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Fast Fractal Coding Method for the Detection of Microcalcification in Mammograms

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Abstract: **The presence of microcalcifications in mammograms can be considered as an early indication of breast cancer. A fast fractal block coding method to model the mammograms for detecting the presence of microcalcifications is presented in this paper. The conventional fractal image coding method takes enormous amount of time during the fractal block encoding procedure. In the proposed method, the image is divided into shade and non shade blocks based on the dynamic range, and only non shade blocks are encoded using the fractal encoding technique. Since the number of image blocks is considerably reduced in the matching domain search pool, a saving of 97.996% of the encoding time is obtained as compared to the conventional fractal coding method, for modeling mammograms. The above developed mammograms are used for detecting microcalcifications and a diagnostic efficiency of 85.7% is obtained for the 28 mammograms used.**

I. INTRODUCTION

Breast cancer is a growth of abnormal cells within the breast. After non-melanoma skin cancer, breast cancer is the most common form of cancer in women. For 2007, the American Cancer Society (ACS) estimates that more than 178,000 new cases of breast cancer will be diagnosed, adding to the 2 million women who have been diagnosed and treated previously for this disease. In addition, the ACS estimates that nearly 40,500 women are expected to die from breast cancer in 2007, making it the second leading cause of cancer death among women (surpassed only by lung cancer) [1]. In India also breast cancer is the second most common cancer in among women.

Mammography is the single most effective way to detect early breast cancer because it can often identify the disease several years before the appearance of symptoms. Mammograms are x-ray pictures of the breast that can show a tumor before it is large enough to be felt. Due to the high incidence of breast cancer among older women, screening is now recommended in many countries, the same also applies to men. Screening methods suggested include breast self-examination and mammography. Mammography has been shown to reduce breast cancer-related mortality by 20-30%. Early detection of cancer saves patients from the more aggressive radical treatments and increases the overall survival rate.

But mammograms are one of the difficult medical images to interpret as the indications of the presence of these cancerous tissues are subtle in nature. The analysis of mammograms by radiologists is time consuming, labor intensive and requires great concentration. When the population of screening mammogram increases, because of the presence of large number of normal ones, the radiologists may miss some of the subtle abnormalities.

An early symptom of breast cancer is the appearance of microcalcifications in the breast. Microcalcifications are small deposits of calcium. The microcalcifications appear as bright spots in the mammogram which may be camouflaged in the mammographic ductal patterns making it difficult to diagnose. The size of microcalcifications is also very small, varying from 0.01 to 1 mm. To help the radiologists in detecting the cancerous regions in the mammograms certain computer aided techniques have been developed. These methods will help the radiologists by giving a "second opinion" while taking the decisions.

Mini *et. al* [2] used a Wavelet based method to eliminate the structures in mammograms produced by normal glandular tissue of varying density based local average subtraction and used Probabilistic Neural Network (PNN) for classification. Several state-of-the-art machine-learning methods like Support Vector Machine (SVM), Kernel Fisher Discriminant (KFD), Relevance Vector Machine (RVM), and committee machines (ensemble averaging and AdaBoost), are investigated in [3] for automated classification of clustered microcalcifications . A set of image structure features for classification of malignancy was used in [4]. The selection of the best features was performed using the multivariate cluster analysis as well as a genetic algorithm (GA)-based search method. Bankman *et.al* [5] presented a new segmentation algorithm and compared it to the multitolerance region growing algorithm and active contours. The new algorithm operates without statistical models, local statistics or thresholds to be selected compared to the other two algorithms. Li *et.al.* [6] developed a methodology based on fractal image modeling to analyze and model breast background structures thus enhancing the presence of microcalcifications.

Fractal image coding was first proposed by Barnsley [7]. Fractals have been used in a lot of image processing applications, compression segmentation, analysis, restoration etc [8]-[13]. Deterministic fractals have extremely high visual complexity with very low information content. They have high degree of redundancy such that they can be recursively made of transformed copies of either themselves or parts of themselves. A.E. Jacquin proposed a novel method for image compression [14], [15] by fractal block coding of images.

