Studies on Some Aspects of the Physics of the Early Universe Using Gravitationally Coupled Scalar Field

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CERTIFICATE

Certified that the work presented in this thesis is a bonafide work done by Miss. Minu Joy, under my guidance in the Department of Physics, Cochin University of Science and Technology and that this work has not been included in any other thesis submitted previously for the award of any degree.

Kochi-682 022 Dr. V. C. Kuriakose

April 10, 2003 Supervising Guide

PREFACE

Quantum fields, so far have been exclusively successful in explaining the high energy world, up to at least the TeV scale. But, beyond the Planck time, the thermal energy of the universe would have been the Planck energy, 10^{19}GeV and we need a quantum treatment of gravity, which remains still elusive. In the absence of a satisfactory theory of quantum gravity it is very difficult to describe the influence of the gravitational field on quantum phenomena. So a semiclassical treatment for the quantum aspects of gravity is adopted in which the gravitational field is treated as a classical background, while the matter fields are quantised in the usual way. Quantum field theory in an external classical gravitational field is usually regarded as a first step towards a more complete theory of quantum gravity.

Quantum fields have profound influence on the dynamical behaviour of the early universe. The inflationary universe scenario broaches the question concerning the role of a scalar field in cosmological evolution and particularly its influence on the development of cosmological inhomogeneities. The influence of quantum fields on the cosmological phase transitions, inflation, particle creation and cosmological perturbations have been investigated by many authors.

This thesis deals with some aspects of the Physics of the early universe, like phase transitions, bubble nucleations and premordial density perturbations which lead to the formation of structures in the universe. A gravitationally coupled scalar field is used as a tool for these studies.

Chapter 1 of the thesis gives an introduction to the phase transitions in the early universe and to the cosmological perturbations. An account of effective potential calculations, renormalisation, spontaneous symmetry breaking, finite temperature field theory, quantum fluctuations and gravitational instability are discussed in this chapter.

In the early universe, symmetries that are spontaneously broken today were restored and during the evolution of the universe there were phase transitions, perhaps many, associated with the spontaneous breakdown of symmetries (SSB). During such a phase transition it is possible for the field to acquire nonzero vacuum expectation values. In general, a symmetry breaking phase transition can be of first or second order. For a first order phase transition the change in the field, ϕ in going from one phase to the other must be discontinuous, while for a second order transition there is no barrier at the transition point and the transition occurs smoothly.

First order phase transitions in the early universe are studied in **Chapter 2** using (3+1) dimensional Bianchi type-I background spacetime. We consider a massive scalar field coupled to the gravitational background and having ϕ^6 self-interaction. ϕ^6 model is known to be nonrenormalisable in (3+1) dimensional flat spacetime. Nonrenormalizability of the field theory does not mean that the theory is not interesting and it does not mean, ofcourse, that finite renormalised prescription for the calculation of one-loop effective potential does not exist. The one-loop effective potential for ϕ^6 theory in a $(3 + 1)$ dimensional Bianchi type-I spacetime is evaluated and it is found that the ϕ^6 model can be regularised using the effective potential method in a (3+1)dimensional curved spacetime.

While constructing a theory of the interaction between quantized matter fields and a classical gravitational field one has to identify the energy-momentum tensor of the quantized fields which acts as the source of the gravitational field. The energy-momentum tensor for the present ϕ^6 self-interacting field is evaluated with (3+1) dimensional Bianchi type-I background spacetime and it is found that the energy-momentum tensor depends on the anisotropy of the spacetime.

The temperature dependence of finite temperature effective potential in quantum field theory leads to phase transitions. Evaluating the one loop effective potential at finite temperature, the finite temperature effects on the phase transitions of early universe are discussed in this chapter. The nature of phase transitions is examined and is verified to be of first order. The crucial dependence of phase transitions of the early universe on spacetime curvature and the gravitational-scalar coupling is also made clear. The phase transitions, induced by the curvature R and the coupling constant ξ are also found to be of first order.

(2+1) dimensional gravity exhibits novel features of interest and there are several important differences between the three and four dimensional problems. Considering the same Lagrangian density with ϕ^6 potential as in chapter 2 and using the momentum cut-off technique, a divergenceless expression for the ϕ^6 potential in a (2+1) dimensional Bianchi type-I background spacetime is obtained in **Chapter 3**. The finite expression for energy momentum tensor is obtained with (2+1) dimensional Bianchi type-I background spacetime also. Evaluating the one loop effective potential at finite temperature the finite temperature effects on phase transitions are studied for this model. The existence of the separate branches of σ_T^2 implies that the phase transition is of first order (where σ_T is the order parameter). It is found that the spacetime curvature and the scalar-gravitational coupling do play a crucial role in determining the nature of phase transitions in this model also.

A first order phase transition proceeds by nucleation of bubbles of broken phase in the background of unbroken phase. The bubbles expand and eventually collide, while new bubbles are continuously formed, until the phase transition is completed. While discussing the bubble collisions one has to consider the interaction between the bubble field and the surrounding plasma. Taking account of this, an exact solution for the damped motion of the bubble in the thin wall regime is obtained in chapter 3.

If the phase transition is strongly first order, the universe may be dominated by the vacuum energy and undergo a period of inflation. The quantum fluctuations in the inflaton field are the most natural choice for the seed perturbations. Small fluctuations in the density results in gravitational instability and gravitational instability causes the growth of perturbations in an expanding universe. The structure we observe in the universe today is the end result of the gravitational amplification of small primeval perturbations. The gravitational instability of a spatially uniform state of dust-like matter described by classical non-relativistic equations has been first investigated by Jeans. If the mass of a body is larger than some minimum mass called the Jeans mass, then the self gravity of matter will start affecting the structure of the body significantly. The possibility of using the instability mechanism of Jeans theory to form self-graviting configurations from a real scalar field is described in **Chapter 4**. A scalar field approach to Jeans mass calculation is considered.

The cosmic fluid can be treated in complete analogy to a scalar field and the description of cosmological perturbations in the universe can be reduced to the study of quantum fluctuations of a gravitationally coupled scalar field. Considering a massive scalar field arbitrarily coupled to a gravitational background, the stress-energy tensor expectation values are computed in this chapter. The vanishing of nondiagonal terms of the expectation value of $T_{\mu\nu}$ allows us to treat the scalar field in complete analogy to a perfect fluid. Then the energy density and pressure associated with the density perturbations in a Robertson-Walker universe are evaluated. The primeval density perturbations produced by the vacuum fluctuations of the scalar field are considered and the Jeans criterion for the structure formation is obtained. Then the expressions for Jeans length and Jeans mass are evaluated for a curved spacetime.

The quantum fluctuations of a scalar field with quartic self-interaction is also considered in this chapter and Jeans length is evaluated. It is found that the self-interaction of the field influences the character of instability and the value of Jeans wave number K_J is altered by the effects of self-interaction.

In **Chapter 5** an anisotropic (3+1) dimensional Bianchi type-I spacetime which is spatially homogeneous is considered as the background metric. As an alternative to the N representation, we can construct an (over)complete normalised set $|\Gamma_{\vec{k}}\rangle$ of coherent state for each mode of the scalar field. The stress-energy tensor expectation values are computed in a coherent state. The density matrix is used to represent the expectation values. Then the energy density and pressure associated with the density perturbations are evaluated. Using these results the exact expression for Jeans wave number is evaluated.

Then the distribution of matter field which is assumed to be locally anisotropic and is coupled to a Bianchi type-I background spacetime is considered. In addition to the quantum fluctuations, perturbations in the background metric are also taken into account in this chapter. The expression for the perturbed energy momentum tensor is obtained. It is found that for the present anisotropic case, the perturbation of pressure in radial and tangential directions are different and therefore Jeans wave number depends on the velocity component of fluctuations in radial and transverse directions.

The results and final conclusions of this thesis work are given in **Chapter 6.**

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1. "First Order Phase Transitions in a Bianchi Type-I Universe", Minu Joy and V. C. Kuriakose, Physical Review D, 62 (2000), 104017.

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4. "Phase Transitions for a ϕ^6 Model in Curved Spacetime", Minu Joy and V. C. Kuriakose, Modern Physics Letters A (To appear).

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4. "Phase Transitions in a Curved Spacetime", XIV DAE Symposium on High Energy Physics, Dec. 18-22, 2000, HCU Hyderabad, India.

5. "Physics of the Early Universe: Field Theoretic Aspects", National Workshop on Field Theory Aspects of Gravity (FTAG -III), Jan. 23-29, 2003, Cochin University of Science and Technology, Cochin, India.

Contents

Chapter 1

Introduction

The quantisation of gravitational field has been pursued with great ingenuity and vigour over the past several decades. But a completely satisfactory quantum theory of gravity still remains elusive. In the absence of a viable theory of quantum gravity one can not describe the influence of the gravitational field on quantum phenomena. So a semiclassical treatment for the quantum aspects of gravity is adopted in which the gravitational field is treated as a classical background field, while the matter fields are quantised in the usual way. Quantum field theory in an external classical gravitational field is usually regarded as a first step towards a more complete theory of quantum gravity.

In the very early universe, at high temperatures and energies, a classical description of matter breaks down and it must be replaced by a description in terms of quantum fields. Although, as yet, there has been no direct observation of the fundamental scalar particle (the Higgs particles), such particles proliferate in modern particle theories. To obtain inflation, we need a material with the unusual property of negative mass. Such a material is a scalar field describing scalar (spin-0) particles. They play a crucial role in bringing about symmetry breaking between the fundamental forces. Scalar fields were introduced by particle physicists long before particle cosmology came into being as a subject, but were pounced upon by the cosmology community because of the range of interesting phenomena in which they may partake.

Quantum fields have profound influence on the dynamical behaviour of the early universe. The potential role of scalar field in Cosmology has been well discussed [1]-[11]. The inflationary universe scenario broaches the question concerning the role of a scalar field in cosmological evolution and particularly its influence on development of cosmological inhomogeneities. The influence of quantum fields on the cosmological phase transitions, inflation, particle creation and cosmological perturbations have been investigated by many authors [8]-[11].

1.1 Scalar Field

Quantum fields are fundamental physical concepts, in terms of which the interactions of the elementary particles are described. Field theory is characterised by a physical quantity called the action S, defined as the functional of the fields and has the general form

$$S = \int dx \mathcal{L}(\Phi(x), \partial_\mu \Phi(x)) \tag{1.1.1}$$

$\mathcal{L}$ is a real scalar local function of fields and their spacetime derivatives and is called the Lagrangian density. Usually it is assumed that the Lagrangian depends only on the field Φ and its first derivatives. The fundamental equations of the classical field theory, the equations of motion, follow from the stationary action principle and have the form

$$\frac{\delta S}{\delta \Phi(x)} = 0 \tag{1.1.2}$$

The Lagrangians of simplest field theory models are polynomials of definite degree in the field Φ which also depend on the first derivatives $\partial_\mu \Phi$ not more than quadratically. As regard the field Φ, it is natural to consider tensors of the low ranks, that is, scalars ϕ, vectors A_μ, second-rank tensors $B_{\mu v}$ and spinors.

For the real scalar field $\phi(x)$ the expression for Lagrangian density has the form,

$$\pounds = \frac{1}{2}\eta^{\mu v}\partial_\mu\phi\partial_v\phi - \frac{1}{2}m^2\phi^2 \tag{1.1.3}$$

Usually the first term in the Lagrangian density in Eq. (1.1.3) is called the kinetic term and the second is called the mass term. The inclusion of the field's interactions in the simplest case is carried out by adding the polynomials over the fields and their derivatives of general degree higher than two to the Lagrangian in Eq. (1.1.3). Including the interaction, leads to the Lagrangian density,

$$\pounds(\phi, \partial_\mu\phi) = \frac{1}{2}\eta^{\mu v}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2 - V(\phi) \tag{1.1.4}$$

where $V(\phi)$ is the interaction potential.

1.2 Gravitationally Coupled Scalar Field

The inclusion of the interaction of scalar field with the gravitational field and the scalar field quantisation in curved spacetime can be performed in an analogous way to the Minkowski spacetime [12]. Furthermore, all the ordinary derivatives ∂_μ must be changed by the general covariant derivatives ∇_μ. It is also necessary to ensure that the Lagrangian is scalar under the general coordinate transformations. Again, the integration in the expression for the action must be performed over the invariant volume. The form of Lagrangian density for the gravitationally coupled scalar field is given by

$$\mathcal{L}(\phi, \partial_\mu \phi) = \frac{1}{2}\sqrt{-g}\{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - [m^2 + \xi R]\phi^2\} \tag{1.2.1}$$

where m is the mass of the field quanta. The coupling between the scalar field and the gravitational field is represented by the term $\xi R\phi^2$, where ξ is a dimensionless parameter and R is the Ricci scalar curvature. The factor $\sqrt{-g}$ has been included to make $\mathcal{L}$ a scalar density and hence to make the action a scalar.

Two values of ξ are of particular interest: the minimally coupled case $\xi = 0$ and the conformally coupled case $\xi = \frac{1}{4}[(n-2)/(n-1)] \equiv \xi(n)$, where n is the spacetime dimension. In the conformally coupled case, if $m = 0$ the action and hence the field equation are invariant, not only under the general coordinate transformation but also under the conformal transformation,

$$\bar{g}_{\mu\nu}(x) = e^{2\sigma(x)} g_{\mu\nu}(x) \qquad\qquad \bar{\phi}(x) = e^{-\sigma(x)}\phi(x)$$

where $\sigma(x)$, the parameter of conformal transformation is an arbitrary scalar field.

Setting the variation of action with respect to ϕ, equal to zero yields the scalar field equation

$$[\Box + m^2 + \xi R]\phi(x) = 0 \tag{1.2.2}$$

where $\Box\, \phi = g^{\mu\nu}\, \nabla_\mu \nabla_\nu \phi = (-g)^{-1/2}\partial_\mu[(-g)^{1/2} g^{\mu\nu}\partial_\nu\phi]$.

1.3 The Effective Potential

The natural way to construct quantum gravity models is to apply quantum field theory models to theories of classical gravitational fields interacting with matter. In this approach, the effective action of the quantum field plays an essential role.

Therefore the investigation of quantum gravity models is concerned with the computation of the effective action and the study of its properties.

Phase transition is characterised by fluctuation in some field variables, usually called the order parameter. But the calculations at the classical level shows that the potential energy of the ordered state is actually less than the energy in the disordered state. Thus in order to treat the problem systematically, a procedure which takes into account all the quantum fluctuations is to be considered and one such procedure is to calculate the effective potential of the system.

This was introduced in quantum field theory by Schwinger [13] and was extended to the study of symmetry breaking phenomena by Jona-Lasinio [14]. An approximation scheme for the evolution of the effective potential was developed by Nambu [15] and his procedure is now known as loop expansion [1],[12]-[17]. The most compact and elegant way to study the symmetry properties of vacuum is the effective potential approach. Jona-Lasinio showed that the minima of the effective potential give without any approximation the true vacuum states of the theory. Effective potential can also be calculated by the diagrammatic method [18]. The virtue of this method is that it enables us to compute higher order corrections while still retaining a great advantage of the semi-classical approximation; the capability to survey all possible vacuum simultaneously. Hence even in the presence of renormalisation effects it helps to study the asymmetric but stable theory described by its Lagrangian.

The effective action (EA) is the classical action with all quantum corrections made to it. For finding the one loop EA [1],[12]-[17] it is necessary to calculate the determinants of the differential operators, dependent on the mean field. For higher order corrections it is necessary to find the Green's function $G(x, y \mid \phi)$, depending on the mean field. Since neither the first problem nor the second can be solved in general case, approximate methods for the EA calculations have been put forward. In one of these methods we compute the effective action for the slowly changing mean fields where we can neglect their derivatives.

Let the mean field $\phi(x)$ change slowly in spacetime. Then the effective action which is essentially the non-local object, may be found as a series in the mean field derivatives. For a slowly varying field we can leave a finite number of terms in these series that lead to the local approximation for the EA.

$$\Gamma[\phi] = \int dx[-V_{eff}(\phi) + \frac{1}{2}Z(\phi)\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + ...] \quad (1.3.1)$$

where $Z(\phi)$ is the generating functional of Green's functions. Every term in the right hand side of Eq. (1.3.1) is a function of the mean field $\phi(x)$. The term $V_{eff}(\phi)$ in the local approximation for EA is called the effective potential.

The calculation of the effective potential is connected with finding the effective action at the constant mean field, $\phi(x) = \phi =$ constant. For calculating effective potential in the arbitrary order of loop expansion, it is necessary to find the determinants of the differential operators with a constant coefficient and the Green's function in the constant mean field.

Let us consider the one loop effective potential for the scalar field theory with the Lagrangian density in Eq. (1.1.4). The effective action in the one loop approximation is

$$\begin{aligned} \Gamma^{(1)}[\phi] &= S[\phi] + \bar{\Gamma}^{(1)}[\phi] = S[\phi] + \frac{i}{2}\frac{h}{2\pi}Tr\ln S_2[\phi] \\ &= S[\phi] + \frac{i}{2}\frac{h}{2\pi}\ln\det S_2[\phi] \end{aligned} \quad (1.3.2)$$

where $S_2[\phi]$ is the Kernel of some differential operator depending on the mean field. For the present Lagrangian, the Kernel of the corresponding operator has the form

$$S_2[\phi] = S_2(x_1, x_2/\phi) = -[\Box + m^2 + V''(\phi(x_1))]\delta(x_1 - x_2) \tag{1.3.3}$$

Then we can rewrite Eq. (1.3.2) as

$$\Gamma^{(1)}[\phi] = S[\phi] + \frac{i}{2}\frac{h}{2\pi} \ln \det \frac{A}{\mu^2} \tag{1.3.4}$$

where

$$A = -[\Box + m^2 + V''(\phi)]. \tag{1.3.5}$$

The operator A has the dimension 2 in mass units and the arbitrary constant μ has the dimension of mass and it is introduced because the expression under the logarithm must be dimensionless.

By definition of the generalised Zeta function [12, 19] we can write

$$\det A = \exp[-\xi'(0/A)] \tag{1.3.6}$$

where $\xi(z/A)$ is the generalised Zeta function connected with the operator A and $\xi'(0/A) = \frac{d\xi(z/A)}{dz}\,|_{z=0}$. Using Eq. (1.3.6) we can write,

$$\Gamma^{(1)}[\phi] = S[\phi] - \frac{i}{2}\frac{h}{2\pi}\xi'(0/\frac{A}{\mu^2}) \tag{1.3.7}$$

Thus the problem consists in finding $\xi(z/A)$. For an arbitrary differential operator A the generalised Zeta function may be represented in the form [12],

$$\xi(z/A) = \frac{1}{\Gamma(z)} \int_0^\infty ids(is)^{Z-1} \int dx K(x, x/s) \tag{1.3.8}$$

In Eq. (1.3.7), substituting the expression for classical action $S[\phi]$,

$$S[\phi] = -\int dx(V(\phi) + \frac{1}{2}m^2\phi^2 + ...) \tag{1.3.9}$$

we can write

$$\Gamma^{(1)}[\phi] = -\int dx(V(\phi) + \frac{1}{2}m^2\phi^2 + ...) - \frac{i}{2}\frac{h}{2\pi}\xi'(0/\frac{A}{\mu^2}) \tag{1.3.10}$$

But from Eq. (1.3.1) we have

$$\Gamma^{(1)}[\phi] = -\int dx V^{(1)}_{eff}(\phi) + ... \tag{1.3.11}$$

Thus from the above two expressions we get

$$V^{(1)}_{eff}(\phi) = V(\phi) + \frac{1}{2}m^2\phi^2 + \frac{i}{2}\frac{h}{2\pi}\xi'(0/\frac{A}{\mu^2}) \tag{1.3.12}$$

and

$$\xi(x, z/\frac{A}{\mu^2}) = \frac{1}{\Gamma(z)}\int_0^{\infty} i\, ds(is)^{Z-1}K(x, x/s) \tag{1.3.13}$$

Thus for the one-loop effective potential computations we have to find $K(x, y/s)$. For a given operator A let us define the differential equation

$$i\frac{\partial K(x, y/s)}{\partial s} = AK(x, y/s) \tag{1.3.14}$$

where operator A acts on the first argument of $K(x, y/s)$. Suppose that the function $K(x, y/s)$ satisfies the following initial condition,

$$K(x, y/s) = \delta(x - y) \tag{1.3.15}$$

For the operator A defined by Eq. (1.3.5) we can rewrite Eq. (1.3.14) as

$$i\frac{\partial K(x, y/s)}{\partial s} = -\frac{1}{\mu^2}[\Box + m^2 + V''(\phi)]K(x, y/s) \tag{1.3.16}$$

The solution of this equation which satisfies the initial condition given by Eq. (1.3.15) has the form

$$K(x,y/s) = -\frac{i\mu^4}{(4\pi is)^2}\exp\left[\frac{i\mu^2}{4s}(x-y)^2 + \frac{i}{\mu^2}(m^2+V''(\phi))s\right] \qquad (1.3.17)$$

Substituting for $K(x,x/s)$ from the above equation in the expression (1.3.13) we get,

$$\begin{aligned}\xi(x,z/\frac{A}{\mu^2}) &= -\frac{i\mu^4}{(4\pi)^2\Gamma(z)}\int_0^\infty \frac{ids(is)^{Z-1}}{(is)^2}\exp\left[\frac{i}{\mu^2}\left(m^2+V''(\phi)\right)s\right] \\ &= \frac{i\mu^4}{(4\pi)^2}\left(\frac{m^2+V''(\phi)}{\mu^2}\right)^{z-2}\frac{\Gamma(z-2)}{\Gamma(z)} \qquad (1.3.18)\end{aligned}$$

Therefore the expression for the one-loop effective potential is

$$V^{(1)}_{eff}(\phi) = V(\phi)+\frac{1}{2}m^2\phi^2-\frac{h\mu^4}{(4\pi)^3}\lim_{Z\to 0}\frac{d}{dz}\left[\left(\frac{m^2+V''(\phi)}{\mu^2}\right)^{z-2}\frac{\Gamma(z-2)}{\Gamma(z)}\right] \qquad (1.3.19)$$

Now using the properties of Gamma functions we have $\frac{\Gamma(z-2)}{\Gamma(z)} = \frac{1}{(z-1)(z-2)}$. Substituting this Eq. (1.3.19) and carrying out elementary transformations we obtain,

$$V^{(1)}_{eff}(\phi) = V(\phi) + \frac{1}{2}m^2\phi^2 + \frac{h}{128\pi^3}(m^2+V''(\phi))\left(\ln\frac{m^2+V''(\phi)}{\mu^2} - \frac{3}{2}\right) \qquad (1.3.20)$$

The effective potential given by the above equation contains arbitrary constants μ^2, λ, etc. For example, let $V(\phi) = \frac{1}{4!}\lambda\phi^4$, where λ is the scalar coupling constant. Then the effective potential depends on the parameters m^2 and λ. These parameters m^2 and λ are not the observable mass and coupling constant. For the introduction of the observable mass and coupling constant it is necessary to define these parameters taking into account the conditions of the measurement. This can be done by the renormalisation technique explained in the next section.

1.4 Renormalisation

Renormalisation is the special procedure for the reconstruction of the theory under consideration so that the divergences are absent and the vertex functions are finite. The divergences may be absorbed by a redefinition of various parameters mass, coupling constant etc. of the theory.

The parameters such as mass and coupling constants which appear in the Lagrangian are not directly measurable quantities. In the classical point-particle theory, for instance, we must "dress up" the bare mass to obtain the physical inertial mass. The latter is, ofcourse, finite while the former, may be infinite. We shall therefore give an operational definition to the fundamental parameters (finite in number). Renormalisation theory will then show that the perturbative expressions for Green functions are finite when expressed in terms of these physical parameters. The origin of divergences lies in the singular character of Green function at short relative distances. Equivalently, in momentum space the Fourier transforms do not vanish fast enough at infinity. In an intermediate step we are then lead to regularise the theory, that is, to replace the original expressions by smoother ones such that the integrals become finite. We shall thus proceed in three steps: (1)regularise, (2)renormalise and (3)eliminate the regularising parameters. Renormalisation will be successful if finite quantities are obtained as a result of this process.

The word 'nonrenormalizable' does not mean that such theories can not be made finite but rather that the proliferation of their divergences and hence counter terms, make them unrealistic in the framework of perturbation theory [17]. After renormalisation they will depend on an infinite set of arbitrary parameters barring any deeper principle allowing to relate them.

The regularisation scheme is a very important element of the theory of renormalisation. Regularisation consists of changing the original divergent Feynman integral into another finite integral, depending on the parameter of regularisation Λ. When the parameter Λ tends to a definite value (regularisation turned off) the regularised integral formally reduces to the original integral. The well-known regularisation schemes are dimensional regularisation, generalised ξ function regularisation and its many loop generalisation operator regularisation, regularisation with the help of higher derivatives, cut-off of the Feynman momenta integrals at the upper limits and cutoff of the proper time integrals at the lower limit.

Our aim is to express the proper Green functions in terms of renormalised Feynman integrals associated with the initial diagrams. This may be achieved by means of three equivalent procedures. In the first approach we add a formal series of counter terms to the initial Lagrangian. This in turn amounts to an order-by-order redefinition of the parameters of the theory: the bare parameters appearing in the Lagrangian are implicit functions of the renormalised ones. The former are most unobservable and divergent as the regularisation is removed, while the latter are the real finite parameters of the theory, mass, coupling constants etc. These two procedures are equivalent to an algorithm of subtraction of the integrand. This operation, due to Bogoliubov, has the merit of providing diagram by diagram, a finite result without any intermediate recourse to a regularisation.

