STUDIES ON
BAG MODELS AND PROPERTIES OF HADRONS

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PARTIAL FULFILMENT OF THE REQUIREMENTS
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CERTIFICATE

Certified that the work reported in the present thesis is based on the bona fide work done by Mr. M.N. Sreedharan Nair, UGC Teacher Fellow, under my guidance in the Department of Physics, Cochin University, and has not been included in any other thesis submitted previously for the award of any degree.

Cochin - 22,
2 November 1981.

K. Babu Joseph,
Supervising Teacher.

DECLARATION

Certified that the work presented in this thesis is based on the original work done by me under the guidance of Dr. K. Babu Joseph in the Department of Physics, Cochin University, and has not been included in any other thesis submitted previously for the award of any degree.

Cochin - 22,
2 November 1981.

M.N. Sreedharan Nair.

PREFACE

The subject matter of this report is the work done by the author in the Physics Department of Cochin University during 1977-'81, as UGC Teacher Fellow.

The thesis is devoted to theoretical studies on the properties of hadrons on the basis of bag models. It contains some applications of the traditional MIT bag model to the spectroscopy and decay of hadrons. The inadequacies of the model are brought out and a new version of the model, called the variable pressure bag model, is developed. Some of the phenomenological applications of this model are discussed and the predictions are compared with experiment.

Chapter 1 is introductory. It contains a very brief account of the current status of elementary particle theory in terms of quarks (and gluons) with special reference to various models of hadronic structure. In chapter 2 the salient features of the MIT bag model are described. Chapter 3 deals with a bag model study of the mass spectrum of charmed mesons. Chapter 4 contains an application of the model to the study of the weak nonleptonic decays of charmed D^0 and D^+ mesons. In chapter 5 a further application of the model, the spectroscopy of gluonic bound states, is discussed. In chapter 6 the variable pressure bag model and its phenomenology are developed. The model is applied to a study of the mass spectrum of ordinary light mesons and baryons and to a detailed analysis of the hadron mass splittings taking into account the SU(3) breaking effects. In chapter 7 the magnetic moments of stable baryons are worked out and compared with their measured values.

A part of these investigations has appeared in the form of the following publications:

1. "A Bag Model Study of D Mesons", Pramana 11, 195 (1978).

2. "A Phenomenological Bag Model with Variable Bag Pressure", Pramana 16, 49 (1981).

3. "Bag Phenomenology of Glueball Spectroscopy", Cochin University Preprint: CUTP-81-1, (1981).

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M.N. Sreedharan Nair.

SYNOPSIS

The thesis deals with studies on the static and dynamic properties of strongly interacting particles, using the phenomenological MIT bag model [1] and also a modified version of the same developed by the candidate [2]. Studies are confined to ordinary baryons and mesons in the low mass regime of hadron spectroscopy, selected mesonic states in the charm sector that are fairly well established experimentally and the somewhat speculative exotic hadronic matter called "glueballs".

First an application of the original MIT bag model to the study of the masses of charmed mesons is considered. Masses of the new narrow resonances D, F, D^* and F^* are estimated in the spherical bag approximation. This constitutes a natural extension to the charm sector, of the studies already made on the low-lying nonstrange and strange hadrons, by the pioneers of the MIT bag model. The mass predictions are in fairly good agreement with observations.

Analysis of the dominant non-leptonic weak decays of D mesons forms another subject of the present investigation. Using PCAC and soft meson theorems amplitudes for a number of two-body final state decays are explicitly evaluated. These are found to be consistent with the measured life times.

As a further application of the traditional MIT model, the phenomenology of glueball spectroscopy is studied. Glueballs are hadrons composed of pure valence gluons in flavour and colour singlet combination. The mass spectrum of the low-lying glueballs is estimated in the framework of the MIT bag model taking into account the colour magnetic interactions of gluons. Arguments are presented for the stability of glue-bags. The newly discovered $\varphi'(1.65)$ is suggested to be a vector glueball with a predicted mass of 1.67 GeV.

Although phenomenological calculations based on the above mentioned model have yielded generally satisfactory results, especially when compared to alternative models of hadron dynamics, the agreement between theory and experiment has not always been quite good. The totally wrong prediction of the pion mass, the consistently low estimates of the baryon magnetic moments and the failure to account for the observed hadron mass splittings [1] are some of the instances to be recalled in this context. These failures, together with the observation that the bag pressure B is not necessarily a universal constant as it is assumed to be, have motivated the present investigator to try to develop a revised version of the model [2].

In the new model, relativistic hydrodynamics is invoked to express the confining pressure B as a function of the energy density of the hadronic bag which makes B a varying parameter rather than a universal constant.

The new bag phenomenology is applied to the ground state baryons and mesons with no charm content. The observed mass spectra are well reproduced with an exact fit to the pion mass. Assuming that the hyperfine mass splittings arise as a result of colour magnetic interactions of quarks the well established mass relations among various hadron multiplets have been verified almost exactly.

The phenomenological content of the variable pressure bag model is further tested by applying the model to obtain predictions of baryon magnetic moments. With the nonstrange quark mass and the baryon bag size being determined from a simultaneous fit to the measured proton gyromagnetic ratio and the nucleon axial vector coupling constant, agreement between theory and experiment has been obtained to an impressive level, The results of this investigation lend credence to the basic additivity assumption of the quark model on the one hand, and go to establish the validity of the variable pressure bag model on the other, at least as far as the static properties of ordinary low mass hadrons are concerned.

References

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CHAPTER 1

INTRODUCTION

It is undisputed that the quark hypothesis has revolutionised our understanding of elementary particle physics. During the past two decades since its formulation, the quark model has provided considerable insight into hadronic phenomena even though no quarks have been found in isolation so far. The nonobservability of quarks is understood in terms of the hypothesis of quark confinement which states that quarks are permanently confined inside hadrons. Models have been developed on the basis of this dogma. The MIT bag model is one of the most successful of such schemes for hadron spectroscopy. In what follows we present a brief description of quarks appearing in different "flavours" and "colours" as hadronic constituents, introduce QCD as the quantum field theory of strong interactions based on the concept of colour and briefly discuss some of the quark confinement schemes with emphasis on bag models.

1.1 Quarks in Flavour and Colour

It is now widely accepted that hadrons are composite objects with fractionally charged quarks as constituents. The quark hypothesis was proposed by Gell-Mann [1] and Zweig [2] with a view to presenting a simpler picture of the world of elementary particles. They were motivated in this by the success of the broken SU(3) symmetry of the strong interaction [3-5]. They suggested that three fundamental spin $\frac{1}{2}$ fields belonging to the basic triplet of SU(3) be considered as the carriers of hadronic quantum numbers: baryon number B, electric charge Q, strangeness S or hypercharge Y, the isospin I and its third component I_3. These fields are called quark fields and their quanta the quarks denoted by u, d, s. The symbols u, d and s stand for the quark properties of being "up", "down" and "strange". These properties are referred to as "flavours". In terms of these quarks the mass spectrum, magnetic moments and decay rates of nonstrange and strange hadrons could be studied, treating mesons (B = 0) as $q\bar{q}$ and baryons (B = 1) as qqq bound systems. Besides these "valence" quarks, there is a zero quantum number "sea" of quarks and antiquarks in the hadrons.

Flavour is unaffected by strong interactions, yet quarks of different flavours do not behave exactly identically in strong interactions because of the difference in their mass which causes the underlying symmetry to break. This, of course, is not the physical mass, as quarks have not been observed as free particles, but it is model dependent, e.g. the mass entering the Lagrangian which describes interactions of quarks. The u and d quarks are light and are supposed to be degenerate in mass. In phenomenological calculations their mass is variantly chosen from a range 0-300 MeV. They have very similar properties and form the basis for the fundamental representation of the isospin SU(2) symmetry group. The s quark is relatively heavier with an assumed mass ranging from 250-500 MeV. Until 1974 all the known hadrons could be understood as composites of u, d, s quarks and the corresponding antiquarks.

With the discovery towards the end of 1974 of the new heavy resonance J/ψ(3.1 GeV) with surprisingly narrow width simultaneously by groups at SLAC [6] and Brookhaven [7] and the subsequent measurement of excited states [8] and radiative transitions [9-11], the proposal for a new hadronic property "charm" and a fourth quark c as its carrier was widely accepted. In fact charm was proposed earlier [12] in connection with the gauge theory of weak interactions [13,14] of hadrons and the above narrow resonances had been anticipated theoretically [15]. The J/ψ(3.1) and ψ'(3.7) have been established as $c\bar{c}$ bound states having hidden charm. The introduction of charm requires the existence of a host of new hadrons possessing explicit charm. Hardly a year and a half after the discovery of J/ψ the first of these charmed particles - the D mesons - were discovered [16,17]. The discovery in the following year at Fermilab of enhancements near 10 GeV [18-20] constituting the Υ family is now accepted as clear cut evidence for the existence of a new quark with flavour "beauty" or "bottom". The b quark has a mass $\sim$ 5 GeV. The existence of a much heavier sixth quark, named "top" quark has been speculated on the basis of a symmetry between quarks and leptons. There are 3 generations of leptons - the electron e, the muon μ and the comparatively newly discovered heavy lepton τ [21] together with the corresponding neutrinos - each generation constituting a left handed doublet representation of SU(2) weak isospin group, referred to as SU(2,W) [22]:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

The quark-lepton symmetry requires the t quark to complete the third generation of quarks:

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L , \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L , \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L ,$$

where

$$d' = d \cos\theta_c + s \sin\theta_c,$$

$$s' = s \cos\theta_c + d \sin\theta_c,$$

$$b' = b ;$$

θ_c being the Cabibbo angle which from experiments is determined to be $\sin\theta_c \simeq 0.22$.

The quantum numbers carried by quarks of different flavours are listed in Table 1.1. Note that the values of B, I_3, S, c, b and Q for the antiquarks are equal and opposite in sign to those for the quarks.

High energy scattering experiments on nucleon targets employing lepton projectiles have revealed a scaling phenomenon [23] in such processes. A reasonable theoretical picture of the phenomenon is provided by Feynman [24]. According to this picture a hadron at such high energies behaves as though it is made up of light point-like spin ½ constituents called "partons". Many details are well described by assuming that these partons carry the interactions and quantum numbers of the quarks. Thus emerges the quark-parton model.

In order to avoid parastatistics for the quarks [25], particularly, to understand baryons as composed of 3 spin ½ quarks there arose the need for assigning an extra degree of

Table 1.1 Quarks, their flavour quantum numbers and electric charges in units of e.

Quark	Flavour	B	I	I_3	S	c	b	Q
u	up	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{2}{3}$
d	down	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{3}$
s	strange	$\frac{1}{3}$	0	0	-1	0	0	$-\frac{1}{3}$
c	charm	$\frac{1}{3}$	0	0	0	1	0	$\frac{2}{3}$
b	beauty	$\frac{1}{3}$	0	0	0	0	-1	$-\frac{1}{3}$
(t	top	$\frac{1}{3}$	..	..	..	..	..	$\frac{2}{3}$)

freedom to quarks known as colour [26,27]. The lowest baryons are made of qqq in relative S-states. For the $\frac{3}{2}^+$ baryons the spin state is also symmetric. To obtain an antisymmetric total wave function for the 3-fermion system it is assumed that each quark appears in 3 different "colours": red, green and blue, and that the qqq wave function is antisymmetric in colour. It is further postulated that colour is unobservable or confined so that the hadrons are colour singlets and have conventional charges. Remarkably enough the colour concept has accounted for rather nicely the observed amplitude of $\pi^0 \rightarrow 2\gamma$ decay [24], the observed value of the ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-) \simeq \sum_q Q_q^2$, and has proved to be consistent with the condition on electric charges Q of all hadron and lepton fields necessary to avoid the so called "triangle anomalies" [28]. The three basic colour fields generated by quarks may be supposed to form a fundamental representation of an internal symmetry group, the colour SU(3) or SU(3,C).

1.2 Gluons [29]

Colour charge is responsible for the strong interaction between quarks. Quarks interact with each other by the exchange of "gluons" which are the quanta of the colour fields. Thus the gluons mediate the strong interaction just as the photon mediates the electromagnetic interaction in QED.

They are massless bosons with spin 1 and carry colour charge unlike the photon which does not carry electric charge. In terms of the triplet of colour fields r, g, b the gluon wave functions are

$$\bar{r}g,\ \bar{r}b,\ \bar{g}r,\ \bar{g}b,\ \bar{b}r,\ \bar{b}g,$$

$$(\bar{r}r - \bar{b}b),\ (\bar{r}r + \bar{b}b - 2\bar{g}g).$$

They thus form an octet representation of SU(3,**C**).

1.3 Quantum Chromodynamics (QCD)

Quantum electrodynamics (QED) is the abelian gauge theory of electromagnetic interaction between charged particles. The gauge group is U(1), which is generated by the charge operator, and the photon field is the gauge field introduced to make the theory invariant under local gauge transformations. Non-abelian gauge theories [30,31] are a generalisation of the abelian U(1) gauge theory. The internal symmetry group is a Lie group with generators F^a satisfying

$$[F^a, F^b] = if^{abc}F^c \qquad (1.1)$$

f^{abc} being the structure constants. The Weinberg-Salam theory [13-14] of electro-weak interactions based on the $SU(2,W) \otimes U(1)$ gauge group with the photon and the intermediate vector bosons $W^{\pm}$, Z^0 forming the gauge bosons is an example of a nonabelian gauge theory. The strong interactions are described by a nonabelian gauge theory [32,33] governed by the gauge group SU(3,C) with the 8 coloured gluons forming the gauge bosons. The theory is known as Quantum Chromodynamics (QCD). It is developed on the same lines as QED. Hence it is an educated guess that hadron dynamics resulting from the properties of quarks and gluons might be based on a standard Lagrangian [34] similar to that of the successful QED:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + i\bar{q}\not{D}q + \bar{q}mq \tag{1.2}$$

where the flavour and colour indices of the quark field q are not written out explicitly. In fact q denotes a column with three colour components. The nonabelian field strength tensor in Eq. (1.2) is given by

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^b_\mu A^c_\nu \tag{1.3}$$

where f^{abc} are the structure constants introduced earlier and g is the small, fundamental quark-gluon coupling constant in the theory. Furthermore,

$$\not{D}q = D_\mu\gamma^\mu q = (\partial_\mu\gamma^\mu - ig\frac{\lambda^a}{2} \gamma^\mu A_\mu)q \tag{1.4}$$

and m is the bare quark mass matrix.

1.3.1 Renormalisability

The postulate that a Lagrangian field theory must be renormalisable reduces possible Lagrangians to a very restricted class. Renormalisable field theories represent the only complete and consistent relativistic dynamical system. Hence it is theoretically rational to postulate the renormalisability, even though the physical meaning of this assumption is unclear. In 1971 't Hooft has proved the renormalisability of nonabelian gauge theories [35].

1.3.2 Asymptotic Freedom [36,37]

Field theories which, for large momenta, approach the free field theory are called asymptotically free. For high energies (or zero quark masses) there is no dimension in $\mathcal{L}_{QCD}$ (g dimensionless). But there is a typical scale $\Lambda \simeq 0.1$ to 1 GeV for QCD. This scale enters as an integration constant in the expression for the renormalised coupling constant $g(Q^2)$ where Q^2 are the typical momenta. For SU(N) gauge theories with N_f fermions (= number of flavours)

$$\frac{g^2(Q^2)}{4\pi} \simeq \frac{6\pi}{(11\ N - 2\ N_f)} \quad \frac{1}{\ln(Q^2/\Lambda^2)} \qquad (1.5)$$

in the limit of large Q^2, provided N_f is not too large. In Eq. (1.5) $\ln \Lambda^2$ is the integration constant. This means that for large momenta (or small distances) the coupling constant goes to zero resulting in asymptotic freedom. Asymptotic freedom is experimentally seen in deep inelastic scattering experiments which reveal the quarks inside the hadron as almost free point-like constituents.

1.3.3 Confinement

Asymptotic freedom permits perturbation theory for large Q^2. But troubles arise for small Q^2, for in this case coupling constant grows large so that the binding becomes stronger and Eq. (1.5) is no longer valid. The situation is referred to as "infrared slavery". It describes the behaviour of QCD for large distances or small momenta at which quarks are made "slaves" by confining them to the inside of hadrons. It is conjectured that nonperturbative approaches to QCD will ultimately prove the confinement of quarks (and gluons).

1.4 Hadrons and Quark Confinement Schemes

The picture that we have of the hadrons is that of an extended object in which the "valence" quarks determine the flavour and spin properties. Colour is the source for the strong interactions which are effected by gluon exchange. The coloured quarks are confined to the hadron which itself is colourless. This allows only $q^m\bar{q}^n$ states (states with m quarks and n anti-quarks) with (m-n) a multiple of 3, or equivalently states with an integer baryon number: B = 0, 1, 2 etc.

Inside the hadron (at short distances) the interaction between the quarks is weak. The quarks are almost free (asymptotic freedom) and for large Q^2 perturbation theory is applicable. Properties concerning the hadron itself involve low Q^2 (large distances) where perturbation theory does not apply. Here we have to consider exact and approximate symmetries like isospin, SU(3), spontaneously broken chiral symmetry etc. Further, one has to explain dual dynamics and understand the screening of the charges of colour symmetry (confinement).

Various field theoretic and phenomenological models have been proposed to understand the properties of hadrons.

1.4.1 Field Theoretic Models

(a) Solitons and Kinks

Among field theoretic approaches we would mention here the classical 1 + 1 dimensional model field theory [38,39] exhibiting bound state structure described by stable finite energy solutions ("solitons") of classical nonlinear field equations involving a colourless quark field. Strong binding may be derived from an invariant, local, renormalisable Lagrangian

$$= \frac{1}{2} (\partial_\mu \varphi)^2 + \frac{1}{2} m^2 \varphi^2 - \frac{1}{4} \lambda \varphi^4 + i \bar{\Psi} \gamma^\mu \partial_\mu \Psi + g \varphi \bar{\Psi} \Psi \quad (1.6)$$

which involves quark field $\Psi(x)$ and a scalar Higgs field $\varphi(x)$. It leads to topologically stable [40] bound state solutions constituting "kink with trapped quark".

(b) The SLAC Bag

The dynamical structure of the SLAC bag [41] is derived from the 2-dimensional model mentioned above. This model is considered in 3+1 dimensions. The semiclassical field theory has finite-size "bubble" ("bag") solutions with quarks confined to the skin of the bubble. The bag with N quarks has size

$$R_N = N^{1/3} R_o, \quad R_o = \frac{\lambda^{1/6}}{2m} \qquad (1.7)$$

and energy

$$E_N = \frac{N^{2/3}}{R_o} \qquad (1.8)$$

In this model the ground state mesons and baryons are composed of colour singlet $q\bar{q}$ and qqq states with all quarks having $j = \frac{1}{2}$. They form a degenerate 35-plet and a degenerate 56-plet respectively, under SU(6), with

$$\frac{R(35)}{R(56)} = \left(\frac{2}{3} \right)^{1/3} \text{ and } \frac{M(35)}{M(56)} = \left(\frac{2}{3} \right)^{2/3} \qquad (1.9)$$

where M stands for mass of the hadron. In the SLAC bag model the confinement is only approximate.

Assuming different potential functions $V(\varphi)$ for the Higgs field, different types of bag structures can be produced. Using a potential with a meta stable vacuum first suggested by

Vinciarelli [42] and applying an appropriate limit procedure it can be shown [43] that the phenomenological **MIT** bag [44] is a limit of a field theoretic bag.

(c) Dual Strings

Introduction of a scalar Higgs field $\varphi(x)$ transforms $\mathcal{L}_{QCD}$ into the Higgs-Yang-Mills Lagrangian [45] which is an approximate model for describing strong coupling phenomena. Nielsen and Olesen [46] had noted the similarity of the Higgs-Yang-Mills Lagrangian to the Ginzberg-Landau Lagrangian for the phenomenological description of superconductivity [47]. This led them to suggest that there might be classical solutions corresponding to the vortex lines in type II superconductors. Nonabelian gauge theories might allow topologically stable solutions with trapped quarks. Solutions of this kind have already been shown to exist [45,46,48].

String-like solutions are significant, for they would provide a physical model for dual strings [49] and might show more easily colour charge screening [50] and hence confinement.

However, the problem of geometrical configuration of baryons in the string model has no satisfactory solution, except in a fat string model [51] incorporating aspects of both strings and bags, where excited baryons take a resonating pattern of qq-q string configurations.

(d) Lattice Models

Lattice field theories with strong coupling [49-51] provide another approach to the problem of quark confinement. Wilson [52] developed a gauge theory of quantized fields on a lattice using the Feynman path integral method of quantum mechanics [53]. A Hamiltonian formulation of Wilson's lattice gauge theory was given by Kogut and Susskind [54]. These studies seem to indicate that string like states in which quark-antiquark pairs are linked by gauge fields form energetically preferred configurations. However, the systematic transition to the continuum is problematic.

1.4.2 Phenomenological Models

A complete and consistent field theoretic description of the structure and properties of hadrons in the framework of QCD is the ultimate goal. But this continues to be a cherished dream of particle physicists, hopefully to be realized some day. Phenomenological models incorporating features of QCD alone can help until that day.

(a) Potential Models

Most of these models [55-60] treat the forces inside a hadron which bind quarks (and antiquarks) non-relativistically with lowest order relativistic corrections in some cases. Usually the potential has two parts: (i) the short-range QCD part plus relativistic corrections (Fermi-Breit interaction) involving spin-dependent terms, (ii) the long-range quark confining potential V(r). For a quantitative understanding of the energy levels of the system one will have to solve the Bethe-Salpeter or Schrodinger equation numerically. One of the well-known models of this type is the Rūjula, Georgi and Glashow model [55]. Non-relativistic potential models are particularly suited for heavy quark systems [61,62].

(b) Bag Models

The most popular and strikingly successful phenomenological model for hadrons, especially in the low mass regime, is the MIT bag model [44,63-66], which is the subject of the present investigation. The model has the unique merit that it allows absolute determination of hadronic properties like mass, magnetic moment, charge radius, axial vector coupling constant, decay amplitudes and so on. It nicely reproduces the masses of most of the light (S-wave) baryons and mesons. With regard to the other hadronic properties it is definitely an improvement over the naive quark model.