In this paper, the method proposed by Jacquin is used in the fractal block coding of the mammograms. The image blocks are classified into shade and non shade blocks based on their visual perception. Only the non shade blocks are coded using the fractal encoding method. Thus, the enormous computation time required in the fractal encoding procedure can be considerably reduced.

II. THEORITICAL BACKGROUND

Let (X, d) be a metric space with d a distortion measure and let μ be an original image that is to be encoded. A transformation on X is a function $f: X \rightarrow X$, which assigns exactly one point $f(x) \in X$ to each point $x \in X$. The transformation $f: X \rightarrow X$, on a metric space (X, d) is called contractive if there is a constant $0 \leq s < 1$ such that

$$d(f(x), f(y)) \leq s.d(x, y) \forall x, y \in X \tag{1}$$

where s is the contractivity factor for f. The inverse problem in iterated transformation theory is the construction of a contractive image transformation τ, defined from the space (X, d) to itself for which μ is an approximate fixed point. i.e. $d(\mu\ , \tau(\mu))$ is as close to zero as possible. The theories of Iterated Function Systems (IFS) and Collage theorem form the basis for fractal image coding techniques.

Theorem 1. An Iterative Function System (IFS) consists of a complete metric space, (X, d) together with a finite set of contractive mappings $\tau_n: X \rightarrow X$, with respective contractive factors s_n, for n=1,2,…..N.

Theorem 2. Let {X; τ_n ,n=1,2,….N} be an iterated function system with contractivity factor s. Then the transformation τ:H(X)→H(X) defined by

$$\tau(B) = \bigcup_{n=1}^{n} \tau_n(B) \tag{2}$$

For all B $\in$H(X), is a contractive mapping on the complete metric space (H(X),d) with contractivity factor s.
That is h (τ (B), τ(C)) ≤s. h (B, C) for all B, C $\in$ H(X).Its fixed point $A \in H(X)$ obeys

$$A = \tau(A) = \bigcup_{n=1}^{n} \tau_n(A) \tag{3}$$

And is given by $A = \lim_{n \rightarrow \alpha} \tau^{on}(B)$ for any $B \in H(X)$.

The fixed point $A \in H(X)$ described in the theorem is called the attractor of IFS.

Collage theorem: Let (X, d) be a complete metric space. Let L $\in$H(X) be given, and let $\varepsilon \geq 0$ be given. Let the IFS {X ;(τ_o), $\tau_o\ \tau_1$ ……. τ_n } with contractivity factor $0 \leq s < 1$, so that

$$h\left(L, \bigcup_{n=1}^{n} \tau_n(L)\right) \leq \varepsilon \tag{4}$$

Where h(d) is the Hausdorff metric. Then

$$h(L, A) \leq \frac{\varepsilon}{1-s} \tag{5}$$

or

$$h(L, A) \leq \frac{1}{1-s} h\left(L, \bigcup_{n=1}^{n} \tau_n(L)\right), \text{ for all } L \in H(X) \tag{6}$$

Since s<1, it can be seen that after a number of iterations, the constructed image $\mu_n = \tau^{on}(\mu_n)$ will be visually close to the original image μ.

The fractal block coding of images exploit the self similarity property of images. Since real world images are not self similar, it is impossible to find a transformation τ for the entire image. But, these images may have local self similarity. Therefore, the image is divided into blocks, and for each block, find the corresponding τ_i. In conventional fractal image coding method the image is divided into non-overlapping blocks called the range blocks and for each range find the matching domain which is twice the size of the range from the same image itself. i.e. the domain which is most similar to the range. The search for the matching domain is time consuming, as the search has to be performed in the entire image.