The effective potential given by (1.3.20) contains the arbitrary constant μ^2 and other constants like λ. Therefore additional relations must be incorporated in to the effective potential and on the basis of it we can express the effective potential in terms of the observable mass and coupling constant. Let m_r^2 and λ_r be the observable values of the mass and coupling constant and they are defined as:

$$\frac{d^2V_{eff}^{(1)}(\phi)}{d\phi^2}\,|_{\phi=\phi_1} = m_r^2 \qquad\qquad \frac{d^4V_{eff}^{(1)}(\phi)}{d\phi^4}\,|_{\phi=\phi_2} = \lambda_r \tag{1.4.1}$$

Here ϕ_1 and ϕ_2 are constants of the mass dimension, reflecting the energy scale on which the mass and coupling constant are measured.

For convenience let us consider the case with, $m = 0$. Then Eq. (1.3.20) becomes,

$$V_{eff}^{(1)}(\phi) = \frac{1}{4!}\lambda\phi^4 + \frac{h\lambda^2}{256\pi^3}\phi^4\left(\ln\frac{\lambda\phi^2}{2\mu^2} - \frac{3}{2}\right) \tag{1.4.2}$$

and let

$$\frac{d^2V_{eff}^{(1)}(\phi)}{d\phi^2}\,|_{\phi=0} = 0 \qquad\qquad \frac{d^4V_{eff}^{(1)}(\phi)}{d\phi^4}\,|_{\phi=\phi_0} = \lambda_r \tag{1.4.3}$$

Substituting Eq. (1.4.2) in (1.4.3) we get

$$\lambda = \lambda_r - \frac{3h\lambda_r^2}{64\pi^3}\phi^2\left(\ln\frac{\lambda_r\phi_0^2}{2\mu^2} + \frac{8}{3}\right) \tag{1.4.4}$$

Now substituting Eq. (1.4.4) in Eq. (1.4.2) we can write

$$V_{eff}^{(1)}(\phi) = \frac{1}{24}\lambda_r\phi^4 + \frac{h\lambda_r^2}{512\pi^3}\left(\ln\frac{\phi^2}{\phi_0^2} - \frac{25}{6}\right) \tag{1.4.5}$$

The above expression is called the Coleman-Weinberg effective potential [20]. The relations (1.4.1) and (1.4.3) are called the normalisation conditions.

1.5 Spontaneous Symmetry Breaking (SSB)

In the quantum frame work, the discussions on symmetries imply the existence of a group of transformations acting on the physical observables and on the dynamical field variables. The symmetry properties of a system is characterised by the behaviour of its ground state. If the conserved quantities and the vacuum are not invariant under symmetry transformations then the symmetry of the system is said to be broken. This can also be stated as, if the state of a system does not respect the symmetry of the Lagrangian of the system, then the system is said to be in the symmetry broken state.

One of the mpst important concepts in modern particle theory is that of spontaneous symmetry breaking (SSB). The idea that there are underlying symmetries of nature that are not manifested in the structure of the vacuum appears to play a crucial role in the unification of forces [10].

Consider the simple theory of a real massive self-interacting scalar field ϕ with the Lagrangian density,

$$\mathcal{L}(\phi, \partial_\mu \phi) = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4 \tag{1.5.1}$$

The minimum of the potential energy density for this field ϕ occurs at $\phi = 0$. Now, consider a theory with negative mass squared with the Lagrangian density,

$$\mathcal{L}(\phi, \partial_\mu \phi) = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4}\phi^4 \tag{1.5.2}$$

For this case the minimum of the effective potential occurs not at $\phi = 0$ but at $\phi_c = \pm\frac{\mu}{\sqrt{\lambda}}$ (Fig. 1.1). This model has a degenerate ground state. In the ground state ϕ is either close to $+\frac{\mu}{\sqrt{\lambda}}$ or to $-\frac{\mu}{\sqrt{\lambda}}$; the two states have the same energy. This theory exhibits SSB. Since the quantum theory must be constructed about a stable extremum of the classical potential, the ground state of the system is either $\phi_c = +\frac{\mu}{\sqrt{\lambda}}$ or $\phi_c = -\frac{\mu}{\sqrt{\lambda}}$, and the reflection symmetry $\phi \to -\phi$, present in the Lagrangian is broken by the choice of a vacuum state. A symmetry of the Lagrangian not respected by the vacuum is said to be spontaneously broken. Even if the field ϕ is zero initially, it soon undergoes a transition to a stable state with the classical field $\phi_c = \pm\frac{\mu}{\sqrt{\lambda}}$, the phenomenon known as SSB.

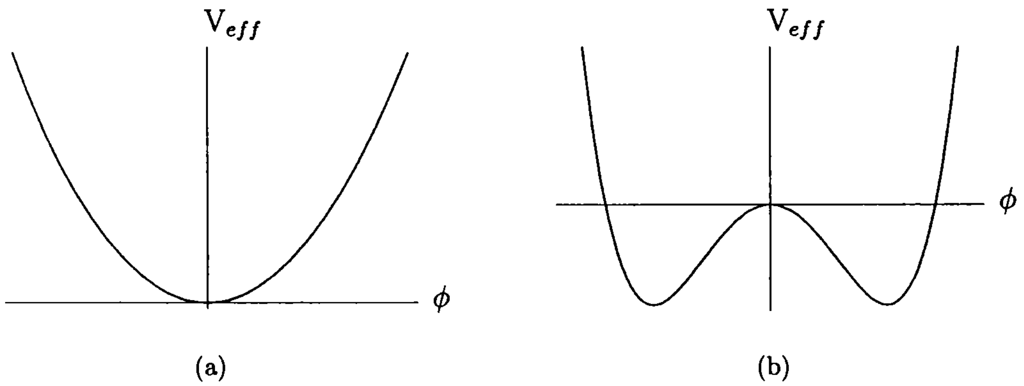


Fig. 1.1: Effective potential $V(\phi)$ in the simplest theories of the scalar field (a)$V(\phi)$ in theory (1.5.1) and (b)$V(\phi)$in theory (1.5.2)

1.6 Finite Temperature Quantum Field Theory

The evolution of particles in vacuum and in a thermal bath are very different. Similarly, the nature of evolution of field changes when coupled to a thermal bath. Let us consider the thermodynamic equilibrium system of the scalar particles ϕ with Lagrangian density given by Eq. (1.5.2).

The only parameter characterising the thermodynamic equilibrium state of the ϕ particles at a temperature T is the density of particles in the momentum space,

$$n_p = \frac{1}{[\exp(\omega_p/T) - 1]} \tag{1.6.1}$$

where $\omega_p = (\mathrm{p}^2 + m^2)^{1/2}$ is the energy of the particle with momentum $\mathbf{p}$ and mass m. At a non-vanishing temperature all physically interesting quantities in the system under consideration are given, not by vacuum averages, but by the Gibbs averages [9] defined by:

$$\langle ... \rangle = \frac{Tr[\exp(-\frac{H}{T})...]}{Tr[\exp(-\frac{H}{T})]} \tag{1.6.2}$$

where H is the Hamiltonian of the system. In particular symmetry breaking parameter in the system is not given by the vacuum expectation value $\sigma = \langle 0| \phi |0\rangle$, but by the temperature dependent quantity $\sigma(T) = \langle \phi \rangle$.

At $T \neq 0$ the component k_0 of the momentum in all Euclidean integrals should be replaced by $2\pi nT$ for bosons and by $(2n + 1)\pi T$ for fermions and instead of the integration over k_0 one should perform a summation over all integer n: $\int dk_0 \rightarrow 2\pi nT \sum_{n=-\infty}^{\infty}$.

It was Kirzhnits and Linde [21], who first suggested that spontaneous symmetry breaking in relativistic theory could be restored above a critical temperature. Later, Weinberg [18] used diagrammatic method to derive a numerical value for the critical temperature in the Kirzhnits and Linde model. Later Dolan and Jackiw [22] used the functional diagrammatic method to study the field theory at finite temperature. They have also calculated expression for critical temperature above which SSB in a relativistic field theory can be removed.

1.7 Phase Transitions

The word phase transitions means sharp transitions of a system from one phase to another, usually accompanied by a change in symmetry. Simplest examples are freezing, melting and boiling in the case of water. The aggregates of elementary particle like quarks and leptons that populate the very early universe can also exist in different phases and undergo analogous changes accompanied by symmetry and energy changes.

This phenomenon is well-known in other branches of physics, especially the physics of condensed matter. For example, in a ferromagnetic substance there is no fundamental preference for one direction of magnetisation over the opposite direction; the underlying theory is completely symmetrical. However, when a sample of ferromagnetic material is cooled through its Curie point, it will acquire a spontaneous magnetisation in one direction or the other, thus breaking the symmetry. The SSB often signals a phase transition. Above the value of some critical parameter, the equilibrium is completely symmetric and below that, there is a transition to an ordered, symmetry broken phase, in which some symmetry breaking order parameter acquire a nonzero value.

Our present understanding of the fundamental particle physics leads to the idea that the universe underwent a sequence of phase transitions. According to the standard hot universe theory, the universe could have expanded from a state in which its thermal energy would have been the Planck energy 10^{19} GeV. At this state of enormously high density, the temperature T is much greater than the critical temperature of a phase transition with symmetry restoration between gravitational, strong and electroweak interactions. Therefore the symmetry between these interactions should have been restored in the very early stages of the evolution of the universe. This is absolutely the farthest point to which we can at the moment imagine extrapolating our theories. Below this temperature the gravitational interaction gets decoupled from the others.

As the temperature decreases to $T \sim T_{c1} \sim 10^{14} - 10^{15}$ GeV, a phase transition takes place, generating a classical scalar field $\phi \sim 10^{15}$ GeV, which breaks the symmetry between the strong and electroweak interactions. When the temperature drops to $T_{c2} \sim 200$ GeV, the symmetry between the weak and the electromagnetic interactions breaks. Finally, at $T \sim 10^2$ MeV, there should be a phase transition which breaks the chiral invariance of the theory of strong interactions and leads to coalescence of quarks into hadrons.

Such a high temperature as required for the grand unification is ofcourse impossible to attain in any currently feasible terrestrial experiment; it is several orders of magnitude beyond even the temperatures in the cores of stars. However, in the generally accepted Hot Big-Bang model, the universe would have gone through this transition very early in its history.

The phenomenon of high-temperature symmetry restoration can be understood in the following way. When the ϕ field is in contact with a thermal bath, the interaction of ϕ particles with particles in the thermal bath will introduce a temperature dependent "plasma mass". On dimensional grounds this must be of the form $m^2_{plasma} = a\lambda T^2$, where a is a numerical constant of order unity. At finite temperature, let m_T^2 be the effective mass of the scalar field about the classical solution $\langle\phi\rangle = 0$. At temperatures where $m_T^2 < 0$, $\langle\phi\rangle = 0$ will be an unstable point, signalling SSB; while at high temperatures where $m_T^2 \geq 0$, the effective mass will be real and $\langle\phi\rangle = 0$ becomes a stable, classical minimum of the potential. There is a critical temperature above which $\langle\phi\rangle = 0$ is a stable minimum and the symmetry is restored.

The phase transitions in quantum field theory are caused by external parameters such as temperature, external electric field, Higgs boson masses, etc. Phase transitions can be induced by external gravitational field also [12].

1.8 Inflation

The scalar field ϕ appearing in unified theories of elementary particles could play the role of a vacuum state with energy density $|V(\phi)|$ [23, 24]. The magnitude of the field ϕ in an expanding universe depends on the temperature. At times of phase transitions, that change ϕ, the energy stored in the field is transformed into thermal energy [6,10]. If, as sometimes happens, the phase transition takes place from a highly supercooled metastable vacuum state, the total entropy of the universe can increase considerably afterwards and the universe can become hot. The corresponding model of the universe was developed by Linde [9]. Guth suggested the exponential expansion [8, 25] of the universe in a supercooled vacuum state $\phi = 0$. If the universe is supercooled, it stays in the false vacuum at $\phi = 0$ until at some stage the ϕ field tunnels across the $V(\phi) > 0$ barrier and falls down the $V(\phi)$ slope to its true vacuum. As the universe cools down, there appears a new minimum besides $\phi = 0$ and it soon becomes an absolute minimum ($\phi = \phi_{\text{min}}$). However, in the new inflationary scenario, the potential, has a barrier near $\phi = 0$ and the transition to the absolute minimum does not proceed too rapidly. In order for sufficient inflation to be achieved after ϕ passes through the barrier, the scalar field is stuck at $\phi = 0$ by this potential barrier at least until the radiation temperature falls below H. In this supercooling regime ρ_r ($\propto T^4$), soon becomes much smaller than ρ_ϕ ($= V(0)$ ~constant) and the universe undergoes inflation.

After the supercooling regime has lasted for sometime, the scalar field develops a non-vanishing expectation value, by quantum or thermal effects. After this stage, there appears a regime in which the scalar field can be treated as a classical field, which is nearly uniform over a scale of the Hubble horizon size at the beginning, the so-called new inflationary regime. The universe continues to expand exponentially due to the slow decrease in the potential energy. Since the perturbations of the scalar field and radiation are coupled gravitationally, the pre-existing perturbations of radiations may induce perturbations of the scalar field which eventually turns into the density perturbations.

The standard scenario for cosmological structure formation is based on inflationary cosmology. According to this model, quantum fluctuations of the scalar field during the expansion era were the perturbing seeds in an initial, globally smooth universe.

1.9 Quantum Fluctuations and Cosmological Perturbations

For a scalar field, perfect homogeneity can not be attained and there will always be some residual fluctuations. So we can split the field as:

$$\phi(x,t) = \phi_0(t) + \delta\phi(x,t) \tag{1.9.1}$$

where $\phi_0(t)$ is the classical background field and $\delta\phi(x,t)$ is the perturbation of the field ϕ.

These fluctuations can be regarded as waves of physical fields with all possible wavelengths, moving in all possible directions. Perturbations of the field lead to density perturbations. There are two distinct theories of how the initial seed fluctuations of cosmological perturbations might have arisen [26]-[28]. One of these models involved the idea of topological defects created during phase transitions in the early universe. The alternative picture involves the inflationary model of the universe, in which the primordial quantum fluctuations get amplified and evolve to become classical seed perturbations [10, 29]. The main purpose of developing cosmological perturbation theories is to examine the properties of primordial density fluctuation necessary to explain the observed structures of the universe and to clarify the origin and the evolutionary behaviour of such density fluctuations.

1.10 Einstein's Field Equations

Mathematically, the problem of describing the growth of small perturbations in the context of general relativity reduces to solving the Einstein's equations linearised about an expanding background. Einstein's field equation is:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \tag{1.10.1}$$

The stress-energy tensor $T_{\mu\nu}$ is the source of gravity and G is Newton's gravitational constant, which we may use instead of the equivalent $M_{pl}^2 = (8\pi G)^{-1}$.

Relativistic fluid flow is described using the stress-energy tensor $T^{\mu\nu}$. This is ultimately defined by asserting that $8\pi G T^{\mu\nu}$ is the source of gravity. From the Einstein's equation we learn that the stress-energy tensor is symmetric, $T^{\mu\nu} = T^{\nu\mu}$.

At any point in spacetime, the energy density is defined as T^{00}, the momentum density is defined as $T^{0i} = T^{i0}$ and the stress tensor is defined as $T_{ij} = T_{ji}$. If the stress is isotropic, the pressure P is defined by $T_{ij} = \delta_{ij}P$.

The Einstein's equation also implies that,

$$\partial_\mu T^{\mu\nu} = 0 \tag{1.10.2}$$

the stress-energy conservation law, encoding both energy and momentum conservation.

We can take the stress-energy tensor to be a smoothly varying function of position, which is equivalent to saying that we are dealing with a fluid. At any point in spacetime, the local rest frame of the fluid is defined as the frame in which the momentum density T^{i0} vanishes. At each spacetime point the energy density in the local rest frame is denoted by ρ :

$$\rho = T^{00}$$

The fluid 4-velocity in this frame is $u^\mu = (1,0,0,0)$. A worldline with this four velocity is said to be comoving with the fluid. A perfect fluid is defined as one that is in the local rest frame, $T_{ij} = P\delta_{ij}$, where P is the pressure. For a perfect fluid, in a generic inertial frame, we have,

$$T^{\mu\nu} = (\rho + P)u^\mu u^\nu + P\eta^{\mu\nu} \tag{1.10.3}$$

During inflation the universe contains fields as opposed to particles and indeed at the quantum level, we can say that this is always the case because a particle can be regarded as a quantised field oscillation. With fields as the source of gravity, general relativity can be derived from the action principle. Since the volume element in generic coordinate is $d^4x\sqrt{-g}$, the action of a system will be of the form,

$$S = \int d^4x\sqrt{-g}\mathcal{L} \tag{1.10.4}$$

where the Lagrangian density $\mathcal{L}$ is a scalar quantity. For a scalar field with Lagrangian density, in a generic coordinate system,

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \tag{1.10.5}$$

the stress-energy tensor of the scalar field is,

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right] \tag{1.10.6}$$

Equation (1.10.3) defines the energy density and pressure. For a homogeneous scalar field, $\phi \equiv \phi(t)$, the energy density ρ is

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{1.10.7}$$

and the pressure

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{1.10.8}$$

1.11 Gravitational Instability

Gravity is the dominant force which governs the large scale dynamics of the universe. The standard theory of cosmological structure formation is based on the idea of gravitational instability [11], [26]-[31] according to which small initial irregularities in the distribution of matter become amplified by the attractive nature of gravity. Small fluctuations in the density results in gravitational instability and gravitational instability causes the growth of perturbations in an expanding universe. The structure we observe in the universe today is the end result of the gravitational amplification of small primeval perturbations.

The gravitational instability of a spatially uniform state of dust-like matter described by classical non-relativistic equations has been first investigated by Sir James Jeans [32]. If the mass of a body is larger than some minimum mass called the Jeans mass, then the self gravity of matter will start affecting the structure of the body significantly.

1.12 Jeans Analysis and Jeans Mechanism of Structure Formation

Jeans considered the problem of formation of galaxies in the universe as a process involving the interplay between gravitational attraction and the pressure force acting on a mass of nonrelativistic fluid [30]. Jeans treatment used Newtonian physics and assumed a static universe [11, 33].

Jeans supposed the universe to be filled with a non-relativistic fluid, with mass density ρ, pressure p, velocity $\mathbf{v}$, and gravitational field $\mathbf{g}$ governed by the equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \mathbf{v}) = 0 \tag{1.12.1}$$

the Euler equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \mathbf{g} \tag{1.12.2}$$

and the gravitational field equations

$$\nabla \times \mathbf{g} = 0 \tag{1.12.3}$$

$$\nabla.\mathbf{g} = -4\pi G\rho \tag{1.12.4}$$

The effects of gravitation were ignored in the unperturbed solution, taken to be that for a static uniform fluid with ρ = constant, p = constant and $\mathbf{v} = 0$. To see whether any initial clumpiness can grow in size by gravitational instability, consider perturbations of the solution. Let us add small perturbations $\rho_1, p_1, \mathbf{v}_1$ and $\mathbf{g}_1$ to the corresponding quantities.

The pressure is related to the energy density by the equation of state. We will consider adiabatic perturbations [11, 31], that is, perturbations for which there are no spatial variations in the equation of state. The adiabatic sound speed, v_s is defined as,

$$v_s^2 \equiv \left(\frac{\partial p}{\partial \rho}\right)_{adiabatic} \tag{1.12.5}$$

and since by assumption there are no spatial variations in the equation of state,

$$v_s^2 = \frac{p_1}{\rho_1} \tag{1.12.6}$$

To first order, the small perturbations satisfy the perturbed version of Eqs. (1.12.1-1.12.4):

$$\begin{aligned}
\frac{\partial \rho_1}{\partial t} &+ \rho \nabla \cdot v_1 = 0 \\
\frac{\partial \mathbf{v}_1}{\partial t} &= -\frac{v_s^2}{\rho} \nabla \rho_1 + \mathbf{g}_1 \\
\nabla \times \mathbf{g}_1 &= 0 \\
\nabla . \mathbf{g}_1 &= -4\pi G \rho_1
\end{aligned} \tag{1.12.7}$$

All the quantities that do not carry a subscript "1" are now understood to refer to the unperturbed solution. Eqs. (1.12.7) can be combined to give a single, second order differential equation for ρ_1 :

$$\frac{\partial^2 \rho_1}{\partial t^2} - v_s^2 \nabla^2 \rho_1 = 4\pi G \rho \rho_1 \tag{1.12.8}$$

The solution takes the form

$$\rho_1 \propto \exp\{i\mathbf{k}.x - i\omega t\} \tag{1.12.9}$$

where ω and $\mathbf{k}$ are related by the dispersion relation

$$\omega^2 = k^2 v_s^2 - 4\pi G \rho \tag{1.12.10}$$

If ω is imaginary, there will be exponentially growing modes; if ω is real, the perturbations will simply oscillate as sound (compressional) waves. From Eq.

(1.12.10) it is clear that for k less than some critical value, ω will be imaginary. This critical value is called the Jeans wave number, k_J and is given by,

$$k_J = \left(\frac{4\pi G\rho}{v_s^2}\right)^{1/2} \tag{1.12.11}$$

The classical Jeans analysis is not directly applicable to cosmology because the expansion of the universe is not taken into account and further because the analysis is Newtonian. So long as the expansion is taken into account, Newtonian treatment of the matter density perturbation is valid on scales well within the horizon and after matter domination. The first satisfactory theory of instabilities of an expanding universe was given by Lifshitz in 1946 [34].

For sufficiently long wave numbers, the waves described by Jeans theory becomes ordinary sound waves, with

$$\omega^2 = k^2 v_s^2 \tag{1.12.12}$$

Gravitational forces will be negligible if the gravitational energy of a sphere of radius k^{-1} is much smaller than its thermal energy:

$$\frac{G(\rho k^{-3})^2}{k^{-1}} \ll \rho v_s^2 k^{-3}$$

Also, the expansion of the universe will have negligible effect if the expansion rate is much less than the frequency:

$$\sqrt{G\rho} \ll |\omega|$$

Both these conditions will be satisfied by the relation (1.12.12), as long as the wave number satisfies the condition

$$k \gg k_J$$

just as in Jeans theory. Thus, even when the expansion of the universe is taken into account, we expect there to be a critical wave number, of the order k_J, above which disturbances can not grow, but only oscillate like sound waves.

It is useful to define the Jeans mass M_J, the total mass contained within a sphere of radius $\lambda_J/2 = \pi/k_J$.

$$M_J = \frac{4\pi}{3}\left(\frac{\pi}{k_J}\right)^3 \rho \tag{1.12.13}$$

Perturbations of mass less than M_J are stable against gravitational collapse, while those of mass greater than M_J are unstable.

When the expansion of the universe is taken into account, the unperturbed solution to Eq. (1.12.7) is

$$\rho = \rho_0 \left[\frac{R_0}{R(t)}\right]^3 \tag{1.12.14}$$

$$\mathbf{v} = \mathbf{r}\left[\frac{\dot{R}(t)}{R(t)}\right] \tag{1.12.15}$$

$$\mathbf{g} = -r\frac{4\pi G\rho}{3} \tag{1.12.16}$$

where the scale factor $R(t)$ satisfies the usual Friedman equation,

$$\dot{R}^2 + k = \frac{8\pi G\rho R^2}{3} \tag{1.12.17}$$

or equivalently,

$$\frac{\ddot{R}}{R} = -\frac{4\pi G\rho}{3} \tag{1.12.18}$$

The first order perturbations in $\rho, \mathbf{v}$ and $\mathbf{g}$ satisfy the following set of equations:

$$\dot{\rho}_1 + 3\frac{\dot{R}}{R}\rho_1 + \frac{\dot{R}}{R}(\mathbf{r}.\nabla)\rho_1 + \rho\nabla.\mathbf{v}_1 = 0 \qquad (1.12.19)$$

$$\dot{\mathbf{v}}_1 + \frac{\dot{R}}{R}\mathbf{v}_1 + \frac{\dot{R}}{R}(\mathbf{r}.\nabla)\mathbf{v}_1 = -\frac{1}{\rho}\nabla p_1 + \mathbf{g}_1 \qquad (1.12.20)$$

$$\nabla \times \mathbf{g}_1 = 0 \qquad (1.12.21)$$

$$\nabla.\mathbf{g}_1 = -4\pi G\rho_1 \qquad (1.12.22)$$

Also, since these are supposed to be adiabatic fluctuations, the pressure perturbation is given by $p_1 = v_s^2\rho_1$. The above equations are spatially homogeneous, so we expect to find plane-wave solutions. Indeed, solutions can be found with the spatial dependence,

$$\rho_1(r,t) = \bar{\rho}_1(t)\exp\left\{\frac{i\mathbf{r}.k}{R(t)}\right\} \qquad (1.12.23)$$

and likewise for $\mathbf{v}_1$ and $\mathbf{g}_1$. The appearance of the factor $1/R$ in the exponential means that the wavelength of these modes is stretched out by the expansion of the universe. Now Eqs. (1.12.19 - 1.12.22) become coupled ordinary differential equations:

$$\dot{\bar{\rho}}_1 + 3\frac{\dot{R}}{R}\rho_1 + i\frac{(\mathbf{k}.\bar{\mathbf{v}}_1)}{R}\rho = 0 \qquad (1.12.24)$$

$$\dot{\bar{\mathbf{v}}}_1 + \frac{\dot{R}}{R}\bar{\mathbf{v}}_1 = -\frac{iv_s^2}{\rho R}\mathbf{k}\,\bar{\rho}_1 + \mathbf{g}_1 \qquad (1.12.25)$$

$$\mathbf{k}\times \bar{\mathbf{g}}_1 = 0 \tag{1.12.26}$$

$$i\mathbf{k}.g_1 = -4\pi GR\,\bar{\rho}_1 \tag{1.12.27}$$

It is convenient to split $\bar{\mathbf{v}}_1$ into two parts; along and perpendicular to the wave vector $\mathbf{k}$:

$$\bar{\mathbf{v}}_1\,(t) = \mathbf{v}_\perp(t) + i\mathbf{k}\epsilon(t) \tag{1.12.28}$$

where

$$\mathbf{k}.v_\perp(t) = 0$$

$$\epsilon \equiv -\frac{i\mathbf{k}.\,\bar{\mathbf{v}}_1}{\mathbf{k}^2} \tag{1.12.29}$$

It is also convenient to express ρ_1 in terms of a fractional change in density δ :

$$\bar{\rho}_1 = \rho(t)\delta(t) \tag{1.12.30}$$

Then Eq. (1.12.25) splits into two uncoupled equations,

$$\dot{\mathbf{v}}_{1\perp} + \frac{\dot{R}}{R}\mathbf{v}_{1\perp} = 0 \tag{1.12.31}$$

$$\dot{\epsilon} + \frac{\dot{R}}{R}\epsilon = \left(-\frac{v_s^2}{R} + \frac{4\pi GR\rho}{\mathbf{k}^2}\right)\delta \tag{1.12.32}$$

and Eq. (1.12.24) simplifies to

$$\dot{\delta} = \frac{\mathbf{k}^2}{R}\epsilon \tag{1.12.33}$$

From the above three equations it is implied that the rotational (or transverse) modes described by $\mathbf{v}_{1\perp}$ decay as $1/R$. On the otherhand, the compressional modes have a more interesting time dependence. Using Eq. (1.12.33) to eliminate ϵ in Eq. (1.12.32) we get,

$$\ddot{\delta} + 2\frac{\dot{R}}{R}\dot{\delta} + \left(\frac{v_s^2\mathbf{k}^2}{R} - 4\pi G\rho\right)\delta = 0 \tag{1.12.34}$$

This goes over to the Jeans dispersion relation Eq. (1.12.10) if we set R constant and define the wave number K as k/R as the physical wave number. The above one is the fundamental differential equation that governs the growth or decay of gravitational condensations in an expanding universe. It is clear that the qualitative behaviour of the solution depends upon whether $\frac{v_s^2\mathbf{k}^2}{R}$ is larger or smaller than $4\pi G\rho$. In an expanding universe as well, Jeans wave number

$$K_J = \left(\frac{4\pi G\rho}{v_s^2}\right)^{1/2} \tag{1.12.35}$$

separates the gravitationally stable and unstable modes.

