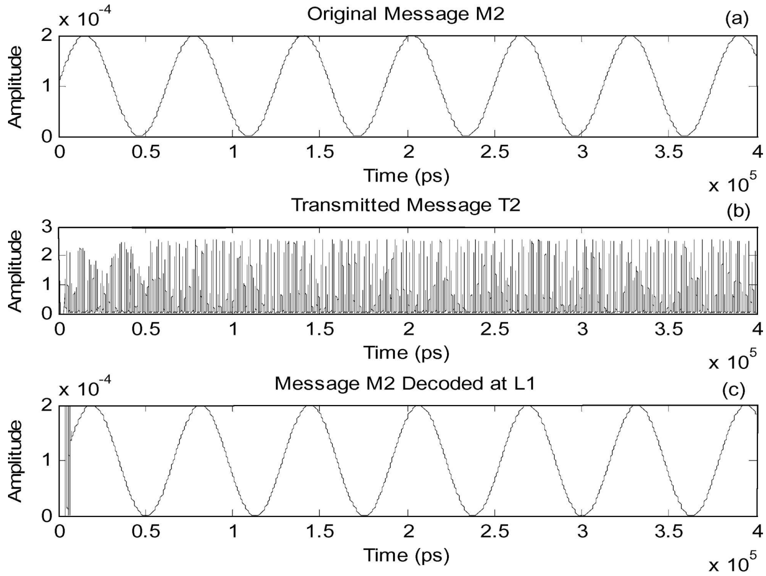

Figure 5. Encoding of analog message M_2 using bidirectional coupling scheme with chaotic modulation: (**a**) original message M_2 of amplitude 1e-4, modulation index of 0.02, (**b**) total transmitted signal T_2 and (**c**) message M_2 decoded at L_1.

Here, P_1 and P_2 represent the outputs of L_1 and L_2; T_1 and T_2 represent the total transmitted signals; M_1' and M_2' represent the messages decoded at L_2 and L_1 respectively. Figures 4a–4c show the original message M_1, the total signal T_1 transmitted from L_1 and the message M_1' decoded at L_2. Figures 5a–5c show the original message M_2, the total signal T_2 transmitted from L_2 and the message M_2' decoded at L_1. From these figures, it is clear that the transmitted signals do not show any qualitative features of the message contained in them. It can also be inferred that the messages are decoded efficiently without any contamination. These results indicate that the messages are successfully encoded and decoded by both lasers simultaneously.

Similarly, digital message of random signals are encoded onto the outputs P_1 and P_2 of lasers L_1 and L_2 by chaotic modulation. The messages M_1 and M_2 are chosen as random bits of amplitudes 2e-4 and 1e-4 respectively. Figures 6a–6c show the original message M_1, the total signal T_1 transmitted from L_1 and the message M_1' decoded at L_2. Figures 7a–7c show the original message M_2, the total signal T_2 transmitted from L_2 and the message M_2' decoded at L_1. It is clear from these figures that this method of encoding is highly efficient for digital messages also.

The variation of the quality of decoding with respect to the variation of coupling scheme from closed loop to open loop is investigated by estimating the variation of similarity between the original message and the decoded message. The correlation coefficient between the original message and the decoded message is estimated as the decoding efficiency which gives instantaneous similarity between the original and decoded messages. The maximum decoding quality would be indicated by a

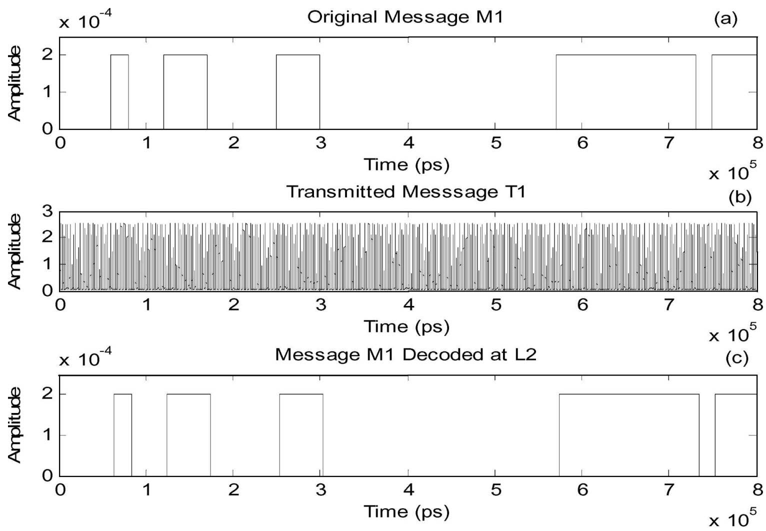

Figure 6. Encoding of digital message M_1 using bidirectional coupling scheme with chaotic modulation: (**a**) original message M_1, random bits of amplitude 2*e*-4, (**b**) total transmitted signal T_1 and (**c**) message M_1 decoded at L_2.