A variant of the MIT bag model is the MIT-Budapest model [66-68] whose mass predictions are not considerably different from those of the MIT model.

The next chapter contains a detailed discussion of the MIT bag model.

CHAPTER 2

THE MIT BAG MODEL

2.1 General Features

This is a relativistic colour-quark model developed by a team of theorists at the Massachusetts Institute of Technology (M.I.T.) to provide a phenomenological description of hadrons. The "bag" is a finite region of space to which quark and gluon fields are confined. Confinement is introduced by hand rather than deduced from basic principles of QCD. This is accomplished by adding a Lorentz covariant term $g_{\mu\nu} B$ to the usual stress tensor $T_{\mu\nu}$ of the quark and gluon fields. Dynamically B is a pressure; it has the dimensions of energy density and is referred to as bag pressure or volume tension. Inside the bag the quarks and gluons behave like a quasi-free quantum gas exerting pressure to the outside which is balanced by the uniform confining pressure B exerted by the vacuum. B sets the scale for confinement phenomena. Its phenomenological value $B \simeq 59$ MeV/fm^3 (or $B^{1/4} \simeq 145$ MeV) determines the masses of light hadrons, the universal slope of Regge trajectories and the Hagedorn temperature. As is generally believed if the bag model emerges as an approximation to QCD then B itself is presumably determined from the underlying theory.

The bag model assigns two distinct phases to the hadronic world: (i) the vacuum which expels quarks and gluons and (ii) the hadrons wherein quarks and gluons move more or less freely. The system may be likened to a (perfect) liquid under constant pressure at the boiling point. The vacuum is the liquid, the bag a bubble and B the latent heat (liberated when a hadron is returned to vacuum). A more interesting analogy [69] which has been the basis of several attempts to derive the bag model from QCD [70] is the Meissner effect in a bulk superconductor with $\vec{E}$ and $\vec{B}$ reversed. The QCD vacuum, like the superconductor is permeated by complex nonperturbative field configurations. Just as the superconductor expels magnetic flux, it expels colour electric flux. On the boundary between a normal region and a superconductor $\vec{E}$ and $\vec{B}$ obey

$$\begin{aligned} \vec{n} \times \vec{E} &= 0 \\ \vec{n} \cdot \vec{B} &= 0 \end{aligned} \tag{2.1}$$

By analogy, at the boundary of a hadron

$$\begin{aligned} \vec{n} \cdot \vec{E} &= 0 \\ \vec{n} \times \vec{B} &= 0 \end{aligned} \tag{2.2}$$

where $\vec{n}$ represents the normal to the surface and a = 1, 2, ...8 is a colour index. Eq. (2.2) written in the covariant form

$$n_\mu F_a^{\mu\nu} = 0 \tag{2.3}$$

is the well-known bag boundary condition. Thus in this picture a hadron is a normal region and the outside world a vast superconductor.

A very interesting and significant feature of the bag model is that the bag is a colour singlet. The boundary condition which ensures confinement demands that no quark or gluon current crosses the bag boundary. Now an extension of the Gauss theorem in electrostatics

$$Q = \frac{1}{4\pi}\int \vec{\nabla}\cdot\vec{E}\, dV = \int \vec{E}\cdot d\vec{s} \tag{2.4}$$

to the chromodynamic case, implies that the confined system has zero colour charge. Hence the introduction of the confining pressure B that counterbalances the flow of colour flux automatically leads to the colour neutrality of the system.

Regarding the chiral property of the bag, the original bag model does not incorporate chiral symmetry or rather its Nambu-Goldstone realisation [71] which is expected of any reasonable, complete phenomenological model for low energy hadron dynamics. Recently there have been attempts to construct "hybrid chiral bags" in which independent Goldstone modes are introduced into the model by hand to implement the symmetry [72].

2.2 Formulations of the Bag Model

2.2.1 The Traditional Model

The bag model is characterised by the energy-momentum tensor

$$T^{\mu\nu}_{bag} = [T^{\mu\nu} - g^{\mu\nu} B]\Theta_V \qquad (2.5)$$

where $T^{\mu\nu}$ represents the usual stress tensor of the field theory and B is the bag pressure, $\Theta_V(x)$ is unity inside the bag and zero outside. $T^{\mu\nu}$ is the energy-momentum tensor for quarks and gluons described by $\mathcal{L}_{QCD}$ in Eq. (1.2):

$$T^{\mu\nu} = \frac{1}{2}\bar{\Psi}\gamma^{\mu}\overleftrightarrow{D}^{\nu}\Psi - G^{a\mu\rho}\partial^{\nu}A^{a}_{\rho} + [-\frac{1}{4}G^{a}_{\rho\sigma}G^{a\rho\sigma}]g^{\mu\nu} \qquad (2.6)$$

where $G^{a}_{\mu\nu}$ is the usual Yang-Mills field strength tensor given by Eq. (1.3) and Ψ is the quark field. Translation invariance requires the conservation of the energy-momentum tensor:

$$\partial_{\mu}T^{\mu\nu}_{bag} = [\partial_{\mu}T^{\mu\nu}]\Theta_V + [n_{\mu}T^{\mu\nu} - Bn^{\nu}]\delta_V = 0 \qquad (2.7)$$

in which δ_V is the derivative of θ_V

$$\partial^\mu \theta_V = n^\mu \delta_V \qquad (2.8)$$

where n^μ is the interior, covariant unit normal to the bag boundary S. Eq. (2.7) gives the equations of motion for the fermion and vector fields inside the bag:

$$(\not\partial + m)\Psi = ig\gamma^\mu F^a \Psi A^a_\mu , \qquad (2.9)$$

and

$$-\partial_\mu G^{a\mu\nu} = ig\overline{\Psi}\gamma^\nu F^a \Psi + gf^{abc} A^b_\mu G^{c\mu\nu} \qquad (2.10)$$

where F^a are given by the eight 3x3 Gell-Mann matrices λ^a:

$$F^a = \frac{\lambda^a}{2} \qquad (2.11)$$

From Eq. (2.7) we also get a set of gauge invariant boundary conditions:

$$i\not n \Psi = \Psi \qquad (2.12a)$$

$$n_\mu G^{\mu\nu}_a = 0 \qquad (2.12b)$$

$$-\frac{1}{4}\sum_a G_{\mu\nu\, a} G^{\mu\nu}_a + \frac{1}{2} n^\mu \partial_\mu (\overline{\Psi}\Psi) = B \qquad (2.12c)$$

In Eq.(2.12c), $\frac{1}{2} n^{\mu}\partial_{\mu}(\bar{\Psi}\Psi)$ represents the quark pressure and $-\frac{1}{4}\sum_{a} G_{\mu\nu a} G^{\mu\nu}_{a} = +\frac{1}{2}\sum_{a}(\vec{E}_a^{\,2} - \vec{B}_a^{\,2})$ represents the gluon pressure. Thus the equation balances the field pressure against the confining pressure B and ensures the stability of the bag. The quark and gluon fields are completely determined by Eqs.(2.9) and (2.10) with the linear boundary conditions (2.12a) and (2.12b).

2.2.2 Lagrangian Formulations

One can think of formulating a bag model for confined quarks by modifying the conventional Dirac Lagrangian by introducing a pressure term B:

$$\mathcal{L} = \mathcal{L}_0 - B = \tfrac{1}{2}\bar{\Psi}\gamma^{\mu}\partial_{\mu}\Psi - \tfrac{1}{2}(\partial_{\mu}\bar{\Psi})\gamma^{\mu}\Psi - B \qquad (2.13)$$

This leads to equation of motion for quark fields and bag boundary conditions inconsistent with each other unless B = 0. However, Chodos and Thorn [73] could develop a Lagrangian formulation of the bag employing surface terms and Lagrange multipliers. The theory is described by the action

$$\begin{aligned} W &= \int_V d^4x\, \mathcal{L} \\ &= \int_V d^4x\, \left(\tfrac{1}{2}\bar{\Psi}\gamma_{\mu}\partial^{\mu}\Psi - \tfrac{1}{2}(\partial^{\mu}\bar{\Psi})\gamma_{\nu}\Psi - B\right) \\ &\qquad - \int_S d^3x\, \bar{\Psi}\Psi\lambda \end{aligned} \qquad (2.14)$$

where $\lambda \equiv \lambda(x)$ is a Lagrange multiplier field defined only on S. V is the 4-dimensional space-time volume swept out by the bag and S is its 3-dimensional boundary. The surface term generates the constraint $\bar{\Psi}\Psi = 0$ on S. As usual equations of motion and boundary conditions are obtained by the variational method. The equation of motion is the Dirac equation in V. The following are the boundary conditions:

$$\tfrac{1}{2}\not{n}\Psi = \lambda\Psi \quad \text{on S} \tag{2.15}$$

$$\bar{\Psi}\Psi = 0 \quad \text{on S} \tag{2.16}$$

and $$n_\mu \partial^\mu(\lambda\bar{\Psi}\Psi) + \mathcal{L}_0 - B = 0 \text{ on S} \tag{2.17}$$

By virtue of Dirac's equation $\mathcal{L}_0 = 0$. Squaring (2.15) one gets $\lambda^2 = \frac{1}{4}$ or $\lambda = \pm\frac{1}{2}$. With $\lambda = \frac{1}{2}$, **Eqs.** (2.15) and (2.17) become the familiar bag boundary conditions.

Another Lagrangian formulation of the bag was proposed by Johnson [74] on physical grounds. Defining the action

$$W = \int d^4x\, \theta(\bar{\Psi}\Psi)[\delta\bar{\Psi}\gamma^\mu\partial_\mu\Psi - \tfrac{1}{2}(\partial_\mu\bar{\Psi})\gamma^\mu\Psi$$

$$-m\bar{\Psi}\Psi - B] \tag{2.18}$$

where θ is now a function of the field variables, and requiring it to be stationary under variations in Ψ, $\bar{\Psi}$ etc., the usual bag boundary conditions for Dirac fields have been shown to emerge.

Starting with a local field theory employing a complex scalar field to represent the quark, Creutz [75] demonstrated the emergence of one version of the bag model in a limit at which quarks got confined in a finite spatial region surrounded by a "skin". Later, the work was extended to include the confinement of Fermi and vector fields in the bag [76], but the conclusion arrived at was that colour non-singlet states could not be eliminated from the theory.

2.3 The Spherical Bag Approximation

The bag model [44] is defined by equations of motion and boundary conditions. The motion of the fields and that of the surface are determined by these equations. But it is difficult to find a general solution to the problem. However, approximate solutions for static spherical boundaries have been found by Chodos et al. [63] for the case of massless quarks and by De Grand et al. [64] for the more general case allowing for non-zero quark mass. In this "cavity" approximation to a hadronic bag, quarks are supposed to occupy the lowest cavity eigen mode for the free Dirac field.

The equation of motion and the boundary conditions for the quark fields in a spherical bag of radius R are

$$(-i\vec{\gamma}\cdot\vec{\nabla} + \gamma_0\omega)\psi_i = 0 \quad \text{inside the bag,} \tag{2.19}$$

$$\hat{r}\cdot\vec{\gamma}\,\psi_i = \psi_i \quad \text{at } r = R \tag{2.20}$$

$$-\frac{\partial}{\partial r}(\bar{\psi}_i\psi_i) = 2B \quad \text{at } r = R \tag{2.21}$$

where i labels the quarks in the bag.

The only static classical solutions of the Dirac equation in a spherical bag satisfying Eqs. (1.26) - (1.28) are the $j = \frac{1}{2}$ solutions.

For massless quarks, with $m_q = 0$ in the Dirac equation, one has two $j = \frac{1}{2}$ solutions $\psi^{(n)}_{j\kappa m}$ corresponding to the two values of the Dirac quantum number $\kappa = -1, +1$:

$$\psi^{(n)}_{\frac{1}{2},-1} = \frac{N(x_{n,-1})}{\sqrt{4\pi}} \begin{pmatrix} i j_0(x_{n,-1}\frac{r}{R})u \\ -j_1(x_{n,-1}\frac{r}{R})\,\vec{\sigma}\cdot\hat{r}u \end{pmatrix} \tag{2.22}$$

$$\psi^{(n)}_{\frac{1}{2},1} = \frac{N(x_{n,1})}{\sqrt{4\pi}} \begin{pmatrix} i j_1(x_{n,1}\frac{r}{R})\vec{\sigma}\cdot\hat{r}u \\ j_0(x_{n,1}\frac{r}{R})u \end{pmatrix} \tag{2.23}$$

where j_0 and j_1 are spherical Bessal functions, u is a two-component Pauli spinor, and the normalisation constant is

$$N(x) = [x^3/(2R^3(x+\kappa)\sin^2 x)]^{\frac{1}{2}} \qquad (2.24)$$

in which x stands for $x_{n,\kappa}$. The linear boundary condition (LBC) (2.20) yields an eigenvalue condition for the mode frequencies $\omega_{n,\kappa}$:

$$j_0(x) = -\kappa\, j_1(x)$$

$$\text{or} \quad \tan x = \frac{\kappa x}{x+\kappa} \qquad (2.25)$$

where the momenta x are related to the mode frequencies by

$$\omega_{n,\kappa} = x_{n,\kappa}/R \qquad (2.26)$$

The modes are specified by n = 1, 2, ... The first few solutions of (2.25) are given by

$$\begin{aligned} \kappa = -1 &: \quad x_{1,-1} = 2.04 \ ; \quad x_{2,-1} = 5.40 \\ \kappa = +1 &: \quad x_{1,1} = 3.81 \ ; \quad x_{2,1} = 7.00 \end{aligned} \qquad (2.27)$$

For massive quarks the lowest cavity eigen state with $j = \frac{1}{2}$ is given by

$$\Psi_{\frac{1}{2},-1} = \frac{N(x)}{\sqrt{4\pi}} \begin{pmatrix} \left(\frac{\omega+m}{\omega}\right)^{\frac{1}{2}} i j_0(xr/R)u \\ -\left(\frac{\omega-m}{\omega}\right)^{\frac{1}{2}} j_1(xr/R)\vec{\sigma}\cdot\hat{r}u \end{pmatrix} \qquad (2.28)$$

in which x stands for $x_{n,-1}$, m is the quark mass, and the normalisation constant N is given by the condition

$$\int_{bag} d^3r\, \Psi^+ \Psi = 1 \qquad (2.29)$$

to be

$$N(x) = \frac{1}{R j_0(x)} \left[\frac{\omega\ (\omega - m)}{2\omega(\omega R - 1) + m} \right]^{\frac{1}{2}} \qquad (2.30)$$

The LBC leads to the constraint

$$j_1(x) = [(\omega + m)/(\omega - m)]^{\frac{1}{2}}\ j_0(x) \qquad (2.31)$$

which generates the transcendental equation for x = x(m,R):

$$\tan x = x/[1 - mR - (m^2R^2 + x^2)^{\frac{1}{2}}] \qquad (2.32)$$

The frequency of the lowest mode (which is numerically equal to the energy of the quark occupying this mode) is given by

$$\omega(m,R) = \frac{1}{R}\ [x^2 + m^2R^2]^{\frac{1}{2}} \qquad (2.33)$$

Each occupied quark mode of mass m in a cavity of radius R contributes a term $\omega(m,R)$ to the energy of the system. As $m \to \infty$, $x(m,R) \to \pi$. This is the value of x in the non-relativistic limit. Also as $R \to \infty$, $\omega \to m$, so that ω can be thought of as the effective mass of the confined quark.

2.4 Masses of Light Hadrons

We shall now consider the spectrum of light baryons and mesons. The quark content of the lowest baryon states is qqq while that of meson states is $q\bar{q}$. The hadron states are classified by the representations of the flavour group which is taken as SU(3) for the non-charm sector. A hadron with static spherical boundary has its interior populated with quark orbitals in colour singlet states.

In the cavity approximation to the bag model [64] the mass of a hadron is a function of R and is a sum of four terms.

$$M(R) = E_q + E_V + E_0 + \Delta E \tag{2.34}$$

The first term is the quark kinetic and rest energy. For a quark with mass m it is given by (2.33). For a hadron composed of N quarks/antiquarks

$$E_q = \sum_{i=1}^{N} \omega(m_i R) \tag{2.35}$$

The second and third terms in (2.34) are consequences of doing field theory in a finite domain. E_V is the energy associated with the confining pressure B. It is referred to as the volume energy and is given by

$$E_V = \frac{4}{3}\pi R^3 B \tag{2.36}$$

E_0 is a phenomenological estimate of the quantum effects associated with fields confined to a finite region of space. It is the well-known zero-point energy of the confined fields. This is assumed to be a negative contribution to the total energy. It thus corresponds to an attractive Casimir stress. It has been shown [64] that the zero-point energy consists of two parts: an infinite term proportional to the volume and a finite part proportional to 1/R. The former is absorbed into a renormalisation of B while the latter is parametrised by including a term

$$E_0 = -Z/R \tag{2.37}$$

in the mass operator, where phenomenology requires Z to be ~ 2.

The last term in Eq. (2.34) is the interaction energy of the quarks arising from their coupling to coloured gluons. This has been estimated [64] to lowest order in the strong interaction coupling constant $\alpha_c = g^2/4\pi$. The quark-gluon interaction has the effect of lifting the degeneracies of the model, in particular, splitting the nucleon from the Δ-resonance and the ρ from the π.

To lowest order in α_c the nonabelian self-coupling does not contribute. The gluons act as eight independent abelian fields. The problem then reduces to one of ordinary electromagnetism with the boundary conditions

$$\hat{r} \cdot \vec{E}^{\,a} = 0 \tag{2.38}$$

$$\hat{r} \times \vec{B}^{\,a} = 0 \tag{2.39}$$

on the bag surface, where $\vec{E}^{\,a}$ and $\vec{B}^{\,a}$ are the colour electric and colour magnetic fields, "a" being the colour index which runs from 1 to 8. The electrostatic interaction energy of a static charge distribution is given by

$$E_e = \tfrac{1}{2}\, g^2 \sum_a{}' \int_{bag} d^3x\, \vec{E}^{\,a}(x) \cdot \vec{E}^{\,a}(x) \tag{2.40}$$

Similarly the magnetostatic interaction energy

$$E_m = -\tfrac{1}{2} g^2 \sum_a \int_{bag} d^3x\, \vec{B}^{\,a}(x) \cdot \vec{B}^{\,a}(x) \tag{2.41}$$

where $g^2 = 4\pi\alpha_c$

The colour magnetic field must satisfy

$$\vec{\nabla} \times \vec{B}_i^{\,a} = \vec{j}_i^{\,a} \qquad r < R \tag{2.42a}$$

$$\vec{\nabla} \cdot \vec{B}_i^{\,a} = 0 \qquad r < R \tag{2.42b}$$

$$\hat{r} \times \sum_i \vec{B}_i^{\,a} = 0 \qquad r = R \tag{2.42c}$$

where $\vec{j}_i^{\,a}$ is the colour current of the i^{th} quark. The colour magnetic field $\vec{B}_i^{\,a}$ generated by the i^{th} quark is obtained by solving (2.42). Substituting this in (2.41)

$$E_m = -3\alpha_c \sum_a \sum_{i>j} (\vec{\sigma}_i \lambda_i^a) \cdot (\vec{\sigma}_j \lambda_j^a)$$

$$\mu(m_i R)\mu(m_j R) \cdot I(m_i R, m_j R)/R^3 \qquad (2.43)$$

where $\vec{\sigma}_i$ and $\vec{\sigma}_j$ are the spin vectors of the quarks i and j; λ^a the Gell-Mann matrices;

$$I(m_i R, m_j R) = 1 + (x_i \sin^2 x_i - 1.5y_i)^{-1}(x_j \sin^2 x_j - 1.5y_j)^{-1}$$

$$\cdot\Big\{-1.5y_i y_j - 2x_i x_j \sin^2 x_i \sin^2 x_j$$

$$+ 0.5x_i x_j [2x_i \, Si(2x_i) + 2x_j \, Si(2x_j)$$

$$-(x_i + x_j)\, Si(2(x_i + x_j))$$

$$-(x_i - x_j)\, Si\,(2(x_i - x_j))]\Big\} \qquad (2.44)$$

with $y_i = x_i - \sin x_i \cos x_i$, x_i being the root of Eq. (2.32) for a given $m_i R$, and

$$\mu(mR) = (R/6)(4\omega R + 2mR-3)/(2\omega R(\omega R-1) + mR) \qquad (2.45)$$

The colour electric interaction energy E_e can also be similarly estimated, but it is found to be insignificant in the case of light hadrons where the quark mass differences are not large.

Substituting for the individual terms in Eq. (2.34) we finally get the hadron mass in terms of the bag radius R and other parameters of the model. The quadratic boundary condition (2.21) now enables us to fix R. This non-linear boundary condition (NLBC) requires that the outward pressure of the fields inside the bag balances the external pressure B locally on the bag's surface ensuring the stability of the bag. For static spherical bags this stability condition is equivalent to minimising the mass M(R) with respect to R. Thus the actual hadron size R_o is determined by $\partial M/\partial R = 0$ and the mass is given by $M(R_o)$.

The mass formula (2.34) is applied to the light baryon and meson spectra (non-charm sector). The parameters of the theory, namely, B, Z, α_c and the strange quark mass m_s (the non-strange quark mass m_n is assumed to be zero) have been determined from the known masses of the particles N, Δ, Ω and ω [64]. They are $B^{1/4} = 0.145$ GeV, Z = 1.84, $\alpha_c = 0.55$ and $m_s = 0.279$ GeV. The results for the baryons and the vector mesons are good, but not so for the pseudoscalars, particularly the predicted pion mass is twice the observed mass.

2.5 Other Static Parameters of the Light Hadrons

Besides hadronic masses, other static properties such as magnetic moments of baryons, axial vector coupling constants and charge radii of the nucleons have also been computed in the bag model [64,77,78]. The predictions for magnetic moments are not quite good. The proton magnetic moment $\mu(P) = 1.9$ nuclear magnetons while its experimental value is 2.7 units. The axial vector coupling constant g_A for neutron β decay has been estimated to be 1.09 whereas its experimental value is 1.25. The charge radii of the nucleons and the pion have been predicted to be consistently lower than the corresponding experimental numbers. The reason for the reduced values of magnetic moments and charge radii might be that the bag size is not large enough.