Chapter 2

First Order Phase Transitions for ϕ^6 model in a Bianchi Type-I Universe

2.1 Introduction

In the previous chapter we have seen that the symmetries that are spontaneously broken today, were restored in the early universe and during the evolution of the universe there were phase transitions, perhaps many, associated with the spontaneous breakdown of symmetries [9, 10]. During such a phase transition it is possible for the field to acquire nonzero vacuum expectation values. Phase transitions can, in general, be of first or second order. For a first order phase transition the change in the field ϕ, in going from one phase to the other must be discontinuous, while for a second order transition there is no barrier at the transition point and the transition occurs smoothly. In a first order phase transition there is a difference between the energy density of the two phases, usually called the latent heat. The nature of phase transition, whether it is first order or not is of particular interest. If the phase transition is strongly first order, the Universe may be dominated by the vacuum energy and undergo a period of inflation. In this case, the transition proceeds by the nucleation of bubbles of the true vacuum. If the phase transition is of higher order or weakly first order, thermal fluctuations may drive the transition.

As mentioned in chapter 1, quantum field theory in an external classical gravitational field is usually regarded as a first step towards a more complete theory of quantum gravity [35]. At high energies the quantum matter fields are free from all the interactions except the conformal one with an external metric. The requirement of the conformal invariance is especially important for the scalar field, as it fixes the value of the non-minimal parameter of the scalar curvature interaction to the conformal value. The effect of quantum conformal factor leads to first order phase transition induced by the curvature where the scalar field plays the role of order parameter [36]-[38].

Investigations on the effects of gravity on quantum fields dates at least since the work of Schrödinger [39]. In the course of cosmological expansion relic gravitons can be created from zero-point quantum fluctuations of the gravitational field. Production of relic gravitons and primordial density fluctuations is covered by the theory known as particle creation in external fields [1, 40]. Zeldovich suggested [41] that the production of elementary particles by the expansion of the universe would bring about isotropy near the Planck time. The back-reaction on the metric of the created particles has been studied by Lukash and Strobinsky [42]. They assumed that the particle creation occurred at a time t_0 large with respect to t_p so that the evolution of the metric at times near t_0 could be treated independently of the created particles. If their results are extrapolated back to $t_0 \sim t_p$ then they indicate that rapid isotropisation should occur. In order to consider directly models in which $t_0 \sim t_p$, one would have to use a renormalised expression for the expectation value of the energy-momentum tensor of the quantised matter fields.

The influence of quantum fields and the gravitational effects on the cosmological phase transitions have been investigated by many authors [2]-[5]. From the analysis based on the one loop renormalised effective potential, it is concluded that the scalar gravitational coupling ξ and the magnitude of the scalar curvature R crucially determine the fate of symmetry. At the classical level the scalar curvature acts as an effective mass of the field and thus influences the phase transition of the system. The effect of anisotropy on the static spacetimes like Mixmaster or Taub Universe on the process of symmetry breaking and restoration has also been discussed earlier [43, 44].

In this chapter we discuss the quantum field effects on phase transition and the temperature dependence of phase transition for a ϕ^6 theory in an anisotropic Bianchi type-I universe. ϕ^6 model has found applications in high energy physics and in condensed matter physics. Apart from its importance in high energy physics as a model scalar field theory, the ϕ^6 model has been used to explain first order phase transitions observed in ferroelectrics and structural phase transitions in crystals [45]. Self interactions up to ϕ^6 exhibit three, well separated lowest levels [46]. Boyanovsky and Masperi [46] have shown that the nature of phase transitions associated with such a field system may be of first order or second order depending on the relative depths of the wells and the strength of coupling.

If the observed universe comes from an initially small, causally connected region, then microphysical processes could have smoothed any initial irregularities. The universe, rather than seeming to require the very special initial conditions of almost perfect homogeneity and isotropy, may be seen as evolving from more generic conditions to the smooth state observed today. Studies of helium formation [47]-[49] indicate that the expansion was effectively isotropic at $t \leq 10^{-1}$ sec.

The isotropy of the observed cosmic black body radiation ($\Delta T/T \leq 10^{-3}$) has served as the basis for a number of investigations implying limits on the time of isotropisation [50]-[53]. Works based on evolutionary models involving classical fluids [52, 53] have concluded that isotropisation of the expansion must have occurred by as early a time as $t \leq 10^{-36}$ sec, a time which is close to the Planck time $t_p \sim 10^{-44}$sec. Although the overall structure of the universe is homogeneous and isotropic on a large scale, it is obviously inhomogeneous and anisotropic on scales characteristic of galaxies and their clusters.

One of the simplest models of an anisotropic universe that describes a homogeneous and a spatially flat universe is the Bianchi type-I cosmology. Unlike the FRW model which has the same scale factor for each of the three spatial directions, the Bianchi type-I cosmology has a different scale factor in each direction, which produces an anisotropy in expansion. Futamase has considered the effective potential in a Bianchi type-I universe [54] which reduces to the spatially flat Robertson-Walker model for zero anisotropy. Huang has discussed the fate of symmetry in a Bianchi type-I universe using an adiabatic approximation for a massless field with arbitrary coupling to gravity [55]. Berkin has also calculated the effective potential in a Bianchi type-I universe, for a scalar field having arbitrary mass and coupling to gravity [56].

ϕ^6 model is known to be nonrenormalisable in (3+1)dimensional flat spacetime. Nonrenormalisability of the field theory does not mean that the theory is not interesting and it does not mean, ofcourse, that finite renormalised prescription for the calculation of one-loop effective potential does not exist [12]. Using the present ϕ^6 model, a finite expression for the one-loop effective potential is obtained in this chapter. The present calculations show that the ϕ^6 model can be regularised using the effective potential method in (3+1) dimensional curved spacetime.

In section 2.2, the one-loop effective potential for ϕ^6 theory in a $(3+1)$ dimensional Bianchi type-I spacetime is evaluated and the properties of quantum field corrections to the symmetry breaking or restoration are discussed. In section 2.3 we obtain a finite expression for the energy momentum tensor for the ϕ^6 theory in this spacetime. The finite temperature effects on the phase transitions of early universe are discussed in section 2.4 and the nature of phase transitions is examined in the next section. The crucial dependence of phase transitions of the early universe on spacetime curvature and the gravitational-scalar coupling are made clear in section 2.6. Section 2.7 is devoted to discussions and conclusions of the present calculations.

2.2 Quantum Field Effects on Symmetry Breaking and Restoration in Bianchi Type-I Spacetime

Consider a massive real self-interacting scalar field ϕ coupled arbitrarily to the gravitational back ground and described by the Lagrangian density $\mathcal{L}$,

$$\mathcal{L} = \sqrt{-g}\left\{\frac{1}{2}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \xi R\phi^2] - \frac{1}{2}\lambda^2\phi^2(\phi^2 - m/\lambda)^2\right\} \tag{2.2.1}$$

where, the classical potential corresponding to this Lagrangian is,

$$V(\phi) = \frac{1}{2}\xi R\phi^2 + \frac{1}{2}\lambda^2\phi^2(\phi^2 - m/\lambda)^2 \tag{2.2.2}$$

This Lagrangian exhibits $\phi \rightarrow -\phi$ symmetry. The equation of motion associated with the Lagrangian given by Eq. (2.2.1) is,

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + (m^2 + \xi R)\phi - 4\kappa\phi^3 + 3\lambda^2\phi^5 = 0 \qquad (2.2.3)$$

in which we put $m\lambda = \kappa$. We can write,

$$\phi = \phi_c + \phi_q \qquad (2.2.4)$$

where ϕ_c is the classical field and ϕ_q is a quantum field with vanishing vacuum expectation value, $\langle\phi_q\rangle = 0$.

As it is mentioned in the previous chapter, the parameters such as mass and coupling constants which appear in the above Lagrangian are not directly measurable quantities. So introducing renormalised parameters,

$$m^2 = m_r^2 + \delta m^2, \qquad \xi = \xi_r + \delta\xi,$$
$$\kappa = \kappa_r + \delta\kappa, \qquad \lambda^2 = \lambda_r^2 + \delta\lambda^2 \qquad (2.2.5)$$

the field equation for the classical field ϕ_c becomes,

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi_c + [(m_r^2 + \delta m^2) + (\xi_r + \delta\xi)R]\phi_c - 4(\kappa_r + \delta\kappa)\phi_c^3$$
$$- 12(\kappa_r + \delta\kappa)\phi_c \langle phi_q^2\rangle + 3(\lambda_r^2 + \delta\lambda^2)\phi_c^5 + 30(\lambda_r^2 + \delta\lambda^2)\phi_c^3 \langle phi_q^2\rangle$$
$$+ 15(\lambda_r^2 + \delta\lambda^2)\phi_c \langle\phi_q^4\rangle = 0 \qquad (2.2.6)$$

and to the one loop quantum effect, the field equation for the quantum field ϕ_q is,

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi_q + (m_r^2 + \xi R)\phi_q - 12\kappa_r\phi_c^2\phi_q + 15\lambda_r^2\phi_c^4\phi_q = 0 \qquad (2.2.7)$$

The effective potential V_{eff} is given by,

$$V_{eff} = \frac{1}{2}[(m_r^2 + \delta m^2) + (\xi_r + \delta\xi)R][\phi_c^2 + \langle\phi_q^2\rangle] - (\kappa_r + \delta\kappa)\phi_c^4 - 6(\kappa_r + \delta\kappa)\phi_c^2\langle\phi_q^2\rangle$$
$$- (\kappa_r + \delta\kappa)\langle\phi_q^4\rangle + \frac{1}{2}(\lambda_r^2 + \delta\lambda^2)\phi_c^6 + \frac{15}{2}(\lambda_r^2 + \delta\lambda^2)\phi_c^4\langle\phi_q^2\rangle$$
$$+ \frac{15}{2}(\lambda_r^2 + \delta\lambda^2)\phi_c^2\langle\phi_q^4\rangle + \frac{1}{2}(\lambda_r^2 + \delta\lambda^2)\langle\phi_q^6\rangle \quad (2.2.8)$$

To make V_{eff} finite, the following renormalisation conditions are used,

$$m_r^2 = \left(\frac{\partial^2 V_{eff}}{\partial\phi_c^2}\right)_{\phi_c=R=0}$$

$$\xi_r = \left(\frac{\partial^3 V_{eff}}{\partial R\partial\phi_c^2}\right)_{\phi_c=R=0}$$

$$\kappa_r = \left(\frac{\partial^4 V_{eff}}{\partial\phi_c^4}\right)_{\phi_c=R=0} \quad (2.2.9)$$

$$\lambda_r^2 = \left(\frac{\partial^6 V_{eff}}{\partial\phi_c^6}\right)_{\phi_c=R=0}$$

To evaluate $\langle\phi_q^2\rangle$, $\langle\phi_q^4\rangle$, and $\langle\phi_q^6\rangle$ we can adopt the canonical quantisation relations:

$$[\phi_q(t,x), \phi_q(t,y)] = [\pi_q(t,x), \pi_q(t,y)] = 0, \quad [\phi_q(t,x), \pi_q(t,y)] = i\delta^3(x-y) \quad (2.2.10)$$

where the conjugate momentum π_q is defined by $\pi_q = \dfrac{\partial\pounds}{\partial(\partial_i\phi)}$. Due to the space homogeneity we can expand the quantum field ϕ_q by the summation or integration over modes in the form,

$$\phi_q(t,x) = C^{-1/2}(t)\int d\mu(k)[a_k\chi_k(t)y_k(x) + a_k^+\chi_k^*(t)y_k^*(x)] \quad (2.2.11)$$

where $y_k(x)$ is a normalised eigen function of the spatial part of field equation, while $\chi_k(t)$ is that of the time part. An explicit functional form of the mode solutions $\chi_k(t)$ and $y_k(x)$ can only be found after specifying the background spacetime.

Let us consider a (3+1) dimensional Bianchi type-I spacetime with small anisotropy which has the line element

$$ds^2 = C(\eta)d\eta^2 - a_1^2(\eta)dx^2 - a_2^2(\eta)dy^2 - a_3^2(\eta)dz^2$$

$$C = (a_1 a_2 a_3)^{2/3} \tag{2.2.12}$$

In this model the mode function can be written in the separated form as $y_k = (2\pi)^{-3/2}\exp(i\kappa.x)$ and then

$$\langle\phi_q^2(\eta)\rangle = \frac{1}{8\pi^3 C(\eta)}\int d^3k\chi_k(\eta)\chi_k^*(\eta), \tag{2.2.13}$$

The wave equation Eq. (2.2.7) will then lead to

$$\ddot{\chi} + \left\{C\left[m_r^2 + (\xi_r - \frac{1}{6})R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \sum_i \frac{k_i^2}{a_i^2}\right] + Q\right\}\chi_k = 0 \tag{2.2.14}$$

where the spacetime curvature function R and the anisotropic function Q are

$$R = 6C^{-1}(\dot{H} + H^2 + Q) \qquad H = \sum_i h_i$$

$$h_i = \frac{\dot{a}_i}{a_i} \qquad Q = \frac{1}{36}\sum_{i<j}(h_i - h_j)^2 \tag{2.2.15}$$

When the metric is slowly varying Eq. (2.2.14) possesses WKB approximation solution:

$$\chi_k = (2W_k)^{-\frac{1}{2}}\exp(-i\int d\eta W_k) \tag{2.2.16}$$

where

$$W_k = \left\{ C \left[m_r^2 + (\xi_r - \frac{1}{6})R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \sum_i \frac{k_i^2}{a_i^2} \right] + Q \right\}^{\frac{1}{2}}$$

Substituting the above solution in Eq. (2.2.13):

$$\langle\phi_q^2\rangle = \frac{1}{16\pi^3 C(\eta)} \int d^3k \left\{ C \left[m_r^2 + (\xi_r - \frac{1}{6})R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \sum_i \frac{k_i^2}{a_i^2} \right] + Q \right\}^{-\frac{1}{2}} \quad (2.2.17)$$

$$= \frac{1}{16\pi} \left\{ \Lambda^2 + \frac{1}{2} A_1 \left[1 + \ln \frac{A_1}{4\Lambda^2} \right] \right\} \quad (2.2.18)$$

where we denote $A_1 = \left[m_r^2 + (\xi_r - \frac{1}{6})R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \frac{Q}{C} \right]$.

Taking the quantum fluctuations to be of Gaussian type [57] and from the relation true for the Gaussian fields [58] we get,

$$\langle\phi_q^4\rangle = 3[\langle\phi_q^2\rangle]^2$$

$$= \frac{3}{64\pi^6 C^2(\eta)} \int d^3k \chi_k(\eta)\chi_k^*(\eta) \int d^3k \chi_k(\eta)\chi_k^*(\eta)$$

$$= \frac{3}{256\pi^2} \left\{ \Lambda^2 + \frac{1}{2} A_1 \left[1 + \ln \frac{A_1}{4\Lambda^2} \right] \right\}^2 \quad (2.2.19)$$

where a momentum cut-off Λ is introduced to regularise the k-integration. From the renormalisation conditions given by Eq. (2.2.9), the renormalisation counter terms are evaluated as,

$$\delta m^2 = \frac{3}{4\pi}(\kappa_r + \delta\kappa)\left[\Lambda^2 + \frac{1}{2}(m_r^2 + \frac{Q}{C})\left(1 + \ln\left[\frac{m_r^2 + \frac{Q}{C}}{4\Lambda^2}\right]\right)\right]$$
$$- \frac{45}{128\pi^2}(\lambda_r^2 + \delta\lambda^2)\left[\Lambda^2 + \frac{1}{2}(m_r^2 + \frac{Q}{C})\left(1 + \ln\left[\frac{m_r^2 + \frac{Q}{C}}{4\Lambda^2}\right]\right)\right]^2 \quad (2.2.20)$$

$$\delta\xi = \frac{3(\kappa_r + \delta\kappa)}{8\pi}(\xi_r - \frac{1}{6})\left(2 + \ln\left[\frac{m_r^2 + \frac{Q}{C}}{4\Lambda^2}\right]\right)$$
$$+ \frac{45(\lambda_r^2 + \delta\lambda^2)}{128\pi^2}(\xi_r - \frac{1}{6})\left[\Lambda^2 + \frac{1}{2}(m_r^2 + \frac{Q}{C})[1 + \ln\frac{m_r^2 + \frac{Q}{C}}{4\Lambda^2}]\right]\left(2 + \ln\frac{m_r^2 + \frac{Q}{C}}{4\Lambda^2}\right)$$
$$(2.2.21)$$

$$\delta\kappa = -\kappa_r - \frac{\lambda_r^2}{60\left[\frac{45\lambda_r^2}{4\pi}\left(2 + \ln\left[\frac{m_r^2 + \frac{Q}{C}}{4\Lambda^2}\right]\right) + \frac{54\kappa_r^2}{\pi(m_r^2 + \frac{Q}{C})}\right]} \quad (2.2.22)$$

$$\delta\lambda_r^2 = -\lambda_r^2 + \frac{\left[-24\lambda_r^2\pi + 702\kappa_r\lambda_r^2\left(2 + \ln\left[\frac{m_r^2 + \frac{Q}{C}}{4\Lambda^2}\right]\right) + \frac{3240\kappa_r^3}{(m_r^2 + \frac{Q}{C})}\right]}{675\left[\frac{45\lambda_r^2}{4\pi}\left(2 + \ln\left[\frac{m_r^2 + \frac{Q}{C}}{4\Lambda^2}\right]\right) + \frac{54\kappa_r^2}{\pi(m_r^2 + \frac{Q}{C})}\right]} \times$$
$$\frac{1}{\left[\Lambda^2 + \frac{1}{2}(m_r^2 + \frac{Q}{C})\left(1 + \ln\left[\frac{m_r^2 + \frac{Q}{C}}{4\Lambda^2}\right]\right)\right]\left[1 - \frac{9\kappa_r}{4\pi}\left(2 + \ln\left[\frac{m_r^2 + \frac{Q}{C}}{4\Lambda^2}\right]\right)\right]}$$
$$(2.2.23)$$

Substituting the renormalisation counter terms, the final expression for $\frac{\partial V_{eff}}{\partial \phi_c}$ obtained from Eq. (2.2.8) is calculated as,

$$\frac{\partial V_{eff}}{\partial \phi_c} = (m_r^2 + \xi_r R)\phi_c + \frac{1}{80B_1}\left[-\lambda_r^2 + \frac{D_1(m_r^2+\frac{Q}{C})}{12E_1(m_r^2+\frac{Q}{C})\ln(m_r^2+\frac{Q}{C})}\right](\xi_r - \frac{1}{6})R\phi_c$$

$$+\frac{\pi}{15B_1}\left[\lambda_r^2 + \frac{4A_1D_1}{3B_1E_1(m_r^2+\frac{Q}{C})\ln(m_r^2+\frac{Q}{C})}\right]\phi_c^3$$

$$+\frac{2\pi D_1}{225B_1E_1(m_r^2+\frac{Q}{C})\ln(m_r^2+\frac{Q}{C})}\phi_c^5 \qquad (2.2.24)$$

where

$$A_1 = m_r^2 + (\xi_r - \frac{1}{6})R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4, \qquad B_1 = \left[\frac{45\lambda_r^2}{2} + \frac{54\kappa_r^2}{(m_r^2+\frac{Q}{C})}\right],$$

$$D_1 = \left[-24\lambda_r^2\pi + 1440\kappa_r\lambda_r^2 + \frac{3240\kappa_r^3}{(m_r^2+\frac{Q}{C})}\right], \qquad E_1 = \left[1 - \frac{9\kappa_r}{2\pi}\right] \qquad (2.2.25)$$

The above equation shows that we can obtain a finite expression for the one loop effective potential using this ϕ^6 model in (3+1) dimensional Bianchi type-I spacetime. Thus it is clear that the ϕ^6 theory in (3+1) dimensions can be regularised in a curved anisotropic spacetime using the effective potential method. It can be noted that once we let the anisotropy in the above equation to be zero, our result is consistent with that of the symmetric homogeneous case.

Now we are in a position to investigate the gravitational and quantum field effects on the cosmological phase transitions. This can be done by considering the case $\phi_c \longrightarrow 0$. In the case of conformal coupling ($\xi_r = \frac{1}{6}$) or vanishing scalar curvature ($R = 0$) we have,

$$\left(\frac{\partial V_{eff}}{\partial \phi_c}\right)_{\phi_c \to 0} \sim m_r^2\phi_c \qquad (2.2.26)$$

which shows that in such situations, the one-loop quantum correction does not change the fate of symmetry. For the other cases, we can find from the above equations that only for some suitable values of scalar gravitational coupling constant the symmetry could be radiatively broken or restored.

The perturbative method of calculating the effective potential can be improved by using Renormalisation Group (RG) approach [59]. Such RG improved effective potential can be calculated in curved spacetime too [60]. The condition expressing the independence of the effective potential from the renormalisation point leads to Renormalisation Group Equation (RGE) [12]. This property in renormalisable theories may be used for construction of the RG improved effective potential, which is much more exact than one loop-effective potential, because it takes into account of all orders of the perturbation theory. However, RG improved potential will not give leading log approximation in the present ϕ^6 model, since it is not multiplicatively renormalisable.

2.3 Energy-Momentum Tensor for ϕ^6 Field in (3+1) Dimensional Bianchi Type-I Spacetime

While constructing a theory of the interaction between quantized matter fields and a classical gravitational field one has to identify the energy-momentum tensor of the quantized fields which acts as the source of the gravitational field. In a semiclassical theory the Einstein equations takes the form

$$G_\mu^\nu = -8\pi \left\langle T_\mu^\nu \right\rangle \tag{2.3.1}$$

where G_μ^ν is the Einstein tensor formed from the classical metric and $\langle T_\mu^\nu \rangle$ denotes the expectation value of the energy-momentum tensor of the quantised particle fields under consideration. Although the use of $\langle T_\mu^\nu \rangle$ as the source of the gravitational field in a semiclassical approximation may not be justified when the probable (in the quantum sense) matter distributions differ greatly from their average, its use appears to be correct in the cosmological context [61]. In general, the expectation values of the formal energy-momentum tensors are not well defined and must be renormalised. Renormalisation of the energy-momentum tensor for free field and ϕ^4 self-interacting field in Bianchi type-I and IX universes were studied by various authors [61]-[63]. The energy-momentum tensor for the ϕ^6 field is

$$T_{\mu\nu} = (1-2\xi)\partial_\mu\phi\partial_\nu\phi + (2\xi - \frac{1}{2})g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi - 2\xi\phi\nabla_\mu\nabla_\nu\phi$$
$$+ 2\xi g_{\mu\nu}\phi\Box\phi - \xi G_{\mu\nu}\phi^2 + (\frac{m^2}{2})g_{\mu\nu}\phi^2 - 2\kappa g_{\mu\nu}\phi^4 + \frac{3}{2}\lambda^2 g_{\mu\nu}\phi^6 \quad (2.3.2)$$

where $G_{\mu\nu}$ is the Einstein tensor.

