S.m.13. SUNANDAKUMARI, V.M.–Theory of differential inequalities with applications to singular perturbation problems and method of lines–1989–Dr. N. Ramanujam

During recent years, the theory of differential inequalities has been extensively used to discuss singular perturbation problems and method of lines to partial differential equations. The present thesis deals with some differential inequality theorems and their aplications to singularly perturbed initial value problems, boundary value problems for ordinary differential equations in Banach space and initial boundary value problems for parabolic differential equations. The method of lines to parabolic and elliptic differential equations are also dealt with.

The thesis is organised into nine chapters. A detailed description of chapters would run as follows:

The first chapter reviews briefly the earlier developments of the 'heories in .he field related to the current work and also discusses the scope of this thesis.

In the second chapter the background material, mainly differential inequality theorems or monotonicity theorems, which shall be needed in the subsequent chapters is presented.

The third chapter is devoted to study the asymptotic behavious of solutions and/or their derivatives of the following boundary value problems for weakly coupled doubly infinite systems of second order ordinary differential equations with a small parameter multiplying the highest dervative. The boundary value problems are the infinite system of differential equations

$$Pu := -\epsilon u'' + f(t, [u]', u, \epsilon) = 0,$$

that is

$$P_i u := -\epsilon u_i'' + f_i(t, u', u, \epsilon) = 0, \; t \in D := (a,b), \; i \in Z$$

subject to any one of the following boundary conditions

$$Ru := (u(a,\epsilon), u(b,\epsilon))^T = (A(\epsilon), B(\epsilon))^T,$$

$$Ru := (u(a,\epsilon), u'(b,\epsilon))^T = (A(\epsilon), B(\epsilon))^T,$$

$$Ru := (u(a,\epsilon), \epsilon u'(a,\epsilon), u'(b,\epsilon))^T = (A(\epsilon), B(\epsilon))^T,$$

where $u = (\ldots, u_{-1}, u_0, u_1, \ldots)^T$

$A(\epsilon) = (\ldots, A_{-1}(\epsilon), A_0(\epsilon), A_1(\epsilon), \ldots)^T$

$B(\epsilon) = (\ldots, B_{-1}(\epsilon), B_0(\epsilon), B_1(\epsilon), \ldots)^T$

ϵ is a small positive parameter such that $0 < \epsilon \leq \epsilon_0$,

T stands for the transpose, Z is the set of all integers, and a prime denotes a differentiation with respect to t. The study is carried out by first obtaining necessary estimates using monotonicity theorems for solutions and/or their derivatives of the above boundary value problems. In turn, these estimates determine the asymptotic behaviour of solutions and/or their derivatives as the small parameter approaches zero.

The results of the third chapter are extended in the fourth chapter for the differential equations obtained by replacing b by ∞.

Chapter 5 deals with certain initial value problems. First the asymptotic study, as the parameter ϵ goes to zero, is carried out for the linear equations of the form

$$\epsilon x'' + \alpha x' + \beta x = r, \; \alpha = \alpha(t,\epsilon), \; \beta = \beta(t,\epsilon)$$

$$r := r(t,\epsilon)$$

and then the results are generalized to the nonlinear systems. Using monotonicity theorems, asymptotic estimates for solutions are constructed, under appropriate assumptions, in terms of solutions of the corresponding reduced problems.

Chapter 6 treats linear and nonlinear parabolic differential equations with a small parameter multiplying the time derivative. More precisely, the differential equations subject to Dirichlet type boundary conditions are considered. Guided by the experience with ordinary differential equations and using parabolic differential inequality theorems, estimates for solutions of the present problems are obtained. In turn, these estimates determine the asymptotoc behaviour of solutions as the small parameter ϵ approaches zero.

In Chapter 7 an attempt is made to study the method of lines (a special case of differential-difference scheme) when applied to nonlinear elliptic differential equations defined on a unit square and semi infinite strip. In fact, a few results are presented on the error estimates and the convergence of the the method of lines to certain boundary value problems. Due to the presence of the small parameter ϵ, a modified version of the existing difference scheme is suggested. Using the second order differential inequalities theory, error estimates and hence the convergence of the method of lines are obtained.

The method of lines, more precisely longitudinal line method, for some problems is discussed in Chapter 8.

The last chapter continues with the study of the method of lines begun in Chapter 7. In this chapter a weakly coupled system of two parabolic differential euations with a small parameter multiplying the highest spatial derivatives is considered and results similar to that given in Chapter 8 are given.