In this paper, instead of checking the entire image for the matching domain, the image is classified into shade and non shade blocks depending on the texture property of the blocks. Only those non shade blocks are coded using the fractal block coding method.

III. CLASSIFICATION OF IMAGE BLOCKS

The image of square size NxN is divided into non overlapping range blocks of size RxR. These range blocks are then classified into shade and non shade blocks. Shade blocks are those blocks that has no major gradients or texture and the gray scale of pixels change slowly or little to human eyes perception. A non shade block has some sudden changes in pixel intensities looking like texture or distinct edges which can be perceived.

Jacquin had classified the image into shade, midrange and edge blocks. Mid range blocks are those blocks whose intensity variations falls between shade and edge blocks. In this paper, only two classifications were used i.e shade and non shade, as mammograms are images having low intensity variations and therefore it is difficult to distinguish between edge and midrange blocks in mammograms. Thus, the classification is limited to shade and non shade blocks.

If the range block is a shade block, no searching is required and only the mean of the pixels is required for decoding. Also, if the domain is a shade block it is not included in the best domain searching pool. The non shade blocks are encoded by the method discussed in the next section.

IV. FRACTAL IMAGE CODING

The image is divided into non overlapping range blocks, R_i. The major task in fractal image coding process is to find the best matching domain block D_i, of size greater than the range generally chosen as twice the range size and thus finding the corresponding τ_i for each R_i.

τ_i can be written as a combination of two transformations G_i and M_i

i.e $\tau_i = G_i \circ M_i$. (7)

where G_i is the geometric part and M_i is the massic part of τ_i.

Geometric part G_i

A domain block of size 2R is mapped by geometric transformation on to a range block by taking the average of the four domain pixel values.

$$\overline{D_i}(k,l) = \frac{\sum_{i=0}^{1}\sum_{j=0}^{1} D_i(k+i, l+j)}{4} \quad (8)$$

Thus the size of the domain is contracted to the size of the range block.

Massic part M_i

These transformations affect the pixels of the transformed domain blocks. The luminance shift is given by

$$\Delta g = \text{mean}(R_i) - \text{mean}(\overline{D_i}) \quad (9)$$

The contrast scaling α is given by

$$\alpha = \min\left(\frac{dr(\text{range})}{dr(\text{domain})}, \alpha_{max}\right), \ \alpha \in [0, 1] \quad (10)$$

where dr is the dynamic range of the respective blocks.Also the averaged domain blocks can have eight different transformations called isometries such as (1) Identity (2) rotation through +90° (3) Rotation through +180° (4) Rotation through -90° (5) Reflection about mid vertical axis (6) Reflection about mid horizontal axis (7)Reflection about first diagonal (8) Reflection about second diagonal.

The domain which minimizes the L2 distortion measure is chosen. The L2 or root mean square distortion between the image blocks R_i and $\overline{D_i}$ is defined as the square root of the sum of the squared difference if the pixel values i.e.:

$$d_{L_2}\left(R_i, \overline{D_i}\right) = \sum_{k,l}\left(R_i(k,l) - \overline{D_i}(k,l)\right)^2 \quad (11)$$

The fractal coefficients for the range blocks are α, Δg and isometry value of the corresponding domain along with the domain locations. The fractal code used to represent the entire image is the union of the parameters of all range blocks as follows:

$$\tau = \bigcup_{i=1}^{n} \tau_i \quad (12)$$

Decoding

In the decoding, the parameters generated in the encoder are used to define the Iterated Function System which should be contractive. The natural decoding scheme consists in iterating the fractal code τ on any initial image μ_0, until the convergence to a stable decoded image is obtained. The mapping of an image under the fractal code is done sequentially. For each cell index i, the transformation τ_i is applied to the current image block over the domain cell D_i, and mapped onto the range cell R_i. The convergence of the algorithm is achieved after 10-12 iterations.