2.6 Exotic Hadrons

The bag model predicts the existence of exotic hadrons such as glueballs and multiquark states [66,79,80]. Multiquark states are hadronic resonances with the quark configuration $q^m\bar{q}^n$ $(m+n > 3)$. Glueballs are all-glue hadrons. Experimental evidences for such rare kinds of hadronic matter are coming up. The bag model is expected to provide reliable estimates of the masses of these unconventional states.

Excited states of conventional low mass hadrons have also been studied in the bag model [81-83]. It is found that with the spherically symmetric bag the excitation energy is inadequate to account for the observed spectrum. Further, the predicted spectrum is marred by the presence of a large number of spurious states corresponding to the translational modes of the bag. However, considering deviations from the spherical symmetry of the bag in lowest order and applying a centre of mass correction to remove spurious states it has been possible to obtain an excited baryon spectrum belonging to a 70 of SU(6)

2.7 Recent Developments

Recently, Milton has made a detailed study [85,86] of the problem of zero-point energy of confined quarks and gluons. He has found that for the quark-gluon bag it is a positive contribution to the mass rather than a negative one as with the original MIT model. The phenomenological implications of this are yet to be tested.

Another notable development during the past two years is the incorporation of chiral symmetry in the bag model [72,87,88] to which we have already alluded in Sec.2.1.

There has been some significant progress in the past two-three years towards the field theoretic foundation of the bag. Particularly there is now a serious attempt [89-94] to derive the bag picture from QCD. It is hoped that the gap between the QCD Lagrangian and the rather successful bag phenomenology will be eventually filled.

The principal assumption in developing the phenomenological bag picture from QCD is to visualise the QCD vacuum as a perfect (or nearly perfect) dielectric substance [66,95,96] and paramagnetic medium in its response to colour gauge fields. By concentration of energy, a small domain (bubble or bag) in normal vacuum phase ($\varepsilon = 1$) may be created in the medium of QCD vacuum where $\varepsilon \rightarrow 0$. Inside the bag, ε is nearly unity, and the quark and the gluon fields behave according to standard perturbative QCD.

CHAPTER 3

A BAG MODEL STUDY OF CHARMED MESONS

3.1 Introduction

The discovery of the narrow resonances J/ψ (3.1) in 1974 [6,7] was a remarkable event in high energy physics. This was followed by the observation of the other members of the "ψ family" in a relatively short period. These heavy hadrons with extremely narrow widths were interpreted as bound $c\bar{c}$ systems, c being the "charmed" quark whose existence was predicted earlier on theoretical grounds [12] and $\bar{c}$ the corresponding antiquark. Their extreme narrowness or long lives implied highly suppressed decays to ordinary light hadrons with no charm content. However, direct experimental evidence for the existence of particles carrying "charm" was lacking until the charmed D^0 and D^+ mesons were discovered in 1976 by the SLAC-LBL group [16,17]. These are narrow resonances observed near 1.87 GeV in e^+e^- annihilation experiments, coupled predominantly to weak hadronic decay channels. Further, evidence for the existence of a meson having both charm and strangeness, called F, became available [97,98] hardly a year after the discovery of the D's.

With a view to exploring the phenomenological content of the MIT bag model in applications falling outside the low mass hadron world we propose to attempt a bag model study of the charmed mesons. The model in its spherical cavity approximation has been quite successful in predicting several of the static properties of light hadrons in reasonable agreement with experimental observations [63,64,99,100]. The model has also been applied with some success to the study of weak nonleptonic decays of baryons and mesons [101,102] and the radiative decays of some of the vector mesons [103], although the bag amplitudes for electromagnetic and weak leptonic decays [104] have not been in good agreement with experiment. In view of the relative simplicity of the ideas on which the bag model is based, its successes should be regarded as remarkable. Clearly the situation warrants efforts to extend the application of the model to the charm regime which serve two purposes:

(i) To enlarge the scope of the model by testing its validity beyond the three-flavour sector of hadron spectroscopy.

(ii) To understand the new hadrons in terms of this model.

We are thus motivated to carry out the work [105] presented in this chapter and the next. In the present chapter we are concerned with the masses of the charmed pseudoscalar mesons: D^0, D^+ and F^+, and their vector counterparts: D^{*0}, D^{*+}, F^{*+}. Our mass predictions are in substantial improvement over earlier estimates [106] and in good agreement with experimental values. The pseudoscalar vector mass splittings are, however, poorly predicted.

The material presented in this chapter is a revised and extended version of Ref.[105]. The present investigation differs from Ref.[105] in two respects:

(i) The study, which was restricted to D mesons only is now extended to cover the mass spectrum of the entire low lying charmed mesons.

(ii) The nonstrange quark mass which was taken to be zero in the earlier work is now given a non-zero value.

With the availability, after the publication of Ref.[105], of confirmed experimental results on F, D* and F* mesons, the extension of the study became necessary. Better overall agreement of the mass predictions with the observed masses has been obtained for the new choice of the nonstrange quark mass.

3.2 Charmed Mesons

The charmed D mesons were first observed by the SLAC-LBL collaboration [16,17] in e^+e^- annihilation at center-of-mass energies 3.9 to 4.6 GeV. Subsequently they were detected in neutrino [106-108], hadron [109] and photon [110-112]-induced reactions. The invariant mass spectra for the sum of all observed D^0 and D^+ decay modes show peaks at 1865 and at 1876 MeV respectively. The D's were produced primarily in association with the D^*'s.

The F^+ and F^{*+} were discovered by the DASP collaboration [97] at DESY in e^+e^- annihilation at c.m. energy 4.414 GeV. From events containing a charged pion, and η and a low energy photon the F and F* masses were found to be 2.03 ± 0.06 and 2.14 ± 0.06 GeV respectively. All of these observations were made at the peaks in the annihilation cross section: 3.772, 4.028, 4.16 and 4.414 GeV which are charmonium resonances above threshold [113]. Even before the discovery of the charmed hadrons, an elaborate SU(4) classification of charmed mesonic and baryonic states and their possible decay modes were worked out by Gaillard, Lee and Rosner [114] by extending the familiar notions of the colour triplet quark model to the four-quark scheme of Glashow, Iliopoulos and Maiani [12].

A charmed meson is composed of a charmed quark (c) and an ordinary light antiquark ($\bar{q}$) forming a spin singlet or a triplet. The D^+ and D^0 form an isospin doublet and the F meson an isospin singlet. They have $J^P = 0^-$. The D^{*+}, D^{*0} and F^{*+} have $J^P = 1^-$. The presence of a fourth quark c besides the familiar u, d, and s quarks implies that the SU(3) nonet of 8+1 mesons will be replaced by a hexadecimet of 15+1 states. The SU(4) multiplet 15 contains, in addition to the ordinary SU(3) resonances with c = 0, six states with open charm, c = $\pm$ 1. Thus in the pseudoscalar case we have

$$c = +1 : \quad D^+, D^0, F^+$$

$$c = -1 : \quad D^-, \bar{D}^0, F^-$$

and in the vector case

$$c = +1 : \quad D^{*+}, D^{*0}, F^{*+}$$

$$c = -1 : \quad D^{*-}, \bar{D}^{*0}, F^{*-}$$

States with hidden charm (c = 0), namely the J/ψ and the η_c [115] in the vector and pseudoscalar cases respectively are, however, excluded from the present investigation.

The masses and quantum numbers of the charmed mesons together with their quark contents are presented in Table 3.1.

Table 3.1 The 0^- and 1^- charmed mesons

Particle	Quark content	J^P	I, I_3	S	Mass (MeV)
D^+	$c\bar{d}$	0^-	$\frac{1}{2}, \frac{1}{2}$	0	1868.3 ± 0.9
D^0	$c\bar{u}$	0^-	$\frac{1}{2}, -\frac{1}{2}$	0	1863.1 ± 0.9
F^+	$c\bar{s}$	0^-	0, 0	+1	2039.5 ± 60
D^{*+}	$c\bar{d}$	1^-	$\frac{1}{2}, \frac{1}{2}$	0	2008.6 ± 1.5
D^{*0}	$c\bar{u}$	1^-	$\frac{1}{2}, -\frac{1}{2}$	0	2006.0 ± 1.0
F^{*+}	$c\bar{s}$	1^-	0, 0	+1	2140.0 ± 60

3.3 The Charmed Bag

Here we consider the parameters of the hadron bag containing a charmed quark c and a nonstrange antiquark $\bar{u}$ or $\bar{d}$ forming the D^0 or D^+ meson. The bag is assumed to be a fixed sphere of radius R (cavity approximation). The field equations and the bag boundary conditions determine the quark and antiquark wave functions [63,64]. For the lowest frequency mode we have

$$\psi(r,t) = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} i\alpha j_0\left(\frac{xr}{R}\right)u \\ \\ -\beta j_1\left(\frac{xr}{R}\right)\vec{\sigma}\cdot\hat{r}u \end{pmatrix} e^{-i\omega t/R} \qquad (3.1)$$

for quarks, and

$$\varphi(r,t) = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} -i\beta j_1\left(\frac{xr}{R}\right)\vec{\sigma}\cdot\hat{r}u \\ \\ \alpha j_0\left(\frac{xr}{R}\right)u \end{pmatrix} e^{i\omega t/R} \qquad (3.2)$$

for antiquarks, where

$$\alpha = [(\omega + m)/\omega]^{\frac{1}{2}}$$

$$\beta = [(\omega - m)/\omega]^{\frac{1}{2}}$$

The symbols have the same significance as explained in Ch.2.

The functions N, x and ω are given respectively by the equations (2.30), (2.32) and (2.33).

We choose all the bag parameters except the charmed quark mass m_c from Ref.64. In that paper results are presented for two different nonstrange quark masses: $m_n = 0$, and $m_n = 108$ MeV. However, we set $m_n = 110$ MeV. The strange quark is given a different mass with a view to breaking the SU(3) degeneracy. In the literature one comes across a wide spectrum of values for the strange quark mass ranging from 100 MeV to 500 MeV or even more. Our choice of the strange quark mass is $m_s = 300$ MeV, which is roughly the same as the MIT value. The bag pressure parameter B which determines the stability of the bag has been found to give best fit to the hadronic masses [64] for the choice $B^{1/4} = 145$ MeV. Following Gaillard, Lee and Rosner [114], and Donoghue and Golowich [116] we fix the charmed quark mass $m_c = 1.5$ GeV. The bag size is determined by minimizing the hadron mass with respect to the bag radius R, a procedure demanded by the quadratic bag boundary condition.

3.4 Charmed Meson Masses

The mass of a hadron of bag radius R is given by the equation

$$M(R) = E_v + E_q + E_o + E_m + E_e \qquad (3.3)$$

The various phenomenological contributions to $M(R)$ are the ones discussed in Ch.2 [Eqs. (2.35) - (2.37) and (2.43)]. The major contribution comes from the relativistic motion of the quarks in the cavity. This may be explicitly written as

$$E_q = N_n \omega(m_n R) + N_s \omega(m_s R) + N_c \omega(m_c R) \qquad (3.4)$$

where N_n, N_s and N_c are respectively the number of nonstrange, strange and charmed quarks/antiquarks contained in the hadronic bag in question. ω is the frequency defined by Eq. (2.33). E_m and E_e in Eq. (3.3), are the 'magnetic' and 'electric' quark-gluon interaction energies. The magnetic spin-spin interaction energy [See Eq. (2.43)] is given by

$$E_m = - \frac{3\alpha_c}{36R} \sum_a \sum_{i>j} (\vec{\sigma}_i \lambda_i^a) \cdot (\vec{\sigma}_j \lambda_j^a)$$

$$\times \mu'(m_i R) \mu'(m_j R) \cdot I(m_i R, m_j R) \qquad (3.5)$$

where

$$\mu'(mR) = \frac{4\omega R + 2mR - 3}{2\omega R(\omega R - 1) + mR} \qquad (3.6)$$

and $I(m_i R, m_j R)$ is a slowly varying function of $m_i R$ and $m_j R$ given by Eq. (2.44). The Casimir invariants $\lambda_i^a \lambda_j^a$ of the colour SU(3) for the colour triplet quarks q_k and colour antitriplet quarks $\bar{q}_1$ forming colour singlet hadrons are given by

$$\sum_a \lambda_i^a \lambda_j^a \ (i \neq j) = -16/3 \text{ for } q\bar{q} \text{ mesons} \qquad (3.7)$$

$$\sum_a \lambda_i^a \lambda_j^a \ (i \neq j) = -8/3 \text{ for } qqq \text{ baryons} \qquad (3.8)$$

Thus we have

$$E_m = \sum_{i>j} \lambda a_{ij} M_{ij} \qquad (3.9)$$

where $\lambda = 2$ for mesons and 1 for baryons. The coefficients a_{ij} are determined by the spin vectors $\vec{\sigma}_i$ and $\vec{\sigma}_j$ of the quarks i and j:

$$a_{ij} = \vec{\sigma}_i \cdot \vec{\sigma}_j \qquad (3.10)$$

and

$$M_{ij} = \frac{8\alpha_c}{36R} \mu'(m_i R)\mu'(m_j R) I(m_i R, m_j R) \qquad (3.11)$$

α_c is the colour coupling constant of quarks. For α_c, we choose the phenomenological value obtained by De Grand et al. [64], namely

$$\alpha_c = 0.55$$

The spin factors a_{ij} (i, j-flavour indices) for the cases of interest, namely, for the charmed pseudoscalar states with flavour-spin content: $(c\uparrow \bar{q}\downarrow)$, and the charmed vector states with flavour-spin content: $(c\uparrow \bar{q}\uparrow)$, are evaluated by noting that here we have two spin-$\frac{1}{2}$ particles with their spins anti-parallel in one case and parallel in the other. Considering, for example, the case of the D meson we have the quark c and the anti-quark $\bar{u}$ or $\bar{d}$ in a spin-0 state. Now the spin vectors

$$\vec{S}_1 = \tfrac{1}{2}\vec{\sigma}_1 \quad \text{and} \quad \vec{S}_2 = \tfrac{1}{2}\vec{\sigma}_2 \qquad (3.12)$$

add upto give

$$\vec{S}_1 + \vec{S}_2 = 0, \qquad (3.13)$$

so that from the identity

$$(\vec{S}_1 + \vec{S}_2)^2 = (\vec{S}_1)^2 + (\vec{S}_2)^2 + 2\vec{S}_1\cdot\vec{S}_2 \qquad (3.14)$$

it follows that

$$\vec{\sigma}_1\cdot\vec{\sigma}_2 = -3 \qquad (3.15)$$

The values of a_{ij} for the various states with specific non-strange (n), strange (s) and charm (c) contents are given in Table 3.2.

Knowledge of the bag radius R is now required for estimating the various contributions to the hadronic mass **M**. This is accomplished by following the standard procedure [64,117] of minimising **M** with respect to R in the zero quark mass limit. In this limit **M** can be expressed as an explicit function

Table 3.2 Values of a_{ij}

	D^+	D^0	F^+	D^{*+}	D^{*0}	F^{*+}
a_{cn}	-3	-3	0	+1	+1	0
a_{cs}	0	0	-3	0	0	+1

of R and the bag parameters:

$$M_k(R) = \frac{4}{3}\pi R^3 B + (Nx(0) - Z + a_k M_{oo})/R \qquad (3.16)$$

in which

N = Total number of quarks and anti-quarks in the hadron bag,

$x(0) = x(mR)$ in the limit $m \to 0$,

$M_{oo} = M_{ij}$ with $m = 0$

$$= \frac{8\alpha_c}{36R}[\mu'(0)]^2 \; I(0,0), \qquad (3.17)$$

$$a_k = \sum_{i>j} \lambda \, (a_{ij})_k \, , \qquad (3.18)$$

where the index k designates the hadron under consideration. Evidently a_k has the same value for all the 0^- states and a different value for the 1^- states. All other factors on the r.h.s. of Eq.(3.16) being identical for the entire class of charmed mesons considered, it follows that we have different bag radii for pseudoscalar and vector multiplets, while the individual members of either multiplet have the same size. Requiring

$$\frac{\partial M}{\partial R} = 0 \qquad (3.19)$$

we get

$$R_k = \left[\frac{Nx(0) - Z + a_k M_{oo}}{4\pi B} \right]^{1/4} \qquad (3.20)$$

The functions $x(mR)$ and $I(m_iR, m_jR)$ are readily evaluated in the limit of vanishing quark mass. They have the numerical values

$$x(0) = 2.04$$

$$I(0,0) = 1.44 \qquad (3.21)$$

Using these and the parameter values

$$Z = 1.84$$

$$B^{1/4} = 0.145 \text{ GeV} \qquad (3.22)$$

we get for the bag radii

$R_0 = 3.3 \text{ GeV}^{-1}$ for the 0^- mesons,

and $R_1 = 4.72 \text{ GeV}^{-1}$ for the 1^- mesons.

Knowing R, the functions $I(m_iR, m_jR)$ and hence M_{ij} are determined for all relevant sets of m_iR and m_jR values. These are listed in Tables 3.3 and 3.4.

Finally we come to the colour 'electric' interactions between quarks in the bag. With quarks of the same kind as in J/Ψ this contribution is zero. It was for this reason that this was not taken into account in computing the bag size which was done in the zero quark mass limit. For quarks of different masses the interaction energy has been determined [64] to be

$$E_e = -\frac{8\alpha_c}{3R}\left(\lambda \sum_{i>j} f(x_i, x_j) - \sum_i f(x_i, x_i)\right) \qquad (3.23)$$

Table 3.3 Values of $I(m_iR, m_jR)$ and M_{ij}, for $m_n = 0$, $m_s = 0.3$ GeV, $m_c = 1.5$ GeV

R (GeV^{-1})	m_iR	m_jR	$I(m_iR, m_jR)$	M_{ij}
	0	0	1.441	0.0788
	0.99	0	1.490	0.0679
3.3	4.95	0	1.593	0.0362
	4.95	0.99	1.669	0.0313

Table 3.4 Values of $I(m_iR, m_jR)$ and M_{ij}, for $m_n = 0.11$ GeV, $m_s = 0.3$ GeV, $m_c = 1.5$ GeV.

R (GeV^{-1})	m_iR	m_jR	$I(m_iR, m_jR)$	M_{ij}
3.3	4.95	0.363	1.6535	0.0353
3.3	4.95	0.990	1.6690	0.0317
4.72	7.08	0.519	1.6634	0.0182
4.72	7.08	1.416	1.7303	0.0160

where $\lambda = 2$ for mesons and 1 for baryons, and

$$f(x_i, x_j) = R \int_0^R \frac{dr}{r^2} \rho_i(r)\, \rho_j(r) \qquad (3.24)$$

$\rho_i(r)$ being the fraction of the quark charge density within a radius r and is a function of m_i, ω_i, x_i and r.

$$\rho_i(r) = \frac{\omega(x_i r - \sin^2 x_i r / x_i r) - m(\sin x_i r \cdot \cos x_i r - \sin^2 x_i r / x_i r)}{\omega(x_i - \sin^2 x_i / x_i) - m(\sin x_i \cos x_i - \sin^2 x_i / x_i)} \qquad (3.25)$$

The functions $f(x_i, x_j)$ have been evaluated numerically on a computer. These are displayed in Tables 3.5 and 3.6.

Putting together various contributions to the bag mass (Eq. 3.3), the masses of the charmed mesons are obtained. Our results are presented in Table 3.7. Experimental masses are given alongside for comparison. (Predicted masses are in the 8th column).

3.5 Discussion

Ref.[105], using the nonstrange quark mass $m_n = 0$, gave a somewhat good prediction for the D meson mass. M_D was found to be 1.805 GeV with the inclusion of a colour electric contribution of 165 MeV. This may be compared with an earlier

Table 3.5 Values of $f(x_i, x_j)$ for charmed pseudoscalar mesons. Parameters: $R = 3.3\ GeV^{-1}$, $m_n = 0.110$ GeV $m_s = 0.3$ GeV, $m_c = 1.5$ GeV.

m_i	ω_i	x_i	m_j	ω_j	x_j	$f(x_i, x_j)$
0.110	0.675	2.198	0.110	0.675	2.198	9.6426
0.110	0.675	2.198	1.500	1.731	2.850	8.8012
0.300	0.784	2.390	0.300	0.784	2.390	8.1865
0.300	0.784	2.390	1.500	1.731	2.850	8.1918
1.500	1.731	2.850	1.500	1.731	2.850	8.3661

Table 3.6 Values of $f(x_i, x_j)$ for charmed vector mesons.
Parameters: $R = 4.72\ GeV^{-1}$, $m_n = 0.110$ GeV, $m_s = 0.3$ GeV, $m_c = 1.5$ GeV.

m_i	ω_i	x_i	m_j	ω_j	x_j	$f(x_i, x_j)$
0.110	0.490	2.255	0.110	0.490	2.255	20.2517
0.110	0.490	2.255	1.500	1.623	2.930	18.9929
0.300	0.607	2.490	0.300	0.607	2.490	17.1517
0.300	0.607	2.490	1.500	1.623	2.930	17.6211
1.500	1.623	2.930	1.500	1.623	2.930	18.3403

Table 3.7 Masses of charmed mesons in the MIT bag model.

Parameters: m_n = 110 MeV, m_s = 300 MeV, m_c = 1500 MeV, $B^{1/4}$ = 145 MeV, Z = 1.84, α_c = 0.55 (All masses are in MeV).

Particle	$M_{Expt.}$	E_q	E_v	E_o	E_m	E_e	$M_{Theor.}$
D^+ D^o	1868 1863	2406	67	-557	-212	180	1884
F^+	2039±60	2515	67	-557	-190	75	1910
D^{*+} D^{*o}	2009 2006	2113	195	-390	36	188	2142
F^{*+}	2140±60	2230	195	-390	32	78	2145

bag model estimate of D meson mass by Szwed [118] employing the bag model with surface tension [68] which yielded a value as low as 1.564 GeV. However, with $m_n = 0$, the bag model predictions for the other charmed mesons turned out to be very bad. The new choice of the non-strange quark mass is consistent with the second set of parameters used by De Grand et al. in their pioneering work on the phenomenological bag model [64].

