

# Analytical Equations for Compact Dual Frequency Microstrip Antenna

Sona O. Kundukulam, Manju Paulson, C. K. Aanandan, P. Mohanan

Microwave Engineering Group, Department of Electronics, Cochin University of Science and Technology, Cochin 682 022, India

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**ABSTRACT:** Design equations are presented for calculating the resonance frequencies for a compact dual frequency arrow-shaped microstrip antenna. This provides a fast and simple way to predict the resonant frequencies of the antenna. The antenna is also analyzed using the IE3D simulation package. The theoretical predictions are found to be very close to the IE3D results and thus establish the validity of the design formulae. © 2002 Wiley Periodicals, Inc. *Int J RF and Microwave CAE* 12: 477–482, 2002. Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mmce.10048

**Keywords:** microstrip; compact antennas; dual frequency; dual polarization

## I. INTRODUCTION

Because of unique and attractive properties such as light weight, low profile, conformal nature, and low production costs, microstrip antennas are fast replacing conventional antennas. The performance of these antennas can be easily predicted using the design formulae for calculating the different resonating modes. IE3D™ (Zeland Software) is an integrated full wave electromagnetic simulation and optimization package for the analysis and design of 3-dimensional microstrip antennas [1]. Using Green's function, current distribution coefficients are calculated for finding the S-parameters, the resonant frequency, and radiation patterns of a microstrip patch. For calculating the resonant frequencies, the present design equations are less time consuming than the above packages.

Empirical relations are available in the literature for calculating the resonant frequencies for a rectan-

gular microstrip antenna [2] and a compact drum shaped antenna [3] for the transverse magnetic ( $TM_{10}$ ) mode frequency and for a broadband dual frequency microstrip antenna [4] for both modes.

In this article we propose the design equations for a compact dual frequency arrow-shaped microstrip antenna [5, 6]. The  $TM_{10}$  and  $TM_{01}$  mode frequencies of this antenna can be calculated for the coaxial feed or electromagnetically coupled feed. The arrow-shaped antennas are compact, having an area reduction greater than 65% and similar radiation characteristics compared to the standard rectangular patch. This antenna resonates at two frequencies corresponding to its width and effective length and this dual frequency dual polarization operation [5] increases its applications in radar and satellite communications. The two orthogonally polarized frequencies can also be excited by perpendicular microstrip feed lines electromagnetically coupled to the antenna. The use of dual ports eliminates the cross talk between the frequencies. The theoretical results obtained from the simple design equations are in good agreement with the IE3D software predictions.

Correspondence to: Dr. C. K. Aanandan; e-mail: aanandan@doe.cusat.edu.

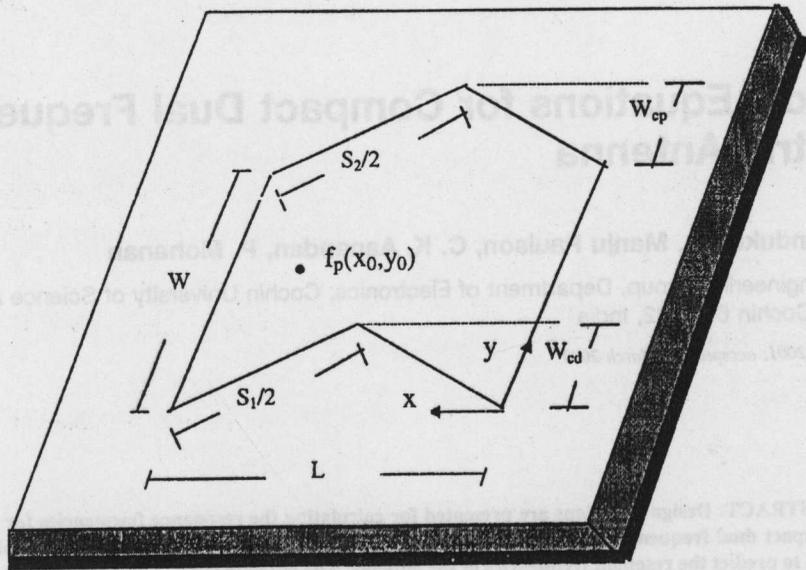


Figure 1. The geometry of the new compact microstrip antenna.