The expectation value of the energy-momentum tensor can be broken into classical and quantum parts. The energy-momentum tensor for the classical part is obtained by substituting ϕ_c for ϕ in Eq. (2.3.2). Considering a (3+1) dimensional Bianchi Type-I background spacetime, the $\eta\eta$ component of the classical renormalised energy-momentum tensor is given by,

$$\langle T_\eta^\eta \rangle^C = \frac{1}{2C}\phi_c^{'}\phi_c^{'} + 2\xi\frac{C^{'}}{C^2}\phi_c\phi_c^{'} + \frac{3\xi}{C}(\frac{C^{'}}{C} + k)\phi_c^2 + \frac{m^2}{2}\phi_c^2 - 2\kappa\phi_c^4 + \frac{3}{2}\lambda^2\phi_c^6 \quad (2.3.3)$$

where k=+1,0,-1 corresponds to the positive, zero or negative spatial curvature respectively. The quantum part of $\langle T_{\mu\nu}\rangle$ is,

$$\langle T_{\mu\nu}\rangle^Q = (1-2\xi)\langle\partial_\mu\phi_q\partial_\nu\phi_q\rangle + (2\xi - \frac{1}{2})g_{\mu\nu}\langle\partial_\alpha\phi_q\partial^\alpha\phi_q\rangle - 2\xi\langle\phi_q \nabla_\mu \nabla_\nu\phi_q\rangle$$

$$+ 2\xi g_{\mu\nu}\langle\phi_q\Box\phi_q\rangle - \xi G_{\mu\nu}\langle\phi_q^2\rangle + (\frac{m^2}{2})g_{\mu\nu}\langle\phi_q^2\rangle - 12\kappa g_{\mu\nu}\phi_c^2\langle\phi_q^2\rangle$$

$$+ \frac{45}{2}\lambda^2 g_{\mu\nu}\phi_c^4\langle\phi_q^2\rangle + \frac{45}{2}\lambda^2 g_{\mu\nu}\phi_c^2\langle\phi_q^4\rangle \quad (2.3.4)$$

By regularising the theory we can obtain a physically finite energy-momentum tensor of the system. The finite expression for the expectation value of the quantum energy-momentum tensor is obtained as,

$$\langle T^\eta_\eta\rangle^Q = \frac{C'^3}{64\pi C^3}(A_1 + 3\frac{Q}{C}) + \frac{C'^2}{256\pi C^3}\frac{(A_1 + \frac{1}{3}\frac{Q}{C})}{A_1} + \frac{1}{32\pi}(\frac{1}{8} - A_1)(m_r^2 + \frac{Q}{C})$$

$$+ \frac{C'^2}{64\pi C^3}A_1 - \frac{3C'^2}{128\pi^3 C^3}(A_1 + \frac{2Q}{C})\left\{\xi_r - \frac{\lambda_r^2(\xi_r - \frac{1}{6})}{80B_1} + \frac{D_1(\xi_r - \frac{1}{6})}{960B_1E_1}\right\}$$

$$+ \frac{3C'A_1}{32\pi C^2}\left[1 - \frac{C'}{C}\right]\left\{\xi_r - \frac{\lambda_r^2(\xi_r - \frac{1}{6})}{80B_1} + \frac{D_1(\xi_r - \frac{1}{6})}{960B_1E_1}\right\}$$

$$+ \frac{A_1}{64\pi}\left\{m_r^2 - \frac{\lambda_r^2(m_r^2 + \frac{Q}{C})}{160B_1} - \frac{D_1(m_r^2 + \frac{Q}{C})}{3840B_1E_1}\right\}$$

$$+ \frac{A_1}{160B_1}\left[\lambda_r^2 + \frac{D_1A_1}{16\pi E_1(m_r^2 + \frac{Q}{C})}\right]\phi_c^2$$

$$+ \frac{A_1D_1}{480B_1E_1(m_r^2 + \frac{Q}{C})\ln(m_r^2 + \frac{Q}{C})}\phi_c^4 \quad (2.3.5)$$

where A_1, B_1, D_1 and E_1 are defined by Eq. (2.2.25). It is clear that the energy-momentum tensor depends on the anisotropy of the spacetime.

2.4 Finite Temperature Effective Potential

In section 1.6, we have seen that the nature of evolution of field changes when coupled to a thermal bath. Under certain conditions, the changes may be absorbed in a temperature dependent potential, the finite temperature effective potential. The temperature dependence of finite temperature effective potential in quantum field theory leads to phase transitions in the early universe [64]. In this case the vacuum expectation value is replaced by the thermal average $\langle\phi\rangle_T = \sigma_T$ taken with respect to a Gibbs ensemble [9].

Considering the same Lagrangian density as above, the zero loop effective potential is temperature independent as given by,

$$V_0(\sigma) = \frac{1}{2}\xi R\sigma^2 + \frac{1}{2}\lambda^2\sigma^2(\sigma^2 - m/\lambda)^2 \tag{2.4.1}$$

The one loop approximation to finite temperature effective potential have been computed by many authors [65]-[69] and is given by,

$$\begin{aligned} V_1^\beta(\sigma) &= \frac{1}{2\beta}\sum_n \int \frac{d^3k}{(2\pi)^3} \ln(k^2 - M^2) \qquad (2.4.2) \\ &= \frac{1}{2\beta}\sum_n \int \frac{d^3k}{(2\pi)^3} \ln(\frac{-4\pi^2 n^2}{\beta^2} - E_M^2) \end{aligned}$$

$$\text{where,} \quad \beta = \tfrac{1}{T} \quad \text{and} \quad E_M^2 = k^2 + M^2, \tag{2.4.3}$$

$$M^2 = m^2 + \xi R - 12\lambda m\sigma^2 + 15\lambda^2\sigma^4$$

The sum over n diverges; it may be evaluated as follows [65]. Define,

$$
\begin{aligned}
v(E) &= \sum_n \ln\left(\frac{4\pi^2 n^2}{\beta^2} + E^2\right), \\
\frac{\partial v(E)}{\partial E} &= \sum_n \frac{2E}{4\pi^2 n^2/\beta^2 + E^2}
\end{aligned} \tag{2.4.4}
$$

Using the following result,

$$
\sum_{n=1}^{\infty} \frac{y}{y^2+n^2} = -\frac{1}{2y} + \frac{1}{2}\pi \coth \pi y \tag{2.4.5}
$$

we get

$$
\frac{\partial v(E)}{\partial E} = 2\beta\left(\frac{1}{2} + \frac{1}{e^{\beta E}-1}\right) \tag{2.4.6}
$$

and integrating,

$$
v(E) = 2\beta\left[\frac{E}{2} + \frac{1}{\beta}\ln(1-e^{-\beta E})\right] + \text{terms independent of E} \tag{2.4.7}
$$

Thus we get

$$
\begin{aligned}
V_1^\beta(\sigma) &= \int \frac{d^3k}{(2\pi)^3}\left[\frac{E_M}{2} + \frac{1}{\beta}\ln(1-e^{-\beta E})\right] \\
&= V_1^0(\sigma) + \bar{V}_1^\beta(\sigma)
\end{aligned} \tag{2.4.8}
$$

where,

$$
V_1^0(\sigma) = \int \frac{d^3k}{(2\pi)^3}\frac{E_M}{2}, \tag{2.4.9}
$$

and

$$
\begin{aligned}
\bar{V}_1^\beta(\sigma) &= \int \frac{d^3k}{(2\pi)^3}\frac{1}{\beta}\ln(1-e^{-\beta E}) \\
&= \frac{1}{4\pi\beta^3}\int_0^\infty x dx \ln\left[1 - \exp -(x^2+\beta^2 M^2)^{1/2}\right]
\end{aligned} \tag{2.4.10}
$$

where we put $x^2/\beta^2 = E_M^2 - M^2$. The integral may be evaluated by expanding $\bar{V}_1^\beta(\sigma)$ as a Taylor series and in the high temperature limit we find that

$$V_1^\beta(\sigma) = \frac{-\pi^2}{90\beta^4} + \frac{M^2}{24\beta^2} - \frac{M^3}{12\pi\beta} - \frac{M^4}{64\pi^2}\ln M^2\beta^2 \qquad (2.4.11)$$

The critical temperature in the present case is

$$T_c = \left[\frac{(m^2+\xi R)}{\lambda m}\right]^{\frac{1}{2}} \qquad (2.4.12)$$

The symmetry breaking present in the model can be removed if the temperature is raised above a certain value called the critical temperature. The order parameter of the theory is temperature dependent.

2.5 Nature of Phase Transitions

The characteristic of a first order phase transition is the existence of a barrier between the symmetric and the broken phases [10]. The temperature dependence of V_{eff} for a first order phase transition obtained using the present ϕ^6 model is shown in Figs. 2.1(a,b) and Figs. 2.2 (a,b,c). When $T \gg T_c$, the effective potential attains a minimum at $\sigma = 0$, which corresponds to the completely symmetric case. When the temperature decreases, a global minimum appears at $\sigma = 0$ and two local minima at $\sigma \neq 0$, which shows the existence of a barrier between the global and local minima. At $T = T_c$, all the minima are degenerate, that means the symmetry is broken. For $T < T_c$ the minima at $\sigma \neq 0$ becomes the global one. If for $T \leq T_c$ the extremum at $\sigma = 0$ remains a local minimum, there must be a barrier between the minimum at $\sigma = 0$ and at $\sigma \neq 0$. Therefore the change in σ in going from one phase to the other must be discontinuous, indicating a first order phase transition. The phase transition starts at T_c by tunnelling, however, if the barrier is high enough the tunnelling effect is very small and the phase transition does effectively start at a temperature $T \ll T_c$ [70]. This shows that the present model can describe first order phase transitions which might have taken place during the evolution of the early universe.

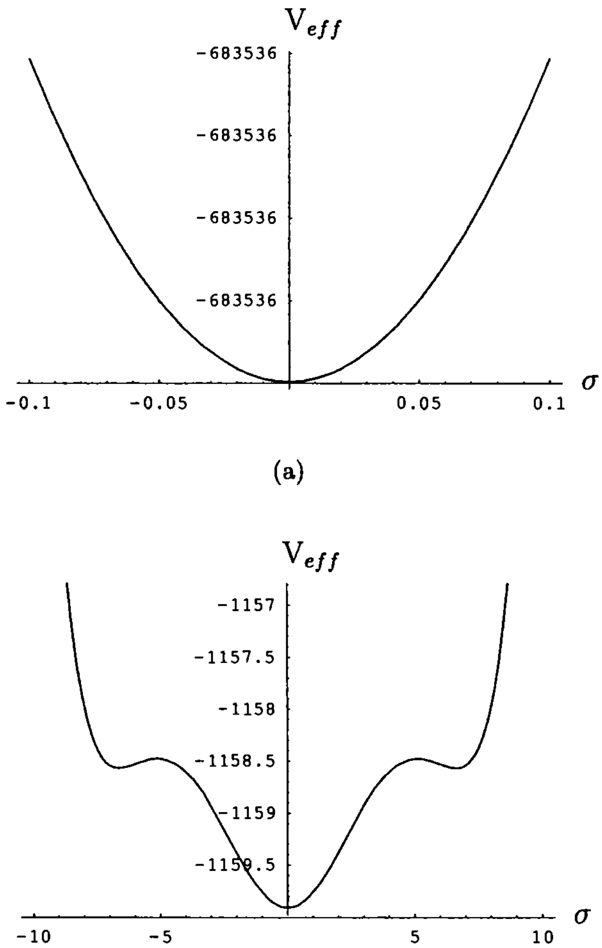


Fig. 2.1: (a) The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.008$, $R = 11.2$, $\xi = 1.6$ and $T = 50$ such that $T >> T_c$, for which the symmetry is completely restored; (b) The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.008$, $R = 0.8$, $\xi = 0.145$ and $T = 10.15$ such that $T > T_c$

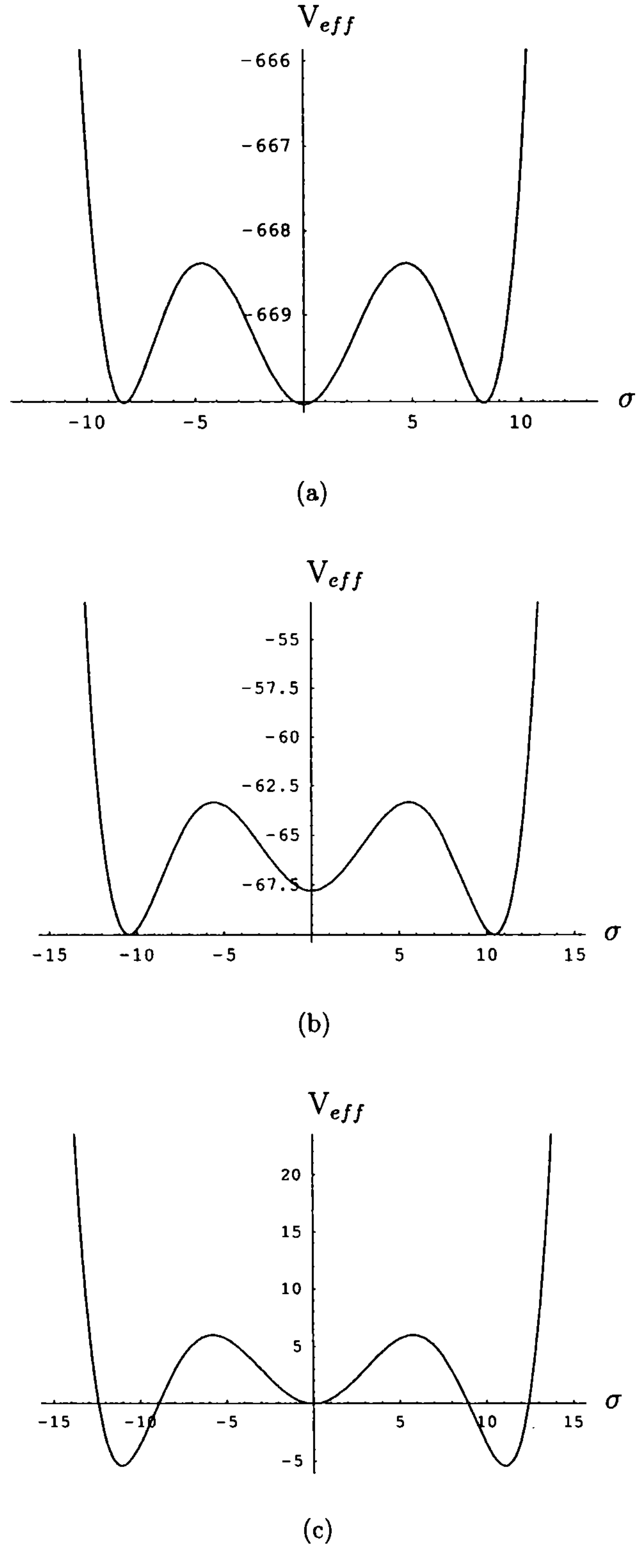


Fig. 2.2: (a) The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.008$, $R = 0.35$, $\xi = 0.004$ and $T = 8.69$ such that $T = T_c$ (b) The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.008$, $R = 0.31$, $\xi = -0.22$ and $T = 5$ such that $T < T_c$ (c) The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.008$, $R = 0.3$, $\xi = -0.3$ and $T = 0$.

2.6 Dependence on Curvature R and Scalar-Gravitational Coupling ξ

Using the present ϕ^6 model, it is proved that the scalar curvature, R can restore broken symmetries for a wide range of parameters from conformal to near minimal couplings, even if the temperature is below the critical temperature. Fig. 2.3 clearly shows that the first order phase transition takes place as R changes.

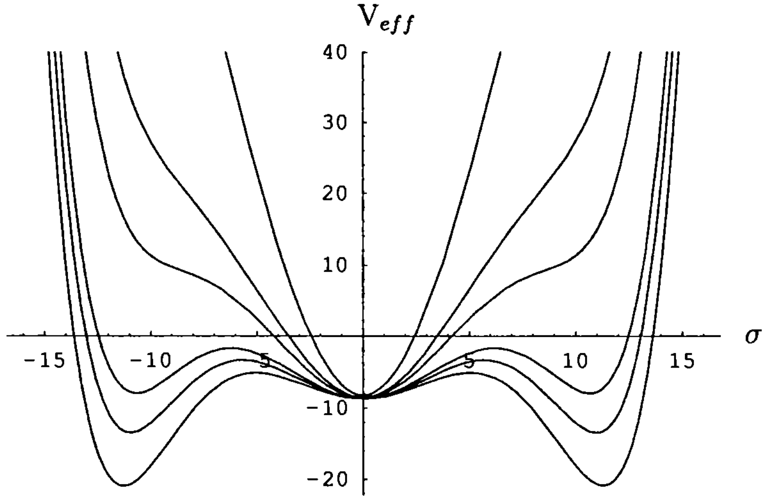


Fig. 2.3: The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.008$, $\xi = 0.1$ and $T = 1$. Starting from top the curves corresponds to the following values of the curvature: R=15, 4, 2.5, 0.5, 0.001, -0.9

The scalar-gravitational coupling constant ξ is found to play a crucial role in symmetry breaking phase transitions. Classically, a positive ξ restores symmetry, while the opposite effects are found for negative coupling [56]. Quantum effects depend on the value of ξ relative to the conformal value $\frac{1}{6}$. The present calculations show that the symmetry is restored as the scalar coupling constant ξ is increased. This phase transition, induced by the coupling constant ξ is also found to be of first order. It is clear from Fig. 2.4 that there is a barrier between the symmetric and broken phases.

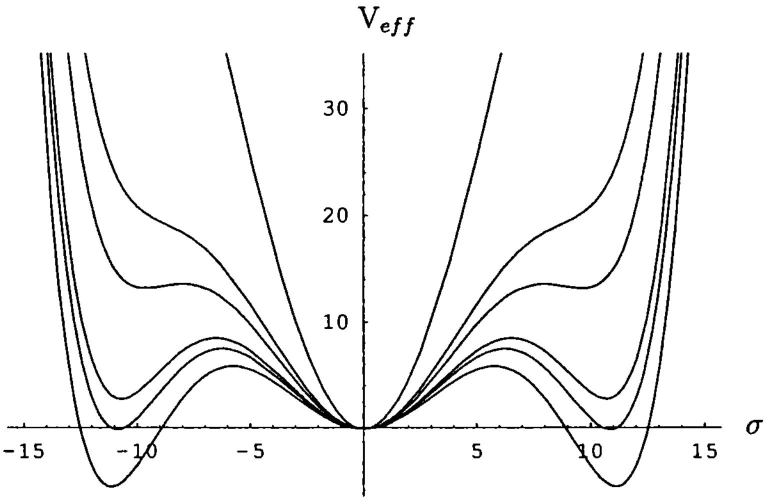


Fig. 2.4: The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.008$, $R = 0.3$ and $T = 3$. Starting from top the curves corresponds to the following values of the curvature: ξ = 6.5, 2.3, 1.25, 0.01, -0.3, -0.7

2.7 Discussion and Conclusions

According to renormalisability considerations, degree of the interaction potential can not be higher than four in (3+1) dimension [12]. The present calculations show that the ϕ^6 theory in (3+1) dimension can be regularised in curved spacetime and one can obtain finite expression for the one loop effective potential. The vacuum expectation values of the stress-energy tensor defined prior to any dynamics in the background gravitational field give us the information about the particle creation and vacuum polarisation [71].

In this chapter we have closely examined and verified the temperature dependence of the phase transitions in the early universe and verified their nature to be of first order as the transition is found to be discontinuous. In most of the works on cosmological phase transitions, the coupling to the background gravitational field is ignored. One deals with the Quantum field theory in flat spacetime at finite temperature and the expansion of the Universe serves only to decrease the temperature. However, at sufficiently early times the spacetime curvature can be expected to be important. Many authors have argued that such effects may be important in the context of cosmological phase transitions in Grand Unified models [12], [72]-[75]. Vilenkin and Ford have shown that spacetime curvature can drastically change the behaviour of the system [76]. O'Connor and co-workers have confirmed the effect of spacetime curvature and arbitrary field coupling on the phase transitions of the early universe [77]. Janson [78], Grib and Mosteparenko [79] and Madsen [80] have independently shown that the interaction with the external gravitational field may lead to SSB. The present work proves that the phase transition taking place during such a SSB is first order. It is found that for $\xi = 0$ or $R = 0$ the system remains in the symmetry broken state for all values of $T \leq T_c$. As the temperature is increased above T_c, the symmetry is restored depending also on the values of ξ and R. It is also found that symmetry can be restored either by increasing the value of ξ or by increasing the value of R keeping the temperature constant. This shows that the scalar-gravitational coupling and the scalar curvature did play a crucial role in determining the nature of phase transitions that took place in the early universe.

These results may be useful for the study of quantum thermal processes in the early universe. To examine the symmetry behaviour of the early universe closely one should take into consideration the effects of spacetime curvature and finite temperature corrections in their full rights.

Chapter 3

Phase Transitions and Bubble Nucleations for ϕ^6 Model in (2+1) Dimensional Curved Spacetime

3.1 Introduction

(2+1) dimensional gravity [81]-[85] exhibits novel features of interest. There are several important differences between the three and four dimensional problems. First of all the divergences in the gravitation action induced by scalar loops in 4 dimensions can, by power counting, be proportional to 1, R, R^2, $R_{\mu\nu}R^{\mu\nu}$ and $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ (or suitable combinations of them). In three dimensions the situation is simplified, as the only candidates are 1 and R [82]. General relativity is a geometric theory of spacetime, and quantizing gravity means quantizing spacetime itself. Ordinary quantum field theory is local, but the fundamental physical observables of quantum gravity are necessarily nonlocal. Ordinary quantum field theory takes causality as a fundamental postulate, but in quantum gravity the spacetime geometry and the causal structure, are themselves subject to quantum fluctuations. Again, perturbative quantum field theory depends on the existence of a smooth, approximately flat background, but there is no reason to believe that the short-distance limit of quantum gravity even resembles a smooth manifold. Faced with these problems, it is natural to look for simpler models that share the important conceptual features of general relativity while avoiding some of the conceptual difficulties. General relativity in $(2+1)$ dimensions is one such scheme of formulation [81]. Another important feature of the conformally invariant scalar theory in three dimensions is that its ϕ^6 coupling can in principle, induce a divergence in the four-point Green's functions necessitating a ϕ^4 coupling, which is not conformally invariant [6].

Field theory of (2+1) dimensions may exhibit several features of interest in condensed matter physics, which are not in (3+1) dimensional field theory, describing high energy physics. (2+1) dimensional ϕ^6 theory finds applications in the study of vortex solution of the abelian Chern-Simons theory [86], blackholes in string theory [1], etc. In this chapter the first order phase transition in a (2+1) dimensional curved spacetime for ϕ^6 model is discussed. In the previous chapter we have obtained a divergenceless expression for the one-loop effective potential for the ϕ^6 model in a (3+1) dimensional Bianchi type-I spacetime. This chapter is organized in the following way. In section 3.2 we evaluate the one-loop effective potential for ϕ^6 theory in a $(2+1)$ dimensional Bianchi type-I spacetime and obtain a divergenceless expression. A finite expression for the energy momentum tensor for the ϕ^6 theory in this spacetime is obtained in section 3.3. The finite temperature effective potential for the same theory is evaluated and the finite temperature effects on the phase transitions are discussed in the next section. In section 3.5 the nature of phase transitions for the present model is examined and is clarified to be of first order. The crucial dependence of phase transitions on spacetime curvature and the gravitational-scalar coupling is made clear in section 3.6. A first order phase transition proceeds by nucleation of bubbles of broken phase in the background of unbroken phase. In section 3.7 the interaction between the bubble field and the surrounding plasma is considered and the expansion of bubbles in such a damping environment is discussed. It is found that there exists an exact solution for the damped motion of the bubble in the thin wall regime. The discussions and conclusions are presented in the final section.