The colour electric interaction is found to yield a significant contribution to the charmed meson mass. It is to be noted that this part of the hadron mass is usually neglected in bag model calculations of masses of ordinary (light) hadrons for which this turns out to be negligibly small (< 5 MeV) as a result of the nearly equal quark masses. For charmed mesons the quark masses are substantially different from each other: $m_n \sim 0$ or 0.1 GeV, while $m_c \sim 1.5$ GeV, and it was conjectured in Ref.[64] that the electric contribution in such cases might be appreciable. Our computation has confirmed this speculation. That this contribution to M is quite sensitive to the difference in the masses of the constituent quarks is evident from the fact that magnitudes of E_e in the case of mesons with nonstrange-charm content are considerably larger than those in the case of mesons with strange-charm quark content (See Table 3.7).

The vector states are split from the pseudo scalar states by the magnetic coupling of gluons to quarks which contributes a spin-spin interaction to the bag energy operator. The colour magnetic interaction has the effect of reducing the energy content of a pseudoscalar bag while lifting up the energy of a vector bag. Thus the D-D* and F-F* mass splittings are in the right direction as observed experimentally. However, the quantitative agreement between the model predictions and the experimental values for the above mass splittings is not quite good.

The agreement between model prediction and experimental result is excellent in the case of D and F* masses while there is discrepancy of about 6-7% in the case of D* and F masses. The overall agreement between theory and experiment can be considered as good regarding the meson masses, while it is not so with the mass splittings.

The reason for this failure is not quite obvious. The nonstrange and charmed quarks have a wide mass separation. They may have to be treated differently as their velocities in the spherical cavity too may differ widely. The heavy quark may not be a truly relativistic object.

CHAPTER 4

WEAK NONLEPTONIC DECAYS OF D MESONS

4.1 Introduction

In this chapter we study [105] the consequences of the fixed sphere bag model when applied to the problem of weak non-leptonic decays of the well established pseudoscalar charmed mesons D^0 and D^+ for which experimental data are now available.

Many of the fundamental ideas about the masses and decay characteristics of the charmed mesons were predicted and discussed in great detail by Gaillard, Lee and Rosner [114] about a year before their actual experimental observation. According to them the D mesons decay primarily into ordinary (non-charmed) mesons through nonleptonic channels.

It is clear that if the charm quantum number is to be conserved by the strong and electromagnetic interactions in analogy with strangeness, then at least one of the new meson states should be stable against those interactions and so should decay weakly. In the conventional picture both D^0 and D^+ (and also F) were predicted to undergo weak decay because of their small electromagnetic mass splitting (less than a pion mass). Their narrow width and the existence of parity violation in the decay process provide confirmation of the theoretical prejudice that the decays proceed via weak interactions.

Besides their hadronic decay modes as reported by the authors of Refs. [119-121], it is now well known that these particles also have weak couplings to semileptonic decay channels [122-125]. The bag model has proved its efficacy in dealing with nonleptonic decays of ordinary hadrons [101,102] and its inadequacy to give predictions for weak semileptonic decays compatible with experimental results [104]. These points coupled with the fact that the primary interest of the present investigator is centred around studies of the mass spectrum of hadrons have restricted him to the study only of hadronic decays of D mesons.

The present study is further restricted to decays into final states containing two particles only, for reasons made explicit in Sec.4.5. The two-body final state hadronic decays of D mesons have been treated by a number of authors [114,126-128] on group theoretic considerations, arriving at relations connecting different decay amplitudes. These approaches cannot give predictions of absolute rates for specific decays. Quigg and

Rosner [129] have used a statistical model to estimate the relative branching ratios. Such a model is expected to give good results only in the limit of very high mass of the decaying particle and a large number of final state particles. The model predicts a higher charged decay multiplicity than is observed experimentally. Kaptanoglu [130] has studied hadronic D-decays utilizing PCAC with an extrapolation to physical region that takes final state interactions into account. He finds that final state interactions can give significant enhancements for some of the decay modes. Maiani [131] has calculated the two body decay rates of charmed mesons using the parton model. His predictions for the branching fractions for $D^0 \to K^-\pi^+$ and $D^+ \to K^0\pi^+$ are compatible with experimental results.

The MIT bag model allows explicit determination of absolute decay widths of specific channels. In the present investigation decay widths of a number of two body final state hadronic decays of D^0 and D^+ mesons are computed in the cavity approximation to the bag model [64] making use of PCAC and soft meson theorems. Reasonable agreement between theory and experiment has been obtained, with a four-quark current-current weak interaction Hamiltonian of the Weinberg-Salam - GIM type [12].

4.2 General Features of Weak Nonleptonic Decays

One of the significant features of weak nonleptonic decays is that these are affected by strong interactions with the result that study of these processes becomes a rather difficult problem. Renormalisation group techniques have been used [132-134] with some success to take account of the strong interaction effects. It was originally conjectured by Wilson [135] that strong interaction corrections would explain the empirical $\Delta I = \frac{1}{2}$ rule. With the advent of QCD as a theory for the strong interactions these corrections became calculable. QCD-based calculations [136,137] using asymptotic freedom arguments, however, have led to results which are in the right direction to explain the $\Delta I = \frac{1}{2}$ rule but which are much too small in magnitude.

Weak decay processes are studied using techniques of current algebra, according to which weak transitions arise from the self-coupling of a single charged V - A current. This coupling contains a purely hadronic piece:

$$\mathcal{L}_h = \frac{G}{\sqrt{2}} J^h_\mu (J^h_\mu)^+ \qquad (4.1)$$

For "old" hadronic decays, the Cabibbo theory [138] gives the interaction in (4.1) in terms of quark fields as

$$\mathcal{L}_h = \frac{G}{\sqrt{2}}[\cos^2\theta_c(\bar{u}d)(\bar{d}u) + (\sin^2\theta_c(\bar{u}s)(\bar{s}u)$$

$$+ \cos\theta_c \sin\theta_c\left\{(\bar{u}d)(\bar{s}u) + (\bar{u}s)(\bar{d}u)\right\}] \tag{4.2}$$

where explicit space-time dependence is suppressed.

Strangeness conserving ($\Delta s = 0$) transitions have amplitudes receiving contributions from two terms in Eq.(4.2):

$$(\bar{u}d)(\bar{d}u) : \quad \Delta s = 0, \quad |\Delta\vec{I}| = 0,2 \tag{4.3}$$

$$(\bar{u}s)(\bar{s}u) : \quad \Delta s = 0, \quad |\Delta\vec{I}| = 0,1 \tag{4.4}$$

From the relative strengths of the couplings of strange and nonstrange quarks it follows that

$$\frac{\text{Ampl.}(\Delta I = 1)}{\text{Ampl.}(\Delta I=0,2)} \simeq \tan^2\theta_c \tag{4.5}$$

Thus the $\Delta I = 1$ coupling, which contributes to pion exchange, is suppressed by a factor $\tan^2\theta_c$, over the $\Delta I = 0,2$ governing nucleon exchange. Coming to the experimental side one notes that strangeness conserving nonleptonic transitions are difficult to detect on the one hand, and on the other, attempts to interpret the results in the light of theoretical predictions are fraught with uncertainties arising from the influence of strong and electromagnetic interactions [139].

Strangeness changing decays are governed by the isospin selection rule:

$$(\bar{u}d)(\bar{s}u) + (\bar{u}s)(\bar{d}u): \qquad \Delta I = \frac{1}{2}, \frac{3}{2} \qquad (4.6)$$

The $\Delta I = \frac{1}{2}$ and 3/2 parts of the interaction have a priori comparable strength. But experimentally the $\Delta I = \frac{1}{2}$ amplitudes are by far dominant. Thus we see the appearance of an empirical selection rule for weak nonleptonic couplings:

$$\Delta I = \frac{1}{2} \qquad (4.7)$$

The situation clearly demands modification of the weak interaction derived from the self coupling of a charged current Eq.(4.1). The modifications suggested are all unacceptable for one reason or another. There is no a priori reason why one of them should be dominant. An alternative hypothesis therefore is that the primary weak coupling is indeed that of Eq.(4.6) but that due to some dynamical mechanism $\Delta I = \frac{1}{2}$ amplitudes are enhanced relative to those with $\Delta I = 3/2$.

The SU(3) transformation properties of the interaction (4.6) introduce constraints among observable amplitudes like the Lee-Sugawara relation [140-142]

$$2A(\Xi^- \rightarrow \Lambda\pi^0) + A(\Lambda \rightarrow n\pi^0) = \sqrt{2}A(\Sigma^+ \rightarrow P\pi^0)$$

which is found to be true experimentally within 10%. Group theoretically one predicts the $K_s \to \pi\pi$ as forbidden. Experimentally, however, the decay rate does not seem particularly suppressed, the K_s life time is comparable to hyperon lifetimes.

The implications of all these for the charmed meson decays will be considered in Sec.4.6.

Soft meson theorems which relate amplitudes such as

$$A \to B + \pi$$

to the amplitude

$$A \to B$$

are often used in the computation of nonleptonic decay amplitudes. Mostly the pion is taken in the limit of vanishing mass. In the case of heavy hadrons like the D mesons not only the pions, perhaps even the kaons can be considered as soft, as ratio of the K mass to the charmed particle mass is fairly small.

4.3 Hadronic D Decays : Basic Assumptions

The present investigation is based on the following assumptions.

(1) The hadronic D decays are parity violating.

This is supported by experimental evidence. The evidence for parity violation reminds one of the old τ-Θ

puzzle, where the two distinct decay modes were observed for **K**:

$$\theta^+ \rightarrow \pi^+ \pi^0 \quad \text{and} \quad \tau^+ \rightarrow \pi^+ \pi^- \pi^+$$

The experimental situation in the case of the D is very similar. A sharp enhancement around 1867 MeV is observed both in the $K_S^0 \pi^\pm$ [120] and in the $K^\mp \pi^\pm \pi^\pm$ spectrum [16]. The identical value of the mass for both these cases suggests that these are two decay modes of the same particle. The situation also leads to the spin-parity assignment of the charmed mesons under consideration, namely, $J^P = 0^-$.

(2) Possible effects arising from CP violation are ignored.

(3) The natural choice for a parity violating and CP conserving interaction is the simple phenomenological GIM model [12] based on the Weinberg-Salam theory of electro-weak interactions [143,144] in which weak hadronic currents are constructed out of four basic quark fields and interact with a single charged massive vector boson.

(4) Final state interactions are not taken into consideration in computing the decay amplitudes.

(5) The effects arising from $D^0 - \bar{D}^0$ mixing are neligible.

The $D^0 - \bar{D}^0$ mixing implies that a state which is initially a pure D^0 becomes a mixture of D^0 and $\bar{D}^0$ at later times. Considering the $D^0 - \bar{D}^0$ system as analogous to $K^0 - \bar{K}^0$ system, one finds that the ratio of the off diagonal to diagonal terms in the D^0, $\bar{D}^0$ mass matrix is $\sim \tan^4\theta \sim 10^{-3}$, so that the mixing effects are very small for the charm changing and strangeness changing ($\Delta s = \Delta c$) amplitudes [145]. In the limit of exact SU(3) the mixing vanishes altogether [127]. The $D^0 - \bar{D}^0$ mixing effect can also give rise to observable CP violating effects in analogy to the $K^0 - \bar{K}^0$ system. No experimental data are available on the subject and these effects are expected to be too small.

(6) SU(3) breaking effects are taken into account by the use of bag eigenfunctions for evaluating matrix elements.

(7) Our predictions do not take into account any kind of special enhancement effects such as those arising from the short distance behaviour imposed on the Hamiltonian [136,137]. The fact is that nobody knows the actual decay mechanism. Once it is known it is easy to probe the strength and precise form of the nonleptonic Hamiltonian. Strong interaction can still introduce modifications.

4.4 Interaction Hamiltonian and its Transformation Properties

In the GIM scheme [12] the weak hadron current is written

$$J^h_\mu = \bar{q} C_h \gamma_\mu (1 + \gamma_5) q \qquad (4.8)$$

where q is the four-component quark column vector (cuds) and the matrix

$$C_h = \begin{pmatrix} 0 & 0 & -\sin\theta_c & \cos\theta_c \\ 0 & 0 & \cos\theta_c & \sin\theta_c \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (4.9)$$

in which θ_c is the Cabibbo angle [138]. Consequently,

$$J^h_\mu = -\sin\theta_c\, \bar{c}\gamma_\mu(1+\gamma_5)d + \cos\theta_c\, \bar{u}\gamma_\mu(1+\gamma_5)d$$

$$+ \cos\theta_c\, \bar{c}\gamma_\mu(1+\gamma_5)s + \sin\theta_c\, \bar{u}\gamma_\mu(1+\gamma_5)s \qquad (4.10)$$

where u and d denote the "up" and "down" quarks, s the strange quark and c the charmed quark.

Here the following observation is in order. The minimal group for gauge theories of the weak and electromagnetic interactions is SU(2) x U(1); proposed by Weinberg and Salam [143,144]. Two models in this group have been remarkably successful in accounting for a large variety of phenomenology. The first is the GIM model [12] which has only left-handed currents. The other (discussed by various authors) [146-151] has both left and right-handed currents. No other SU(2) x U(1) model has proved to agree with experiment [152]. The second type of models has been described as "unnatural" for reasons connected with the mixing between quarks [153] (to avoid strangeness changing neutral currents). Further, we do not go for a six-quark model of the Kobayashi - Maskawa type (KM) [154] as we are assuming the D-decays to be CP invariant.

We take the non-leptonic Hamiltonian in the current-current form

$$H_W = -\frac{G}{\sqrt{2}} J^h_\mu (J^h_\mu)^+ \qquad (4.11)$$

We consider here parity-violating, charm-changing and strangeness-changing decays for which the following selection rules hold.

$$\Delta s = \Delta c = \pm 1, \text{ so that } \Delta s/\Delta c = +1$$

$$-\Delta s = \Delta c = 1, \text{ so that } \Delta s/\Delta c = -1 \qquad (4.12)$$

These when applied to (4.11) yield the following interactions

$$H_W^{\Delta s/\Delta c=1} = -\frac{G}{\sqrt{2}} \cos^2\theta_c \left\{ \bar{d}\gamma_\mu(1+\gamma_5)u\bar{c}\gamma_\mu(1+\gamma_5)s \right.$$

$$\left. + \bar{s}\gamma_\mu(1+\gamma_5)c\bar{u}\gamma_\mu(1+\gamma_5)d \right\} \qquad (4.13)$$

$$H_W^{\Delta s/\Delta c=-1} = \frac{G}{\sqrt{2}} \sin^2\theta_c \left\{ \bar{d}\gamma_\mu(1+\gamma_5)c\bar{u}\gamma_\mu(1+\gamma_5)s \right.$$

$$\left. + \bar{s}\gamma_\mu(1+\gamma_5)u\bar{c}\gamma_\mu(1+\gamma_5)d \right\} \qquad (4.14)$$

The Cabibbo angle θ_c being relatively small ($\sim 15^{\circ}$), the interaction (4.13) is Cabibbo-favoured while (4.14) is Cabibbo suppressed.

With only the left-handed currents the Hamiltonian (4.11) is evidently of the conventional (V - A) x (V - A) form. In SU(3) the flavour changing currents form an octet representation. Hence the Hamiltonian should have pieces transforming as the irreducible representations 1, 8, 10 and 27 contained in the direct product representation $8 \otimes 8$. Of these the flavour changing charged members of interest belong to the 8 and 27. In SU(4) the total hadronic current transforms as the direct product $4 \otimes \bar{4}$ which decomposes into $1 \oplus 15$. Hence the flavour changing current-current interaction should have the transformation property of

$$15 \otimes 15 = 1_S \oplus 15_S \oplus 15_A \oplus 20_S \oplus 45_A \oplus \overline{45}_A \oplus 84_S \qquad (4.15)$$

Our Hamiltonian being symmetric [which property becomes explicit when a Fierz transformation is performed on one of the factors of the Hamiltonian], the representations of interest are the 1_S, 15_S, 20_S and 84_S. The presence of charm and strangeness-changing currents excludes the singlet. The strangeness-changing neutral currents belong to the 15_S. This is absent for the GIM

Hamiltonian. Hence our Hamiltonian can be written as a mixture of 20 and 84 of SU(4). The significance of this point will be discussed later.

(For a derivation of the Hamiltonian see Appendix II).

4.5 Decay Amplitudes

In order to determine the two-body decay amplitudes soft meson techniques are invoked. This involves the transformation of matrix elements of the form $\langle B\pi | H_W | A \rangle$ into the form $\langle B | H_W | A \rangle$. This may be a drastic approximation since we let the meson momentum vanish in the soft meson limit, and the latter may imply omission of terms of appreciable magnitude. However, it should be remembered that a bag model calculation of nonleptonic decay amplitudes cannot but depend on such techniques as we are still not in a position to handle more than two bags at a time in this sort of computation. This also explains partly, why we are considering only two-body final state decays, apart from the fact that two-body decays of charmed particles are more energetic and are therefore less affected by final state interactions.

Using the PCAC hypothesis

$$\partial_\mu A_{k\mu}(x) = f_\pi m_\pi^2 \pi_k, \tag{4.16}$$

we can write in the soft pion limit [155]

$$\langle B\pi_a | H_W^{pv}(0) | A \rangle = -\frac{1}{f_\pi}\langle B | [F_a^5, H_W^{pv}(0)] | A \rangle$$

$$= -\frac{1}{f_\pi}\langle B | [F_a, H_W^{pc}(0)] | A \rangle, \qquad (4.17)$$

in which 'pv' stands for 'parity violating' and 'pc' stands for 'parity conserving'.

Thus

$$\langle B\pi^+ | H_W^{pv}(0) | A \rangle = -\frac{1}{\sqrt{2}f_\pi}\langle B | [F_-^5, H_W^{pv}(0)] | A \rangle$$

$$= -\frac{1}{\sqrt{2}f_\pi}\langle B | [F_-, H_W^{pc}(0)] | A \rangle \qquad (4.18)$$

In the present investigation we consider the following two-body hadronic decay channels of D^0 and D^+:

$D^0 \rightarrow K^-\pi^+ \qquad D^+ \rightarrow \bar{K}^0\pi^+$

$\rightarrow \bar{K}^0\pi^0$

$\rightarrow \bar{K}^0\eta^0$

which are Cabibbo favoured and which respect the selection rule $\Delta s/\Delta c = +1$, and

$$D^0 \to K^+\pi^- \qquad D^+ \to K^0\pi^+$$

$$\to K^0\eta^0 \qquad \to K^+\eta^0$$

which are Cabibbo suppressed and subject to the selection rule $\Delta s/\Delta c = -1$. For these specific decays, soft meson limit provides the following transformations.

$$\langle K^-\pi^+ | \overset{\Delta s/\Delta c=1}{H_W^{pv}(0)} | D^0 \rangle = -\frac{1}{\sqrt{2}f_\pi} \langle \bar{K}^0 | \overset{\Delta s/\Delta c=1}{H_W^{pc}(0)} | D^0 \rangle$$

$$\langle \bar{K}^0\pi^0 | \overset{\Delta s/\Delta c=1}{H_W^{pv}(0)} | D^0 \rangle = -\frac{1}{f_\pi} \langle \bar{K}^0 | \overset{\Delta s/\Delta c=1}{H_W^{pc}(0)} | D^0 \rangle$$

$$\langle K^+\pi^- | \overset{\Delta s/\Delta c=-1}{H_W^{pv}(0)} | D^0 \rangle = -\frac{1}{\sqrt{2}f_\pi} \left\{ \langle K^0 | \overset{\Delta s/\Delta c=-1}{H_W^{pc}(0)} | D^0 \rangle - \langle K^+ | \overset{\Delta s/\Delta c=-1}{H_W^{pc}(0)} | D^+ \rangle \right\} \qquad (4.19)$$

$$\langle \bar{K}^0\eta^0 | \overset{\Delta s/\Delta c=1}{H_W^{pv}(0)} | D^0 \rangle = \frac{\sqrt{3}}{2f_K} \langle \bar{K}^0 | \overset{\Delta s/\Delta c=1}{H_W^{pc}(0)} | D^0 \rangle$$

$$\langle K^0\eta^0 | \overset{\Delta s/\Delta c=-1}{H_W^{pv}(0)} | D^0 \rangle = -\frac{\sqrt{3}}{2f_K} \langle K^0 | \overset{\Delta s/\Delta c=-1}{H_W^{pc}(0)} | D^0 \rangle$$

where, in the last two cases the soft kaon limit has been taken.

Again,

$$\langle \bar{K}^0\pi^+ | H_W^{pv}(0) | D^+ \rangle^{\Delta s/\Delta c=1} = \frac{1}{\sqrt{2}f_\pi} \langle \bar{K}^0 | H_W^{pc}(0) | D^0 \rangle^{\Delta s/\Delta c=1}$$

$$\langle K^0\pi^+ | H_W^{pv}(0) | D^+ \rangle^{\Delta s/\Delta c=-1} = -\frac{1}{\sqrt{2}f_\pi} \Big\{ \langle K^+ | H_W^{pc}(0) | D^+ \rangle^{\Delta s/\Delta c=-1} - \langle K^0 | H_W^{pc}(0) | D^0 \rangle^{\Delta s/\Delta c=-1} \Big\} \qquad (4.20)$$

$$\langle K^+\eta^0 | H_W^{pv}(0) | D^+ \rangle^{\Delta s/\Delta c=-1} = -\frac{\sqrt{3}}{2f_K} \langle K^+ | H_W^{pc}(0) | D^+ \rangle^{\Delta s/\Delta c=-1}$$

The matrix elements are evaluated using the quark, antiquark wave functions Ψ and φ given by Eqs. (3.1) and (3.2) respectively, and assuming the quark structures

$$D^0 = \frac{1}{\sqrt{2}} (\overset{\uparrow\downarrow}{\bar{u}c} - \overset{\downarrow\uparrow}{\bar{u}c})$$

$$D^+ = \frac{1}{\sqrt{2}} (\overset{\uparrow\downarrow}{\bar{d}c} - \overset{\downarrow\uparrow}{\bar{d}c}) \qquad (4.21)$$

and also taking the parity conserving parts of the Hamiltonians (4.13) and (4.14). Using the same notation as illustrated by (4.21) other particles of interest such as the pions, the kaons and their antiparticles are also expressed by quark fields with spin direction (See ref.[156]). For the various processes under consideration we get the amplitudes

$$
\begin{aligned}
A(D^0 \to K^-\pi^+) &= \left(\frac{1}{\sqrt{2}} \cos^2\theta_c\right)k \\
A(D^0 \to \bar{K}^0\pi^0) &= (\cos^2\theta_c)k \\
A(D^0 \to K^+\pi^-) &= (-\sqrt{2}\ \sin^2\theta_c)k \\
A(D^0 \to \bar{K}^0\eta^0) &= \left(-\frac{\sqrt{3}}{2}\cos^2\theta_c\right)k' \\
A(D^0 \to K^0\eta^0) &= \left(-\frac{\sqrt{3}}{2}\sin^2\theta_c\right)k' \\
A(D^+ \to \bar{K}^0\pi^+) &= \left(\frac{1}{\sqrt{2}} \cos^2\theta_c\right)k \\
A(D^+ \to K^0\pi^+) &= (\sqrt{2}\ \sin^2\theta_c)k \\
A(D^+ \to K^+\eta^0) &= \left(-\frac{\sqrt{3}}{2}\sin^2\theta_c\right)k'
\end{aligned}
\tag{4.22}
$$

where

$$
\begin{aligned}
k &= \frac{G}{\sqrt{2}} \cdot \frac{1}{f_\pi} \cdot I, \\
k' &= \frac{G}{\sqrt{2}} \cdot \frac{1}{f_K} \cdot I,
\end{aligned}
\tag{4.23}
$$

The pion decay constant $f_\pi = 94$ MeV, the **kaon** decay constant $f_K \sim 1.3\ f_\pi$ and $G = 10^{-5}/M_P^2 = 1.132 \times 10^{-5}\ \text{GeV}^{-2}$.

