S.m.12. MATHEW, P.M.–On the Lattice of Topologies and Boolean Spaces–1988.–Dr. T. Thrivikraman

Correspondence between order and topoogy was investigated by several mathematicians in different contexts—G. Birkhoff (1936) and R Vaidyanathasvamy (1960) being the forerunners.

In this thesis, the study is on the relation between topology and order in the following contexts.

i) Relation between some known order properties and topological concepts—determination of maximal and minimal topologies having those properties.

ii) Order theoretic aspects of the lattice of all topologies on a set.

iii) Studies on some classes of ideals in a Boolean ring on order theoretic terms and their topological interpretation using Stone representation theorem.

Andima S.J and Thron W.J. defined (1978) a space upward directed if any two elements in the basic set has an upper bound under the induced order and characterised maximal upward directed toplogies order theoretically. In Chapter 1 of this thesis, the equivalence of the notions of ultraconnected and upward directed is indicated and the property-downward directed is discussed and its equivalence to hyperconnectedness among principal topologies is established.

Aslo an internal characterisation of hyperconnected spaces using semi-topological concepts is established and the result of T. Noiri (1980) that hyperconnectedness is a semi-topological property is derived as a corollary. Maximal hyperconnected topologies are characterised and it is shown that hyperconnected door topologies are maximal hyperconnected and minimal door. Further, maximal ultraconnected topologies on a set are determined and it is shown that ultraconnected $T_{1/2}$ topologies are maximal ultraconnected and minimal $T_{1/2}$.

Chapter 2 is a study of F-connected spaces, spaces which are hyperconnected and ultraconnected. F-connectedness is shown to be not hereditary not productive and not semitopological. The maximal compact F-connected topologies are determined. It is noted that any dual atom in the lattice of topologies is the least upper bound of two minimal door topologies of which one is maximal ultraconnected and the other maximal hyperconnected.

In chapter 3, a study of the concept of covers in the lattice L(X) of all topologies on a set X is presented. Also a discussion of this concept in the system of closure relations is undertaken. A method to construct a completely regular topology coarser than a given topology which is the unique completely regular topology covered by the original topology characteristic for preservation of continuous maps is also given in this chapter.

An immediate consequence of Stone representation theorem is that the Boolean algebra of regular open sub-sets of a compact Hausdorff, O-dimensional space is a completion of its clopen subsets. In chapter 4, it is shown that the Boolean algebra of normal ideals in a Boolean ring is a minimal completion of the algebra of its semiprincipal ideals, which is an extension of the above result to the case of Boolean rings. Further a natural limit to the applicability of Stone's theorem is indicated.