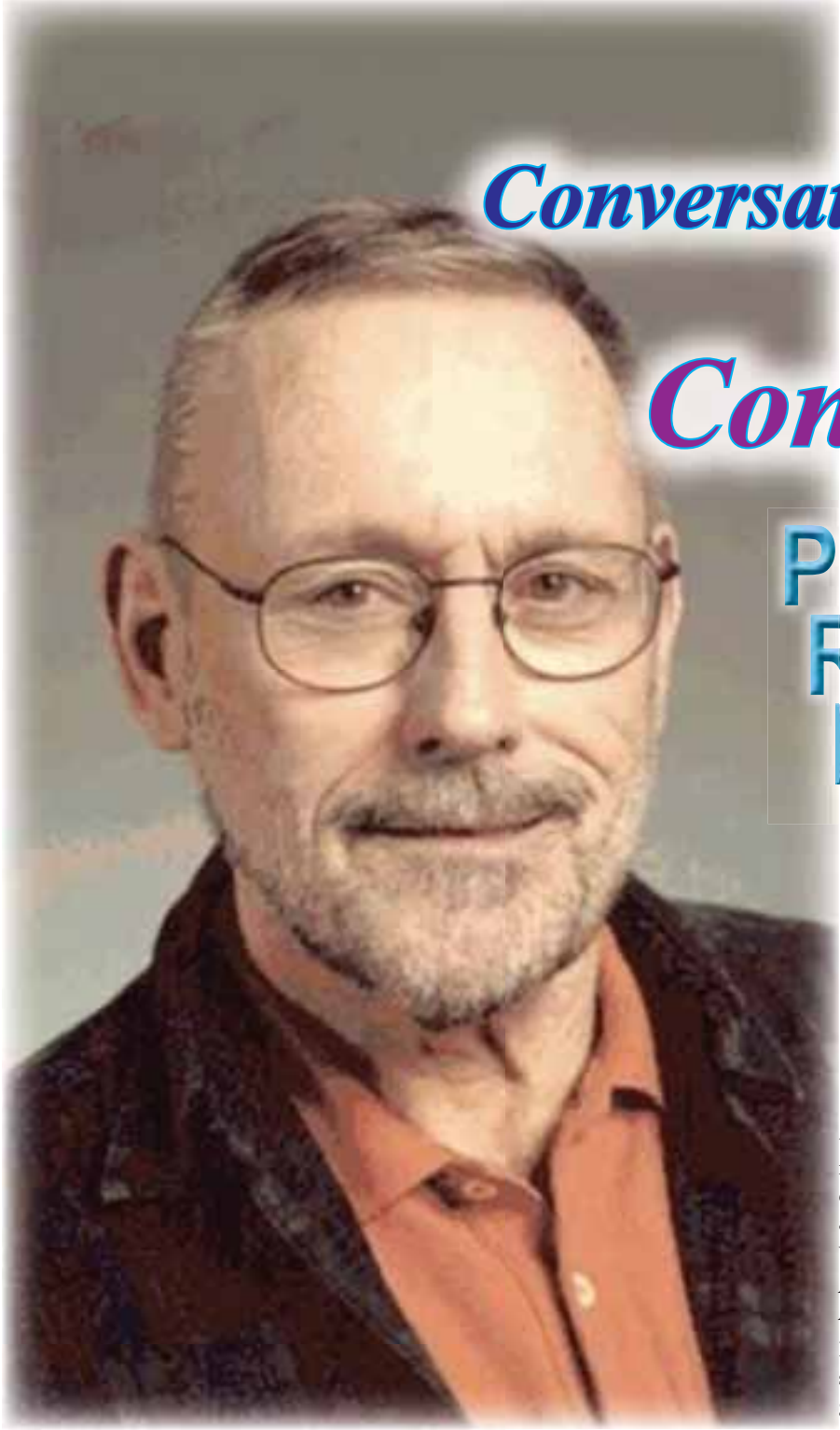
Conversation with a Chaos Consultant

Professor Robert L. Devaney



AmbatVijayakumar (AV): How was your entry into mathematics?

