S.m.5. THRESIAMMA, T.K.–Discrete Commutative Difference Operator Theory–1986–Dr. Wazir Hasan Abdi and Dr. T. Thrivikraman.

This thesis is an attempt to study basic and bibasic commutative difference operators on the lines of J.L. Burchnall and T.W. Chaundy's "Commutative differential operators" and on the discrete sets $(p^m x_0)$, $(q^n y_0)$, $(q^m x_0, q^n y_0)$, $(q^m x_0, q^n y_0)$, $(p^m x_0, q^n y_0)$, $(q^m x_0, p^n y_0)$ $m,n \in Z$ where p,q are positive real constants called bases, $p = q = 1$. Also bibasic pseudo-analytic functions are introduced using two bases p and q.

Though commutative differential operators play an important role in analysis, no basic or bibasic theory is available. This thesis is an attempt in this direction.

In the first chapter an outline of the theory of commutative differential operators done by Burchnall and Chaundy in the classical case is given. A historical survey of the study of q-difference equations, q-analytic function theory of C.J. Harman (1972), bibasic analytic functions of Khan, M.A., bianalytic functions of K.K. Velukutty (1982), Discrete Pseudoanalytic functions of Mercy K. Jacob (1983) and the recent works already done by others have been stated. A list of the results established in the thesis is also given.

The second chapter deals with definitions of basic difference operators, characteristic identity of two commutative basic difference operators and the specific nature of commutative difference operators. If

$P_m = \sum a_r \theta^r$

$Q_n = \sum b_r \theta^s$

then it is proved that if a_r and b_s are constants or q-periodic functions of x then P_m and Q_n are commutative. But if they are variable functions of x they are commutative. Hence the conditions for which these commute are obtained. It is observed that there are some relationships between the coefficients $a_r(x)$ and $b_s(x)$ which make the operators commute with each other. Some examples are constructed. Then it is proved that the difference operators P and Q are commutative if and only if $F(P,Q)=0$.

Some special commutative operators are developed in the third chapter. Taking $\varpi = x\theta$, θ^r is factorised by means of ϖ. If two operators have a common factor, by transference of that factor we obtain new operators. And it is shown that if P' and Q' are the new operators obtained by transference of common factor of P and Q respectively then $F(P,Q) = F(P', Q')$. The result $f(\varpi)x^a = f([a])x^a$ is derived, and the inverse $f^{-1}(\varpi)x^a = f(1 \div [a])\ x^a$ is found. Using these results some q-difference equations are solved.

In the fourth chapter basic adjoint operators are defined and their properties are listed. If P and Q are commutative, their adjoint also are commutative. The same results are obtained for transference also. Then it is shown that if a linear operator commutes with an operator P it is a polynomial in that operator.

The bibasic commutative difference operator are studied by considering functions of x and y in R^2. Definition of D_{px} and D_{qy} with bases p and q and their properties studied in the fifth chapter. Some special bibasic commutative difference operators are taken and some bibasic difference equations are solved.

The last chapter deals with bibasic pseudoanalytic functions and their properties. Pseudoanalytic functions of Mercy, bibasic analyic functions of Khan, bianalyic functions of K.K. Velukutty, and q-analytic functions of Harman are special cases of this. Bibasic analogues of Z^n and exp $b(z)$ of classical power and exponential functions etc., are also given as examples of such functions.