V. IMPLEMENTATION

The image is divided into non overlapping range blocks of size 4x4. To classify these blocks the dynamic range of the block is found by

$$\text{Dynamic Range} = \frac{\text{Max Pixel value}}{\text{Min Pixel value}} \quad (13)$$

If the dynamic range is less than 0.05, the block was classified as shade block. Thus, if a range is a shade block, its location and mean of the pixel values are stored.

If the range is a non shade block, it has to be encoded by the fractal encoding procedure discussed in section IV. For this block the matching domain has to be found out such that $R_i \cap D_i = \varphi$. This is because; microcalcifications in mammograms appear as single or isolated clusters. Therefore there may not be a matching domain corresponding to the range containing the microcalcification unless that region itself is included in the search area.

The search for the matching domain is performed from the next adjacent pixel on wards so that no microcalcification regions are missed. The domain which minimizes the equation (11) is selected. For the chosen domain, find Δg and α from equation (9) and (10) respectively. The domain is assumed to have four isometries: identity, +90, +180 and -90 as this would suffice in modeling the mammograms and detecting the microcalcifications. Store the domain locations, Δg ,α and its isometry value for the corresponding range block. This will correspond to the τ_i of the chosen range block R_i. This process is repeated for all the range blocks.

While decoding, the modeled image is obtained from any arbitrary initial image of the same size by applying τ_i to the domain locations iteratively. Convergence is obtained after

10-12 iterations. The modeled image will be visually close to the original image.

The background region of the breast is now modeled using the fractal method. To enhance the presence of microcalcifications, the difference between the original image and the modeled image is found out. The noise in the residue image is removed by applying threshold in a two step process.

i. Initial threshold T_o is taken as 3.5 times the standard deviation of the image.

ii. The second standard deviation is found from those pixels of the difference image whose gray level values are below T_o. The new threshold T_1 is arbitrarily selected as 3.5 times this standard deviation.

The image is made binary by equating the pixels whose gray level is less than 6.5, obtained by trail and error, to 0 and others to 255. The locations of the microcalcifications alone will be detected from the difference image.

VI. RESULTS AND DISCUSSIONS

The mammograms for the experiment are obtained from the freely available database provided by the Mammographic Image Analysis Society (MIAS) Digital Mammogram Database [16]. The images in the database are digitized at 50-micron pixel edge, which are then reduced to 200-micron pixel edge and clipped or padded so that every image is having 1024 x 1024 pixels. The accompanied 'Ground Truth' contains details regarding the character of the background tissue, class and severity of the abnormality and x, y coordinate of its centre and radii.28 mammograms with microcalcifications and 61 normal ones were used in the study.

The regions of interest (ROI) in the mammograms containing microcalcification were chosen as 64x64, 128x128 and 256 x256. The range sizes were varied from 16x16, 8x8, and 4x4 to 2x2. When the range was increased beyond 8x8 visible blocking artifacts were present in the modeled image. The presence of microcalcifications were enhanced when the modeled image is subtracted from the original image even for a range size of 16x16, since the difference image was made binary as discussed in section V. If the dynamic range of the block, given in equation (13), is chosen as less than 0.05 for a small range size, e.g. 2x2, almost all the range blocks will be classified as shade blocks, thus requiring much less time to encode. The encoding time is increased when the block size increased, because it may be classified as a non shade block which has to be modeled by fractal encoding method. Thus the optimum block size for the proposed method is chosen as 2x2.