## II. COAXIALLY FED ARROW-SHAPED MICROSTRIP ANTENNA

### A. Design

The geometry of the dual frequency arrow-shaped microstrip antenna is shown in Figure 1. The structure consists of a patch of length  $L$ , width  $W$ , an intruding triangle of height  $W_{cp}$ , and a protruding triangle of height  $W_{cd}$ , with slanted lengths  $S_1$  and  $S_2$  etched on a substrate of dielectric constant  $\epsilon_r$  and thickness  $h$ . The antenna is coaxially fed at  $f_p(x_0, y_0)$  to excite the  $TM_{10}$  and  $TM_{01}$  mode frequencies.

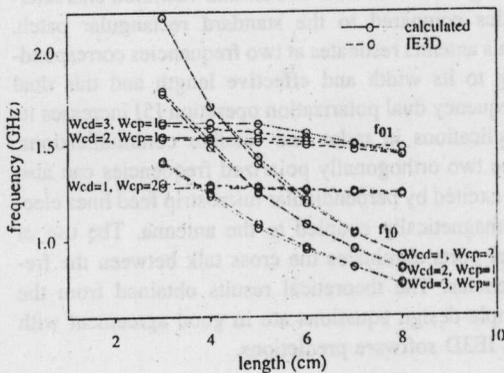


Figure 2. The variation of the  $TM_{10}$  and  $TM_{01}$  mode frequencies with the length  $L$  for different  $W_{cp}$  and  $W_{cd}$  values ( $W = 5$  cm).

### B. Resonant Frequency Calculations

The frequencies  $f_{10}$  and  $f_{01}$  can both be calculated by modifying the standard equation for a rectangular patch.

$$f_{10} = \frac{c}{2(S_{eff} + 2\Delta l_1)\sqrt{\epsilon_1}}, \quad (1)$$

$$f_{01} = \frac{c}{2(W_{eff} + 2\Delta l_2)\sqrt{\epsilon_2}}, \quad (2)$$

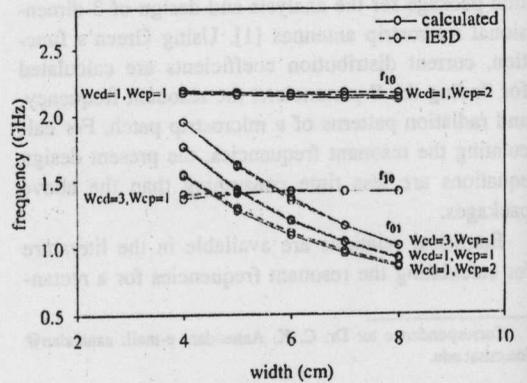


Figure 3. The variation of the  $TM_{10}$  and  $TM_{01}$  mode frequencies with the width  $W$  for different  $W_{cp}$  and  $W_{cd}$  values ( $L = 3$  cm).

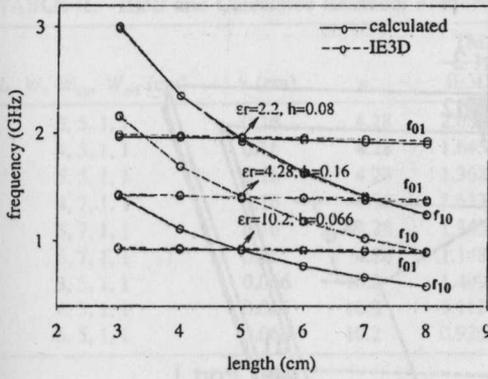


Figure 4. The variation of the  $TM_{10}$  and  $TM_{01}$  mode frequencies with the length  $L$  for different dielectric constant ( $\epsilon_r$ ) and thickness ( $h$ ) values ( $W = 5$  cm,  $W_{cp} = 1$  cm,  $W_{cd} = 1$  cm).

$$\epsilon_1 = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} (1 + 12h/W)^{-1/2}, \quad (3)$$

$$\epsilon_2 = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} (1 + 12h/S)^{-1/2}, \quad (4)$$

$$\Delta l_1 = \frac{0.412h(\epsilon_1 + 0.3)(W/h + 0.258)}{(\epsilon_1 - 0.258)(W/h + 0.8)}, \quad (5)$$

$$\Delta l_2 = \frac{0.412h(\epsilon_2 + 0.3)(S/h + 0.258)}{(\epsilon_2 - 0.258)(S/h + 0.8)}, \quad (6)$$

where  $S = (S_1 + S_2)/2$ .