Robert Devaney: As an undergrad, I was planning to be a medical doctor, and my advisor suggested I major in something other than biology since that would probably make me look better to medical schools. So I majored in math, because that was "easy" for me. I then took a topology course that I loved and became hooked on math and gave up the idea of medical school.

AV: You have been an administrator/ author/researcher. Which one did you enjoy the most?

Robert Devaney: I'm primarily a researcher – that's my passion. Yes, I have served as chair of my department and am now president of the MAA, but these are not positions I love. I do these things only because I need to "pay back" my University and the MAA for how good they have been to me. As an author, I love my field of research and feel that it is important to communicate how interesting and exciting this field is to others (high school students, undergrads, and grad students, for each group of which I have written books).

AV: How is the MAA different from the American Mathematical Society (AMS)?

Robert Devaney: The AMS primarily supports mathematics research, while the goal of the MAA is to support mathematics exposition, primarily at the undergrad level. So the MAA produces journals that are mainly expository. It is a prime mover in supporting undergraduate research in mathematics, an endeavor that has really taken off in recent years. It also runs Project NExT that brings together recent PhDs in Mathematics for a series of meetings aimed at helping them improve their teaching skills. And it runs numerous meetings that bring together both mathematics researchers and educators, with sessions aimed at both groups.

Professor Robert L. Devaney of the Department of Mathematics and Statistics, Boston University, USA and currently the President of Mathematical Society of America (MAA) was in Cochin to deliver lectures in the 'International Workshop and Conference on Fractals and Wavelets' (IWCFW) held during 9-16 November 2013 at Rajagiri School of Engineering and Technology, Kakkanad. His talks were so exciting that I renewed my love with chaos and fractals, and decided to know more about its discovery and significance in today's mathematics.

Robert L. Devaney received his undergraduate degree from the College of the Holy Cross in 1969 and his PhD from the University of California at Berkeley in 1973 under the direction of Stephen Smale. His main area of research is dynamical systems, primarily complex analytic dynamics, but also including more general ideas about chaotic dynamical systems. Lately, he has become intrigued with the incredibly rich topological aspects of dynamics, including indecomposable continua, Sierpinski curves, and Cantor bouquets.

He is the (co)-author or editor of fourteen books in this area of mathematics. These include *An Introduction to Chaotic Dynamical Systems; A First Course in Chaotic Dynamical Systems; Chaos, Fractals, and Dynamics: Computer Experiments in Modern Mathematics,* and a series of four books collectively called *A Tool Kit of Dynamics Activities,* aimed at high school students and teachers as well as college faculty.

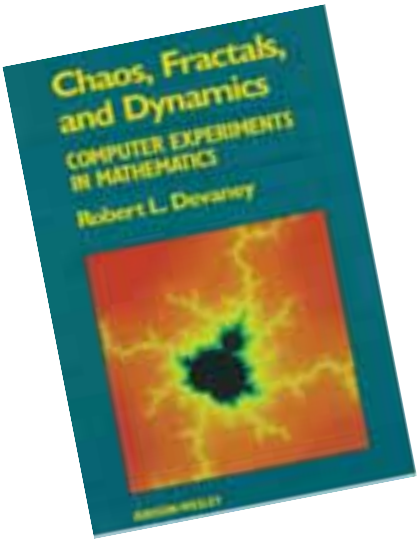
Professor Devaney has delivered over 1,500 invited lectures on dynamical systems and related topics in over 35 countries on six continents worldwide. In 2002, he received the National Science Foundation Director's Award for Distinguished Teaching Scholars and in the same year the ICTCM Award for Excellence and Innovation with the Use of Technology in Collegiate Mathematics. In 2004, he was named the Carnegie/CASE Massachusetts Professor of the Year. In 2005, he received the Trevor Evans Award from the Mathematical Association of America for an article entitled "Chaos Rules" published *in Math Horizons*. In 2009 he was inducted into the Massachusetts Mathematics Educators Hall of Fame. And, in 2012, he was named a Fellow of the American Mathematical Society.