The bag overlap integral I in (4.23) is

$$I = \frac{2N_1N_2N_3N_4}{4\pi} \int_0^R r^2 dr\, [\alpha_1\alpha_4\, j_o(y_1)j_o(y_4) + \beta_1\beta_4\, j_1(y_1)j_1(y_4)] \times [\alpha_2\alpha_3\, j_o(y_2)j_o(y_3) + \beta_2\beta_3 j_1(y_2)j_1(y_3)] \qquad (4.24)$$

where

$$\alpha_i = [(\omega_i + m_i)/\, \omega_i]^{\frac{1}{2}},$$

$$\beta_i = [(\omega_i - m_i)/\, \omega_i]^{\frac{1}{2}},$$

and $\qquad y_i = x_i\, r/R\,,\ i = 1,2,3,4.$

This integral has been evaluated numerically on a computer for the following input parameters:

$$m_1 = m_c = 1.5 \text{ GeV}$$

$$m_2 = m_3 = m_n = 0 \qquad (4.25)$$

$$m_4 = m_s = 0.3 \text{ GeV}.$$

and an average bag radius R = 3.3 GeV^{-1}. N_i, x_i and ω_i ($i = 1,2,3,4$) are solutions of Eqs.(2.30), (2.32) and (2.33) respectively corresponding to given m_i, R values. The value of the integral is obtained as

$$I = 1.497 \times 10^{-2} \tag{4.26}$$

Substituting for I, the absolute magnitudes of the decay amplitudes are computed and these are listed in Table 4.1.

4.6 Possible Sextet Enhancement

The well known concept of octet enhancement in SU(3) as applied to weak hadronic processes of ordinary hadrons prompts one to think of extending the same to nonleptonic charm decays governed by SU(4). The current-current Hamiltonian contains products of four quark/antiquark operators such as $(\bar{u}d)(\bar{s}u)$ which transform like a $\pi^- K^+$ system. It thus has pieces corresponding to $I = \frac{1}{2}$ and $I = 3/2$. Experimentally we have the $\Delta I = \frac{1}{2}$ rule and theoretically [136,137,126] the idea of octet enhancement which explains it. As we have seen, the Hamiltonian has transformation properties of an octet and a 27 representation. The $I = 3/2$ piece belongs to the 27 and the $I = \frac{1}{2}$ piece to the 8. Hence enhancing the 8 will automatically generate the approximate $\Delta I = \frac{1}{2}$ rule.

In SU(4), in place of 8 and 27 we have the 20 and the 84 representations respectively [See Sec.4.4]. Thus eliminating the 27 in SU(3) amounts to eliminating the 84 in SU(4) and the SU(3) octet enhancement is equivalent to the SU(4) 20-plet

Table 4.1 Amplitudes for two-body decays

Decay mode	10^7 x amplitude
$D^0 \rightarrow K^- \pi^+$	8.5
$D^0 \rightarrow \bar{K}^0 \pi^0$	12.00
$D^0 \rightarrow K^+ \pi^-$	-1.22
$D^0 \rightarrow \bar{K}^0 \eta^0$	-8 0
$D^0 \rightarrow K^0 \eta^0$	-0.5
$D^+ \rightarrow \bar{K}^0 \pi^+$	8.5
$D^+ \rightarrow K^0 \pi^+$	1.22
$D^+ \rightarrow K^+ \eta^0$	-0.5

enhancement over the 84. The SU(3) contents of 20 are obtained by the decomposition

$$20 = 8 \oplus 6 \oplus \bar{6}$$

where 8 represents the charm conserving ($\Delta c=0$) transitions and 6 and $\bar{6}$ the $\Delta c = +1$ and $\Delta c = -1$, charm changing transitions respectively. Thus the SU(3) octet enhancement ultimately implies a sextet dominance for the charm changing decays.

If sextet dominance is assumed then the number of parameters needed to describe the $\Delta c = \pm 1$ transitions is reduced and it leads to certain relations between different decay modes. For example, all the charm changing two-body final state decays of pseudoscalar mesons can be expressed in terms of one common parameter [127]. However it is argued that the 20 enhancement in SU(4) will be minimal [145].

The most spectacular prediction of the sextet dominance model is the vanishing of $A(D^+ \rightarrow \bar{K}^0\pi^+)$, which is in total disagreement with experiment. Compare the model prediction

$$A(D^+ \rightarrow \bar{K}^0\pi^+) = 0, \quad A(D^0 \rightarrow K^-\pi^+) \neq 0 \qquad (4.27)$$

with the experimental branching ratios for these two decay modes [120]:

$$\begin{aligned} BR(D^+ \rightarrow \bar{K}^0\pi^+) &= 1.5 \pm 0.6\ \% \\ BR(D^0 \rightarrow K^-\pi^+) &= 2.2 \pm 0.6\ \% \end{aligned} \qquad (4.28)$$

4.7 Discussion of the Results

It is instructive to compare our predictions with other theoretical estimates and also with available experimental data. For $D^0 \rightarrow K^- \pi^+$ amplitude we get 8.5×10^{-7}. It corresponds to a decay rate 0.2×10^{12} per sec which may be compared with the value of 5×10^{12} per sec obtained by Gaillard et al.[114] as well as the amplitude 3.9×10^{-6} obtained by Donoghue and Golowich [116]. Gaillard et al. assumes the simplest imaginable picture of D meson decays where the c quark decays into an s quark by bremsstrahlung of a virtual W^+ boson which then materialises into a $u\bar{d}$ pair. The light antiquark $\bar{u}$ for D^0, and $\bar{d}$ for D^+ is a passive "spectator" to the decay. Donoghue and Golowich have employed renormalisation group techniques in their calculation. There is order of magnitude agreement among the various predictions, which are all consistent with a total measured life time $\sim 10^{-13}$sec [157].

Table 4.2 compares the measured two-body decay rates with a number of predictions including that of the present bag model computation. The part of this table excluding the last column is taken from Ref.[158]. The results of the present calculation appear to be in better agreement with data. The disagreement in the other cases is fairly large for both the ratios considered. The smallness of $\bar{K}^0 \pi^0$ in all these models

Table 4.2 Comparison of two-body data with several predictions

	Experimental data	Ref. [159]	Ref. [160]	Ref. [161]	Ref. [162]	Present calculation
$\frac{B(D^0 \to \bar{K}^0\pi^0)}{B(D^0 \to K^-\pi^+)}$	0.73±.35	0.024	0.025	0.028	0.024	1.86
$\frac{B(D^0 \to K^-\pi^+)}{B(D^+ \to \bar{K}^0\pi^+)}$	1.29±.30	1.67	1.67	1.85	1.71	1.00

which assume QCD enhancements is explained as due to "colour suppression". Colour suppression means cancellation of amplitudes arising from the 20 and 84 when only QCD enhancement is assumed [163]. It has been further proposed that the assumption of additional sextet enhancement beyond QCD can bring these branching ratios within the experimental values. The authors of Ref.[163] suggest that the deviations of these estimates from experimental data may arise from final state interactions. Deshpande et al. [164] have suggested that final state soft gluon exchange may have the effect of reducing the suppression of the amplitude for $\bar{K}^0\pi^0$. The situation is obviously unenviable and one is tempted to suspect the reliability of these models.

Our result for the first listed ratio, however, differs substantially from all the above model predictions but only reasonably from the experimental data. The fact that the theoretical estimate is still higher than the experimental value by a factor of 2 casts doubts on the validity of PCAC being applied to obtain reduced matrix elements. PCAC may not be a very good approximation in the present context.

Donoghue and Holstein [165] in their paper on charmed nonleptonic decay in asymptotically free theories predict on the basis of renormalisation group techniques a sextet enhancement for $\Delta c = \Delta s$ decays. Consequently they get zero amplitude for $D^+ \to \bar{K}^0\pi^+$ mode. But we have obtained equal amplitudes for this mode and the $D^0 \to K^-\pi^+$:

$$A(D^+ \to \bar{K}^0\pi^+) = A(D^0 \to K^-\pi^+)$$

contradicting the speculation regarding sextet enhancement. Cabibbo and Maiani [166], employing a simple quark recombination scheme show that

$$A(D^+ \to \bar{K}^0\pi^+) = 0.8\, A(D^0 \to K^-\pi^+)$$

which compares well with the measured branching ratios [Eq.(4.28)]. The bag model result reported here is thus in qualitative agreement with the experimental data.

CHAPTER 5

BAG PHENOMENOLOGY OF GLUEBALL SPECTROSCOPY [167]

5.1 Introduction

One of the interesting predictions of the bag model is the existence of pure gluonic hadrons. But in the phenomenological bag model developed by De Grand et al. [64] the existence of "valence" gluons in hadron states is ignored. However, the effects of coloured glue is taken into account by considering the quark-quark interaction as mediated by gluons in which gluons are treated as classical fields generated by the quark colour currents. The interaction contributes to hyperfine splittings of the hadron masses. Hadrons which are composite objects with "valence" gluons only are referred to as "gluonia" or "glueballs". The present work aims at a phenomenological study of the glueball spectroscopy in the context of the MIT bag model taking into account the colour-spin interaction of gluons.

There have been some attempts in the past to understand the dynamical manifestations of gluons in hadron physics. These include the work of Freund and Nambu [168] on the existence of a flavour-singlet and colour-singlet meson as a consequence of the breaking of the Zweig-Iizuka rule [169] and that of Fritzsch and Minkowski [170] on the possibilities for the spectrum of "glue mesons". According to Freund and Nambu the poles on the Regge-type trajectories, the Pomeron with unit intercept and slope $\sim 0.5\ GeV^{-2}$ and its daughters are nothing but bound gluestates with masses around 1.5 GeV.

The formalism of lattice QCD has been employed by Kogut, Sinclair and Susskind [171] to obtain mass predictions for a few light glueball states in the mass range 1 - 1.5 GeV. Their method consists in making a strong coupling expansion of the Hamiltonian and extrapolating the results to zero coupling using the Padé approximant technique.

Robson [172] has enumerated a number of glueball states with $J^{PC} = 0^{++}, 2^{++}, 1^{--}, 3^{--}$ etc. and has presented a discussion on their masses and decay widths within the Freund-Nambu model. He has also considered the problem of mixing of the glueball states with the ordinary isoscalar mesons of comparable mass. The paper contains a suggestion that the isoscalar meson S^* is a glueball.

The work of Jaffe and Johnson [173] relating to the bag phenomenology of the glueball spectrum has led to the general conclusion that there are many gluonic hadrons in the energy range 900-1600 MeV. They have used the same bag parameters as for the quarks/antiquarks - bound systems [64], but ignored the important contribution of the colour-spin interaction of gluons to the glueball mass. Calculations are made in the spherical cavity approximation to the bag.

It should be emphasized that being a relativistic theory of hadronic structure the MIT bag model must be especially suited for the study of the bound states of massless gluons which are extreme relativistic objects. The spectacular success of the quark-bag model in the lowest mass regime is a reminder to this observation. The spherical bag approximation is expected to yield masses of spin-0, spin-1 and possibly spin-2 glueballs to the same degree of reliability as the masses of the light 0^- and 1^- mesons and $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons obtained in Ref.[64]. One is thus deeply motivated for a more detailed bag model investigation of the very rich spectrum of the glueballs.

5.2 Stability of the Glue Bags

As mentioned above Jaffe and Johnson have made use of the fixed sphere (static spherical cavity) bag model for their study of the glueball spectrum. The validity of this approach might be questioned on the grounds of its inadequacy for the description of higher order angular momentum states which no longer have spherical symmetry. There is a more serious objection raised against assuming spherical shape for the gluonic bag the recognition of which later made Jaffe and Johnson to suspect the validity of their own work, namely, the possibility of the volume tension B balancing the gluon field pressure $\frac{1}{2}(\vec{E}^2 - \vec{B}^2)$ going negative thereby destroying the stability of the bag. Obsessed with the awkwardness of a negative bag pressure, Robson investigated alternatives to spherical shape which led him to propose a toroidal configuration [174] for glue bags. The toroidal bag, however, does not have the essential simplicity and elegance of the spherical bag picture that proved to be so successful with the low mass states of the confined quarks. Further, apart from suggesting that the ground state glueball mass might be around 1 GeV, the toroidal bag model does not make any definite quantitative predictions.

It seems that the question of stability can be adequately answered by the following argument and the simple spherical bag picture can be retained at least in the case of the lowest angular momentum states of bound glue: In addition to the gluon field pressure one has also to consider the Casimir stress which gives rise to the zero-point energy of the field fluctuations. In the MIT bag model [64] this is determined phenomenologically and it makes a negative contribution to the bag Hamiltonian implying that the Casimir stress is attractive. Milton [85] has recently made theoretical investigations of the problem of Casimir effect using Green's function technique. He has found that the Casimir stress in the case of spherical cavity is attractive for quark fields while it is repulsive for gluon fields. We argue that the positive Casimir stress acting in conjunction with the gluon field pressure $\frac{1}{2}(\vec{E}^2 - \vec{B}^2)$ against the confining bag pressure B can ensure the stability of the spherical cavity. In estimating the glueball masses we include the positive zero-point energy contribution as suggested by Milton, the phenomenological implications of which have not been tested so far.

5.3 Bagged Gluon Fields

Gluon fields are nonabelian colour gauge fields A^a_μ whose action W inside the spherical bag, invariant under the colour gauge group SU(3), is given by

$$W = \int dt \int_{bag} d^3r \left\{ -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} \right\} \tag{5.1}$$

where the field strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{5.2}$$

in which f^{abc} are the structure constants of the colour gauge group SU(3) and g is the gluon-gluon (equal to quark-gluon) coupling constant. The bag model imposes two boundary conditions on the gauge fields

$$n^{\mu}F^{a}_{\mu\nu} = 0 \tag{5.3}$$

$$-\tfrac{1}{2}F^{a*}_{\mu\nu}\, F^{a}_{\mu\nu} = 2B, \tag{5.4}$$

where $n^{\mu} = (0,\hat{n})$, $\hat{n}$ being the unit normal to the spherical surface and B is the bag pressure.

The action principle $\delta W = 0$ yields the field equations

$$D^{ab}_{\mu}F^{b\mu\nu} = 0 \tag{5.5}$$

in which

$$D^{ab}_{\mu} \equiv \delta^{ab}\partial_{\mu} - gf^{abc}A^{c}_{\mu} \tag{5.6}$$

is the gauge covariant derivative.

The colour currents arising from the gluon fields are given by

$$J^{a}_{\mu}(x) = f^{abc}\, F^{b}_{\mu\nu}\, A^{c}_{\nu}, \tag{5.7}$$

$$a = 1,2,\ldots,8.$$

They give 8 colour generators

$$F^{a} = \int_{bag} d^{3}r \;\; J^{a}_{0}(x) \tag{5.8}$$

which measure the colour charges of the gluonic hadrons. The linear boundary condition (5.3) ensures that the colour flux does not cross the bag boundary. Now Gauss's theorem leads to the vanishing of the total colour charges F^a

$$F^a = \int_{bag} d^3r \, J_0^a(\vec{r},t) = 0 \qquad (5.9)$$

Consequently only colour singlet gluonic hadrons can exist.

Glueballs are colour singlet and flavour singlet composites of spin-1 gluons. In what follows we consider bound gg and ggg states. We first consider free gluon fields confined to a spherical cavity (bag). They should satisfy the equation of motion

$$(\vec{\nabla}^2 + \omega^2)\vec{A}^a = 0 \qquad (5.10)$$

inside the cavity. With g = 0, the linear boundary condition (5.3) on $\vec{A}^a$ gives two conditions

$$\hat{n}\cdot\vec{A}^a = 0 \qquad (5.11)$$

$$\hat{n} \times (\vec{\nabla} \times \vec{A}^a) = 0 \qquad (5.12)$$

on the bag surface. The cavity allows electric and magnetic multipole solutions [175] of opposite parity for particular J,m values. The electric multipole fields or transverse magnetic (TM) fields have the form

$$\vec{A}^{E}_{Jm} = \left\{ \left(\frac{J+1}{2J+1} \right)^{\frac{1}{2}} \mathcal{J}_{J-1}(\omega r) \quad \vec{Y}_{J,J-1,m} \right.$$

$$\left. - \left(\frac{J}{2J+1} \right)^{\frac{1}{2}} \mathcal{J}_{J+1}(\omega r) \, \vec{Y}_{J,J+1,m} \right\} e^{-i\omega t} \qquad (5.13)$$

with parity $(-1)^J$ and the magnetic multipole fields or transverse electric (TE) fields have the form

$$\vec{A}^{M}_{Jm} = \mathcal{J}_J(\omega r) \; \vec{Y}_{J,J,m} \quad e^{-i\omega t} \qquad (5.14)$$

with parity $(-1)^{J+1}$, where $\vec{Y}_{Jlm}$ are vector spherical harmonics and $\mathcal{J}_J(\omega r)$ are Bessel functions of order J. Besides each gluon has an intrinsic negative parity. Thus depending on the cavity eigenmodes occupied, gluons may be supposed to appear in two varieties: electric gluons g_e with $J^P = 1^+$ occupying TM modes and magnetic gluons g_m with $J^P = 1^-$ occupying TE modes. Their eigen frequencies are determined by the linear constraints (5.11) and (5.12). For a spherical bag surface with radius R these give for the E and M fields respectively

$$\mathcal{J}_{J-1}(\omega R) + \mathcal{J}_{J+1}(\omega R) = \mathcal{J}_J(\omega R) = 0 \qquad (5.15)$$

$$\mathcal{J}_{J-1}(\omega R) = \frac{J}{J+1} \; \mathcal{J}_{J+1}(\omega R) \qquad (5.16)$$

From these the lowest eigen frequencies ω_0 are found to be given by

$$g_m: \quad x_m = x_0(TE) = 2.74 \tag{5.17}$$

$$g_e: \quad x_e = x_0(TM) = 4.49 \tag{5.18}$$

where $x = [\omega^2 R^2 - m^2 R^2]^{\frac{1}{2}}$

$= \omega R$, since the gluons are massless.

5.4 Allowed Glueball States

The total ground state wave function of a glueball state is the symmetric product of the spatial, spin and colour wave functions of the gg or ggg combinations, as the case may be. The charge conjugation C of the bound system is determined by the symmetry property of the colour wave function. For gg states with δ^{ab} coupling, C = +. For ggg states formed by symmetrical d-type coupling, C = —, and by antisymmetrical f-type coupling C = +, where the negative intrinsic charge parity of gluons is taken into account. There seems to be some ambiguity in the literature with regard to the spin-parity assignments of glueball states. This probably arises from the fact that some authors [173,176,66] have ignored the intrinsic space and charge parities of the gluon and are therefore in error.

5.4(i) Two-gluon (gg) bound states

Gluons are colour octets. In colour space these can be combined to form colour singlets. For example, a system of two gluons give rise to the direct product representation $8 \otimes 8$ which has the following decomposition into irreducible representations of SU(3):

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 10 \oplus 27$$

With two vector (spin-1) gluons we can construct states with spin-2, spin-1 and spin-0. As we are considering only bound systems in S-wave states the spatial part of the wave function is always symmetric. For the two gluon system the colour wave function is also symmetric. It follows that the colour states should combine symmetrically giving an overall symmetric function. Thus with the two types of gluons g_m and g_e with $J^P = 1^-$ and $J^P = 1^+$ respectively, the two-gluon hadronic states given in Table 5.1 can be generated.

5.4(ii) Three-gluon (ggg) bound states

For L = 0 ground states of colour singlet 3g glueballs the spatial wave function is symmetric, with positive parity. However the different intrinsic parities of g_m and g_e give rise to different space parity for $g_m g_m g_m$, $g_m g_m g_e$ etc. combinations. Three spin-1 gluons can generate four different spin states: spin-3, spin-2, spin-1 and spin-0 states, possessing definite

Table 5.1 Allowed gg glueball ground states

Gluon content	J^{PC}	Gluon content	J^{PC}
$g_m g_m$	0^{++}	$g_m g_e$	0^{-+}
	1^{++}		1^{-+}
	2^{++}		2^{-+}
$g_e g_e$	0^{++}		
	1^{++}		
	2^{++}		

symmetry properties. The symmetry of the 3-gluon colour state will be fixed so that the total wave function is symmetric. For example consider the $g_m g_m g_m$ state with total spin = 0. This has an antisymmetric spin function, and so an antisymmetric colour function with f^{abc} colour coupling. Both these states have negative space parity so that the overall parity of the 3 g_m bound state taking into account the negative intrinsic parity of g_m, becomes negative. The f-type colour coupling gives a charge parity C = +. Thus this state has $J^{PC} = 0^{-+}$. Some of the allowed 3-gluon states are listed in Table 5.2.