The effective length  $S_{eff}$  and width  $W_{eff}$  are calculated as follows for ( $L < W$ ):

$$\left. \begin{aligned} S_{eff} &= S_1 - (0.0001/L) + 0.01W - 0.68(W_{cd} - 0.01) - 0.03(W_{cp} - 0.01) \\ W_{eff} &= W + 0.58W_{cp} - 0.43W_{cd} \end{aligned} \right\} \text{for } W_{cd}/W \leq 0.5,$$

$$\left. \begin{aligned} S_{eff} &= 0.5(S_1 + L) + 0.4W_{cd} - 0.175W - 0.03(W_{cp} - 0.01) \\ W_{eff} &= 0.78W + 0.025W_{cd} + 0.49W_{cp} \end{aligned} \right\} \text{for } W_{cd}/W > 0.5; \quad (7)$$

TABLE I. IE3D and Calculated Resonant Frequencies with Percentage of Error

$L, W, W_{cp}, W_{cd}$ (cm)	$h$ (cm)	$\epsilon_r$	$TM_{10}$ Mode Frequency (GHz)			$TM_{01}$ Mode Frequency (GHz)		
			IE3D	Calcd	Error (%)	IE3D	Calcd	Error (%)
3, 4, 0.5, 1	0.16	4.28	2.159	2.1611	0.09	1.886	1.8831	0.15
3, 4, 2, 1	0.16	4.28	2.172	2.1895	0.8	1.547	1.5366	0.67
3, 5, 1, 3	0.16	4.28	1.412	1.4017	0.73	1.605	1.6175	0.78
3, 6, 2, 1	0.16	4.28	2.163	2.155	0.36	1.111	1.0898	1.9
3, 6, 1, 5	0.16	4.28	0.9661	0.9512	1.54	1.372	1.3618	0.74
4, 5, 1, 1	0.16	4.28	1.691	1.6893	0.1	1.424	1.4134	0.74
4, 7, 1, 1	0.16	4.28	1.687	1.6694	1.04	1.035	1.0261	0.86
5, 6, 0.5, 1	0.16	4.28	1.376	1.375	0.07	1.245	1.2415	0.2
7, 5, 1, 4	0.16	4.28	0.7455	0.748	0.34	1.474	1.4738	0.01
7, 4, 2, 1	0.16	4.28	1.038	1.0375	0.04	1.468	1.4652	0.19
5, 5, 1, 1	0.32	2.2	1.903	1.9002	0.14	1.903	1.9066	0.18
7, 5, 1, 1	0.32	2.2	1.374	1.4003	1.91	1.862	1.8625	0.02
8; 5, 1, 1	0.32	2.2	1.217	1.2356	1.5	1.847	1.8427	0.23
3, 5, 1, 1	0.08	2.2	2.982	3.0031	0.7	1.987	1.962	1.25
6, 5, 1, 1	0.08	2.2	1.62	1.6394	1.19	1.954	1.933	1.06
8, 5, 1, 1	0.08	2.2	1.234	1.2435	0.76	1.925	1.8962	1.49
4, 5, 1, 1	0.066	10.2	1.112	1.1026	0.08	0.9318	0.9183	1.44
3, 5, 1, 4	0.066	10.2	0.7419	0.7283	1.9	1.067	1.044	1.87
3, 5, 1, 1	0.066	10.2	1.425	1.4126	0.87	0.9389	0.9218	1.85