3.2 One-loop Effective Potential for ϕ^6 Theory in (2+1) Dimensional Bianchi Type-I Spacetime

Let us consider a massive self interacting scalar field ϕ coupled arbitrarily to the gravitational back ground and described by the Lagrangian density $\mathcal{L}$

$$\mathcal{L} = \sqrt{-g}\left\{\frac{1}{2}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \xi R\phi^2] - \frac{1}{2}\lambda^2\phi^2(\phi^2 - m/\lambda)^2\right\} \tag{3.2.1}$$

The equation of motion associated with the Lagrangian (3.2.1) is,

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + (m^2 + \xi R)\phi - 4\kappa\phi^3 + 3\lambda^2\phi^5 = 0 \tag{3.2.2}$$

in which we put $m\lambda = \kappa$. Writing $\phi = \phi_c + \phi_q$, where ϕ_c is the classical field and ϕ_q is a quantum field with vanishing vacuum expectation value, $\langle\phi_q\rangle = 0$, the field equation for the classical field ϕ_c is given by,

$$\begin{aligned} g^{\mu\nu}\nabla_\mu\nabla_\nu\phi_c + [(m_r^2 + \delta m^2) + (\xi_r + \delta\xi)R]\phi_c - 4(\kappa_r + \delta\kappa)\phi_c^3 - 12(\kappa_r + \delta\kappa)\phi_c\left\langle\phi_q^2\right\rangle \\ + \; 3(\lambda_r^2 + \delta\lambda^2)\phi_c^5 + 30(\lambda_r^2 + \delta\lambda^2)\phi_c^3\left\langle\phi_q^2\right\rangle + 15(\lambda_r^2 + \delta\lambda^2)\phi_c\left\langle\phi_q^4\right\rangle = 0 \end{aligned} \tag{3.2.3}$$

where the bare parameters m, ξ, κ and λ are replaced by the renormalised terms. To the one loop quantum effect, the field equation for the quantum field ϕ_q is,

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi_q + (m_r^2 + \xi_r R)\phi_q - 12\kappa_r\phi_c^2\phi_q + 15\lambda_r^2\phi_c^4\phi_q = 0 \tag{3.2.4}$$

The effective potential V_{eff} is given by,

$$\begin{aligned} V_{eff} = & \frac{1}{2}[(m_r^2 + \delta m^2) + (\xi_r + \delta\xi)R][\phi_c^2 + \langle\phi_q^2\rangle] - (\kappa_r + \delta\kappa)\phi_c^4 \\ & - 6(\kappa_r + \delta\kappa)\phi_c^2\langle\phi_q^2\rangle - (\kappa_r + \delta\kappa)\langle\phi_q^4\rangle + \frac{1}{2}(\lambda_r^2 + \delta\lambda^2)\phi_c^6 \\ & + \frac{15}{2}(\lambda_r^2 + \delta\lambda^2)\phi_c^4\langle\phi_q^2\rangle + \frac{15}{2}(\lambda_r^2 + \delta\lambda^2)\phi_c^2\langle\phi_q^4\rangle + \frac{1}{2}(\lambda_r^2 + \delta\lambda^2)\langle\phi_q^6\rangle \end{aligned} \tag{3.2.5}$$

To make V_{eff} finite, the following renormalisation conditions are used,

$$m_r^2 = \left(\frac{\partial^2 V_{eff}}{\partial\phi_c^2}\right)_{\phi_c=R=0}, \qquad \xi_r = \left(\frac{\partial^3 V_{eff}}{\partial R\partial\phi_c^2}\right)_{\phi_c=R=0},$$

$$\kappa_r = \left(\frac{\partial^4 V_{eff}}{\partial\phi_c^4}\right)_{\phi_c=R=0}, \qquad \lambda_r^2 = \left(\frac{\partial^6 V_{eff}}{\partial\phi_c^6}\right)_{\phi_c=R=0} \tag{3.2.6}$$

Consider a (2+1) dimensional Bianchi type-I spacetime with small anisotropy which has the line element

$$ds^2 = C(\eta)d\eta^2 - a_1^2(\eta)dx^2 - a_2^2(\eta)dy^2, \qquad C = a_1 a_2 \tag{3.2.7}$$

In this model the mode function of the quantum field ϕ_q can be written in the separated form as $u_k = C^{-1/4}(2\pi)^{-1}\exp(i\kappa.x)\chi_k(\eta)$. The wave equation Eq. (3.2.4) will then lead to

$$\ddot{\chi}_k + \left\{C\left[m_r^2 + (\xi_r - \frac{1}{8})R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \sum_i \frac{k_i^2}{a_i^2}\right] + Q\right\}\chi_k = 0 \tag{3.2.8}$$

where the spacetime curvature function R and the anisotropic function Q are

$$R = 8C^{-1}(\overset{\bullet}{H} + H^2 + Q), \quad H = \sum_i h_i, \quad h_i = \frac{\dot{a}_i}{a_i}, \quad Q = \frac{1}{16}\sum_{i<j}(h_i - h_j)^2 \quad (3.2.9)$$

When the metric is slowly varying, Eq. (3.2.8) possesses WKB approximation solution:

$$\chi_k = (2W_k)^{-\frac{1}{2}} \exp(-i \int d\eta W_k) \quad (3.2.10)$$

where, $$W_k = \left\{ C \left[m_r^2 + (\xi_r - \tfrac{1}{8})R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \sum_i \tfrac{k_i^2}{a_i^2} \right] + Q \right\}^{\frac{1}{2}}$$

Using the above solution we get:

$$\begin{aligned} \langle \phi_q^2 \rangle &= \frac{1}{8\pi^2 C(\eta)} \int d^2k \left[m_r^2 + (\xi_r - \frac{1}{8})R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \sum_i \frac{k_i^2}{a_i^2} + \frac{Q}{C} \right]^{-1/2} \\ &= \frac{1}{16\pi} \left[\Lambda + \frac{A_2}{2\Lambda} - {A_2}^{1/2} \right] \end{aligned} \quad (3.2.11)$$

where $A_2 = (m_r^2 + (\xi_r - \frac{1}{8})R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \frac{Q}{C})$.

And we get,

$$\begin{aligned} \langle \phi_q^4 \rangle &= \frac{3}{16\pi^4 C} \int d^2k \chi_k(\eta)\chi_k^*(\eta) \int d^2k' \chi_{k'}(\eta)\chi_{k'}^*(\eta) \\ &= \frac{3}{16\pi^2} \left[2A_2 + \Lambda^2 - 2\Lambda {A_2}^{1/2} \left(1 + \frac{A_2}{2\Lambda^2} \right) \right] \end{aligned} \quad (3.2.12)$$

where a momentum cut-off Λ is introduced to regularize the k-integration. From the renormalisation conditions given by Eq. (3.2.8) the renormalisation counter terms are evaluated and substituting the renormalisation counter terms, we obtain $\frac{\partial V_{eff}}{\partial \phi_c}$ from Eq. (3.2.5) as,

$$\frac{\partial V_{eff}}{\partial \phi_c} = (m_r^2 + \xi_r R)\phi_c + \frac{\lambda_r^2[(m_r^2 + \frac{Q}{C})^{1/2} - {A_2}^{1/2}]}{2B_2}\phi_c$$

$$- \frac{D_2[(m_r^2 + \frac{Q}{C}) + A_2]}{20\pi B_2 E_2}\phi_c + \frac{1}{4B_2}\left[\frac{\lambda_r^2}{(m_r^2 + \frac{Q}{C})^{1/2}} - \frac{D_2}{5\pi E_2}\right](\xi_r - \frac{1}{8})R\phi_c$$

$$+ \frac{\pi}{3B_2}\left[2\lambda_r^2 - \frac{D_2 {A_2}^{1/2}}{5\pi E_2}\right]\phi_c^3 + \frac{2D_2}{75 B_2 E_2}\phi_c^5 \quad (3.2.13)$$

where,

$$A_2 = (m_r^2 + (\xi_r - \frac{1}{8})R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \frac{Q}{C}),$$

$$B_2 = \left[\frac{-45\lambda_r^2}{(m_r^2 + \frac{Q}{C})^{1/2}} + \frac{108\kappa_r^2}{(m_r^2 + \frac{Q}{C})^{3/2}}\right],$$

$$D_2 = \left[-\lambda_r^2\pi + \frac{117\kappa_r\lambda_r^2}{(m_r^2 + \frac{Q}{C})^{1/2}} + \frac{270\kappa_r^3}{(m_r^2 + \frac{Q}{C})^{3/2}}\right] \quad \text{and} \quad E_2 = \left[\frac{9\kappa_r}{\pi} - (m_r^2 + \frac{Q}{C})^{1/2}\right] \quad (3.2.14)$$

Thus it is clear that we can obtain a finite expression for the one loop effective potential for the ϕ^6 model in (2+1) dimensional Bianchi type-I spacetime. In the previous chapter it is shown that ϕ^6 potential can be regularized in (3+1) dimensional curved spacetime. In this section a divergenceless expression for the ϕ^6 one-loop effective potential in a (2+1) dimensional Bianchi type-I background spacetime is obtained.

3.3 Energy-Momentum Tensor for ϕ^6 Field in (2+1) Dimensional Bianchi Type-I Spacetime

Considering a (2+1) dimensional Bianchi type-I background spacetime, the finite expression for the expectation value of the quantum energy-momentum tensor is obtained by adopting momentum cut-off regularization technique:

$$\langle T^{\eta}_{\eta}\rangle^{Q} = \frac{C'^2[3A_2 - \frac{Q}{C}]}{768\pi C^4 A_2^{3/2}} - \frac{1}{48\pi}A_2^{3/2} - \frac{C'^2 A_2^{1/2}}{256\pi C^3} - \frac{C'^2(A_2 + \frac{Q}{C})}{128\pi C^3 A_2^{1/2}}$$

$$+ \frac{C'^2}{16\pi C^3}\left\{\xi_r + \frac{(\xi_r - \frac{1}{8})\lambda_r^2}{4B_2} - \frac{D_2(\xi_r - \frac{1}{8})}{20\pi B_2 E_2}\right\}\frac{(A_2 + \frac{Q}{C})}{A_2^{1/2}}$$

$$+ \frac{C'}{16\pi C^2}\left[\frac{C'}{C} - 12\right]\left\{\xi_r + \frac{(\xi_r - \frac{1}{8})\lambda_r^2}{4B_2} - \frac{D_2(\xi_r - \frac{1}{8})}{20\pi B_2 E_2}\right\}A_2^{1/2}$$

$$+ \frac{A_2^{1/2}}{8\pi}\left\{m_r^2 + \frac{\lambda_r^2(m_r^2 + \frac{Q}{C})^{1/2}}{2B_2} - \frac{D_2(m_r^2 + \frac{Q}{C})}{20\pi B_2 E_2}\right\}$$

$$- \frac{A_2^{1/2}}{2B_2}\left\{\lambda_r^2 - \frac{3A_2^{1/2}D_2}{20\pi E_2}\right\}\phi_c^2 - \frac{A^{1/2}D}{20B_2E_2}\phi_c^4 \qquad (3.3.1)$$

where A_2, B_2, D_2 and E_2 are defined by Eq. (3.2.14). For this case also it is clear that the energy-momentum tensor depends on the anisotropy of the spacetime.

A knowledge of $T_{\mu\nu}$ is important for two reasons. It can be used to assess the importance of quantum effects on the dynamics of the gravitational field itself, that is the back-reaction problem. Also, it is frequently a more useful probe of the physical situation than a particle count. In regions of strong gravity, vacuum polarisation effects, akin to those in QED can lead to important phenomena even in the absence of actual particle creation.

3.4 Finite Temperature Behaviour

To evaluate the finite temperature effective potential, the vacuum expectation value is replaced by the thermal average $\langle\phi\rangle_T = \sigma_T$. Considering the same Lagrangian density as above, the zero loop effective potential is temperature independent,

$$V_0(\sigma) = \frac{1}{2}\xi R\sigma^2 + \frac{1}{2}\lambda^2\sigma^2(\sigma^2 - m/\lambda)^2 \tag{3.4.1}$$

The one loop approximation to finite temperature effective potential [65]-[68] is given by,

$$\begin{aligned} V_1^\beta(\sigma) &= \frac{1}{2\beta}\sum_n \int \frac{d^2k}{(2\pi)^2}\ln(k^2 - M^2) \\ &= \frac{1}{2\beta}\sum_n \int \frac{d^2k}{(2\pi)^2}\ln(\frac{-4\pi^2n^2}{\beta^2} - E_M^2) \end{aligned} \tag{3.4.2}$$

where,

$$\beta = \frac{1}{T}, \quad E_M^2 = k^2 + M^2, \quad M^2 = m^2 + \xi R - 12\lambda m\sigma^2 + 15\lambda^2\sigma^4 \tag{3.4.3}$$

Proceeding as in the previous chapter, in the high temperature limit [8] it is obtained that

$$V_1^\beta(\sigma) = \frac{1}{4\pi\beta^3}\xi(3) - \frac{M^2}{8\pi\beta}\ln(M\beta) \tag{3.4.4}$$

where $\xi(z)$ is the Riemannian Zeta function [87], $\xi(z) = \sum_{n=1}^{\infty}\frac{1}{n^Z}$, $[Re\ Z > 1]$.

In the (2+1) dimensional case also, the symmetry breaking present in this ϕ^6 model can be removed if the temperature is raised above a certain value called the critical temperature. The expression for critical temperature in this case is obtained as

$$T_c = (m^2 + \xi R) \exp\left(\frac{2(m^2 + \xi R)}{3T_c \lambda m}\right) \quad (3.4.5)$$

The order parameter of the theory is temperature dependent. The temperature dependence of finite temperature effective potential leads to phase transitions.

3.5 First Order Phase Transitions

On shifting the field from ϕ to $\phi+\sigma$ in the equation (3.2.2) and taking the Gibbs average of the corresponding equation we get:

$$\Box\sigma_T + (m^2 + \xi R)\sigma_T - 4\kappa{\sigma_T}^3 - 12\kappa\sigma_T \left\langle \phi^2 \right\rangle - 12\kappa\sigma_T^2 < \phi > +15\lambda^2\sigma_T \left\langle \phi^4 \right\rangle$$
$$+ 30\lambda^2\sigma_T^2 \left\langle \phi^3 \right\rangle + 30\lambda^2\sigma_T^3 \left\langle \phi^2 \right\rangle + 15\lambda^2\sigma_T^4 \left\langle \phi \right\rangle + 3\lambda^2\sigma_T^5 = 0 \quad (3.5.1)$$

Using the standard finite temperature Green's-function methods we can find that in the high temperature limit,

$\left\langle \phi^2 \right\rangle = \frac{1}{4\pi} \int \frac{dk}{(Exp(k/T) - 1)}$, $\left\langle \phi^4 \right\rangle = 3[\left\langle \phi^2 \right\rangle]^2$ and $\left\langle \phi^3 \right\rangle = \left\langle \phi \right\rangle = 0$. Thus equation (3.5.1) becomes

$$\Box\sigma_T + (m^2 + \xi R)\sigma_T - 4\kappa\sigma_T^3 - 12\kappa\sigma_T \left\langle \phi^2 \right\rangle + 15\lambda^2\sigma_T \left\langle \phi^4 \right\rangle$$
$$+ 30\lambda^2\sigma_T^3 \left\langle \phi^2 \right\rangle + 3\lambda^2\sigma_T^5 = 0 \quad (3.5.2)$$

Assuming that σ_T is a constant we obtain,

$$\sigma_T\left[(m^2+\xi R)-4\kappa\sigma_T^2-12\kappa\left\langle\phi^2\right\rangle+15\lambda^2\left\langle\phi^4\right\rangle+30\lambda^2\sigma_T^2\left\langle\phi^2\right\rangle+3\lambda^2\sigma_T^4\right]=0 \tag{3.5.3}$$

This equation has degenerate solutions: $\sigma_T = 0$, and

$$\sigma_T^2=\frac{\left\{(4m-30\lambda\left\langle\phi^2\right\rangle)\pm(4m^2-12\xi R-96\lambda m\left\langle\phi^2\right\rangle+900\lambda^2(\left\langle\phi^2\right\rangle)^2+180\left\langle\phi^4\right\rangle)^{\frac{1}{2}}\right\}}{6\lambda} \tag{3.5.4}$$

Each of these solutions defines a possible phase of the field system with its characteristic excitations. On heating the field system from absolute zero, the two branches of σ_T^2 given by the above equation coincide at a temperature for which

$$\left(4m^2-12\xi R-96\lambda m\left\langle\phi^2\right\rangle+900\lambda^2(\left\langle\phi^2\right\rangle)^2+180\left\langle\phi^4\right\rangle\right)=0 \tag{3.5.5}$$

yielding a common value of σ_T. The existence of the separate branches of σ_T^2 implies that the phase transition is of first order [10, 88]. Numerical results obtained using equation (3.5.4) clearly shows that the order parameter does not vanish even for very high values of the temperature. Fig. 3.1 gives the variation of the two branches of σ_T^2 with respect to temperature. It is found that the two branches coincides at a particular value of T given by equation (3.5.5). From the figure it is clear that there is a discontinuity for the variation of the order parameter with temperature, indicating a first order phase transition.

The temperature dependence of V_{eff} obtained for the present ϕ^6 model in the (2+1) dimensional background spacetime is shown in Fig. 3.2. It is found that for $T \gg T_c$ the effective potential attains a minimum at $\sigma = 0$, which corresponds to the completely symmetric case. When the temperature decreases, a global

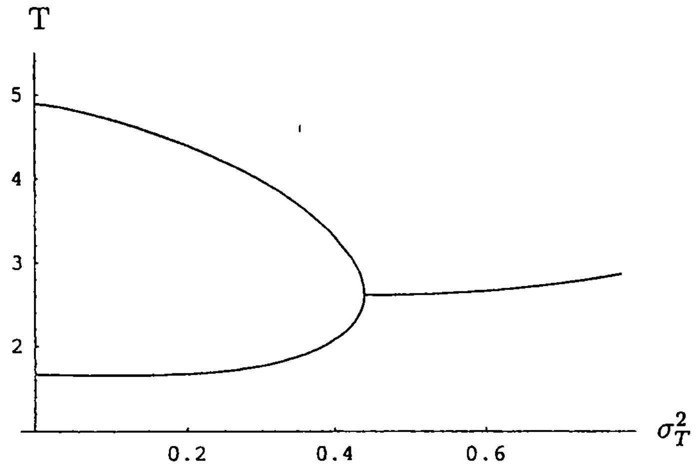


Fig. 3.1: Variation of the two branches of σ_T^2 with respect to temperature. The two curves coincide after the temperature which satisfies equation (3.5.5), where $m = 3.9371$, $\lambda = 0.8$, $R = 0.9$ and $\xi = 0.2$

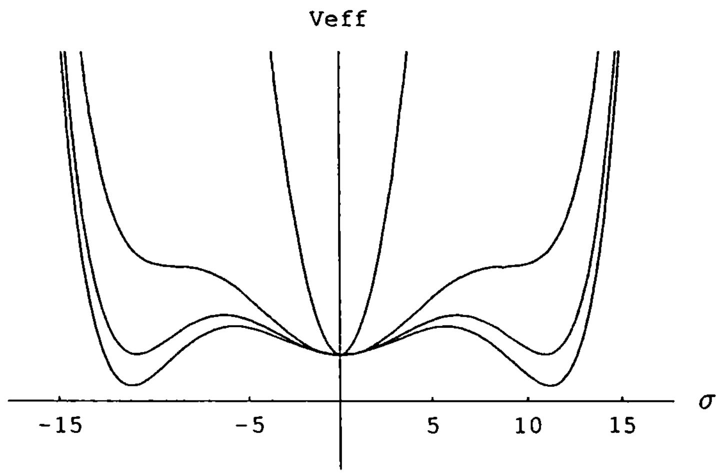


Fig. 3.2: The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.008$. Starting from the top the curves corresponds to the following values of the parameters: (i)$R = 3.3$, $\xi = 2.54$, $T = 25$ such that $T >> T_c$, $(ii)R = 1.93$, $\xi = 0.198$ and $T = 18.5$ such that $T > T_c$, $(iii)R = 0.42$, $\xi = 0.02$ and $T = 9$ such that $T = T_c$, $(iv)R = 0.35$, $\xi = -0.3$ and $T = 5$ such that $T < T_c$

minimum appears at $\sigma = 0$ and two local minima at $\sigma \neq 0$, which shows the existence of a barrier between the global and local minima. At $T = T_c$, all the minima are degenerate, which implies that the symmetry is broken. For $T < T_c$ the minima at $\sigma \neq 0$ become global minima. If for $T \leq T_c$ the extremum at $\sigma = 0$ remains a local minimum, there must be a barrier between the minimum at $\sigma = 0$ and at $\sigma \neq 0$. Therefore the change in σ in going from one phase to the other must be discontinuous, indicating a first order phase transition [10], [70, 88].

3.6 Dependence on Curvature R and Scalar-Gravitational Coupling ξ

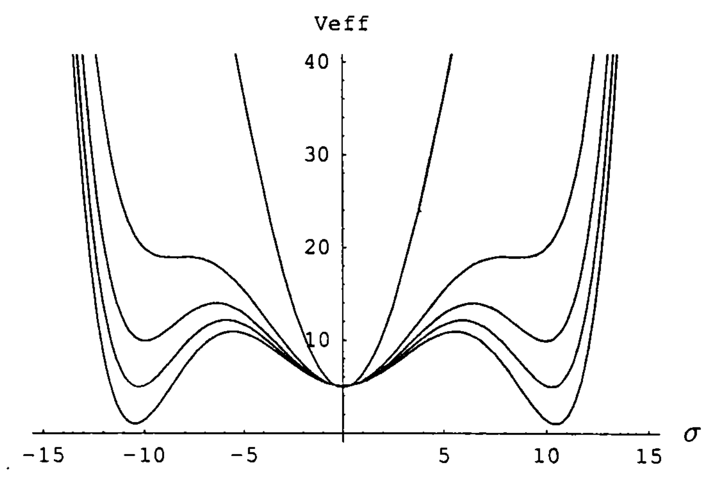


Fig. 3.3: The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.009$, $\xi = 0.1$ and $T = 4$. Starting from top the curves corresponds to the following values of the curvature: R=20, 3, 0.99, 0.02, -0.72.

Fig. 3.3 clearly shows the crucial role of scalar curvature R in determining the fate of symmetry and the phase transitions for the present model. From the figure it is clear that the first order phase transition takes place as R changes. It is found that for $R = 0$ or $\xi = 0$ the system remains in the symmetry broken state for all values of $T \leq T_c$. As the temperature is increased above T_c, the symmetry is restored depending on the values of R and ξ also. It is also found that symmetry can be restored either by increasing the value of R or by increasing the value of ξ keeping the temperature constant, even below the critical temperature. It is clear from Fig. 3.4 that there is a barrier between the symmetric and broken phases. Thus the phase transition, induced by the coupling constant ξ is also of first order. This shows that the scalar-gravitational coupling and the scalar curvature do play a crucial role in determining the nature of phase transitions.

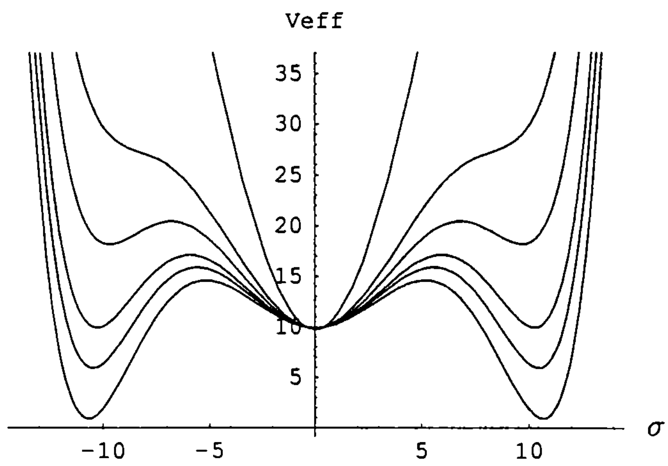


Fig. 3.4: The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.009$, $R = 0.2$ and $T = 5$. Starting from top the curves corresponds to the following values of the curvature: ξ = 9, 2, 0.85, 0.025, -0.35, -0.8.

3.7 Bubble Nucleation and Expansion

A first order phase transition proceeds by nucleation of bubbles of broken phase in the background of unbroken phase [8]. Decay of metastable vacuum state with

$\phi = 0$ proceeds via quantum tunnelling [10] with the nucleation of bubbles of the asymmetric phase. The bubbles expand and eventually collide, while new bubbles are continuously formed, until the phase transition is completed.

Consider a massive self interacting complex field Φ coupled arbitrarily to the gravitational back ground, with the Φ^4 potential

$$V(\Phi) = \frac{1}{2}\xi R |\Phi|^2 + \frac{1}{2}\lambda^2 |\Phi^2| (|\Phi|^2 - m/\lambda)^2 \tag{3.7.1}$$

with a minimum at $|\Phi| = 0$ and a set of minima at $|\Phi| = \left[\frac{m \pm \sqrt{-\xi R}}{\lambda}\right]^{1/2}$, connected by U(1) transformation. When the temperature is below T_c, a false vacuum is found at $\Phi = 0$ and true vacuum at $\Phi = \Phi_0 \neq 0$. As the temperature increases, the false vacuum will decay to the true vacuum state via bubble nucleation. During a first order phase transition as in Fig. 3.2, at a temperature $T < T_c$, where $|V(\Phi)|$ at a minimum with $\Phi = \Phi_0 \neq 0$ is much lower than the barrier height in $V(\Phi)$ between $\Phi = 0$ and $\Phi = \Phi_0$, the thin wall approximation [8] is valid. The equation of motion for this system is

$$\partial_\mu \partial^\mu \Phi = -\frac{\partial V}{\partial \Phi} \tag{3.7.2}$$

Let us consider the minimally coupled case $\xi = 0$. For the thin wall regime, the approximate solution of Eq. (3.7.2) is obtained as

$$|\Phi| = \left\{\frac{m}{2\lambda}\left(\tanh\left[m(\chi - R_0)\right] + 1\right)\right\}^{1/2} \tag{3.7.3}$$

where R_0 is the bubble radius at nucleation time and $\chi^2 = |\vec{x}|^2 - t^2$. The kink like shape of this solution, obtained numerically is shown in Fig. 3.5.