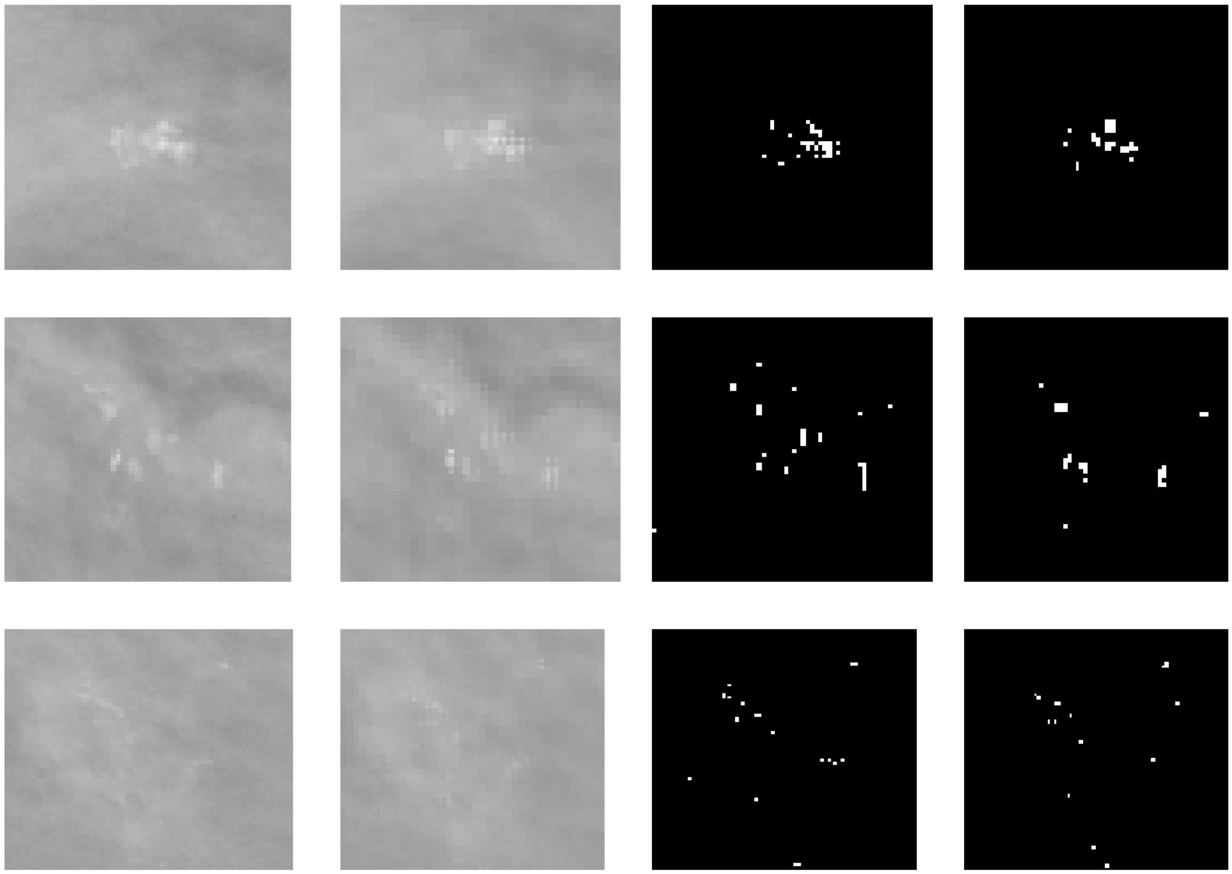
The method is compared with the conventional fractal image encoding method with quad tree partitioning which checks the entire domain pool. In the conventional encoding method, the image is divided into non overlapping range blocks. For each range, find the domain twice the size of the range anywhere in the image, such that $R_i \cap D_i$ is φ. The domain whose error is less than that in equation (11) is chosen. The parameters Δg, α and the isometry values of the chosen domain are computed and stored. If no domain in the domain pool satisfies the error condition, the range is quad tree partitioned and for each of the four range blocks the above domain search is performed. This quad tree partitioning is done twice to find the matching domain. Even then if no matching domain is found, the domain with minimum error is selected. Here as the block size is reduced the time required for encoding will increase, because the number of blocks increases and all these blocks are to be encoded by fractal coding method. The results are tabulated in table 1.