AV: Can you please elaborate the MAA Project 'NExT'. I think this is quite relevant in the Indian context also.

Robert Devaney: Project NExT (New EXperiences for Teachers) is a program run by the MAA for the past 20 years. It brings in a new cohort of about 70 recent PhDs in mathematics for several years of intensive training regarding new methods of teaching college-level mathematics. I have given a number of workshops to these individuals over the years, most dealing with the ODE course. It is very interesting that, nowadays, many of the former Project NExT fellows have moved on to become leaders in the MAA. Last year, we brought in ten new governors of our 29 sections around the US. If I recall correctly, seven of them were former NExTers.

AV: Does University faculty involve in school education?

Robert Devaney: This was definitely not the case when I started working with HS teachers in the late 1980's. Indeed, many of my colleagues thought that I had left mathematics when I started running workshops for HS teachers. But this was a million dollar grant which back then, was



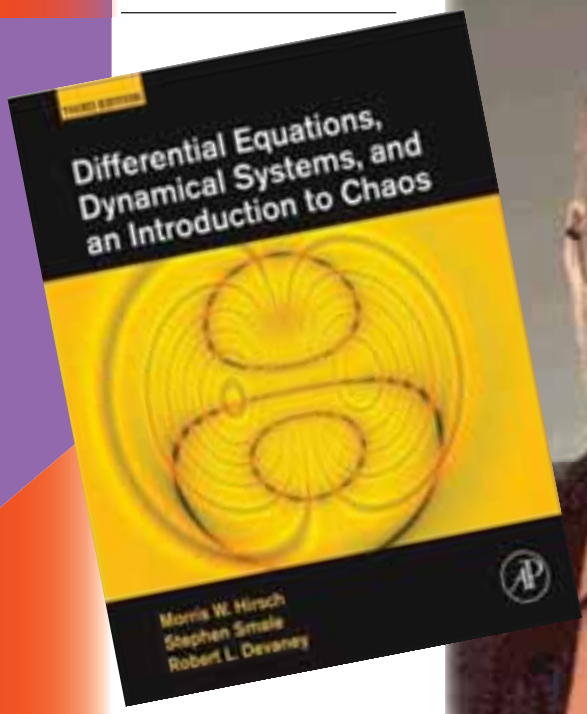
a big thing. So people came around. Now a number of my colleagues are involved in working with HS teachers/students, so this change is good.

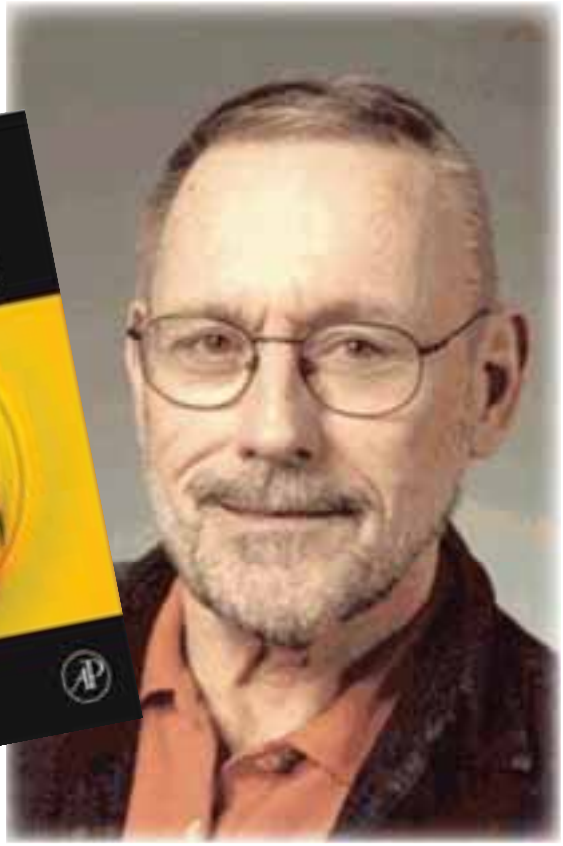
AV: But for fractals, what would you have chosen?