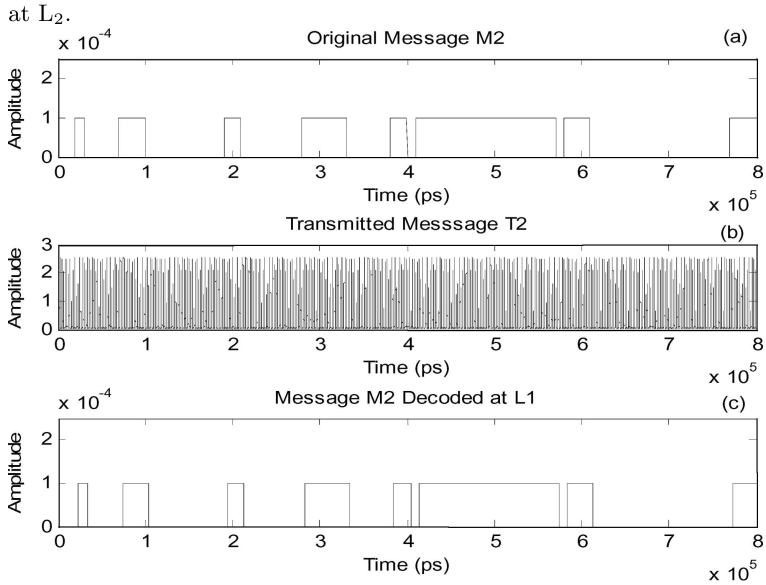

Figure 7. Encoding of digital message M_2 using bidirectional coupling scheme with chaotic modulation: (**a**) original message M_2, random bits of amplitude 1*e*-4, (**b**) total transmitted signal T_2 and (**c**) message M_2 decoded at L_1.

value of 1 for the correlation between the original message and the decoded message. When ideal decoding is achieved, the instantaneous similarity between the original message and the decoded message is maximum which is indicated by the correlation

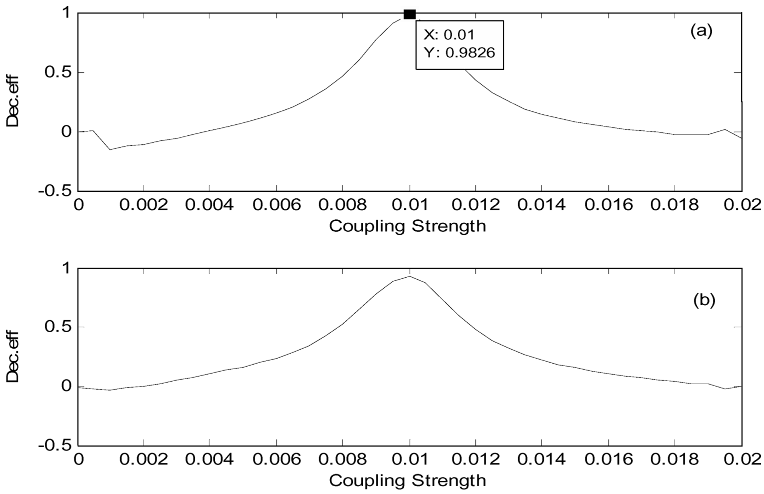

Figure 8. (**a**) Variation of decoding efficiency of message M_1 with increase in coupling strength for bidirectional coupling scheme with chaotic modulation, (**b**) variation of decoding efficiency of message M_2 with increase in coupling strength for bidirectional coupling scheme with chaotic modulation.

value of 1. As the decoding quality decreases the instantaneous similarity between the original message signal and the decoded message will decrease and this will be indicated by the decrease in the correlation between these signals. Figures 8a and 8b show the variation of decoding efficiency of messages M_1 and M_2 respectively at lasers L_2 and L_1. One can find that the decoding efficiency of both messages increases with coupling strength and reaches its maximum value at a coupling strength of 0.01 where the coupling becomes a symmetric closed loop. The decoding quality is lower for all asymmetric coupling strengths. However, for variations up to $\pm 5\%$ of coupling strength, the decoding efficiency is more than 90%.

4. Conclusions

The efficiency of open loop and closed loop bidirectional coupling schemes in synchronizing two directly modulated semiconductor lasers with delayed optoelectronic feedback is investigated and the ideal window of the coupling and self-feedback proportion is identified. A symmetric bidirectional closed loop coupling where the total required delayed feedback is provided equally by the delayed outputs of each of the two lasers is very effective in synchronizing them. This scheme is found to be useful for efficient bidirectional secure communication between the two lasers. Both analog and digital messages are successfully encoded and decoded at both ends simultaneously. Even though correlation coefficient and SEDC indicate complete synchronization at this coupling, it should be noted that SEDC can only indicate the nature of stability and not the exact state of stability like conditional or transverse Lyapunov exponents. Further studies on the stability of synchronization provided by this coupling will be highly useful for the successful application of this encryption technique. However, existing results of this numerical analysis indicate that this method can be suitably modified for secure communication between other chaotic systems which can incorporate a self-feedback and a coupling. This method can also be modified for synchronizing arrays of chaotic systems.

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