While discussing the bubble collisions [89]-[91], one has to consider the interaction between the bubble field and the surrounding plasma. As the bubble wall sweeps through a specific point, the Higgs field ϕ acquires an expectation

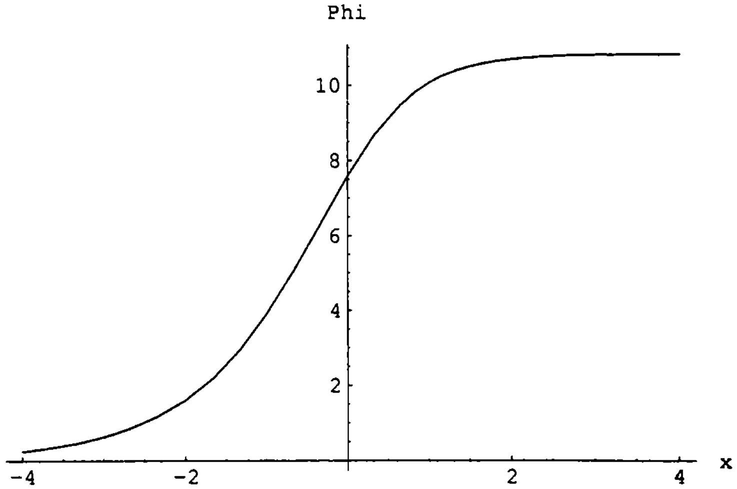


Fig. 3.5: Shape of the solution given by Eq. (3.7.3)

value and the field coupled to it acquires mass. Thus particles without enough energy to acquire the corresponding mass inside the bubble will bounce-off the wall (thus imparting negative momentum to it), while the rest will get through. Obviously, the faster the wall propagates the stronger this effect will be, since the momentum transfer in each collision will be larger and thus a force proportional to the velocity with which the wall sweeps through the plasma appears. Thus considering the damping effect of the surrounding plasma on the motion of the walls we insert a frictional term in the equation of motion,

$$\partial_\mu \partial^\mu \Phi + \gamma \left|\dot{\Phi}\right| e^{i\theta} = -\frac{\partial V}{\partial \Phi} \tag{3.7.4}$$

where $\left|\dot{\Phi}\right| = \dfrac{\partial |\Phi|}{\partial t}$, θ is the phase of the field and γ stands for the friction coefficient.

To find the solution of Eq. (3.7.4) in the thin wall limit, first we suppose that solution for which the wall has the form of a travelling wave do exist [90]. Writing Φ in polar form $\Phi = \rho e^{i\theta}$, we can rewrite Eq. (3.7.4) and for a single bubble configuration we take the phase of the bubble θ to be constant. Then the equation for the modulus of the field is

$$\partial_\mu \partial^\mu \rho + \gamma \dot{\rho} = -\frac{\partial V(\rho)}{\partial \rho} \tag{3.7.5}$$

Because the wall thickness is much smaller than the radius of the bubble, we can go to (1+1) dimensions to get an approximate expression for the terminal velocity of the bubble under this equation in the thin wall limit. Inserting the ansatz $\rho = \rho(x - x_0(t))$ leads to

$$(1 - \dot{x}_0^2)\rho'' + (\ddot{x}_0 + \gamma \dot{x}_0)\rho' = \frac{\partial V(\rho)}{\partial \rho} \tag{3.7.6}$$

where $\rho' = \frac{\partial \rho}{\partial x}$. Multiplying by ρ' and integrating over $-\infty \leq x \leq +\infty$ we get,

$$(\ddot{x}_0 + \gamma \dot{x}_0)\left(\int_{-\infty}^{+\infty} \rho'^2 dx\right) = \int_{-\infty}^{+\infty} V' dx = \Delta V \tag{3.7.7}$$

where ΔV is the potential energy difference between the false and the true vacuum phases. For the initial conditions $x_0(t=0) = R_0,\ \dot{x}_0(t=0) = 0$, the solution of Eq. (3.7.7) is

$$x_0(t=0) = \frac{1}{\gamma}\alpha t + \frac{\alpha}{\gamma^2}(e^{-\gamma t} - 1) + R_0 \tag{3.7.8}$$

where $\alpha \equiv \Delta V / (\int \rho'^2 dx)$. Thus for values of $t >> \gamma^{-1}$ the bubble walls will have reached their terminal velocity

$$v_{ter} = \frac{\Delta V}{\gamma(\int \rho'^2 dx)} \tag{3.7.9}$$

To get an approximate expression for ρ valid within this regime, it suffices to rewrite Eq. (3.7.5) with an ansatz $\rho = \rho(r - r_0(t))$, where r is the usual radial coordinate. Using $\ddot{r}_0 = 0,\ \dot{r}_0 \approx v_{ter}$, we get,

$$(1 - v_{ter}^2)\frac{\partial^2 \rho}{\partial r^2} + (\frac{2}{r} + \gamma v_{ter})\frac{\partial \rho}{\partial r} = \frac{\partial V(\rho)}{\partial \rho} \tag{3.7.10}$$

According to Eq. (3.7.9) the terminal velocity roughly goes like

$$v_{ter} = \frac{\Delta V}{\gamma(\frac{\rho_{tv}^2}{\delta_m^2}\delta_m)} = \frac{\Delta V \delta_m}{\gamma \rho_{tv}^2} \tag{3.7.11}$$

where δ_m is the bubble wall thickness and ρ_{tv} is the true vacuum value of the field. At the values of r for which the first derivative of the field is important ($r \sim R$ for thin wall bubble), we have $R\gamma v_{ter} \sim \delta_m << 1$, and the second term in the second parenthesis of Eq. (3.7.10) is negligible when compared to the first. Again, since the radius of thin walled bubble is very large, we can also neglect the term $(\frac{2}{r})\frac{\partial \rho}{\partial r}$ in the standard thin wall approximation. Thus we get,

$$(1 - v_{ter}^2)\frac{\partial^2 \rho}{\partial r^2} = \frac{\partial V(\rho)}{\partial \rho} \tag{3.7.12}$$

where r is the radial coordinate and v_{ter} is the terminal velocity of the bubble walls. For the present ϕ^6 potential the solution for Eq. (3.7.12) is obtained as

$$\rho = \left\{ \frac{m}{2\lambda} \left(\tanh\left[\frac{m(r - v_{ter}t - R_0)}{\sqrt{1 - v_{ter}^2}} \right] + 1 \right) \right\}^{1/2} \tag{3.7.13}$$

which is simply a Lorentz-contracted moving domain wall. Thus it is clear that there exists an exact solution for the damped motion of the bubble in the thin wall regime. Fig. 3.6 gives the kink like shape of this solution obtained numerically.

The bubbles of the new phase nucleated within the old one subsequently expand and collide with each other. This would take place via Kibble mechanism [92]. In order to justify Kibble mechanism, one must follow the evolution of amplitude and phase of Φ [93]-[95]. The regions of the old phase trapped within the new one give birth to topological defects [10].

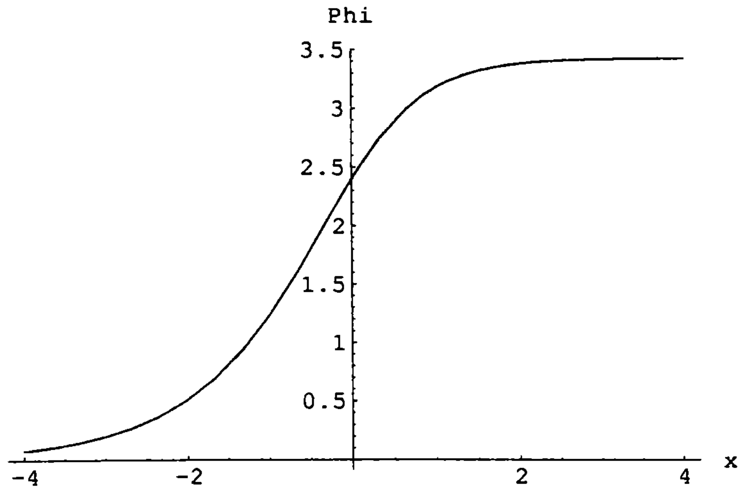


Fig. 3.6: Shape of the solution given by Eq. (3.7.13) considering the damping effect

3.8 Discussion and Conclusions

In this chapter a divergenceless expression for the ϕ^6 one-loop effective potential and energy-momentum tensor in a (2+1) dimensional Bianchi type-I background spacetime is obtained. The temperature dependence of phase transitions for the ϕ^6 model is closely examined and the nature of phase transitions for the present model is verified to be of first order.

In the present work, considering a ϕ^6 potential we have proved that the gravitational effects are of particular interest in a (2+1) dimensional Bianchi type-I spacetime. The phase transition taking place in a (2+1) dimensional Bianchi type-I background spacetime, during such a SSB is of first order.

As the bubbles of the low temperature phase expand, they expel heat into their surroundings, heating the high temperature phase up to T_c. At this point the pressure of the high temperature phase prevents further expansion of the low temperature phase. After all, T_c is the temperature at which the two phases have equal pressures and can coexist.

To get the false vacuum and to study the bubble nucleation Ferrera *et al.* [90, 91] has introduced a ϕ^3 term in the ϕ^4 potential. But in the present ϕ^6 model, false vacuum at $\phi = 0$ and true vacuum at $\phi = \phi_0 \neq 0$ occur naturally at temperatures $T < T_{c.}$ Considering the interaction between the bubble field and plasma an exact solution for the damped motion of the bubble in the thin wall regime is obtained for the present model. Whether or not the universe recovers from a first order phase transition and any relics are left behind depends upon the nucleation, expansion and collision of bubble and on the process of eventual transition to the new phase.

Chapter 4

Scalar Field Approach to Jeans Mass Calculations

4.1 Introduction

The formation of large scale structure still remains as an unsolved problem in Cosmology. Gravity is the dominant force which governs the large scale dynamics of the universe [26]-[31]. The standard theory of cosmological structure formation is based on the idea of gravitational instability [27, 33] according to which small initial irregularities in the distribution of matter become amplified by the attractive nature of gravity. Small fluctuations in the density results in gravitational instability and gravitational instability causes the growth of perturbations in an expanding universe [96]-[98]. The structure we observe in the universe today is the end result of the gravitational amplification of small primeval perturbations. There are two distinct theories of how the initial seed fluctuations might have arisen [96]. One of these models involved the idea of topological defects created during phase transitions [10] in the early universe. The alternative picture involves the inflationary model of the universe, in which the primordial quantum fluctuations get amplified and evolve to become classical seed perturbations [26, 30, 99].

Normal physical processes can act coherently only over sizes smaller than Hubble radius. Thus any physical process leading to density perturbations at some early epoch, $t = t_i$, could only have operated at scales smaller than $H^{-1}(t_i)$. But most of the relevant astrophysical scales were much bigger than $H^{-1}(t_i)$ for reasonably early epochs. Thus if we want the seed perturbation to be originated in the early universe, then it is difficult to understand how any physical process could have contributed to it. But at sufficiently small t, if $\lambda(t) << H^{-1}(t_i)$ then the physical processes can lead to an initial density perturbation. It is possible to make this if the scale factor, $a(t)$ increases rapidly (eg., exponentially) with t for a short period of time as in the case of inflation. The most natural choice for the seed perturbations is the quantum fluctuations in the inflaton field $\phi(x, t)$ [11, 100].

The gravitational instability of a spatially uniform state of dust-like matter described by classical non-relativistic equations has been first investigated by Jeans [32]. If the mass of the matter distribution is larger than some minimum mass called the Jeans mass, then the self gravity of matter will start affecting the structure of the body significantly. Perturbations for which the wave number is smaller than the Jeans wave number can grow to form different structures in the universe.

The theory of linearised density perturbations in an expanding universe can be reduced to the study of a real scalar field in an external classical background. Perturbation in a universe filled by scalar field minimally coupled to gravity is clearly described by Mukhanov, Feldman and Brandenberger [101]. They have calculated the growth rates of perturbations and the analysis is applied to study the evolution of fluctuations in inflationary universe models. Density fluctuations of a cosmological quantum real scalar field in a coherent state is studied and Jeans instability mechanism is generalized in this context by Bianchi, Grasso and Ruffini [102]. Gravitational instability of spatially uniform state of a relativistic scalar field on time dependent back ground is discussed by Khlopov, Malomed and Zeldovich [103] and the instability is demonstrated to be similar to the Jeans instability. They have shown that the effects of self-interaction of the field may drastically alter the character of the instability. Considering a complex scalar field with a positive mass and quartic self-interacting term in the potential, Jetzer and Scialom [104] have derived the Jeans wave number in the Newtonian regime.

The possibility of using the instability mechanism of Jeans theory to form self-graviting configurations from a real scalar field is described in this chapter. A scalar field approach to Jeans mass calculation is discussed. The expression for Jeans mass for a (3+1) dimensional spatially flat Robertson-Walker universe is evaluated. The cosmic fluid is treated in complete analogy to a scalar field and it is considered that the description of cosmological perturbations in the universe can be reduced to the study of quantum fluctuations of a gravitationally coupled scalar field. Considering a massive scalar field arbitrarily coupled to a gravitational background, the stress-energy tensor expectation values are computed in section 4.2. The vanishing of nondiagonal terms of the expectation value of $T_{\mu\nu}$ allows us to treat the scalar field in complete analogy to a perfect fluid. The energy density and pressure associated with the density perturbations are evaluated in the next section. In section 4.4 the primeval density perturbations produced by the vacuum fluctuations $\delta\phi$ of the scalar field are considered and the Jeans criterion for the structure formation is evaluated. These results are used to evaluate Jeans wave number and Jeans mass for the present case in section 4.5. In section 4.6 scalar field with quartic self-interaction is considered and the Jeans wave number is evaluated. The discussions and conclusions are presented in the last section.

4.2 Stress-Energy Tensor Expectation Values of the Gravitationally Coupled Scalar Field

Consider a massive scalar field ϕ coupled to the gravitational background and described by the Lagrangian density

$$\pounds = \sqrt{-g}\left\{\frac{1}{2}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - (m^2+\xi R)\phi^2\right\} \quad (4.2.1)$$

and energy-momentum tensor,

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}L \quad (4.2.2)$$

where $L = (-g)^{-1/2}\pounds$. In the gravitationally coupled case,

$$T_{\mu\nu} = (1-2\xi)\partial_\mu\phi\partial_\nu\phi + (2\xi - \frac{1}{2})g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - 2\xi\phi\nabla_\mu\nabla_\nu\phi + 2\xi g_{\mu\nu}\phi\Box\phi - \xi G_{\mu\nu}\phi^2 + (\frac{m^2}{2})g_{\mu\nu}\phi^2 \quad (4.2.3)$$

Consider a Robertson-Walker spacetime with background metric

$$ds^2 = dt^2 - a_i^2(t)\delta_{ij}dx^i dx^j \quad (4.2.4)$$

Taking the conformal time transformation, $\partial t = a\partial\eta$ and denoting $\frac{\partial\phi}{\partial\eta} = \dot{\phi}$, we can write the diagonal components of stress-energy tensor as,

$$T_{\eta\eta} = \frac{\dot{\phi}^2}{2a^2} - \frac{1}{a^2}(2\xi - \frac{1}{2})\sum_{i=1}^{3}(\partial_i\phi)^2 + 6\xi\frac{\dot{a}}{a^3}\phi\,\dot{\phi} + \frac{3\xi}{a^2}(\frac{\dot{a}^2}{a^2}+\kappa)\phi^2 + (\frac{m^2}{2})\phi^2 \quad (4.2.5)$$

and for $i = 1, 2, 3,$

$$T_{ii} = (1-2\xi)(\partial_i\phi)^2 - (2\xi - \frac{1}{2})\left[\dot{\phi}^2 - \sum_{j=1}^{3}(\partial_j\phi)^2\right] + 6\frac{\xi}{a^2}(\frac{\ddot{a}}{a}+\kappa)\phi^2 - a^2(\frac{m^2}{2})\phi^2 \tag{4.2.6}$$

For the the minimally coupled case, $\xi = 0$ the nonzero components of energy-momentum tensor can be obtained as,

$$T_{\eta\eta} = \frac{\dot{\phi}^2}{2a^2} + \frac{1}{2a^2}\sum_{i=1}^{3}(\partial_i\phi)^2 + (\frac{m^2}{2})\phi^2 \tag{4.2.7}$$

and

$$T_{ii} = (\partial_i\phi)^2 + \frac{1}{2}\left[\dot{\phi}^2 - \sum_{j=1}^{3}(\partial_j\phi)^2\right] - a_i^2(\frac{m^2}{2})\phi^2 \tag{4.2.8}$$

Each mode of the quantised scalar field can be expanded in even and odd parity modes,

$$\phi(x) = (2\pi)^{-3/2}\sum_{\vec{k}}[q_{\vec{k}(\eta)}cos\vec{k}.\vec{x} + q_{-\vec{k}(\eta)}sin\vec{k}.\vec{x}] \tag{4.2.9}$$

Substituting the above expression in equations (4.2.7) and (4.2.8) and applying $(2\pi)^{-3/2}\int d^3x$ to the result yields the spatially averaged components,

$$\bar{T_{\eta\eta}} = \frac{1}{32\pi^3 a^2}\sum_{\vec{k}}\left[\left(\frac{\partial q_{\vec{k}}}{\partial\eta}\right)^2 + a^2\left(\frac{\sum_{i=1}^{3}k_i^2}{a^2} + m^2\right)q_{\vec{k}}^2\right] \tag{4.2.10}$$

and

$$\bar{T_{ii}} = \frac{1}{32\pi^3}\sum_{\vec{k}}\left[\left(\frac{\partial q_{\vec{k}}}{\partial\eta}\right)^2 + \left(2k_i^2 - a^2\left(\frac{\sum_{i=1}^{3}k_i^2}{a^2} + m^2\right)\right)q_{\vec{k}}^2\right] \tag{4.2.11}$$

where $\sum_{\vec{k}}$ extends over both even and odd parity modes.

The annihilation operator $a_{\vec{k}}$ is defined by

$$a_{\vec{k}} = -i\frac{d\beta_{\vec{k}}(\eta)}{d\eta}\hat{q}_{\vec{k}} + i\beta_{\vec{k}}(\eta)\hat{p}_{\vec{k}} \qquad \text{where} \quad \hat{p}_{\vec{k}} = -i\partial/\partial\hat{q}_{\vec{k}} \tag{4.2.12}$$

The complex function $\beta_{\vec{k}}(\eta)$ is a solution to the classical equation of motion corresponding to the Lagrangian density, Eq. (4.2.1), such that

$$\beta^*_{\vec{k}} \frac{d\beta_{\vec{k}}}{d\eta} - \beta_{\vec{k}} \frac{d\beta^*_{\vec{k}}}{d\eta} = i \tag{4.2.13}$$

The expectation values of the diagonal components of the stress energy tensor are obtained as,

$$\langle \bar{T}_{\eta\eta} \rangle = \frac{1}{32\pi^3 a^2} \sum_{\vec{k}} \left[\left| \frac{d\beta_{\vec{k}}}{d\eta} \right|^2 + \omega^2_{\vec{k}}(\eta) \left| \beta_{\vec{k}} \right|^2 \right] \tag{4.2.14}$$

and

$$\langle \bar{T}_{ii} \rangle = \frac{1}{32\pi^3} \sum_{\vec{k}} \left[\left| \frac{d\beta_{\vec{k}}}{d\eta} \right|^2 + \left(2k_i^2 - \omega^2_{\vec{k}}(\eta) \right) \left| \beta_{\vec{k}} \right|^2 \right] \tag{4.2.15}$$

The vanishing of the nondiagonal terms of the expectation values of the components of $T_{\mu\nu}$ allows us to treat the scalar field in complete analogy to a perfect fluid.

4.3 Energy Density and Pressure Associated with the Quantum Field Fluctuations

The equation of motion associated with Lagrangian in Eq. (4.2.1) is given by

$$g^{\mu\nu} \nabla_\mu \nabla_\nu \phi + (m^2 + \xi R)\phi = 0 \tag{4.3.1}$$

For a scalar field, perfect homogeneity can not be attained and there will always be some residual fluctuations. So we can split the field as:

$$\phi(x,t) = \phi_0(t) + \delta\phi(x,t) \tag{4.3.2}$$

where $\phi_0(t)$ is the classical background field and $\delta\phi(x,t)$ is the perturbation of the field ϕ. $\delta\phi(x,t)$ satisfies the field equation:

$$g^{\mu\nu}\nabla_\mu \nabla_\nu(\delta\phi) + (m^2 + \xi R)(\delta\phi) = 0 \tag{4.3.3}$$

where we have assumed that $\delta\phi$ is small. Each mode of the quantized scalar field fluctuations can be expanded in even and odd parity modes as,

$$\delta\phi(x.t) = (2\pi)^{-3/2}\sum_{\vec{k}}[a_k^+\chi_k(t)\cos\vec{k}.\vec{x} + a_{-k}\chi_{-k}^*(t)\sin\vec{k}.\vec{x}] \tag{4.3.4}$$

Substituting Eq.(4.3.4) in Eq.(4.3.3) we obtain,

$$\ddot{\chi_k} + \left\{C[m^2 + \xi R] + k^2\right\}\chi_k = 0 \tag{4.3.5}$$

Considering the minimally coupled $\xi = 0$ case we can write the diagonal components of energy-momentum tensor for the quantum fluctuations as,

$$\delta T_{\eta\eta} = \frac{1}{2a^2}\left(\partial\eta(\delta\phi)\right)^2 + \frac{1}{2a^2}\sum_{i=1}^{3}\left(\partial i(\delta\phi)\right)^2 + \frac{m^2}{2}(\delta\phi)^2 \tag{4.3.6}$$

and for $i = 1,2,3$,

$$\delta T_{ii} = \frac{1}{2}\left(\partial\eta(\delta\phi)\right)^2 + \frac{1}{2}\sum_{i=1}^{3}\left(\partial i(\delta\phi)\right)^2 - a^2\frac{m^2}{2}(\delta\phi)^2 \tag{4.3.7}$$

The expectation values of the diagonal components of the energy-momentum tensor of quantum field fluctuations are obtained as,

$$\langle\delta\bar{T}_{\eta\eta}\rangle = \frac{1}{32\pi^3a^2}\sum_{\vec{k}}\left[\left|\frac{\partial\chi_{\vec{k}}}{\partial\eta}\right|^2 + \omega_{\vec{k}}^2(\eta)\left|\chi_{\vec{k}}\right|^2\right] \tag{4.3.8}$$

and

$$\langle \delta \bar{T}_{ii} \rangle = \frac{1}{32\pi^3} \sum_{\vec{k}} \left[\left| \frac{\partial \chi_{\vec{k}}}{\partial \eta} \right|^2 + \left(2k^2 - \omega_{\vec{k}}^2(\eta) \right) \left| \chi_{\vec{k}} \right|^2 \right] \tag{4.3.9}$$

where

$$\omega_{\vec{k}}^2(\eta) = a^2 \left[m^2 + \frac{k^2}{a^2} \right] \tag{4.3.10}$$

and $\sum_{\vec{k}}$ extends over both even and odd parity modes.

To completely fix the representation, a boundary condition must be imposed on $\chi_{\vec{k}}(\eta)$. In most models there exists a regime $\eta = \eta_{WKB}$ defined by the WKB condition

$$\omega_{\vec{k}}^{-1} \frac{d\omega_{\vec{k}}}{d\eta} << \omega_{\vec{k}} \tag{4.3.11}$$

in which we require

$$\lim_{\eta \to \eta_{WKB}} \chi_{\vec{k}}(\eta) = (2\omega_{\vec{k}})^{-1/2} \exp\left(i \int^{\eta} \omega_{\vec{k}} d\bar{\eta} \right) \tag{4.3.12}$$

and

$$\lim_{\eta \to \eta_{WKB}} \frac{d\chi_{\vec{k}}(\eta)}{d\eta} = i\omega_{\vec{k}} \chi_{\vec{k}}(\eta) \tag{4.3.13}$$

The interpretation of $\langle n_{\vec{k}} \rangle$ as a particle number is valid only in a WKB regime defined by $\omega_{\vec{k}}^{-1} \frac{d\omega_{\vec{k}}}{d\eta} << \omega_{\vec{k}}$. This condition will be valid for modes with wavelength smaller than Hubble radius (oscillation period << expansion time scale). In such a regime we require the WKB limit equations (4.3.12) & (4.3.13) for $\chi_{\vec{k}}$. Evaluating equations (4.3.8) and (4.3.9) in the WKB limit we get a simplified expression for the diagonal components of the expectation value of energy-momentum tensor of quantum fluctuations as,

$$\lim_{\eta \to \eta_{WKB}} \langle \delta T_{\eta\eta}^{\vec{k}} \rangle = \frac{\omega_{\vec{k}}(\eta)}{32\pi^3 a^2} \tag{4.3.14}$$

and

$$\lim_{\eta \to \eta_{WKB}} \left\langle \delta T_{ii}^{\bar{k}} \right\rangle = \frac{1}{32\pi^3} \frac{k_i^2}{\omega_{\bar{k}}} \tag{4.3.15}$$

The vanishing of the nondiagonal terms of the expectation values of the components of $T_{\mu\nu}$ allows us to treat the scalar field in complete analogy to a perfect fluid. The similarity of the gravitational instabilities of a free scalar field and dust-like matter was pointed out by Turner [105].

4.4 Primeval Density Perturbations and the Jeans Criterion

The vacuum fluctuations of the scalar field $\delta\phi$ generate a primeval density perturbation in the early universe. Such a primeval perturbation is regarded as a viable candidate for the origin of large scale structure. Let us consider the matter in the universe to be a scalar field minimally coupled to gravity and described by the Lagrangian density in Eq. (4.2.1). In the conformal time coordinates the field equation is

$$\ddot{\phi} + 2\tilde{H}\,\dot{\phi} - \nabla^2\,\phi + a^2 V'(\phi) = 0 \tag{4.4.1}$$

where $V' = \frac{\partial V}{\partial \phi}$ and $\tilde{H} = \frac{\dot{a}}{a}$ using conformal time.