5.5 Glueball Masses

We now briefly describe our method of estimating glueball masses. We consider the L = 0, ground states of the gluonic hadrons to be spherical bags of finite size. The bag size is fixed by minimizing the mass with respect to radius. The mass is determined by diagonalising the phenomenological Hamiltonian for the glue-bag. Our Hamiltonian has the four ingredients:

(i) The Kinetic energy E_g of the constituent gluons.

The kinetic energy of a gluon is given by

$\omega = \frac{x}{R}$, where x is given by Eqs.(5.17) and (5.18).

Thus

$$E_g = \sum_i N_i \omega_i \ , \qquad (5.19)$$

where N_i is the number of gluons of the i^{th} type with kinetic energy ω_i.

Table 5.2 Some of the allowed ggg glueball ground states

Gluon content	J^{PC}	Gluon content	J^{PC}
$g_m g_m g_m$	0^{-+}	$g_m g_e g_e$	0^{-+}
	1^{--}		1^{-+}
			2^{--}
$g_m g_m g_e$	0^{++}	$g_e g_e g_e$	0^{++}
	1^{+-}		1^{++}
	2^{++}		

(ii) The volume energy $E_V = \frac{4}{3}\pi R^3 B$. (5.20)

(iii) The zero-point energy E_0 associated with the gluon field fluctuations.

(iv) The colour magnetic interaction energy E_M of gluons arising from their colour magnetic moments.

Regarding E_0 and E_M, we differ from the calculation of Jaffe and Johnson [173]. In their work the zero-point energy gave a negative contribution to the mass. As in the standard quark-bag model [64] this is assumed to have the form $-Z/R$, where Z has the phenomenological value 1.84. On the other hand we use Milton's result [85], namely,

$$E_0 = + 0.51/R \tag{5.21}$$

which has been derived using Green's function technique and applying a suitable renormalisation. Colour magnetic interaction is altogether ignored in Ref.[173], although some authors have speculated that this might yield a significant contribution to the glueball mass [176]. We recognize that the lowest order gluon-gluon interaction is essentially a first order interaction involving the colour coupling constant $\alpha_c = g^2/4\pi$, like the lowest order quark-quark interaction by gluon exchange. The latter has been studied by De Grand et al.[64] who have treated the colour gauge fields to lowest order in α_c as 8 independent abelian fields. In a naive extension of this approach to the gluon-gluon interaction, one has to solve a set of Maxwell's equations as in [64] but with the quark colour current being replaced by one arising from the vector gluon field itself. This procedure is expected to lead to an interaction term similar to the one for massless quarks.

The colour electric charges of gluons must be proportional to the colour SU(3) generators. These are thus given by gF^a. In analogy with the electromagnetic case we may define colour magnetic moments $\vec{\mu}^a$ in terms of colour charges:

$$\vec{\mu}^a \sim gF^a\vec{S} \qquad (5.22)$$

$F^a(a = 1,2,\ldots 8)$ being the 8 generators of SU(3). For S-wave states only the Fermi interaction need be considered. In this case the interaction energy between colour magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ is proportional to $\vec{\mu}_1 \cdot \vec{\mu}_2$. Thus for the colour magnetic energy of gluons in a spherical cavity of radius R, in relative S-states we write

$$E_M = - \frac{c}{R} \sum_{i>j} (F_i^a \vec{S}_i) \cdot (F_j^a \vec{S}_j) \qquad (5.23)$$

which is similar in form to the corresponding expression in quark-bag dynamics. The numerical parameter c is to be determined phenomenologically. It must have the same value for all glueball states. This is of course a crude approach to a formidable problem, but the equivalence shown in the case of quark-quark interaction, between the semi classical approach of De Grand et al.[64] and a consistent field theoretic formalism [177] developed by Close and Horgan on the basis of QCD perturbation theory inside a spherical hadron [178] has encouraging implications in its favour. Hence we write for the mass of a glueball of radius R

$$M = E_V + E_g + E_o + E_M$$

$$= \frac{4}{3}\pi R^3 B + \sum_i N_i x_i/R + b/R$$

$$- (c/R)\sum_a \sum_{i>j} (F_i^a \vec{S}_i)\cdot(F_j^a \vec{S}_j) \qquad (5.24)$$

where the x_i are given by Eqs.(5.17) and (5.18) and b = 0.51 [85].

For two gluons belonging to colour octet to form a colour singlet we have

$$\sum_{i=1}^{2} F_i^a |2g\rangle = 0 \qquad (5.25)$$

Squaring

$$\langle 2g|\sum_a [(F_1^a)^2 + (F_2^a)^2 + 2F_1^aF_2^a] |2g\rangle = 0$$

$$\text{or,} \langle 2g|\sum_a [\sum_{i=1}^{2}(F_i^a)^2 + \sum_{i\neq j} F_i^aF_j^a]|2g\rangle = 0 \qquad (5.26)$$

The eigen value of the Casimir operator $(F^a)^2$ for a colour SU(3) octet is given by

$$f_c^2 = 3.$$

It follows that

$$\sum_a F_i^a F_j^a = -3 \qquad i \neq j \tag{5.27}$$

Similarly for a system of three gluons belonging to colour octet to form a colour singlet

$$\sum_a F_i^a F_j^a = -3/2 \qquad i \neq j \tag{5.28}$$

The spin factor $\sum'_{i>j} \vec{S}_i \cdot \vec{S}_j$ for each of the allowed 2g and 3g cases is evaluated by noting that each gluon has spin 1 and by taking into account the appropriate resultant spin value. For example, consider 3 spin-1 gluons forming a spin-1 state. Two of the gluons may be supposed to be in a spin-0 state which then combine with the third to form the required spin-1 state. With gluons 1 and 2 in the spin-0 state we have

$$|\vec{S}_1 + \vec{S}_2| = 0, \text{ where } |\vec{S}_i| = 1, \text{ for } i = 1,2,3.$$

Now

$$(\vec{S}_1 + \vec{S}_2)^2 = (\vec{S}_1)^2 + (\vec{S}_2)^2 + 2\vec{S}_1 \cdot \vec{S}_2 \tag{i}$$

The eigen values of the spin operators are given by

$$(\vec{S}_1 + \vec{S}_2)^2 = 0$$

$$(\vec{S}_i)^2 = 2, \text{ for } i = 1,2,3.$$

$$\therefore 0 = 2 + 2 + 2\, \vec{S}_1 \cdot \vec{S}_2$$

$$\text{or } \vec{S}_1 \cdot \vec{S}_2 = -2 \qquad \text{(ii)}$$

The resultant spin vector

$$\vec{S} = (\vec{S}_1 + \vec{S}_2) + \vec{S}_3$$

has magnitude 1, so that

$$(\vec{S})^2 = 1(1 + 1) = 2$$

Now

$$(\vec{S})^2 = (\vec{S}_1 + \vec{S}_2)^2 + (\vec{S}_3)^2 + 2(\vec{S}_1 + \vec{S}_2)\cdot\vec{S}_3$$

i.e., $2 = 0 + 2 + 2\,(\vec{S}_1 + \vec{S}_2)\cdot\vec{S}_3$

$$(\vec{S}_1 + \vec{S}_2)\cdot\vec{S}_3 = 0 \qquad \text{(iii)}$$

Combining (ii) and (iii)

$$\vec{S}_1\cdot\vec{S}_2 + (\vec{S}_1 + \vec{S}_2)\cdot\vec{S}_3 = -2 + 0 = -2$$

$$\text{or } \sum_{i>j} \vec{S}_i\cdot\vec{S}_j = -2 \qquad \text{(iv)}$$

The spin factors for various states are tabulated in Table 5.3.

Table 5.3 Values of $\vec{S}_i \cdot \vec{S}_j$ for various resultant spin values of bound 2-gluon and 3-gluon states

No.of gluons in the state	Resultant spin	$\sum_{i>j} \vec{S}_i \cdot \vec{S}_j$
2	0	-2
	1	-1
	2	1
3	0	-3
	1	-2
	2	0
	3	3

B and c are the only parameters of our theory. For B we choose the standard bag model value [64], namely $B^{1/4} = 0.145$ GeV. In order to fix the value of c, we identify the observed hadron state $S^*(980)$ as the lowest lying 0^{++} glueball with gluon content $(g_m g_m)$ and use its mass as input in (5.24). The resulting predictions for the various glueball masses are listed in Tables 5.4 and 5.5.

The following arguments are presented in favour of our choice of S^* as a glueball.

Glueballs constitute a new kind of hadronic matter. No observed particle has been unambiguously established as a glueball, although there have been suggestions in this regard now and then. There are two schools of thought regarding the position of glueballs in the mass spectrum of hadrons. One holds that they are extremely massive and so are not seen at present energies. This is a mere guess, and is not backed by any order of magnitude estimates. The other view is that the masses of glueballs are not too high and that they must occur "right among ordinary hadrons". This is something more than a guess. Existing predictions [168,170-173] are in line with this point of view. In particular, Jaffe and Johnson [173] have predicted that the lowest glueball has a mass 960 MeV and that it is a scalar particle having $J^{PC} = 0^{++}$. There are three particles: $\varepsilon(700)$, $S^*(980)$ and $\delta(980)$ around this energy

Table 5.4 Masses (in GeV) of low-lying 2 gluon bound states in the bag model. Parameters: $B^{1/4} = 0.145$ GeV, $c = 0.375$.

Gluon content	J^{PC}	R (GeV^{-1})	E_v	E_g	E_o	E_M	Total M
$g_m g_m$	0^{++}	5.10	0.246	1.075	0.100	-0.441	0.980
$g_m g_e$	0^{-+}	5.61	0.327	1.289	0.090	-0.401	1.306
$g_e g_e$	0^{++}	6.00	0.401	1.496	0.085	-0.375	1.607
$g_m g_m$	1^{++}	5.44	0.298	1.007	0.094	-0.207	1.192
$g_m g_e$	1^{-+}	5.87	0.375	1.232	0.087	-0.192	1.502
$g_e g_e$	1^{++}	6.23	0.448	1.441	0.082	-0.180	1.790
$g_m g_m$	2^{++}	5.98	0.396	0.916	0.085	+0.188	1.585
$g_m g_e$	2^{-+}	6.32	0.467	1.144	0.081	+0.178	1.870
$g_e g_e$	2^{++}	6.61	0.535	1.358	0.077	+0.170	2.140

Table 5.5 Masses (in GeV) of low lying 3 gluon bound states in the bag model. Parameters: $B^{1/4} = 0.145$ GeV, $c = 0.375$.

Gluon content	J^{PC}	R (GeV^{-1})	E_V	E_g	E_c	E_M	Total M
$g_m g_m g_m$	0^{-+}	5.91	0.393	1.378	0.086	-0.283	1.574
$g_m g_m g_e$	0^{++}	6.30	0.463	1.582	0.081	-0.268	1.858
$g_m g_e g_e$	0^{-+}	6.60	0.532	1.776	0.077	-0.256	2.130
$g_e g_e g_e$	0^{++}	6.86	0.598	1.963	0.074	-0.246	2.390
$g_m g_m g_m$	1^{--}	6.08	0.416	1.352	0.084	-0.185	1.667
$g_m g_m g_e$	1^{+-}	6.41	0.486	1.555	0.080	-0.176	1.946
$g_m g_e g_e$	1^{-+}	6.68	0.553	1.754	0.076	-0.168	2.214
$g_e g_e g_e$	1^{++}	6.93	0.616	1.944	0.074	-0.162	2.472
$g_m g_m g_e$	2^{++}	6.59	0.530	1.513	0.077	0	2.120
$g_m g_e g_e$	2^{--}	6.86	0.595	1.711	0.074	0	2.380

value with controversial identity. The ε(700) is no more alive according to the latest particle data. The δ(980) is an isovector particle bearing explicit flavour. Glueballs are flavour singlets and so isoscalars. $S^*(980)$ is an isoscalar. It is thus a probable candidate for the lowest lying 0^{++} glueball $(g_m g_m)$. Another significant point in support of this choice is the relatively narrow width (~ 40 MeV) of S^*. Low mass glueballs are expected to be somewhat narrow. This prediction is based on the argument that these states can undergo √Zweig-violating decays into ordinary $q\bar{q}$ mesons [172,179].

The scalar mesons have always been a subject of controversy. A conventional quark model classification [180] puts the S^* along with the already dead ε(700), the well established narrow resonance δ(980) and the relatively obscure broad resonance κ (~ 1300) in an SU(3) nonet with L = 1. The problems arising out of this assignment are discussed in detail by Jaffe [181] and Estabrooks [182] who rule out this possibility. On the other hand Jaffe [181] proceeds to identify these resonances with his 0^+ $q^2\bar{q}^2$ multiquark states. But, Estabrooks [182] advances arguments to contradict Jaffe's claim. Thus the situation apparently favours our choice of S^* as a glueball state.

5.6 Discussion of the Results and Conclusions

The effect of colour magnetic interaction of gluons has been incorporated in the glueball mass estimate, for the first time. The inclusion of this effect has the consequence that there is an overall enhancement of masses. However, the present calculation once again affirms that the glueball masses are not too high as speculated by some. The low lying glueballs are found to be in the mass range $\sim 1 - 2.5$ GeV.

We propose that the new narrow resonance $\varphi'(1.65)$ reported to have been discovered recently [183,184] and suspected as a radial excitation of φ, could be the 1^{--} vector glueball state with the glue content $(g_m g_m g_m)$ and predicted mass 1.667 GeV. The Reggeization of glueball spectroscopy [167,66] has predicted the 1^{--} to be degenerate with the 0^{++} and 2^{++} states having roughly 1.3 GeV mass which is clearly at variance with our predictions. Precise measurements of its relative couplings to $K\bar{K}$ and $\rho\pi$ channels will establish the SU(3) flavour singlet behaviour of φ'.

The 0^{++} and 2^{++} gg states are split in our model as a result of the colour magnetic interaction of gluons. The degeneracy of the scalar and tensor mesons encountered in the earlier calculations [172,173,66] is thus lifted. In QCD the chromomagnetic forces are expected to break such degeneracy. While the spherical bag predictions are subject to modifications by effects of non-spherical deformations, still they must be closer to QCD expectations.

CHAPTER 6

A PHENOMENOLOGICAL BAG MODEL WITH VARIABLE BAG PRESSURE

6.1 Introduction

Here we attempt to develop a new version [185] of the phenomenological MIT bag model [64] by introducing certain modifications of fundamental importance. In doing this we are motivated partly by the drawbacks of the existing model as manifested in its poor predictions of masses of some of the light hadrons, particularly, the pion, the magnetic moments of baryons and the mass relations among certain hadron multiplets; and partly by the observation that the confining pressure B is not necessarily a universal constant but must be determined by the material content of the bag.

6.2 Variable Bag Pressure

The introduction of the bag pressure term $g_{\mu\nu} B$ in the Lagrangian density to represent a volume tension to balance the outward thrust of the quark gas inside the bag is hailed as an original invention of the MIT team. However, the origin of this term has remained more or less obscure, and in phenomenological applications of the bag model it is treated as a universal parameter, whose value is determined by fitting known masses. We wish to point out that there are indications to the effect that B is not a universal constant. The motivation of the MIT theorists for introducing the pressure term in the bag Lagrangian is apparently the familiar solution of the Einstein general relativity equation with a 'cosmological constant' to generate a closed universe. But todate there is no evidence to give hope that an extension of the cosmological theories to the realm of the microcosm can generate the bag with a pressure term in any order of magnitude agreement with its phenomenologically determined value. It seems that the only hope at present for a field-theoretic derivation of the colour-quark bag rests on QCD. Johnson [84] made a proposal for the form of the ground state wavefunction of QCD, where his suggestion for handling QCD quantitatively, leads to the phenomenology of the MIT bag model The bag pressure term in this case is found to be related to the fundamental scale parameter Λ of QCD.

In the theory of hadronic structure recently proposed by Callan, Dashen and Gross [70], qualitative aspects of a bag-like picture that emerges from the properties of QCD vacuum are discussed. According to them, the normal QCD vacuum is populated with instantons, and merons forming a dense phase, and is colour-repellant. Above a critical field strength the colour fields (due to quarks and/or gluons) find themselves in finite regions (bags) from where instantons are expelled leaving a dilute phase. The bag pressure P is then computed as the zero-field difference in free energy density between the dense and dilute phases. In a semiquantitative analysis it is shown that B is inversely proportional to the permeability of the dilute phase, which is approximately a constant, although, strictly speaking, it is essentially a density-dependent factor. The theory, however, has not been developed into a calculational device.

As a plausible means to relate the bag pressure to the energy density in a hadron, we will seek an equation of state for hadronic matter. Relativistic hydrodynamics when applied to the bag model [186,187] leads to an equation of state

$$P = \frac{1}{3}\varepsilon - \frac{4}{3}B \qquad (6.1)$$

where ε is the total energy density. The condition for the stability of the system, namely,

$$P = 0 , \qquad (6.2)$$

determines B in terms of the total energy density:

$$B = \frac{1}{4}\varepsilon \qquad (6.3)$$

This equation relates of course, to the extreme relativistic case where the quarks have negligible mass. In general, one has

$$B \leq \frac{1}{4} \varepsilon \tag{6.4}$$

Writing

$$\varepsilon = \rho + B \tag{6.5}$$

in which ρ represents the contribution from sources other than the volume tension, equation (6.3) becomes

$$B = \frac{1}{3} \rho \tag{6.6}$$

For evaluating the properties of the quark gas, Chapline and Nauenberg [186] make use of the fact that the quark-gluon coupling constant α_c in the MIT bag theory is small enough to permit an agreeable perturbation approximation. However, following the suggestion of Freedman and McLerran [188], they use a renormalised quark-gluon coupling constant depending on the Gibbs energy per quark rather than a fixed coupling constant as in the MIT model. This approach naturally lends further support to the choice of a variable bag pressure.

6.3 The Bag Hamiltonian

The Hamiltonian of the original MIT bag model was developed and its diagonalization was discussed by Chodos et al. [63] and De Grand et al. [64]. As this has already been considered in detail in chapters 2 and 3, here we present only the final form of the energy equation that enables one to estimate the mass of a hadron of bag size R:

$$M(R) = \left\{ N_n \omega(m_n R) + N_s \omega(m_s R) + N_c \omega(m_c R) \right\} + \frac{4}{3}\pi R^3 B + \sum_{i>j} \lambda a_{ij} M_{ij} - Z/R \qquad (6.7)$$

where the symbols have the same significance as explained in earlier chapters. The indices i and j run over the quark/antiquark flavours present in the hadron. The requirement of stability against expansion that follows from the quadratic bag boundary condition for the spherical case fixes the bag radius R.

The parameters B, Z, α_s and the quark masses are determined by a fit to known hadron masses. De Grand et al. [64] consider two such fits, keeping the Δ, N, ω and Ω masses correct, which yield the following parameter values:

$$\text{(i)}\; m_u = m_d = 0, \quad m_s = 0.279 \text{ GeV}$$
$$B^{1/4} = 0.145 \text{ GeV}, \quad Z = 1.84, \quad \alpha_s = 0.55 \qquad (6.8)$$

$$\text{(ii)}\; m_u = m_d = 0.108 \text{ GeV}, \quad m_s = 0.353 \text{ GeV}$$
$$B^{1/4} = 0.125 \text{ GeV}, \quad Z = 1.95, \quad \alpha_s = 0.75. \qquad (6.9)$$

The former set gives a somewhat better prediction of the static properties of hadrons and seems to be rather universally accepted for bag model computations. With these parameter values, De Grand et al. [64] reproduced the mass spectrum of the uncharmed hadrons in fairly good agreement with observations, with a spectacular deviation arising only in the case of the pion mass, whose predicted value turns out to be twice the experimental value. A re-adjustment of the parameters fitting the correct pion mass badly spoils the other predictions.

6.4 The New Bag Phenomenology

We start by relating the bag pressure B to the energy density ρ arising from all contributions to the hadronic mass except the volume energy, through equation (6.6)

$$B = \frac{1}{3}\rho$$

We wish to point out here that this relation follows from the requirement of hydrodynamical stability, namely $P = 0$, and that it is consistent with the traditional stability condition $\partial M/\partial R = 0$ used in the original MIT model to determine the bag size. Writing

$$A = \sum_i N_i \omega(m_i R) + \sum_{i>j} \lambda\, a_{ij} M_{ij} - Z/R$$

$$= \frac{4}{3}\pi R^3 \rho \qquad (6.10)$$

We have

$$M = \frac{4}{3} \pi R^3 (\rho + B)$$

$$= \frac{4}{3} A \qquad (6.11)$$

An important point over which we differ from the MIT bag model calculations is with regard to the bag radius. In the MIT model the bag radius is different for different hadrons, as typified by a value of 5 GeV^{-1} for the nucleon and 3.26 GeV^{-1} for the kaon. Among the baryons the differences are not much, the variation being from 4.91 GeV^{-1} (Ξ) to 5.48 GeV^{-1} (Δ). As for the mesons, except for the kaon and the pion, the bag radius has a value slightly less than that of a typical baryon, but the K and π mesons have comparatively much smaller size, 3.26 and 3.34 GeV^{-1} respectively. It is interesting to note that, to get the right mass for the pion, the bag size has to be further reduced considerably. What is curious is the fact that there is no straightforward relationship between the masses of hadrons and their sizes, like the size varying as some power of the mass However, the radii are such that one can speak of an average baryon size (R_B) and an average meson size (R_M), with a SLAC bag model-type relation [41], namely,

$$R_M \sim R_B \times (2/3)^{1/3}$$

being obeyed approximately, where the factor 2/3 represents the ratio of the number of quarks and antiquarks in a meson to that in a baryon. It may further be noted that the bag model results are not very sensitive to small variations in the bag size. This is evident from Table 6.1. We are therefore motivated to introduce the idea of a constant bag radius with one value for baryons and a different value for mesons. In Eq. (6.10) the zero-point energy term now becomes a constant. Denoting this term by ε, we have

$$M = \frac{4}{3} \left[\sum_j N_j \omega(m_j R) + \sum_{i>j} a_{ij} M_{ij} - \varepsilon \right] \qquad (6.12)$$

in which ε has the same value for all baryons but has another fixed value for the mesons.