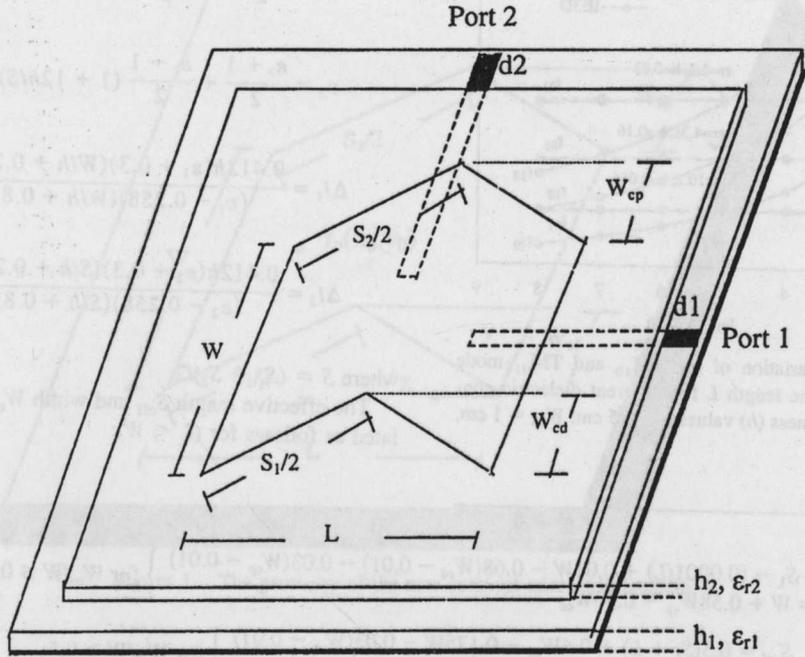


Figure 5. The geometry of the dual port compact microstrip antenna.

and for ( $L \geq W$ ):

$$S_{eff} = S_1 + 2.3(L - 2W - 0.0046/L)W_{cd} + 0.00006/L - 0.1(W_{cp} - 0.01), \text{ for } W_{cd}/W < 1;$$

$$W_{eff} = W + 0.58W_{cp} - 0.43W_{cd} + 0.0023(L - W)/W, \text{ for } W_{cd}/W \leq 0.5;$$

$$W_{eff} = 0.78W + 0.025W_{cd} + 0.49W_{cp} + 0.0025W_{cd}/W + 0.17(L - W - 0.01), \text{ for } W_{cd}/W > 0.5. \quad (8)$$

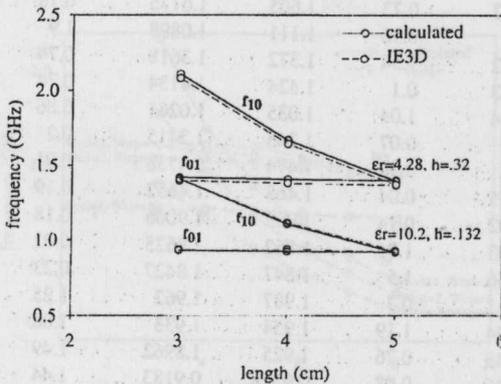


Figure 6. The variation of the  $TM_{10}$  and  $TM_{01}$  mode frequencies with the length  $L$  for different dielectric constant ( $\epsilon_r$ ) and thickness ( $h$ ) values for the dual port antenna ( $W = 5$  cm,  $W_{cd} = 1$  cm,  $W_{cp} = 1$  cm).

### C. Comparison of Calculated and IE3D Results

The results obtained by using eqs. (1) and (2) and an IE3D simulation for various lengths of arrow-shaped antennas for different combinations of  $W_{cd}$  and  $W_{cp}$  values are shown in Figure 2. Note from the graph that the  $f_{10}$  mode frequency varies rapidly and the  $f_{01}$  frequency remains almost constant for particular  $W_{cd}$  and  $W_{cp}$  values. The theoretical and simulated results using IE3D are in very good agreement.

Figure 3 shows the variation of both the frequencies versus the width of the patch. The width variations mainly affect the  $f_{01}$  mode frequency, keeping the other almost constant. The results of the variation of the resonant frequencies with different  $h$  and  $\epsilon_r$  combinations are shown in Figure 4. The theoretical results almost follow the IE3D results in all cases.

Table I shows the calculated frequencies, IE3D simulated frequencies, and their corresponding percentage of error for different combinations of  $W_{cd}$ ,  $W_{cp}$ ,  $h$ , and  $\epsilon_r$  values. In all these cases, both of the results are found to be in good agreement with an error of less than 2%.