Perturbing Eq. (4.4.1) and linearising the equation of motion for the scalar field perturbation $\delta\phi$ is obtained as,

$$(\ddot{\delta\phi}) + 2\tilde{H}\,(\dot{\delta\phi}) - v_s^2\,\nabla^2\,(\delta\phi) + m^2 a^2 (\delta\phi) = 0 \tag{4.4.2}$$

where we have $\nabla^2 = -\dfrac{k^2}{a^2}$. v_s is the speed of propagation of perturbation [11, 101, 106]. $v_s^2 = 1$ for scalar field matter and $v_s^2 = \delta p / \delta \rho$ for ideal gas matter [106]. As it is already mentioned we consider subhorizon modes only. Until a few Hubble times after the horizon exit, the last term of the above equation is negligible [11]. To be more clear, compare the last term with the one before it. At the epoch of horizon exit, $k = aH$, because of the slow-roll condition [8] $M_{pl}^2 V'' << V$ is equivalent to $m^2 << H^2$. Thus for cosmological perturbations we can rewrite the above equation as

$$(\ddot{\delta\phi}) + 2\tilde{H}\,(\dot{\delta\phi}) + \frac{v_s^2 k^2}{a^2}(\delta\phi) = 0 \tag{4.4.3}$$

The perturbations $\delta\phi(x,t)$ in the scalar field leads to the perturbations in energy density $\delta\rho(x,t)$. So using Eq. (4.4.3) we can write as

$$\ddot{\delta} + 2\tilde{H}\,\dot{\delta} + \frac{v_s^2 k^2}{a^2}\delta = 0 \tag{4.4.4}$$

where $\delta = \dfrac{\delta\rho}{\rho}$ is the density contrast parameter. Since the scalar field is coupled to the gravitational background field we have to consider the effects of gravitational field potential also. Including the gravitational field potential, the above equation becomes,

$$\ddot{\delta} + 2\tilde{H}\,\dot{\delta} = \left(4\pi G\rho - \frac{v_s^2 k^2}{a^2}\right)\delta \tag{4.4.5}$$

The primeval density perturbations satisfy the adiabatic condition [11] for density contrast. For cosmological perturbations v_s is the speed of sound.

The above equation tells us how or whether gravitational instability leads to the growth of condensation in the expanding universe. The right hand side of the equation shows the competing effects of gravity and the pressure gradient force. At very long wave length $k \rightarrow 0$, the equation reduces to the zero pressure case. At very short wave length, large k, the pressure term dominates and δ tends to oscillate as a sound wave. The pressure and gravity terms balance when the wave length is equal to the Jeans length given by,

$$\lambda_J = v_s(\pi/G\rho)^{1/2} \tag{4.4.6}$$

which corresponds to the classical Jeans criterion for the structure formation.

Consider the relation:

$$\langle T^{\mu\nu}\rangle = (\rho_1 - p_1)u^\mu u^\nu - p_1 g^{\mu\nu} \tag{4.4.7}$$

where, $u^\mu = (1,0,0,0)$ and p_1 and ρ_1 are the first order fluctuation amplitudes of the corresponding quantities. The energy density and pressure associated to the quantum field perturbation are,

$$\rho_1 = \langle T^0_0\rangle \qquad \text{and} \qquad p_1 = -\langle T^i_i\rangle \tag{4.4.8}$$

From the definition of the sound velocity of adiabatic perturbations [11], [31], [106]

$$v_s^2 = \frac{p_1}{\rho_1} = \frac{k_i^2}{\omega_{\bar{k}}^2(\eta)} \tag{4.4.9}$$

In a nonrelativistic regime $\frac{k_i}{a} << m$ and we can write,

$$v_s^2 = \frac{k_i^2}{a^2 m^2} \tag{4.4.10}$$

4.5 Jeans Length and Jeans Mass

Jeans explained that starting from a homogeneous and isotropic fluid, small fluctuations in the density ρ_1 and velocity v_1 could evolve with time. He showed that density fluctuations can grow in time if the stabilizing effect of pressure is much smaller than the tendency of the self-gravity of a density fluctuation to induce collapse. When the pressure inside the perturbed fluid is greater than the self-gravity, the perturbation will propagate like an acoustic wave with velocity v_s. Jeans calculations were done in the context of a static background fluid [32]. The theory of instabilities of an expanding universe was given by Lifshitz in 1946

[34]. He also concluded that there exists a critical wave number k_J above which disturbances can not grow but only oscillate like sound waves.

During most phases of the expansion of the universe, we can approximate the expansion factor by $a(t) \propto t^n$ with a suitable n which is less than unity [26]. For the matter dominated case [31], the scale factor $a(t) \propto t^{2/3}$ and for the spatially flat $\kappa = 0$ case,

$$\rho = \frac{1}{6\pi G t^2} \tag{4.5.1}$$

Using Eq. (4.4.10) we can write as

$$v_s \propto t^{-2/3} \tag{4.5.2}$$

Eq. (4.4.5) then takes the form,

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} + \left(\frac{\Lambda^2}{t^{8/3}} - \frac{2}{3t^2}\right)\delta = 0 \tag{4.5.3}$$

where

$$\Lambda^2 = t^{8/3}\frac{v_s^2 k^2}{a^2} \tag{4.5.4}$$

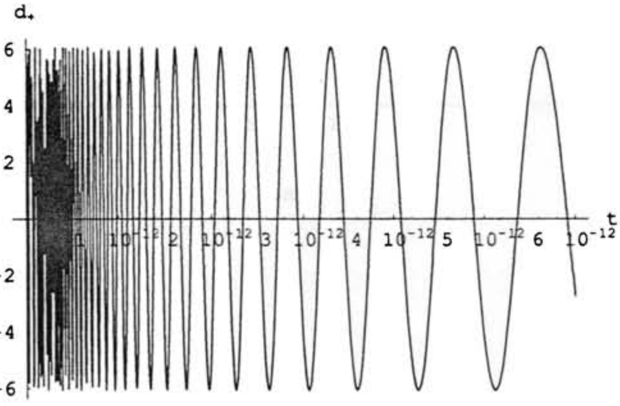
The solutions of Eq. (4.5.3) are,

$$\delta_\pm \propto t^{-1/6} J_{\mp 5/2}(3\Lambda t^{-1/3}) \tag{4.5.5}$$

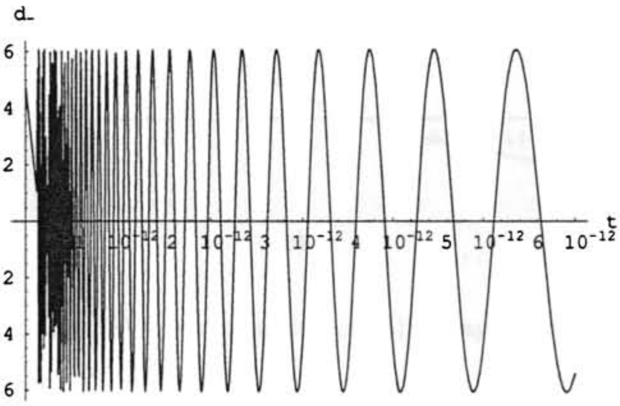
where $J_n(x)$ are Bessel functions of the first kind. The Bessel function $J_n(x)$ oscillates for $x >> 1$ as shown Figs. 4.1 (a & b). For $x < 1$, the solutions δ_+ and δ_- behave as in Figs. 4.2 (a & b). Both the growing as well as damped modes are present. It is evident from the Fig. 4.2 that the growing modes dominate over the decaying modes.

The critical condition $x = 1$ gives,

$$t^{1/3} \approx \Lambda \tag{4.5.6}$$

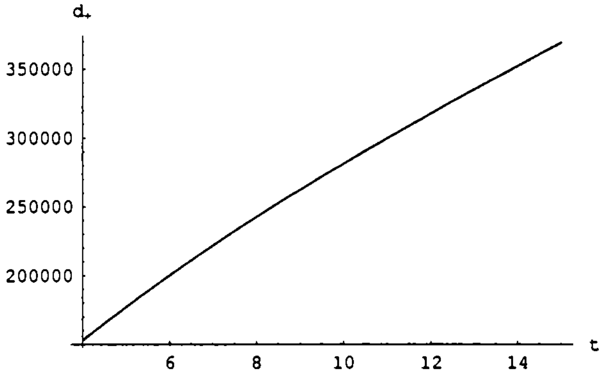


(a)

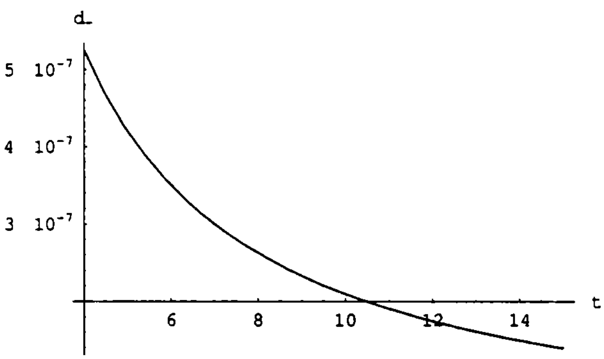


(b)

Fig. 4.1: (a) Oscillating mode δ_+; (b) Oscillating mode δ_-



(a)



(b)

Fig. 4.2: (a) Growing mode δ_+; (b) Decaying mode δ_-

Eq. (4.5.4) & (4.5.6) together imply that

$$t^{-2} \sim 6\pi G\rho \sim \frac{v_s^2 k^2}{a^2} \tag{4.5.7}$$

Substituting the expression for sound velocity in Eq. (4.5.7) the classical Jeans length for the perturbations is obtained as,

$$\lambda_J = 2\pi/K_J$$

where

$$K_J^2 \sim m\sqrt{\pi G \rho} \tag{4.5.8}$$

where $K = \dfrac{k}{a}$, the physical wave number [26, 30]. The Eq. (4.5.3) has sinusoidal solutions for $K > K_J$ (Fig. 4.1) and exponential (growing as well as damping as in Fig. 4.2) solutions for $K < K_J$. If $K >> K_J$ we get sinusoidal disturbances that do not grow but simply propagate like sound waves. In this case the gravitational forces and the expansion of the universe may be neglected [30]. The only disturbances that have any prospects of growth are those for which $K < K_J$.

The Jeans mass for the perturbations [30, 31] is then given by,

$$\begin{aligned} M_J &= \frac{4}{3}\pi\rho\left(\frac{2\pi}{K_J}\right)^3 = \frac{32}{3}\pi^{13/4}\rho^{1/4}\left(\frac{1}{m\sqrt{G}}\right)^{3/2} \\ &= \frac{32}{3}\pi^{13/4}\rho^{1/4}\left(\frac{m_{pl}}{m}\right)^{3/2} = 10^2\rho^{1/4}\left(\frac{m_{pl}}{m}\right)^{3/2} \end{aligned} \tag{4.5.9}$$

The fluctuations will have a chance to grow under its self-gravitation if the mass of perturbed matter is greater than M_J. It may grow under its self-gravitation to form a galaxy.

4.6 Density Perturbations and Jeans Wave number for a Scalar Field with Quartic Self-Interaction

Now let us consider the quantum fluctuations of a scalar field with quartic self interaction described by the Lagrangian density,

$$\pounds = \sqrt{-g}\left\{\frac{1}{2}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - (m^2+\xi R)\phi^2] - \lambda\frac{\phi^4}{4!}\right\} \tag{4.6.1}$$

The velocity of perturbations is obtained in this case as

$$v_s^2 = \frac{1}{3}\frac{k_i^2}{a^2(m^2+\frac{\lambda}{2}\phi_0^2)} \tag{4.6.2}$$

It is clear that the velocity of fluctuation depends on the self-interaction of the scalar field.

For this self-interacting field the Jeans wave number K_J is obtained as,

$$K_J^2 = \sqrt{18\pi G\rho(m^2+\frac{\lambda}{2}\phi_0^2)} \tag{4.6.3}$$

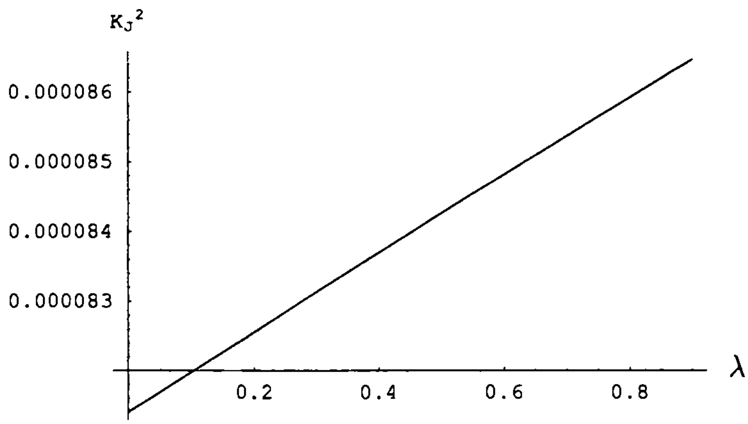


Fig. 4.3: Effects of self-interaction on the value of K_J

The instability growth rate monotonically falls off when K^2 increases from 0 to K_J^2. The above equation implies that the self-interaction of the field influences the character of instability and the value of Jeans wave number K_J is altered by the effects of self-interaction. Fig. 4.3 shows the effects of self-interaction on the value of K_J. The critical value of the wave number K_J, above which the disturbances can not grow, is higher for the self-interacting field with $\lambda > 0$.

4.7 Discussion and Conclusions

The key idea in studying the formation of structure in the universe is that of gravitational instability. It depends on the nature of the universe as a whole, for example on how rapidly it is expanding and on how much material is in it to provide the gravitational attraction and it also depends on the form of the initial irregularities.

Jeans considered the problem of formation of galaxies in the universe as a process involving the interplay between gravitational attraction and the pressure force acting on a mass of nonrelativistic fluid. So long as the pressure forces are negligible an overdense region is expected to accrete material from its surroundings by the gravitational attraction and thus becoming even more dense. The denser it becomes the more it will accrete, resulting in an instability which can ultimately cause the collapse of a fluctuation to a gravitationally bound object. The knowledge of Jeans wavelength $\lambda_J = 2\pi/K_J$ provides an estimate of the size of the objects which can be formed by gravitational collapse.

It is generally assumed that at early times the particle content of the universe formed an ideal gas. In the present work, the cosmic fluid is treated in complete analogy to a scalar field and the description of cosmological perturbations in the universe is reduced to the study of quantum fluctuations of a gravitationally coupled scalar field. The primeval density perturbations produced by the vacuum fluctuations, $\delta\phi$, of the scalar field is considered and the Jeans criterion for the structure formation are evaluated.

The possibility of using the instability Jeans mechanism to form self-gravitating configurations from real scalar field is discussed in this chapter. Bianchi and co-workers [102] have clearly discussed the physical meaning of how the Jeans instability occurs in a scalar field. Khlopov and co-workers have discussed the gravitational instability of a free scalar field and for a self-interacting scalar field. The Jeans wave number is obtained from the solution of dispersion relation of the perturbations of a scalar field [103]. Jetzer and Scialom have considered the linear scalar mode perturbations and they have obtained the expression for Jeans wave number starting from the general relativistic wave equations and solving the dispersion relation [104]. In the present work the same result is obtained by a different approach. Scalar field approach to Jeans mass calculation is discussed. The application of classical Jeans theory to scalar field is conditioned by the vanishing of the expectation values of the nondiagonal components of the energy-momentum tensor. The scalar field is treated in complete analogy to a perfect fluid and the energy density and pressure associated to the gravitational perturbations are evaluated. The exact expression of Jeans wave number for the perturbations is obtained. The present work shows that the value of K_J, that is the critical value of the wave number above which the disturbances can not grow, is altered by the effects of self-interaction of the field.

Briefly, quantum fluctuations in an expanding universe can lead to energy density perturbations. It is usually assumed that there exist small primordial perturbations which slowly increase in amplitude due to gravitational instability to form the structures we observe at the present time on the scales of galaxies and galaxy clusters. The simple criterion needed to decide whether the fluctuation will grow with time is that the typical length scale of a fluctuation should be greater than the Jeans length λ_J for the fluid.

Chapter 5

Jeans Mass Calculations for an Anisotropic Case

5.1 Introduction

A scalar field approach to Jeans mass calculations are performed in the previous chapter and the possibility of using the instability mechanism of Jeans theory, to study the formation of structure in the universe is discussed. The superdense matter may be anisotropic, at least in certain density ranges. It is of considerable interest to determine the extent to which local anisotropy can alter the structure of massive objects [107]. The role played by local pressure anisotropy in the onset of instabilities have been studied and it has been shown that small anisotropies may drastically change the stability of the system [108]. Herrera and Santos [109] have studied the Jeans instability criterion for interstellar gas that can produce anisotropies during its evolution. In this chapter distribution of matter which is coupled to a Bianchi type-I background spacetime is considered and the density perturbation in such a distribution is studied. The possibility of using the instability mechanism of Jeans theory to study the formation of structure, for an anisotropic case is studied in this chapter.

The quantum state of the scalar field near the initial singularity is inaccessible to an observer at the present time just as the state of the quantised scalar field inside the event horizon of a black hole is inaccessible to an observer at infinity [110]. Hawking's suggestion [111] is that this ignorance of the actual state of the quantised field is best expressed by taking a random superposition of all allowed states in the inaccessible region [112]. It is assumed that all the phase information is lost so that the system can no longer be described by a pure quantum mechanical state. It is possible, however, to construct a density matrix from which expectation values may be calculated [113]. Berger constructed a coherent state representation [112] valid even near the singularity for each mode of the quantised scalar field in a classical spatially homogeneous and anisotropic background cosmology. The stress-energy tensor expectation values are calculated in a coherent state representation and it has been found that the values so obtained coincide with the classical values expected for the zero-point energy. Coherent states are minimum uncertainty states [114]. For the coherent state the uncertainty is minimum in amplitude and phase and hence it is the closest possible quantum mechanical state to a classical field.

As an alternative to the N representation, a coherent state representation is constructed for each mode of the scalar field in this chapter. The stress-energy tensor expectation values are computed in a coherent state in section 5.2. The density matrix is used to represent the expectation values. Then the energy density and pressure associated with the density perturbations are evaluated. Using these results the exact expressions for Jeans wave number for the present case is evaluated in section 5.3. Then the distribution of matter which is assumed to be locally anisotropic is considered in the next section and the density perturbations in such a distribution is studied. In section 5.5 the metric and matter field perturbations are considered and from the calculations of section 5.6 it is found that for the present anisotropic case, the perturbation of pressure in radial and tangential directions are different. Discussions and conclusions are presented in section 5.7

5.2 Energy Density and Pressure Associated to the Perturbations with an Anisotropic Background Spacetime

To probe the quantum effects in Cosmology, coherent states (cs) are being used. Gravitation and other primordial perturbations created from zero point fluctuations in the process of cosmological evolution should be in a coherent state. cs representation for each mode of a quantised scalar field in a classical spatially homogeneous anisotropic background is constructed in this section. The expectation values of stress-energy tensor, $T_{\mu\nu}$ of a free scalar field are calculated in cs. Then the energy density and pressure associated to the perturbation are evaluated.

5.2.1 Scalar Field Gravitationally Coupled to Bianchi Type-I Spacetime

Consider a massive scalar field ϕ coupled arbitrarily to the gravitational background and described by the Lagrangian density

$$\mathcal{L} = \sqrt{-g}\left\{\frac{1}{2}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - (m^2 + \xi R)\phi^2]\right\} \tag{5.2.1}$$

with energy-momentum tensor,

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}L \tag{5.2.2}$$

where $L = (-g)^{-1/2}\pounds$. In the gravitationally coupled case,

$$T_{\mu\nu} = (1-2\xi)\partial_\mu\phi\partial_\nu\phi + (2\xi - \frac{1}{2})g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - 2\xi\phi\nabla_\mu\nabla_\nu\phi + 2\xi g_{\mu\nu}\phi\Box\phi - \xi G_{\mu\nu}\phi^2 + \frac{m^2}{2}g_{\mu\nu}\phi^2 \tag{5.2.3}$$

Consider a (3+1) dimensional Bianchi type-I spacetime which is spatially homogeneous and has the line element

$$ds^2 = dt^2 - \sum_{i=1}^{3} a_i^2(t)(dx^i)^2 \tag{5.2.4}$$

as the background metric. Taking the conformal time transformation, $\partial t = C^{1/2}\partial\eta$ where $C = (a_1a_2a_3)^{2/3}$ and denoting $\frac{\partial\phi}{\partial t} = \dot{\phi}$, we can write the diagonal components of stress-energy tensor :

$$T_{\eta\eta} = \frac{\dot{\phi}^2}{2C} - (2\xi - \frac{1}{2})\left(\sum_{i=1}^{3}\frac{1}{a_i^2}(\partial_i\phi)^2\right) + 2\xi\frac{\dot{C}}{C^2}\phi\,\dot{\phi} + \frac{3\xi}{C}(\frac{\dot{C}^2}{C^2} + \kappa)\phi^2 + (\frac{m^2}{2})\phi^2 \tag{5.2.5}$$

and for $i = 1, 2, 3$:

$$T_{ii} = (1-2\xi)(\partial_i\phi)^2 - (2\xi - \frac{1}{2})\left[\frac{a_i^2}{C}\dot{\phi}^2 - a_i^2\left(\sum_{j=1}^{3}\frac{1}{a_j^2}(\partial_j\phi)^2\right)\right] + 6\frac{\xi}{C}\left(\frac{\ddot{C}}{2C} - \frac{\dot{C}^2}{4C^2} + \kappa\right)\phi^2 - a_i^2(\frac{m^2}{2})\phi^2 \tag{5.2.6}$$

Considering the minimally coupled case, $\xi = 0$ we get,

$$T_{\eta\eta} = \frac{\dot{\phi}^2}{2C} + \frac{1}{2}\left(\sum_{i=1}^{3}\frac{1}{a_i^2}(\partial_i\phi)^2\right) + (\frac{m^2}{2})\phi^2 \tag{5.2.7}$$

and

$$T_{ii} = (\partial_i\phi)^2 + \frac{1}{2}\left[\frac{a_i^2}{C}\dot{\phi}^2 - a_i^2\left(\sum_{j=1}^{3}\frac{1}{a_j^2}(\partial_j\phi)^2\right)\right] - a_i^2(\frac{m^2}{2})\phi^2 \qquad (5.2.8)$$

Each mode of the quantized scalar field can be expanded in even and odd parity modes,

$$\phi(x) = (2\pi)^{-3/2}\sum_{\vec{k}}[q_{\vec{k}}(\eta)\cos\vec{k}.\vec{x} + q_{-\vec{k}}(\eta)\sin\vec{k}.\vec{x}] \qquad (5.2.9)$$

Since the background metric is spatially homogeneous we require the quantum state of the system to be also spatially homogeneous. Thus we need consider only the spatially homogeneous modes of the expressions in Eqs. (5.2.7) & (5.2.8). Substituting the above expression in Eqs. (5.2.7) & (5.2.8) and applying $(2\pi)^{-3/2}\int d^3x$ to the result yields the spatially averaged components,

$$\bar{T}_{\eta\eta} = \frac{1}{32\pi^3 C}\sum_{\vec{k}}\left[\left(\frac{\partial q_{\vec{k}}}{\partial\eta}\right)^2 + \omega_{\vec{k}}^2(\eta)q_{\vec{k}}^2\right] \qquad (5.2.10)$$

and

$$\bar{T}_{ii} = \frac{1}{32\pi^3 C}a_i^2\sum_{\vec{k}}\left[\left(\frac{\partial q_{\vec{k}}}{\partial\eta}\right)^2 + \left(\frac{2k_i^2}{a_i^2}C - \omega_{\vec{k}}^2(\eta)\right)q_{\vec{k}}^2\right] \qquad (5.2.11)$$

where

$$\omega_{\vec{k}}^2(\eta) = C\left(\sum_{i=1}^{3}\frac{k_i^2}{a_i^2} + m^2\right) \qquad (5.2.12)$$

and $\sum_{\vec{k}}$ extends over both even and odd parity modes.