The quark mass m , bag radius R, the strong interaction coupling constant α_c and the zero-point energy ε are the parameters of the theory. We make use of the known values of the axial vector coupling constant g_A and the proton magnetic moment μ(P) to make a phenomenological estimate of the non-strange quark mass $m_n (m_n = m_u = m_d)$ and the bag size R. In the bag model

$$g_A = \frac{5}{9} \left[\frac{2\omega^2 R^2 + 4\omega R \cdot mR - 3mR}{2\omega R(\omega R - 1) + mR} \right] \qquad (6.13)$$

From β-decay of the neutron we find $g_A = 1.25$. With this value of g_A as input, (6.13) is solved numerically so as to satisfy the transcendental equation (2.32) which may be rewritten as

$$\tan (\omega^2 R^2 - m^2 R^2)^{\frac{1}{2}} = - (\omega^2 R^2 - m^2 R^2)^{\frac{1}{2}} / (\omega R + mR - 1) \qquad (6.14)$$

Table 6.1 Variation of mass with R in the case of the nucleon, with parameters: $m_n = 0$, $Z = 1.84$, $\alpha_c = 0.55$

R (GeV^{-1})	M (GeV)
4.8	0.934
4.9	0.932
5.0	0.931
5.1	0.932
5.2	0.933
5.3	0.936
5.4	0.939

Putting the values of ωR and mR thus obtained in the expression for the proton gyromagnetic ratio

$$g_P = 2M_P\mu_P = \frac{1}{3} RM_P \left[\frac{4\alpha + 2\lambda - 3}{2\alpha(\alpha - 1) + \lambda} \right] \qquad (6.15)$$

where $\alpha = \omega R$ and $\lambda = mR$, and using the experimental value of $g_P = 2.79$, we obtain $R = 8.88\ \mathrm{GeV}^{-1}$, and a quark kinetic energy

$$\omega_n = 0.294\ \mathrm{GeV},$$

that corresponds to a quark mass

$$m_n = 0.114\ \mathrm{GeV}.$$

The strange quark mass is then determined from the $\Lambda - N$ mass separation which yields

$$\omega_s = 0.427\ \mathrm{GeV}$$

corresponding to a bare quark mass

$$m_s = 0.302\ \mathrm{GeV}.$$

Finally we determine the values of α_c and ε by fitting the known masses of Λ and Ξ . The values obtained are

$$\alpha_s = 0.94 \quad \text{and} \quad \varepsilon = 68\ \mathrm{MeV}.$$

This value of α_c is greater than the MIT value of 0.55.

Large values of α_c are certainly permitted in bag-type confinement schemes. Renormalisation group arguments [37] show that the quark-gluon coupling constant can become very large for small momentum transfers. This large increase in quark-gluon coupling at small momentum transfers is conjectured as resulting in quarks being confined to a finite region of space.

A very recent work by Close and Monaghan [189] on a new approach to interactions in the MIT bag has explicitly shown that cavity perturbation theory is valid for $\alpha_c \lesssim 5$. This clearly justifies the phenomenological value of $\alpha_c = 0.94$ herein obtained which is equivalent to 3.6 in the convention followed by the authors of Ref. [189].

6.5 Mass Spectrum of Light Hadrons

6.5.1 Baryons

The parameter values mentioned in Sec. 6.4 are used to work out the mass spectrum of the low-lying baryons. Using the mass formula (6.12) and the spin factors a_{ij} (Table 6.2) the following relations can be written:

$$M_N = \frac{4}{3}\,[3\omega_n - 3\,M_{nn} - \varepsilon\,]$$

$$M_\Lambda = \frac{4}{3}\,[2\omega_n + \omega_s - 3M_{nn} - \varepsilon\,] \qquad (6.16a)$$

$$M_\Sigma = \frac{4}{3}\,[2\omega_n + \omega_s + M_{nn} - 4M_{ns} - \varepsilon\,]$$

Table 6.2 Values of a_{ij}

Particle	a_{nn}	a_{ns}	a_{ss}
N	-3	0	0
Δ	$+3$	0	0
Λ	-3	0	0
Σ	$+1$	-4	0
Σ^*	$+1$	$+2$	0
Ξ	0	-4	$+1$
Ξ^*	0	$+2$	$+1$
Ω	0	0	$+3$
π	-3	0	0
ρ	$+1$	0	0
ω	$+1$	0	0
K	0	-3	0
K^*	0	$+1$	0
φ	0	0	$+1$

$$M_{\Xi} = \frac{4}{3}\,[\omega_n + 2\,\omega_s - 4M_{ns} + M_{ss} - \varepsilon\,]$$

$$M_{\Delta} = \frac{4}{3}\,[3\omega_n + 3M_{nn} - \varepsilon\,]$$

$$M_{\Sigma^*} = \frac{4}{3}\,[2\omega_n + \omega_s + M_{nn} + 2M_{ns} - \varepsilon\,] \qquad (6.16b)$$

$$M_{\Xi^*} = \frac{4}{3}\,[\omega_n + 2\,\omega_s + 2M_{ns} + M_{ss} - \varepsilon\,]$$

$$M_{\Omega} = \frac{4}{3}\,[3\omega_s + 3M_{ss} - \varepsilon\,]$$

The values of M_{ij} are determined using equation (3.11). These are listed in Table 6.3. The results of our calculation of baryon masses, along with the experimental values as well as the MIT results for two sets of parameters are presented in Table 6.4.

6.5.2 Mesons

We have the following relations for the meson masses:

$$M_{\pi} = \frac{4}{3}\,[2\omega_n - 6M_{nn} - \varepsilon\,]$$

$$M_{K} = \frac{4}{3}\,[\omega_n + \omega_s - 6M_{ns} - \varepsilon\,]$$

$$\left.\begin{matrix} M_{\omega} \\ M_{\rho} \end{matrix}\right\} = \frac{4}{3}\,[2\omega_n + 2M_{nn} - \varepsilon\,] \qquad (6.17)$$

$$M_{K^*} = \frac{4}{3}\,[\omega_n + \omega_s + 2M_{ns} - \varepsilon\,]$$

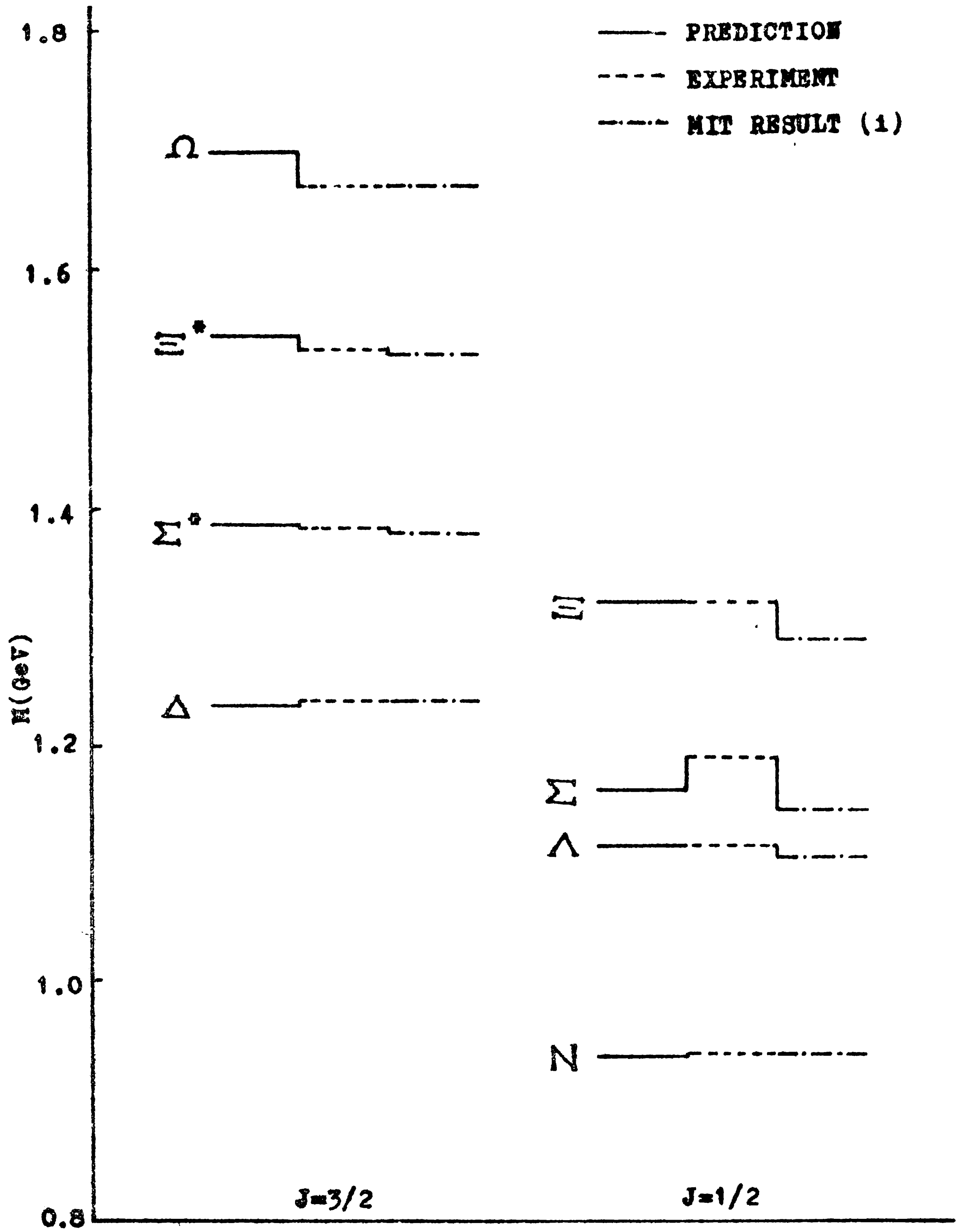
$$M_{\varphi} = \frac{4}{3}\,[2\omega_s + 2M_{ss} - \varepsilon\,]$$

Table 6.3 M_{ij} values for baryons.
Parameters: m_n = 0.114 GeV, m_s = 0.302 GeV, R = 8.88 GeV^{-1}

m_iR	m_jR	ω_iR	ω_jR	$I(m_iR, m_jR)$	M_{ij}
1.01	1.01	2.61	2.61	1.5575	0.0393 α_c
1.01	2.68	2.61	3.79	1.6143	0.0298 α_c
2.68	2.68	3.79	3.79	1.6956	0.0229 α_c

Table 6.4 Masses (in GeV) of the low-lying baryons, for the parameters: m_n = 0.114 GeV, m_s = 0.302 GeV, R = 8.88 GeV^{-1}, α_c = 0.94, ε = 0.068 GeV. The MIT results are for the two sets of parameters (i) and (ii) mentioned in the text.

Particle	Expt.	MIT(i)	MIT(ii)	Present calculation
N	0.938	0.938	0.938	0.937
Λ	1.116	1.105	1.103	1.116
Σ	1.189	1.144	1.145	1.163
Ξ	1.321	1.289	1.286	1.321
Δ	1.236	1.236	1.233	1.233
Σ^*	1.385	1.382	1.381	1.386
Ξ^*	1.533	1.529	1.528	1.542
Ω	1.672	1.672	1.672	1.702



MASS SPECTRUM OF BARYONS

The parameters appropriate to the baryon spectrum do not fit with the meson spectrum. Hence we seek another set of parameters for the low-lying mesons. There is no reason to change the quark masses. But it is reasonable to assume that the ratio of the meson to baryon bag radius is dependent on the ratio of the number of quarks and antiquarks in mesons and baryons. Thus following the SLAC approach [41], we assume

$$R_M = R_B(2/3)^{1/3} \tag{6.18}$$

With $R_B = 8.88\ \text{GeV}^{-1}$,

$$R_M = 7.75\ \text{GeV}^{-1}.$$

This decrease in bag size causes an increase in quark kinetic energies. Thus, for mesons we obtain

$$\omega_n = 0.321\ \text{GeV}.$$

and $\omega_s = 0.448\ \text{GeV}.$

Using the same value of the colour coupling constant α_c as for baryons, and fitting the experimental kaon mass we find

$$\varepsilon = 0.178\ \text{GeV}.$$

These parameters generate quite an agreeable meson spectrum. The pion mass has not improved considerably over the MIT value.

However a new value of α_c determined from the chromomagnetic splitting of ω and π, namely

$$\alpha_c = 1.25$$

which is within the bound suggested by Close and Monaghan [189] and the corresponding value of ε,

$$\varepsilon = 0.175 \text{ GeV}$$

needed to fit the mass of ω bring about substantial improvement over the MIT result. Our values of the meson masses for the two sets of parameters along with the MIT results are presented in Table 6.6. (Table 6.5 contains the relevant M_{ij} values). The K mass has come out somewhat poorer. But significantly, there is overall improvement which includes an exact fit with the pion mass.

6.6 Hadron Mass Splittings

Mass splittings among the light hadrons arise mainly from SU(3) breaking effects introduced through quark mass difference between nonstrange and strange flavours. Also there are the short range chromomagnetic forces depending on quark spins and masses that produce hyperfine mass splittings. These effects have already been taken into account in the bag formula for hadronic mass, where the chromomagnetic spin-spin interaction is assumed to arise from exchange of colour octet vector gluons

Table 6.5 M_{ij} values for mesons

Parameters: $m_n = 0.114$ GeV, $m_s = 0.302$ GeV,

$R = 7.75$ GeV^{-1}

$m_i R$	$m_j R$	$\omega_i R$	$\omega_j R$	$I(m_i R, m_j R)$	M_{ij}
0.88	0.88	2.49	2.49	1.5233	0.0482 α_c
0.88	2.34	2.49	3.47	1.5832	0.0391 α_c
2.34	2.34	3.47	3.47	1.6028	0.0308 α_c

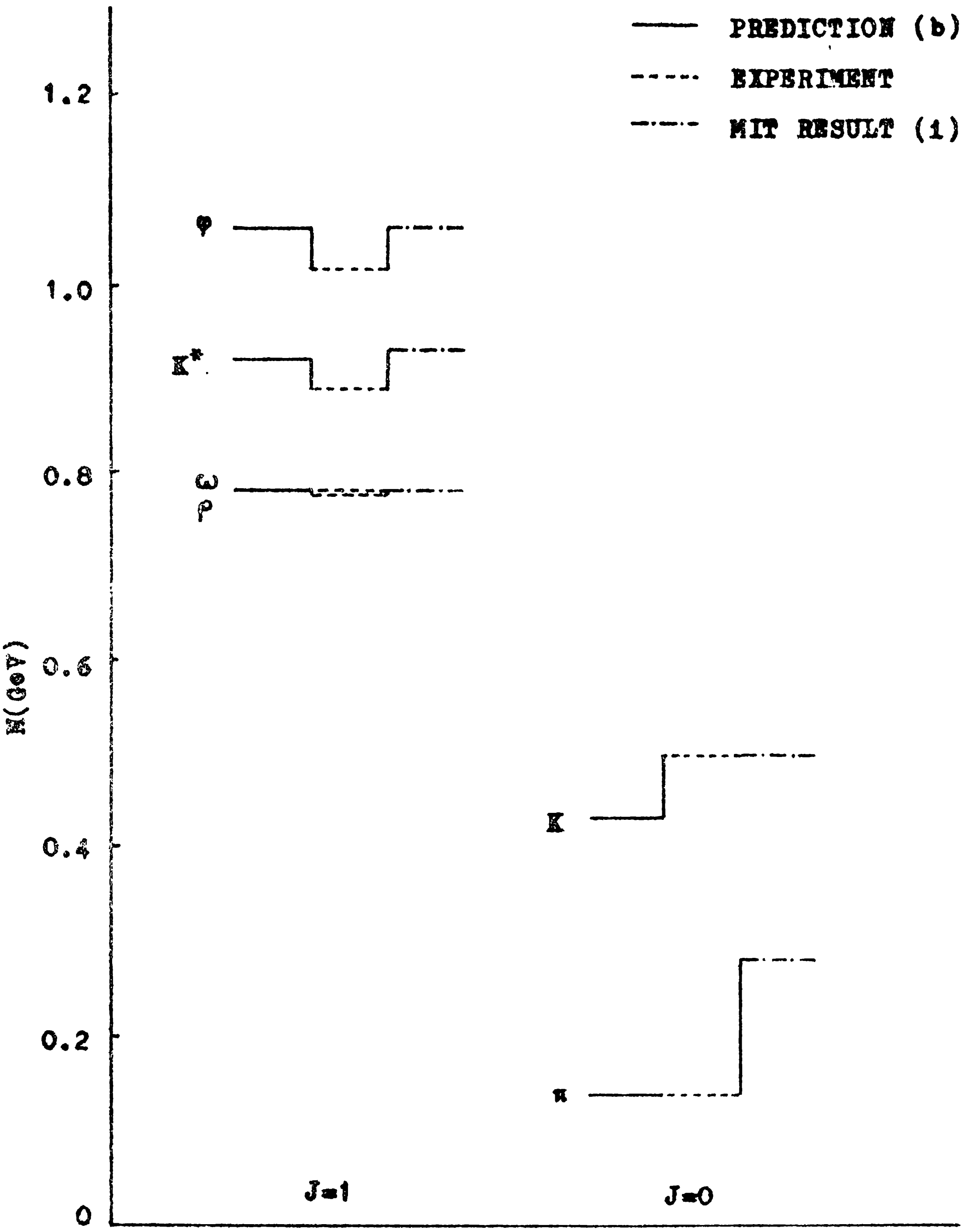
Table 6.6 Masses (in GeV) of the low-lying mesons, for the parameters:

(a) $m_n = 0.114$ GeV, $m_s = 0.302$ GeV, $R = 7.75$ GeV^{-1}, $\alpha_c = 0.94$, $\varepsilon = 0.178$ GeV.

(b) $m_n = 0.114$ GeV, $m_s = 0.302$ GeV, $R = 7.75$ GeV^{-1}, $\alpha_c = 1.25$, $\varepsilon = 0.175$ GeV.

The MIT results are for the sets of parameters (i) and (ii) mentioned in the text.

Particle	Expt.	MIT(i)	MIT(ii)	Present calculation (a)	Present calculation (b)
π	0.139	0.280	0.175	0.258	0.139
K	0.495	0.497	0.371	0.495	0.432
ω	0.783	0.783	0.783	0.740	0.783
ρ	0.77±.01	0.783	0.783	0.740	0.783
K^*	0.892	0.928	0.925	0.885	0.921
φ	1.020	1.068	1.063	1.034	1.062



MASS SPECTRUM OF MESONS

between the quarks. Thus in the mass spectrum presented above degeneracy between baryon decuplet and octet, as well as vector and pseudoscalar mesons and that among the various isospin multiplets are lifted. The mass splitting among the members of an isomultiplet is due to the mass difference between u and d quarks and the electromagnetic interaction between the quarks. These effects are not incorporated in the present calculations; the u and d quarks are assumed to have the same mass. The absolute value of the chromomagnetic interaction energy can be computed in the bag model in terms of the colour coupling constant α_c unlike in other models which invariably employ, in addition, a phenomenologically determined universal mass parameter depending on unknown details of the wavefunction.

From the energy (mass) equation (6.12) or equivalently from (6.16) and (6.17) the various SU(3) and SU(6) mass formulae can be deduced. The equal spacing rule for the baryon decuplet:

$$\Omega - \Xi^* = \Xi^* - \Sigma^* = \Sigma^* - \Delta \qquad (6.19)$$

(where particle symbols stand for the masses of the corresponding particles) that follows from the linear Gell-Mann-Okubo mass formula [5,190] implies that

$$2(M_{ns} - M_{ss}) = M_{nn} - M_{ss} = 2(M_{nn} - M_{ns})$$

$$\text{or} \quad M_{ns} = \frac{1}{2}(M_{nn} + M_{ss}) \qquad (6.20)$$

The Gell-Mann-Okubo formula for the baryon octet:

$$2N + 2\Xi = 3\Lambda + \Sigma , \qquad (6.21)$$

also implies the relation (6.20) among the colour magnetic interaction terms. This relation is very well satisfied by the new bag phenomenology. Also the SU(6) relation [191]:

$$\Sigma^* - \Sigma = \Xi^* - \Xi \qquad (6.22)$$

is satisfied exactly, while the relation [192]:

$$2\Delta - 2N = 3\Sigma^* + \Sigma - 3\Lambda , \qquad (6.23)$$

is satisfied within 2.5%.

For the vector mesons, the linear Gell-Mann-Okubo mass formula, taking into account the octet singlet mixing, reads

$$2\varphi + \omega + \rho = 4K^* \qquad (6.24)$$

This relation is well satisfied by the present model. The equation follows exactly from the already mentioned relation (Eq.(6.20)) among the magnetic interaction terms. However, the empirical relation

$$(K^* - K)_{us}/(\rho - \pi)_{uu} = 0.667 \qquad (6.25)$$

is only poorly satisfied. In the present model, the ratio of the vector-pseudoscalar splittings for the differing flavour combinations is obtained as

$$M_{ns}/M_{nn} \simeq 0.8 \qquad (6.26)$$

It is interesting to compare these results with those of the original MIT calculations [64]. In the MIT model the linear Gell-Mann-Okubo mass formula for baryon implies, as in our case $\frac{1}{2}(M_{nn} + M_{ss}) = M_{ns}$ and is satisfied well, if it be assumed that R does not vary from state to state which, of course, is not the case. The SU(6) relation (6.22) requires

$$M_{ns} = M_{ss} \qquad (6.27)$$

and the equal spacing rule for the decuplet demands

$$M_{ns} = M_{ss} = M_{nn} \qquad (6.28)$$

Neither of these formulae is satisfied.

An important aspect of the present model is the simple qualitative explanation it provides for the mass splittings among baryons. The decuplets are heavier than the corresponding octets for the fact that, even though the quark rest and kinetic energy contributions are identical, the colour magnetic interaction contribution is negative in the case of the octets, while it is positive for the decuplets. Besides, the contributions are different in magnitude as a result of SU(3) symmetry-breaking effects and the different spin orientations of the constituent quarks. Thus the octet-decuplet splitting turns out to be a pure hyperfine splitting. The situation is not so straightforward in the MIT model as the zero-point energy contributions are different for different hadrons.