TABLE 1
Average Mean Square Error and Cross Correlation Between the original mammogram and the modeled image obtained by Fractal coding with Conventional method (range size 8x8) and with Block Classification (range size 2x2), ROI 64x64

Mammograms	Method	Avg. Mean Square Error	Avg. Correlation	Encoding Time (minutes)
Normal	Conventional Fractal Coding	10.4195	0.9734	26.7561
	Fractal coding with Shade and Non shade blocks	2.6921	0.9826	0.2520
Abnormal	Conventional Fractal Coding	10.9933	0.9694	25.7263
	Fractal coding with Shade and Non shade blocks	1.494	0.9815	0.79936

TABLE II
Detection Sensitivity for Conventional Fractal coding with range size 8x8 and Fractal Coding by block classification with range size 2x2

Mammo grams	Method	# of Sam ples	TP	FP	FN	% Dete ction	Time In minutes
Normal	Conventional Fractal Coding	61	56	5	-	91.566	26.7561
	Fractal Coding with Shade & Non Shade Blocks	61	58	3	-	95.08	0.2520
Abnormal	Conventional Fractal Coding	28	23	-	5	82.1428	25.7263
	Fractal Coding with Shade & Non Shade Blocks	28	24	-	4	85.71	0.79936



(a) (b) (c) (d)

Fig. 1. (a) Original mammogram (b) Decoded Mammogram by block classification (c) Detected Microcalcification by block classification with range 2x2 (d) Detected Microcalcifications by Conventional Fractal Coding method with range 8x8 (the region of interest in both case is 64x64)

The Mean Square Error (MSE) between the original and the modeled mammogram is

$$\mathrm{MSE}=\frac{\sum \left(f(i,j)-F(i,j)\right)^2}{N^2} \tag{14}$$

where f and F are the original and the modeled image respectively, of size NxN. The signal to noise ratio between the original and modeled image is found to vary from 21.6540dB to 38.6775dB for the abnormal mammograms and for normal mammograms it varied from 23.5301dB to 38.1445dB for fractal coding with shade and non shade block classification. The conventional method of fractal coding took an average of 26.2412 minutes to encode, while the proposed method needed only 0.52568 minutes when encoding normal and abnormal mammograms.

Thus a saving of 97.996% of the encoding time is obtained in the proposed method. Fig. 1 shows the comparison of the microcalcifications detected in the mammograms by the conventional fractal coding method and the proposed fractal coding method by classification into shade and non shade blocks. In both the methods almost the same locations of the microcalcifications were enhanced.

The microcalcification detection results are expressed in terms of three parameters: True Positive (TP), False Positive (FP) and False Negative (FN). A TP is obtained when a normal/abnormal mammogram is correctly detected as normal/abnormal. When a normal mammogram is incorrectly classified as abnormal; it is defined as a FP.A false positive is counted if two or more erroneous detections are made within an empty closed, region of 0.5cm in width [17].

A FN is obtained when an abnormal mammogram is incorrectly classified into normal class. The table II shows the detection results. A detection accuracy of 85% is obtained for the proposed method as compared to 82% using the conventional fractal encoding method for the 28 abnormal mammograms.

VII. CONCLUSION

A fast fractal encoding method for detecting the presence of microcalcifications in mammograms is presented in this paper. The image blocks are divided into shade and non shade blocks based on the dynamic range of the block. If the dynamic range is made very less and the block size is also too small eg.2x2, almost all blocks in the image will be shade blocks. Thus it takes much lesser time to encode. But as the block size increases, blocking artifacts will be present in the modeled image. The blocking artifacts present in the modeled image did not affect the detection of microcalcifications even with block size of 8x8. In the classification, midrange blocks as proposed by Jacquin were not included, as it did not make any difference in the block coding of mammograms. Since screening mammography is more frequent in European countries, the proposed method can be used by the radiologists to diagnose the presence of breast cancer at an early stage.

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