5.2.2 Energy-Momentum Tensor Expectation Values in Coherent State

As an alternative to the N representation, we can construct an (over)complete normalized set $|\Gamma_{\vec{k}}\rangle$ of coherent state for each mode of the scalar field. The behaviour of the classical scalar field near the cosmological singularity [115, 116] is best followed quantum mechanically by constructing such a representation [113, 117]. Coherent states are defined to be eigen states of the annihilation operator,

$$a_{\vec{k}}\,|\Gamma_{\vec{k}}\rangle = \Gamma_{\vec{k}}\,|\Gamma_{\vec{k}}\rangle \tag{5.2.13}$$

where $\Gamma_{\vec{k}}$ is the time dependent complex number and $a_{\vec{k}}$ is defined by

$$a_{\vec{k}} = -i\frac{d\beta_{\vec{k}}(\eta)}{d\eta}\hat{q}_{\vec{k}} + i\beta_{\vec{k}}(\eta)\hat{p}_{\vec{k}}$$

where

$$\hat{p}_{\vec{k}} = -i\partial/\partial\hat{q}_{\vec{k}} \tag{5.2.14}$$

The complex function $\beta_{\vec{k}}(\eta)$ is a solution to the classical equation of motion corresponding to the Lagrangian density such that

$$\beta^*_{\vec{k}}\frac{d\beta_{\vec{k}}}{d\eta} - \beta_{\vec{k}}\frac{d\beta^*_{\vec{k}}}{d\eta} = i \tag{5.2.15}$$

Taking the expectation values of the diagonal components in the coherent state [118] we get,

$$\langle \bar{T}_{\eta\eta}\rangle_{cs} = \frac{1}{32\pi^3 C}\sum_{\vec{k}}\left\{\left[\left(\frac{d\beta^*_{\vec{k}}}{d\eta}\right)^2 + \omega^2_{\vec{k}}(\eta)\beta^{*2}_{\vec{k}}\right]\Gamma^2_{\vec{k}} + \left[\left(\frac{d\beta_{\vec{k}}}{d\eta}\right)^2 + \omega^2_{\vec{k}}(\eta)\beta^2_{\vec{k}}\right]\Gamma^{*2}_{\vec{k}}\right.$$

$$\left. + \left[\left|\frac{d\beta_{\vec{k}}}{d\eta}\right|^2 + \omega^2_{\vec{k}}(\eta)\left|\beta_{\vec{k}}\right|^2\right](2\Gamma^2_{\vec{k}} + 1)\right\} \tag{5.2.16}$$

and

$$\langle \bar{T}_{ii} \rangle_{cs} = \frac{1}{32\pi^3 C} a_i^2 \sum_{\vec{k}} \Bigg\{ \left[\left(\frac{d\beta^*_{\vec{k}}}{d\eta} \right)^2 + \left(\frac{2k_i^2}{a_i^2} C - \omega^2_{\vec{k}}(\eta) \right) \beta^{*2}_{\vec{k}} \right] \Gamma^2_{\vec{k}}$$

$$+ \left[\left(\frac{d\beta_{\vec{k}}}{d\eta} \right)^2 + \left(\frac{2k_i^2}{a_i^2} C - \omega^2_{\vec{k}}(\eta) \right) \beta^2_{\vec{k}} \right] \Gamma^{*2}_{\vec{k}}$$

$$+ \left[\left| \frac{d\beta_{\vec{k}}}{d\eta} \right|^2 + \left(\frac{2k_i^2}{a_i^2} C - \omega^2_{\vec{k}}(\eta) \right) \left| \beta_{\vec{k}} \right|^2 \right] (2\Gamma^2_{\vec{k}} + 1) \Bigg\} \tag{5.2.17}$$

The coherent state for the scalar field is the product over modes of the coherent state for each mode. We assume the modes to be noninteracting so that the density matrix for the field is just the product of the density matrices for each mode. Thus we find a density matrix

$$\rho = \int \left(\Pi_{\vec{k}} \frac{d^2\Gamma_{\vec{k}}}{\pi \langle n_{\vec{k}} \rangle} \right) \exp \left(- \sum_{\vec{k}} \frac{\left| \Gamma_{\vec{k}} \right|^2}{\langle n_{\vec{k}} \rangle} \right) \left| \{\Gamma_{\vec{k}}\} \right\rangle \left\langle \{\Gamma_{\vec{k}}\} \right| \tag{5.2.18}$$

where

$$\left| \{\Gamma_{\vec{k}}\} \right\rangle \equiv \Pi_{\vec{k}} \left| \Gamma_{\vec{k}} \right\rangle$$

The density matrix given by Eq. (5.2.18) may be used to evaluate expectation values through $\langle A \rangle = tr(\rho A)$, where $\langle A \rangle$ is the expectation value of any operator A. Using the density matrix, the stress-tensor expectation values are evaluated as:

$$\langle T^{\vec{k}}_{\mu\nu} \rangle = tr(T^{\vec{k}}_{\mu\nu} \rho_{\vec{k}}) = \int d^2\Gamma_{\vec{k}} \left| \alpha(\Gamma_{\vec{k}}) \right|^2 \left\langle \Gamma_{\vec{k}} \right| T^{\vec{k}}_{\mu\nu} \left| \Gamma_{\vec{k}} \right\rangle \tag{5.2.19}$$

where

$$\left|\alpha(\Gamma_{\bar{k}})\right|^2 = \frac{1}{\pi \left\langle n_{\bar{k}}\right\rangle} e^{-\left|\Gamma_{\bar{k}}\right|^2/\left\langle n_{\bar{k}}\right\rangle} \tag{5.2.20}$$

Thus

$$\left\langle T^{\bar{k}}_{\eta\eta}\right\rangle = \frac{1}{32\pi^3 C}(2\left\langle n_{\bar{k}}\right\rangle + 1)\left[\left|\frac{d\beta_{\bar{k}}}{d\eta}\right|^2 + \omega^2_{\bar{k}}(\eta)\left|\beta_{\bar{k}}\right|^2\right] \tag{5.2.21}$$

and

$$\left\langle T^{\bar{k}}_{ii}\right\rangle = \frac{a_i^2}{32\pi^3 C}(2\left\langle n_{\bar{k}}\right\rangle + 1)\left[\left|\frac{d\beta_{\bar{k}}}{d\eta}\right|^2 + \left(\frac{2k_i^2}{a_i^2}C - \omega^2_{\bar{k}}(\eta)\right)\left|\beta_{\bar{k}}\right|^2\right] \tag{5.2.22}$$

The coherent states are parametrised by initial conditions for the scalar field. These states become the usual minimum-uncertainly wave packets if (and only if) the time scale for the evolution of the background spacetime is much greater than the periods of oscillation of the modes of the scalar field, $H >> T$ [119]. In such a regime we require the WKB limit equations, given by Eqs. (4.3.11)-(4.3.13) for $\beta_{\bar{k}}$. Evaluating Eqs. (5.2.21) and (5.2.22) in the WKB limit we get a simplified expression for the diagonal components of the expectation value of energy-momentum tensor,

$$\lim_{\eta->\eta_{WKB}}\left\langle T^{\bar{k}}_{\eta\eta}\right\rangle = \frac{1}{16\pi^3 C}(\left\langle n_{\bar{k}}\right\rangle + 1/2)\omega_{\bar{k}}(\eta) \tag{5.2.23}$$

and

$$\lim_{\eta->\eta_{WKB}}\left\langle T^{\bar{k}}_{ii}\right\rangle = \frac{k_i^2}{16\pi^3\omega_{\bar{k}}}(\left\langle n_{\bar{k}}\right\rangle + 1/2) \tag{5.2.24}$$

Thus the energy density and pressure associated to the perturbation are obtained as,

$$\rho_1 = \frac{1}{16\pi^3 C}\left(\langle n_{\bar{k}}\rangle + 1/2\right)\omega_{\bar{k}}(\eta) \tag{5.2.25}$$

and

$$p_1 = \frac{k_i^2}{48\pi^3 a_i^2 \omega_{\bar{k}}}\left(\langle n_{\bar{k}}\rangle + 1/2\right) \tag{5.2.26}$$

Using the definition Eq. (5.2.12) of $\omega_{\bar{k}}$ and the metric Eq. (5.2.4), it is clear that the trace of $\langle T^{\bar{k}}_{\mu\nu}\rangle$ is formally zero for a massless scalar field. Regularization of the vacuum stress-energy term may yield a trace anomaly [1].

From the definition of the sound velocity of adiabatic perturbations [11, 31, 106] we get:

$$v_s^2 = \frac{p_1}{\rho_1} = \frac{k_i^2}{3a_i^2\left(\sum\limits_{i=1}^{3}\frac{k_i^2}{a_i^2} + m^2\right)} \tag{5.2.27}$$

In a nonrelativistic regime $\frac{k_i}{a_i} << m$ and we can write,

$$v_s^2 = \frac{1}{3}\frac{k_i^2}{a_i^2 m^2} \tag{5.2.28}$$

For the Bianchi type-I spacetime with scale factors a_i, let $V = a_1 a_2 a_3$ be the 'volume scale factor' [10]. Then the mean scale factor $\bar{a} \propto V^{1/3}$ and $C \propto \bar{a}^2$. Let us put $\bar{a}$ instead of a_i in Eq. (5.2.28). Then,

$$v_s^2 = \frac{1}{3}\frac{k_i^2}{\bar{a}^2 m^2} \tag{5.2.29}$$

Rewriting the metric for the Bianchi type-I spacetime, using spherical polar coordinates,

$$ds^2 = c^2dt^2 - R_1^2(t)\frac{dr^2}{(1-kr^2)} - R_2^2(t)r^2d\theta^2 - R_3^2(t)r^2\sin^2\theta d\phi^2 \tag{5.2.30}$$

with

$$\frac{R_1^2(t)}{(1-kr^2)} = a_1^2(t)$$

$$R_2^2(t)r^2 = a_2^2(t)$$

$$R_3^2(t)r^2\sin^2\theta = a_3^2(t) \tag{5.2.31}$$

Taking $c = 1$ we get,

$$R_0^0 = \frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R2} + \frac{\ddot{R}3}{R_3}$$

$$R_1^1 = \frac{\ddot{R}_1}{R_1} + \frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{2k}{R_1^2}$$

$$R_2^2 = \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} + \frac{2k}{R_1^2} - \frac{1}{r^2}\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right]$$

$$R_3^3 = \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} + \frac{2k}{R_1^2} - \frac{1}{r^2}\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right]$$

and

$$R = 2\left[\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3}\right] + 2\left[\frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3}\right] + \frac{6k}{R_1^2} - \frac{2}{r^2}\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right] \tag{5.2.32}$$

The Einstein equation $G_\nu^\mu = kT_\nu^\mu$ with $k = 8\pi$ and T_ν^μ =diag$(\rho, -p_r, -p_\theta, -p_\phi)$

then yield the set of equations,

$$8\pi\rho = -\left[\frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3}\right] - \frac{3k}{R_1^2} + \frac{1}{r^2}\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right]$$

$$8\pi p_r = -\left[\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} + \frac{k}{R_1^2}\right] + \frac{1}{r^2}\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right]$$

$$8\pi p_\theta = -\left[\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{k}{R_1^2}\right]$$

$$8\pi p_\phi = -\left[\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{k}{R_1^2}\right] \tag{5.2.33}$$

Let $\bar{R}$ be the mean scale factor of the Bianchi type-I universe with metric in Eq. (5.2.30). Then

$$H = \frac{\dot{\bar{R}}}{\bar{R}} \quad \text{and} \quad \frac{\ddot{\bar{R}}}{\bar{R}} = -q(t)[H(t)]^2 \tag{5.2.34}$$

and Eqs. (5.2.33) become

$$8\pi\rho = -3\left[\frac{\dot{\bar{R}}^2}{\bar{R}} + \frac{k}{\bar{R}^2}\right]$$

and

$$8\pi p = -\left[2\frac{\ddot{\bar{R}}}{\bar{R}} + \frac{\dot{\bar{R}}^2}{\bar{R}} + \frac{k}{\bar{R}^2}\right] \tag{5.2.35}$$

For a system behaving like dust $p = 0$ and $\rho = \rho_0\frac{R_0^3}{R^3}$ where the subscript '0' denotes the corresponding quantity in the present epoch. Using the above equations we can write

$$\rho = \frac{3}{8\pi G}\left(H^2 + \frac{k}{\bar{R}^2}\right) \tag{5.2.36}$$

where $\frac{3k}{8\pi G\bar{R}^2}$ is the contribution due to anisotropy, ρ_{AN} [10].

5.3 Jeans Wave Number Calculations

Let us consider the simple case with $a_1 = a_2 = a_3 = \bar{a}$ and $k = 0$. Then the equation that tells us how or whether gravitational instability leads to the growth of condensation in the expanding universe is given by Eq. (4.4.5) and for the present case it can be written as,

$$\ddot{\delta} + 2\frac{\dot{\bar{a}}}{\bar{a}}\dot{\delta} + \left(\frac{v_s^2 k^2}{\bar{a}^2} - 4\pi G\rho\right)\delta = 0 \tag{5.3.1}$$

where $\delta = \dfrac{\rho_1}{\rho}$, the density contrast parameter.

During most phases of the expansion of the universe, we can approximate the expansion factor by $\bar{a}(t) \propto t^n$ with a suitable n which is less than unity. For the matter dominated case, the scale factor $\bar{a}(t) \propto t^{2/3}$.

Proceeding as in the previous chapter we get the final expression for Jeans wave number as

$$K_J^2 \sim m\sqrt{\pi G\rho} \tag{5.3.2}$$

5.4 Energy Density and Pressure Associated with the Anisotropic Matter Field Distribution

Now let us consider the distribution of matter field which is assumed to be locally anisotropic and is coupled to a Bianchi type-I background spacetime. In addition to the quantum fluctuations, perturbations in the background metric are also taken into account in this section. Let us consider a real scalar matter field distribution ϕ which is assumed to be locally anisotropic, gravitationally coupled to an anisotropic background spacetime.

For the anisotropic matter the energy momentum tensor is $T^{\mu}_{\ v}$ =diag$(\rho, -p_r, -p_\theta, -p_\phi)$. Let us consider the case with $p_\theta = p_\phi$, which we denote by $p_\perp$ called the tangential pressure and p_r the radial pressure. So we can write,

$$p_r = -\langle T^r_{\ r}\rangle \qquad \text{and} \qquad p_\perp = -\langle T^\theta_{\ \theta}\rangle = -\left\langle T^\phi_{\ \phi}\right\rangle \tag{5.4.1}$$

5.5 Metric and Matter Field Perturbations

As it is already mentioned in the previous chapter we can split the field into unperturbed and perturbed parts:

$$\phi(x,t) = \phi_0(t) + \delta\phi(x,t) \tag{5.5.1}$$

To model the universe more realistically the perturbation in the background metric also is to be included. The perturbation $\delta\phi(x,t)$ leads to the perturbations in energy density $\delta\rho(x,t)$ and hence in the metric of spacetime. In this case, it is convenient to split the metric into two parts, the first being the background metric and the other describing how the background spacetime deviates from the idealized background model.

$$g_{\mu v} =^{(0)} g_{\mu v} + \delta g_{\mu v} \tag{5.5.2}$$

The metric perturbations are of three distinct types: scalar, vector and tensor perturbations. Both vector and tensor perturbations exhibit no instabilities. Vector perturbations decay kinematically in an expanding universe whereas tensor perturbations lead to gravitational waves which do not couple to energy density and pressure inhomogeneities. However, scalar perturbations may lead to growing inhomogeneities [11, 101] which, in turn have an important effect on the dynamics of matter.

The energy-momentum tensor can also be also decomposed into background and perturbed parts.

$$T^{\mu}_{\ \nu} =^{(0)} T^{\mu}_{\ \nu} + \delta T^{\mu}_{\ \nu} \tag{5.5.3}$$

where $\delta T^{\mu}_{\ \nu}$ is linear in matter and metric perturbations $\delta\phi$ and $\delta g_{\alpha\beta}$. Substituting Eq. (5.5.1) and Eq. (5.5.2) in the above equation we obtain the background energy-momentum tensor for the minimally coupled case, in conformal time as

$$^{(0)}T^{\eta}_{\ \eta} = \frac{1}{2C}\dot{\phi}_0^2 + \frac{1}{2}\sum_{i=1}^{3}\frac{1}{a_i^2}(\partial_i\phi_0)^2 + \frac{m^2}{2}\phi_0^2 \tag{5.5.4}$$

and

$$^{(0)}T^{i}_{\ i} = -(\partial_i\phi_0)^2 - \frac{1}{2C}\dot{\phi}_0^2 + \frac{1}{2}\left[\sum_{i=1}^{3}\frac{1}{a_i^2}(\partial_i\phi_0)^2\right] + \frac{m^2}{2}\phi_0^2 \tag{5.5.5}$$

and the first-order perturbation

$$\delta T^{\eta}_{\ \eta} = \frac{1}{C}\dot{\phi}_0(\dot{\delta\phi}) + \frac{1}{2}\delta g^{\eta\eta}\dot{\phi}_0^2 - \frac{1}{2C^2}\delta g_{\eta\eta}\dot{\phi}_0^2 - \frac{1}{2}\delta g^{ii}(\partial_i\phi_0)^2 + \frac{1}{2C}\frac{1}{a_i^2}\delta g_{\eta\eta}(\partial_i\phi_0)^2 + \frac{1}{a_i^2}(\partial_i\phi_0)(\partial_i\delta\phi) + m^2\phi_0\delta\phi \tag{5.5.6}$$

and

$$\delta T^{i}_{\ i} = -\frac{1}{a_i^2}(\partial_i\phi_0)(\partial_i\delta\phi) - \frac{1}{C}\dot{\phi}_0(\dot{\delta\phi}) - \frac{1}{2}\delta g^{00}\dot{\phi}_0^2 + \frac{1}{2C}\frac{1}{a_i^2}\delta g_{ii}\dot{\phi}_0^2 - \frac{1}{2}\delta g^{ii}(\partial_i\phi_0)^2 - \frac{1}{2}\frac{1}{a_i^4}\delta g_{ii}(\partial_i\phi_0)^2 + m^2\phi_0\delta\phi \tag{5.5.7}$$

From Eq. (5.5.7) it is clear that the perturbations in pressure is different in different directions and $\delta p_r \neq \delta p_\perp$. This implies that the velocity of perturbations will be different in radial and transverse directions.

5.6 Jeans Wave Number Calculations for an Anisotropic Medium

Any region with very slightly higher density will gravitationally attract matter from surrounding regions and thereby increasing in density. Correspondingly any regions of density lower than average will have matter removed by the gravitational attraction of neighbouring regions.

From the definition of sound velocity of adiabatic perturbations we get the expression for velocity perturbations in the radial direction as

$$v_{sr}^2 = \frac{\delta p_r}{\delta \rho} \tag{5.6.1}$$

and in the tranverse direction

$$v_{s\perp}^2 = \frac{\delta p_\perp}{\delta \rho} \tag{5.6.2}$$

where δp_r and $\delta p_\perp$ denote the first order fluctuation amplitudes of the corresponding quantities.

Let us take the scale factor as

$$a_i(t) = t^{2/3}[1 + b_i(x_i)] \tag{5.6.3}$$

The term $b_i(x_i)$ causes the anisotropy in the scale factors, which is taken as time independent. The above form of scale factor gives a direction independent Hubble constant as it is expected in the actual case. When $b_i(x_i) = 0$ the above form leads to the isotropic case. The Hubble constant and deceleration parameter are

$$H = \frac{\dot{a}}{a} = \frac{2}{3t} \qquad \text{and} \qquad q = -\frac{\ddot{a}}{a}\frac{1}{H^2} = \frac{1}{2}$$

For the $k = 0$ - the spatially flat case [30],

$$\rho = \frac{1}{6\pi G t^2} \tag{5.6.4}$$

For the present anisotropic case the equation that tells us how or whether gravitational instability leads to the growth of condensation in the expanding universe is given by

$$\ddot{\delta} + 2H\dot{\delta} + \left(v_{si}^2 K_i^2 - 4\pi G\rho\right)\delta = 0 \tag{5.6.5}$$

where, K_i is the physical wave number [26, 30] of perturbations in the i^{th} direction.

For a general specific heat ratio γ, pressure varies as ρ^γ and the speed of sound is

$$v_{sr} = \left(\frac{\gamma p_r}{\rho}\right)^{\frac{1}{2}} = \left(\frac{\gamma \rho^\gamma}{\rho}\right)^{\frac{1}{2}} \propto \rho^{\frac{\gamma-1}{2}}$$

and

$$v_{s\perp} = \left(\frac{\gamma p_\perp}{\rho}\right)^{\frac{1}{2}} = \left(\frac{\gamma \rho^\gamma}{\rho}\right)^{\frac{1}{2}} \propto \rho^{\frac{\gamma-1}{2}} \tag{5.6.6}$$

And Eq. (5.6.4) implies that

$$t^{-2} \propto \rho \tag{5.6.7}$$

Eqs. (5.6.6) and (5.6.7) show that

$$v_s \propto t^{1-\gamma} \tag{5.6.8}$$

Then Eq. (5.6.5) takes the form,

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} + \left(\frac{\Lambda^2}{t^{2(\gamma-1)}} - \frac{2}{3t^2}\right)\delta = 0 \tag{5.6.9}$$

where

$$\Lambda^2 = t^{2(\gamma-1)} v_{si}^2 K_i^2 \tag{5.6.10}$$

and γ is the specific heat ratio.

The solutions of Eq. (5.6.9) are,

$$\delta_\pm \propto t^{-1/6} J_{\mp 5/2}(3\Lambda t^{1/3}) \tag{5.6.11}$$

where $J_n(x)$ are Bessel functions of the first kind. The Bessel function $J_n(x)$ oscillates for $x >> 1$. For $x < 1$, both the growing as well as damped modes are present and the growing modes dominate over the decaying modes.

The critical condition $x = 1$ gives,

$$t^{1/3} \approx \Lambda \tag{5.6.12}$$

Eqs. (5.6.10) & (5.6.12) together imply that

$$t^{-2} \sim 6\pi G\rho \sim v_{si}^2 K_i^2 \tag{5.6.13}$$

which corresponds to the Jeans criterion. Thus we get the Jeans length for the perturbations in the i^{th} direction as,

$$\lambda_{Ji} = 2\pi / K_{J_i}$$

where

$$K_{J_i}^2 \sim \frac{6\pi G\rho}{v_{si}^2} \tag{5.6.14}$$

Equation (5.6.14) shows that the Jeans length depends on the velocity component of fluctuations in the radial and transverse directions and thus on the direction of wave propagation.

5.7 Discussion and Conclusions

Instability is the first step to an understanding of where the structure in the galaxy distribution [120] came from; it grew by gravity out of smaller structures that existed earlier. Once the universe becomes matter dominated, small primeval density inhomogeneities grow via the Jeans or gravitational instability into the rich array of structures present today [10]. Chan, Herrera and Santos have explained that different degrees of instability will lead to different patterns of evolution in the collapse of self-graviting objects [108].

Chan *et al.* [108] have shown that small anisotropies may, in principle, drastically change the stability of the system. Herrera and Santos [109] have shown that for systems with anisotropic pressures, instabilities may develop for masses of several orders of magnitude smaller than the corresponding Jeans mass for an ideal locally isotropic gas.

In this chapter we have been discussing the possibility of using the Jeans instability mechanism to form self-gravitating configurations from an anisotropic field distribution. We consider the distribution of matter field which is assumed to be locally anisotropic and is coupled to an anisotropic background spacetime. The energy density and pressure associated with the anisotropic matter field distribution are evaluated. Vanishing of the expectation values of the nondiagonal components of $T_{\mu\nu}$ allows to treat the scalar field in complete analogy to fluid distribution. Considering the metric and matter field perturbations it is found that for the present anisotropic case, the perturbation of pressure in radial and tangential directions are different. This implies that the velocity of perturbations will be different in radial and tangential directions. The Jeans wave number for the present case is evaluated. It is found that the Jeans length depends on the velocity component of fluctuations in radial and transverse directions and thus on the direction of propagation of the fluctuations.

Chapter 6

Results and Conclusions

Quantum aspects of the gravitational interaction plays an essential role in theoretical high energy physics. The questions of the quantum gravity are naturally connected with early universe and Grand Unification Theories. In spite of numerous efforts, the various problems of quantum gravity remain still unsolved. In this condition, the consideration of different quantum gravity models is an inevitable stage to study the quantum aspects of gravitational interaction. The important role of gravitationally coupled scalar field in the physics of the early universe is discussed in this thesis. The major results and conclusions of the present work are summarised in this chapter.

The temperature effects in the theory of induced gravity coupled to matter fields are discussed in chapters 2 and 3. Bianchi type-I background spacetime is considered in which the computation of the effective potential can be performed exactly.

(i)Considering a ϕ^6 self-interacting scalar field gravitationally coupled to an anisotropic background spacetime, a divergenceless expression for one loop effective potential is obtained and it is proved that ϕ^6 potential can be regularised in curved spacetime.

(ii)Finite expressions of the energy momentum tensor for the ϕ^6 theory in (3+1) and (2+1) dimensional spacetimes are obtained. The vacuum expectation values of the stress-energy tensor defined prior to any dynamics in the background gravitational field give us the information about the particle creation and vacuum polarisation.

(iii)The temperature dependence of phase transitions for the ϕ^6 model is closely examined and verified. The nature of phase transitions for the ϕ^6 model is discussed and is found to be of first order.

(iv)The crucial role played by scalar-gravitational coupling and curvature in determining the nature of phase transitions are also studied. The interaction with the external gravitational field may lead to SSB. The models described in chapters 2 nd 3 prove that the phase transitions taking place during such a SSB is first order in nature.

(v)A first order phase transition proceeds by nucleation of bubbles of broken phase in the background of unbroken phase. Considering the interaction between the bubble field and the surrounding plasma the expansion and collisions of bubbles in such a damping environment is discussed. It is found that there exists an exact solution for the damped motion of the bubble in the thin wall regime.

These results are useful for the study of quantum thermal processes in the early universe. To examine the symmetry behaviour of the early universe closely one should take into consideration the effects of spacetime curvature and finite temperature effects in their full rights. Whether or not the universe recovers from a first order phase transition and any relics are left behind depends upon the nucleation, expansion and collision of bubble and on the process of eventual transition to the new phase.

If the phase transition is strongly first order, the universe may be dominated by the vacuum energy and undergo a period of inflation. The quantum fluctuations in the inflaton field are the most natural choice for the seed perturbations. The quantum fluctuations during the period of exponential expansion generate classical energy density perturbations which look like small amplitude plane waves of all wavelengths superimposed with a spectrum which is in good agreement with the requirements for successful structure formation.

In chapter 4, the cosmic fluid is treated in complete analogy to a scalar field and the description of cosmological perturbations in the universe is reduced to the study of quantum fluctuations of a gravitationally coupled scalar field.

(vi)The primeval density perturbations produced by the vacuum fluctuations $\delta\phi$ of the scalar field are considered and the Jeans criterion for the structure formation is evaluated.

(vii)The possibility of using the Jeans instability mechanism to form self-gravitating configurations from a gravitationally coupled scalar field distribution is studied. A scalar field approach to Jeans mass calculation is discussed and the expression for Jeans mass for a (3+1) dimensional spatially flat Robertson-Walker universe is evaluated.

(viii)It is found that the self-interaction of the field influences the character of instability and the value of Jeans wave number K_J is altered by the effects of self-interaction.

(ix)The results obtained in chapter 4 are generalised to an anisotropic case in chapter 5. For an anisotropic matter distribution the velocity of perturbations will be different in radial and tangential directions and it is found that K_J depends on the velocity of fluctuations in radial and transverse directions.

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