We observed that the  $TM_{10}$  mode frequency varies with the slanted length  $S_1$  and  $TM_{01}$  mode frequency with the effective width (dependent on  $W_{cd}$ ,  $W_{cp}$ ,  $W$ ) of the patch. The frequency ratio between these two modes can be changed by varying the dimensions  $W_{cd}$  and  $W_{cp}$ .

TABLE II. IE3D and Calculated Resonant Frequencies for Dual Port Antenna with Percentage of Error

$L, W, W_{cp}, W_{cd}$ (cm)	$h$ (cm)	$\epsilon_r$	TM <sub>10</sub> Mode Frequency (GHZ)			TM <sub>01</sub> Mode Frequency (GHZ)		
			IE3D	Calcd	Error (%)	IE3D	Calcd	Error (%)
3, 5, 1, 1	0.16	4.28	2.08	2.107	1.61	1.395	1.4175	1.3
4, 5, 1, 1	0.16	4.28	1.645	1.669	1.94	1.379	1.4057	1.46
5, 5, 1, 1	0.16	4.28	1.368	1.3926	1.98	1.363	1.3963	1.61
4, 7, 1, 1	0.16	4.28	1.633	1.6433	1.48	1.013	1.028	0.63
5, 7, 1, 1	0.16	4.28	1.343	1.3582	1.29	1.008	1.021	1.13
6, 7, 1, 1	0.16	4.28	1.148	1.1557	1.2	1.003	1.057	0.67
3, 5, 1, 1	0.066	10.2	1.409	1.4084	0.91	0.9364	0.9278	0.04
4, 5, 1, 1	0.066	10.2	1.112	1.103	0.8	0.9364	0.922	0.81
5, 5, 1, 1	0.066	10.2	0.9263	0.9143	0.04	0.9172	0.9176	1.29

When solved for a rectangular patch ( $W_{cd} = 0$ ,  $W_{cp} = 0$ ), the design equations shown above give exactly the same results as those obtained from the design equation of a standard rectangular microstrip patch.

### III. ELECTROMAGNETICALLY COUPLED DUAL PORT ARROW-SHAPED MICROSTRIP ANTENNA

#### A. Design

The above antenna is reconfigured using two perpendicular microstrip feed lines to eliminate cross talk between the two polarizations and to achieve excellent isolation between the ports. The geometry of the proposed antenna is shown in Figure 5. The antenna is etched on a dielectric substrate of thickness  $h_2$  and dielectric constant  $\epsilon_{r2}$  and fed by proximity coupling using two 50  $\Delta$  perpendicular microstrip lines etched on a substrate of thickness  $h_1$  and dielectric constant  $\epsilon_{r1}$ .

#### B. Resonant Frequency Calculation

The equations given above for the coaxially fed arrow-shaped microstrip antenna is modified to obtain the frequencies for the dual ports. Here the thickness of the substrate is modified because of the effect of another substrate with the microstrip feed line. Hence, the value of  $h$  used in the above equations should be replaced by an effective thickness  $h_{eff} = h_1 + h_2$  and the dielectric constant  $\epsilon_r = \epsilon_{r1} = \epsilon_{r2}$ , where  $h_1$  and  $h_2$  are the thicknesses of the two layers.

#### C. Comparison of Calculated and IE3D Results

The variation of the two resonant frequencies with the length for different values of  $h_{eff}$  and  $\epsilon_r$  as calculated from eqs. (1) and (2) are shown in Figure 6. The IE3D curves are given in the figure to validate the closed-

form expressions. Here the theoretical results are in good agreement with the IE3D values with a maximum error of less than 2% as shown in Table II.

### IV. CONCLUSION

The design equations for calculating the TM<sub>10</sub> and TM<sub>01</sub> mode frequencies of an arrow-shaped dual frequency microstrip antenna are presented. The results are compared with IE3D simulation results, which show that both are in good agreement with an error of less than 2%. These equations provide a fast and simple method for the design of this compact microstrip antenna.

### ACKNOWLEDGMENTS

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