Robert Devaney: What I love about fractals/chaos is the fact that these are very accessible topics for HS students (and even lower level students). As I mentioned to you, I organize Math Field Days two or three times each year at my university. Each of these brings 500-600 high school students and teachers to our campus for a day of activities aimed at showing the students how beautiful and exciting contemporary mathematics is. I give two lectures at these events, one on the chaos game (fractals) and the other on the Mandelbrot and Julia sets (chaos). The students and their teachers really love these events, as this is their only exposure to what's going on in contemporary mathematics. And they love the pictures that I show them! However, I am not involved in training students for International Math Olympiads and American Math competitions.

AV: What is Mandelbrot set explorer?

Robert Devaney: This is an online introduction to the Mandelbrot set that is aimed at high school/undergraduate students. The goal is to introduce them to the mathematics behind the Mandelbrot set and associated Julia sets. Included are applets that allow them to zoom in to the Mandelbrot set, compute various Julia sets. This is an online tool to help people understand the Mandelbrot set and associated Julia sets. (For details see: http://math.bu.edu/DYSYS/explorer/)





Since the Mandelbrot set deals with just quadratic functions of the form z^2 + c, this makes this topic quite accessible to high school students. Included are nine explorations where they use software that I have developed to investigate the Mandelbrot sets. They can use the software to make pictures of various regions in the Julia and Mandelbrot sets and also to make movies that show how the Julia sets change as the parameter is varied.

AV: Any Fields Medal for research works in fractals or related themes?

Robert Devaney: Well, Yoccoz received the Fields Medal for his work in complex dynamics which is related to fractal geometry/chaos theory. Curt McMullen also received this medal for his work in complex dynamics. In addition, former Fields medalists John Milnor and Bill Thurston were heavily involved in this field. While this is technically not Fractal Geometry, it is still very close to this area as everything we "see" in this field is a fractal.

AV: What was Mandelbrot's role in Fractal Geometry?

Robert Devaney: I'm not really a fractal geometer, so I cannot comment on Benoit Mandelbrot's role in this field, though he is, of course, known as the father of fractal geometry. Rather, I am most familiar with his work in complex dynamics, which is my research area. He is most famous for the discovery of the "Mandelbrot set." There is an interesting history here. Mandelbrot's uncle, Szolem Mandelbrojt, was a mathematician working in Europe during the period when Fatou and Julia (and others) initiated the study of complex dynamics. But the field died out in the 1930's, basically because nobody could "see" the remarkably beautiful objects that were being investigated. Apparently Szolem knew Fatou and Julia's work and was aware that there were interesting pictures involved. At that time, Benoit was working at the IBM Thomas J Watson research center and hence had access to the world's most powerful computers at the time. So when his uncle told him about the work of Fatou and Julia, Benoit decided to plot the picture of "fundamental dichotomy." That occurs for the family of complex polynomials z^2 + c, as follows. If the orbit of the critical point 0 escapes to infinity under iteration, then the "Julia set" (the place where the map is chaotic) is a Cantor set, but if this orbit does not escape, the Julia set is a connected set. So Julia sets for quadratic polynomials consist of either one piece or uncountably many pieces. So Benoit decided to plot those complex c-values for which the Julia set is connected, and out came, in my opinion, the most beautiful and intricate object in all of mathematics, the Mandelbrot set.

Back in the 1970's, my research centered around iteration of maps of the unit interval and the plane, but not complex analytic maps of the plane. When I heard Mandelbrot speak about his work at Harvard in the early 1980's, I immediately got involved in complex dynamics. So did many other people, including such eminent mathematicians as Dennis Sullivan, Adrien Douady, William Thurston, and John Milnor, as well as future Fields medalists Jean-Christophe Yoccoz and Curt McMullen. The new field of complex dynamics was born, thanks to Benoit Mandelbrot.

AV: What do you feel about being the president of MAA?

Robert Devaney: In the US we have many mathematics organizations: the AMS (primarily for researchers), SIAM (primarily for applied research mathematicians), NCTM – the National Council of Teachers of Mathematics (for high school teachers), and the MAA.

I belong to all of these societies and have been heavily involved with all of them over the years, being an active mathematics researcher who works often with high school teachers (and their students). For example, I have spoken at more than 70 regional and national NCTM meetings since 1990, and I have also served on numerous AMS committees over the years, including the Council of the AMS. What I really like about the MAA is that it is devoted to excellent exposition of mathematics, especially at the undergraduate level. But this extends well beyond the undergraduate community. For example, the MAA organizes the national AMC (American Mathematics Competition), which produces national mathematics exams for the top students in middle and high school (over 350,000 take these exams each year).

The MAA also is heavily involved in promoting and enhancing undergraduate research in mathematics, an endeavor that has definitely raised the number of good students who choose to pursue careers in the mathematical sciences. And the MAA runs Project NExT (New Experiences in Teaching), which brings together a cohort of recent math PhD's to get new ideas about teaching mathematics. This is really necessary since most graduate

receive very little training regarding the teaching of mathematics while they serve as teaching assistants during their graduate school years.

AV: What are your main contributions as an author?

Robert Devaney: I am most pleased with a paper with Michal Krych that introduced the notion of Cantor bouquets in the complex dynamics of entire functions, primarily the exponential map. This was one of the first papers that dealt with the dynamics of entire functions, which is an area that has blossomed in the past twenty years. For the last ten years I have been investigating singularly perturbed complex maps. My paper with Dan Look and David Uminsky on the 'Escape Trichotomy' for these maps really made this field take off. Recently, I wrote an article that appeared in the Bulletin of the AMS that summarizes my work in this area over the past decade.

I am also pleased with the reaction to my books on the subject of dynamical systems. And my book Differential Equations (coauthored with Blanchard and Hall), aimed at sophomore level students with a calculus background, has really taken off in the US. In this book, we take a dynamical systems approach to differential equations (meaning we emphasize using qualitative and numerical methods for solving ODEs, rather than the traditional approach that looks for specific analytic solutions of the equation). After this book first appeared in 1995, many of the classic texts on ODEs have adopted our approach.

AV: How is the book Chaotic Dynamical Systems different?

Robert Devaney: Well, that book is getting old now. I wrote it back in the 1980's. It was one of the first books at the advanced undergraduate/beginning graduate level to deal with discrete dynamical systems, with an emphasis on understanding things like bifurcations and chaos. In addition, this book included topics from complex dynamics such as the Mandelbrot and Julia sets, which was a field that was just emerging in the 1980's.

More recently, with my colleagues Paul Blanchard and Dick Hall, we have written a text on Differential Equations. Before that time, most texts emphasized finding only closed form solutions of differential equations, something that usually cannot be done. Instead, we emphasized using the computer and computer graphics, together with such images as phase lines and phase planks, bifurcation diagrams, nullclines, and other geometric techniques to understand at least qualitatively the behavior of solutions of ODEs.

"Many of my colleagues thought that I had left mathematics when I started running workshops for HS teachers....."

(I had known that Prof. Devaney collects a coffee mug from any college he visits. I thanked him profusely and gifted him a coffee mug with a picture of Albert Einstein.)

Dr. AmbatVijayakumar is with the Department of Mathematics, Cochin University of Science and Technology, Cochin-682022; Email: vambat@gmail.com/vijay@cusat.ac.in