Note that the $\Sigma - \Lambda$ mass difference also arises from colour magnetic interaction. Here the energy contributions are different as a result of the mass difference between the non-strange and strange quarks and the difference in relative spin orientations of the quarks. In Λ, the spin-zero combination of the u and d quarks leads to the result

$$\sum_{i>j} a_{ij} M_{ij} = -3M_{nn} \tag{6.29}$$

In Σ, u and d quarks form a spin-1 state with the consequence that

$$\sum_{i>j} a_{ij} M_{ij} = M_{nn} - 4M_{ns} \tag{6.30}$$

In the exact SU(3) limit,

$$M_{nn} = M_{ns} \tag{6.31}$$

and the $\Sigma - \Lambda$ degeneracy is restored.

6.7 Conclusion

We have introduced two major modifications in the phenomenological MIT bag model by choosing a variable bag pressure term B and a fixed bag size R. The bag size is fixed so as to reproduce the proton magnetic moment, and hence it differs considerably from the average radius in the original bag model. In our theory B is not a universal constant parameter as in the MIT model; it is determined by the density of hadronic matter constituting each particle. Its value thus varies from hadron to hadron. We have been able to reproduce the mass spectrum of the light hadrons, with an exact fit with the pion mass. Our analysis of the hadron mass splittings, taking into account the chromomagnetic hyperfine interactions of quarks, leads to well-established mass relations among hadrons. The comparatively better results obtained in the present approach are a clear manifestation of the fact that the universal character of the bag pressure term is not an essential requirement at least as far as the bag phenomenology is concerned. We wish to emphasize here that whatever success the bag model has achieved is essentially due to its being a quark model, and that the model permits a lot of flexibility in the choice of its parameters.

CHAPTER 7

MAGNETIC MOMENTS OF BARYONS

7.1 Introduction

The variable pressure bag model [185] developed in the previous chapter is used to make predictions of magnetic moments of baryons. The nonrelativistic quark model has been fairly successful in predicting the baryon magnetic moments in qualitative agreement with experiment. In particular, the famous relation

$$\mu(P)/\mu(N) = -3/2 \qquad (7.1)$$

is a remarkable achievement of the naive quark model [193,194]. This quark model result which is based on SU(6) symmetry compares very well with the experimental number -1.46.

The fundamental assumptions leading to the relation (7.1) are:

1. The well-known additivity assumption: the magnetic moment of a baryon is the sum of the magnetic moments of the constituent quarks.

2. The spins of the quarks determining the directions of the quark magnetic moments are given by the nonrelativistic SU(6) wave functions.

3. The magnetic moment of a quark is proportional to its electric charge.

Thus one can write for the baryon magnetic moment

$$\vec{\mu}_B = \sum_q \vec{\mu}_q \tag{7.2}$$

where the quark magnetic moment $\vec{\mu}_q$ is given by [156,195]

$$\vec{\mu}_q = \lambda_q e_q \langle \vec{\sigma}_q \rangle \tag{7.3}$$

in which e_q is the quark charge in units of the proton charge e, $\frac{1}{2}\vec{\sigma}_q$ is the quark spin operator and λ_q is the quark scale moment μ_q divided by e_q. Eq. (7.3) can also be written as

$$\vec{\mu}_q = \mu_q \langle \vec{\sigma}_q \rangle \tag{7.4}$$

Eq. (7.3) leads to the following expressions for the various baryon magnetic moments in terms of λ_q [195,196]:

$$\mu(P) = \frac{8}{9}\lambda_u + \frac{1}{9}\lambda_d$$

$$\mu(N) = -\frac{2}{9}\lambda_u - \frac{4}{9}\lambda_d$$

$$\mu(\Lambda) = -\frac{1}{3}\lambda_s$$

$$\mu(\Sigma^+) = \frac{8}{9}\lambda_u + \frac{1}{9}\lambda_s$$

$$\mu(\Sigma^0) = \frac{4}{9}\lambda_u - \frac{2}{9}\lambda_d + \frac{1}{9}\lambda_s \qquad (7.5)$$

$$\mu(\Sigma^-) = -\frac{4}{9}\lambda_d + \frac{1}{9}\lambda_s$$

$$\mu(\Xi^0) = -\frac{2}{9}\lambda_u - \frac{4}{9}\lambda_s$$

$$\mu(\Xi^-) = \frac{1}{9}\lambda_d - \frac{4}{9}\lambda_s$$

In the exact SU(3) limit

$$\lambda_u = \lambda_d = \lambda_s$$

and the usual SU(3) relations among baryon magnetic moments follow. The quark model, however, does not permit the explicit evaluation of absolute values of the quark moments. The factors λ_u, λ_d, and λ_s are determined from the known experimental values of the magnetic moments of P, N and Λ. These are then used to predict the magnetic moments of the other baryons. The bag model, on the other hand, allows the absolute magnitudes of the quark moment contributions μ_u, μ_d and μ_s being estimated using the cavity eigenfunctions of the confined quarks.

But quantitatively the performance of the original MIT bag model [64] in this context has not been quite commendable. In fact, one of the serious drawbacks of the MIT model as pointed out by its promoters is its poor prediction of baryon magnetic moments. The proton magnetic moment was found to be as low as 1.9 nuclear magneton (n.m.) as against the experimental value of 2.79 n.m., though the ratio of $\mu(P)$ to $\mu(N)$ was obtained as -3/2 in agreement with the quark model prediction. Consistently low values were obtained for the magnetic moments of other baryons as well. The smallness of μ_B was conjectured as due to the smallness of the bag size. A number of later works [101,197,198] have demonstrated that a large bag size is an essential requirement for obtaining better accord with magnetic moment data. The variable pressure bag model with its relatively large size which is consistent with the observed proton magnetic moment is expected to do better. The purpose of the present investigation is to see how far this expectation is borne out by facts. Better agreement with magnetic moment data, if obtained, would provide a further point in support of the relevance of the new version of the bag model in phenomenological applications.

7.2 Quark Moments in the Bag Model

The magnetic moment of a single quark of mass m confined to a bag of radius R is given by

$$\mu_q = \int_0^R (d^3x \tfrac{1}{2} \vec{r} \times :q^+(x) \vec{\alpha} e_q q(x):)_z \qquad (7.6)$$

in which q(x) is the quark wave function given by Eq.(2.28) and e_q is the electric charge of quark q. On evaluation the integral yields

$$\mu_q = \frac{R}{6} \left(\frac{4\omega R + 2mR - 3}{2\omega R(\omega R-1) + mR}\right) e_q \qquad (7.7)$$

Remembering that

$$e_u = 2e/3 \ ,$$

$$e_d = e_s = -e/3 \ ,$$

we write

$$\mu_u = \frac{Re}{9} \left(\frac{4\omega_u R + 2m_u R - 3}{2\omega_u R(\omega_u R-1) + m_u R}\right) \qquad (7.8a)$$

$$\mu_d = -\frac{Re}{18} \left(\frac{4\omega_d R + 2m_d R - 3}{2\omega_d R(\omega_d R-1) + m_d R}\right) \qquad (7.8b)$$

$$\mu_s = -\frac{Re}{18} \left(\frac{4\omega_s R + 2m_s R - 3}{2\omega_s R(\omega_s R-1) + m_s R}\right) \qquad (7.8c)$$

Since we have assumed that

$$m_u = m_d \; (=m_n) \; ,$$

we have

$$\mu_d = - \tfrac{1}{2}\mu_u \qquad (7.9)$$

In the present calculations the bag radius is the same for all baryons. Hence μ_u, μ_d, and μ_s do not vary from baryon to baryon as in the original MIT bag calculations [64]. We choose the same bag parameters as we considered in chapter 6 for the baryon mass spectrum.

Hence we have

$$m_n R = 1.01, \quad m_s R = 2.68 \; ,$$

$$\omega_n R = 2.61, \quad \text{and}\; \omega_s R = 3.79 \; , \qquad (7.10)$$

so that

$$\mu_u = 0.9915e = 0.9915e \times 2M_p/e \text{ n.m.}$$

$$= 1.86 \text{ n.m.},$$

$$\mu_d = - \tfrac{1}{2}\mu_u = -0.93 \text{ n.m.},$$

$$\mu_s = - 0.68 \text{ n.m.} \qquad (7.11)$$

In nonrelativistic free quark models (models which do not take into account the finite size of the hadron) the quark moments are independent of the hadron. The phenomenological values of λ_q [195] are

$$\lambda_u = 2.778 \text{ n.m.}$$

$$\lambda_d = 2.9313 \text{ n.m.}$$

$$\lambda_s = 1.84 \pm 1\% \text{ n.m.} \qquad (7.12)$$

λ_u and λ_d are very nearly equal and λ_s is roughly $(2/3)\lambda_u$. Taking the average value of λ_u and λ_d, and assuming quark moments μ_q as Dirac moments for point-particles of mass m_q,

$$\mu_q = e_q \lambda_q = e_q/2m_q \qquad (7.13)$$

(in natural units), one finds that

$$m_u = m_d = 0.330 \text{ GeV, and}$$

$$m_s = 0.510 \text{ GeV.}$$

In the MIT bag model which is a relativistic model

$$\lambda_q = \frac{R}{6}\,\frac{4\alpha + 2\lambda - 3}{2\alpha(\alpha - 1) + \lambda} \qquad (7.14)$$

where $\lambda = mR$, $\alpha = R\omega(mR)$. λ_q, thus depends on the quark energy $\omega(mR)$ and the bag size R, both of which are different for different hadrons and hence the assumption that λ_q is independent of the hadron is no longer true. However, in the variable pressure bag model in which the present calculations are done, since the bag size and $\omega(mR)$ which depends only on the quark mass and the fixed value of R are the same for all baryons, values of λ_u, λ_d and λ_s do not vary from baryon to baryon. It seems that the scale of the magnetic moments (the value of λ_q) is determined primarily by the structure afforded by a finite bag size rather than by quark mass. In fact Donoghue et al. [101] have shown in the case of proton that its magnetic moment is not appreciably affected by reasonable changes in quark masses. An increase in the nonstrange quark mass from 0 to 130 MeV results in as small a change in $\mu(P)$ as -0.75%. But the dependence of the magnetic moment on the bag radius is crucial.

7.3 Baryon Magnetic Moments

From the magnetic moments of individual quarks, the baryon magnetic moments are computed by making use of the additivity assumption of the quark model. Thus ignoring the possibility of quark anomalous magnetic moments, we express the baryon magnetic moment as the sum of the quark moments, assuming further that the orbital angular momenta of the quarks are zero. Hence we have the operator relation

$$\vec{\mu}_B = \sum_i \vec{\mu}_q(m_i R) \qquad (7.15)$$

i being flavour index. The value of μ_B is estimated in terms of μ_q from a knowledge of the flavour and spin wave functions of the baryon, and assuming the validity of the quark model relations (7.3) and (7.4). Denoting the combined spin and flavour wave functions of the quarks by u, d, s we have for the magnetic moment of the proton

$$\mu(P) = (uud, (\sum_i \vec{\mu}_q(m_i R))_z uud) \qquad (7.16)$$

The expression on the righthandside of Eq.(7.16) contains products of expectation values of the quark scale moment and quark spin operators: $\lambda_q e_q$ and σ_q^3, respectively. The expectation values of σ_q^3 for the various baryons are presented in Table 7.1. These are determined by assuming the following structure for the spin functions:

$$\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \qquad (7.17)$$

Using the values of $\langle \sigma_q^3 \rangle$, Eq. (7.16) can be brought to the form

$$\mu(P) = \frac{1}{3}(4\mu_\mu - \mu_d) \qquad (7.18a)$$

Similarly expressions for magnetic moments of the other baryons can be obtained in terms of μ_u, μ_d and μ_s as follows:

$$\mu(N) = \frac{1}{3}(4\mu_d - \mu_u) \qquad (7.18b)$$

$$\mu(\Lambda) = \mu_s \qquad (7.18c)$$

$$\mu(\Sigma^+) = \frac{1}{3}(4\mu_u - \mu_s) \qquad (7.18d)$$

Table 7.1 $\langle \sigma_q^3 \rangle$ for the stable baryons

Baryon	$\langle \sigma_u^3 \rangle$	$\langle \sigma_d^3 \rangle$	$\langle \sigma_s^3 \rangle$
P	4/3	-1/3	0
N	-1/3	4/3	0
Λ	0	0	1
Σ^+	4/3	0	-1/3
Σ^o	2/3	2/3	-1/3
Σ^-	0	4/3	-1/3
Ξ^o	-1/3	0	4/3
Ξ^-	0	-1/3	4/3
Ω	0	0	3

$$\mu(\Sigma^{0}) = \frac{1}{3}(2\mu_u + 2\mu_d - \mu_s) \tag{7.18e}$$

$$\mu(\Sigma^{-}) = \frac{1}{3}(4\mu_d - \mu_s) \tag{7.18f}$$

$$\mu(\Xi^{0}) = \frac{1}{3}(4\mu_s - \mu_u) \tag{7.18g}$$

$$\mu(\Xi^{-}) = \frac{1}{3}(4\mu_s - \mu_d) \tag{7.18h}$$

$$\mu(\Omega) = 3\mu_s \tag{7.18i}$$

μ_u, μ_d and μ_s have the bag model values by Eq. (7.11). Inserting these values in Eqs. (7.18) the magnetic moments of the various baryons are evaluated and these are listed in Table 7.2 along with the experimental data [157] and the predictions of the original MIT bag model [64]. For the proton and the neutron we get the values

$$\mu(P) = 2.79 \text{ n.m.}$$

$$\mu(N) = -1.86 \text{ n.m.}$$

in excellent agreement with experimental data. For the ratio of these moments we get

$$\mu(P)/\mu(N) = -3/2$$

in exact agreement with the quark model prediction. The experimental data presented here is based on the latest report of the particle Data Group [157], while the data given in our published

Table 7.2 Magnetic moments of baryons in units of nuclear magneton

Baryon	Expt.	MIT result	Present result
P	2.79	1.90	2.79
N	-1.91	-1.27	-1.86
Λ	-0.61	-0.484	-0.68
Σ^+	2.33±.15	1.843	2.707
Σ^0	..	0.589	0.847
Σ^-	-1.41±.25	-0.684	-1.013
Ξ^0	-1.20±.06	-1.064	-1.527
*Ξ^-	-1.85±.75	-0.437	-0.597
Ω	..	-1.452	-1.84

*A very recent measurement has yielded the value $\mu(\Xi^-) = -0.75$. However, this is a preliminary result based on a partial sample of the data, as reported by Devlin [199].

work [185] which forms the basis of this chapter is taken from their earlier report. Note the change in the values of $\mu(\Sigma^+)$ and $\mu(\Sigma^-)$, and the new addition $\mu(\Xi^0)$.

7.4 Discussion

It is clear from Table 7.2 that in general the magnetic moments of baryons resulting from the present calculation are in better agreement with experiment than the corresponding MIT results [64]. In Ref.[64] magnetic moments of baryons are listed in units of proton magnetic moment $\mu(P)$, with the absolute value of $\mu(P)$ being 1.9 n.m. only. The $\mu(P)/\mu(N)$ ratio has however been obtained as -3/2. This is not surprising. It should be noted that this particular result has nothing to do with bag phenomenology. Any model of hadrons with the fractionally charged quarks as constituents should give this result as long as the ratio of the electrical charges of the u and d quarks is -2, and the basic assumptions mentioned in the introduction are followed.

Our prediction for $\mu(\Lambda)$ is -0.68 n.m. while the experimental number is -0.61 n.m. Here we recall the suggestion made by Lipkin [200] as to how magnetic moment of Λ can be reproduced by setting the quark mass difference

$$m_s - m_u = M_\Lambda - M_P \qquad (7.19)$$

In the bag model, instead of the bare quark mass m, one should use the effective quark mass $\omega(m,R)$. Thus setting

$$\omega(m_s R) - \omega(m_n R) = M_\Lambda - M_P$$

or simply,

$$\omega_s - \omega_n = M_\Lambda - M_P \qquad (7.20)$$

we find

$$\omega_s = 0.472 \text{ GeV}.$$

This corresponds to a bare quark mass $m_s = 0.357$ GeV. It gives $\mu_s = -0.62$ instead of -0.68. Accordingly $\mu(\Lambda)$ becomes -0.62 n.m., where the agreement with experiment is now within 2‰. The above procedure for fixing m_s also brings about changes in the magnetic moment predictions for other baryons directed towards the respective experimental values. Lipkin [200] has obtained another quark model prediction of the Λ magnetic moment in exact agreement with experiment using a different input for SU(3) breaking in quark masses, namely,

$$m_s/m_u = (M_{\Sigma^*} - M_\Sigma)/(M_\Delta - M_P) \qquad (7.21)$$

Before examining the implication of such a relation in the context of the bag model, we would correct the above equation as

$$m_u/m_s = (M_{\Sigma^*} - M_\Sigma)/(M_\Delta - M_P) \qquad (7.22)$$

In a colour-quark model, the mass splittings $M_{\Sigma^*} - M_\Sigma$ and $M_\Delta - M_P$ arise from "colour magnetic" interactions of quarks.

It follows that the above mass splittings are proportional to the quark magnetic moments μ_s and μ_u respectively, which in turn must be inversely proportional to the respective quark masses. Hence (7.22) must be the correct relation for the quark mass ratio.

The equivalent of relation (7.22) for the bag model is obtained by replacing m_u and m_s by the effective quark masses ω_u and ω_s:

$$\omega_u/\omega_s = (M_{\Sigma^*} - M_{\Sigma})/(M_{\Delta} - M_P) \qquad (7.23)$$

Using the observed mass splittings we find $\omega_s = 0.448$ GeV, which corresponds to a bare quark mass $m_s = 0.328$ GeV. These inputs lead to a bag model prediction: $\mu(\Lambda) = 0.65$ n.m., which is worse than the prediction resulting from the quark mass difference relation (7.19). In MIT-type bag models the spin splittings are, in general, related to the colour magnetic interaction terms M_{ij} defined in chapter 3 [Eq. (3.11)]. In the variable pressure bag model, we have, in particular, the mass equations (6.16), from which it follows that

$$(M_{\Sigma^*} - M_{\Sigma})/(M_{\Delta} - M_P) = M_{ns}/M_{nn}$$

$$= \frac{\mu'(m_sR)I(m_nR, m_sR)}{\mu'(m_nR)I(m_nR, m_nR)} \qquad (7.24)$$

where $\mu'(mR)$ is given by Eq. (3.6). The slowly varying functions $I(m_iR, m_jR)$ appearing on the r.h.s. of Eq. (7.24) must be very nearly equal. Hence we may write

$$(M_{\Sigma^*} - M_{\Sigma})/(M_{\Delta} - M_P) = \mu'(m_sR)/\mu'(m_nR)$$

$$= \mu_s'/\mu_u' \qquad (7.25)$$

Using the bag parameters $R = 8.88\ \text{GeV}^{-1}$ and $m_n = 0.114$ GeV which reproduce the proton magnetic moment we get $\mu_u' = 1.0048$. This, together with the hadron mass splittings gives $\mu_s' = 0.661$, which yields for the strange quark moment $\mu_s = -0.611$ n.m. Thus we are led to the prediction

$$\mu(\Lambda) = -0.61 \text{ n.m.},$$

in exact agreement with its precisely measured value [203].

The above result provides a striking confirmation of the fact that the mass splittings $M_{\Sigma^*} - M_{\Sigma}$ and $M_{\Delta} - M_P$ arise purely as a result of the colour magnetic interactions of quarks and the exact agreement obtained by Lipkin between theory and agreement as regards $\mu(\Lambda)$ cannot be considered as accidental contrary to the skepticism expressed by Minami [202]. The latter author used a Lipkin-type relation for the mass difference of the u and d quarks:

$$m_d - m_u = M_N - M_P \qquad (7.26)$$

and using quark model he found that

$$m_u \simeq 335.8 \text{ MeV and } M_d \simeq 337.1 \text{ MeV},$$

with $\mu(P) = 2.793$ n.m. as input. This SU(3) breaking effect was then used to predict $\mu(N)$ which turned out to be -1.858 n.m. This is worse than the SU(3) prediction of -1.862 n.m. Further, he finds that the smaller the value of m_u/m_d the larger is the deviation of calculated value of $\mu(N)$ from the experimental number. In short, the SU(3) prediction cannot be improved upon with $m_d > m_u$, while observed P-N mass separation demands m_d to be greater than m_u. Hence it is argued that the success of the Lipkin's relation for quark mass difference can perhaps be an accident. We would like to point out that the extension of the said Lipkin's relation to the present context is not justified in view of the fact here the P-N splitting is caused not by the quark mass difference alone. The energy associated with the electromagnetic field generated by the quarks should definitely have a non-negligible effect. The situation is, however, different in the Λ-P case where the Lipkin relation for the quark mass difference holds. The Λ-P mass splitting is caused almost entirely by SU(3) breaking forces that arise from the difference in u and s quark masses, as the colour magnetic interaction effects on Λ and P masses cancel each other and electromagnetic interaction effects are negligible.

Based on the basic assumptions of the nonrelativistic SU(6) quark model Fritzsch [195] makes the following predictions of the hyperon magnetic moments using the measured magnetic moments of P, N and Λ :

$$\mu(\Sigma^+) = 2.69 \text{ n.m.}, \quad \mu(\Sigma^-) = -1.09 \text{ n.m.},$$
$$\mu(\Xi^0) = -1.44 \text{ n.m.}, \quad \mu(\Xi^-) = -0.495 \text{ n.m.} \tag{7.27}$$

There is very good agreement between these and the results of our calculation (See Table 7.2). Also it is instructive to compare these predictions with those of De Rũjula et al. [55]:

$$\mu(\Sigma^+) = 2.67 \text{ n.m.}, \quad \mu(\Sigma^-) = -1.05 \text{ n.m.},$$
$$\mu(\Xi^0) = -1.39 \text{ n.m.}, \quad \mu(\Xi^-) = -0.46 \text{ n.m.} \tag{7.28}$$

Large discrepancy between theoretical predictions and the experimental value is noted in the case of the strange hyperon

$$\mu(\Xi^-)_{exp.} = -1.85 \pm 0.75 \tag{7.29}$$

$$\mu(\Xi^-)_{theor.} = -0.495 \text{ (Ref.[195])}$$
$$= -0.46 \text{ (Ref.[55])} \tag{7.30}$$
$$= -0.597 \text{ (present calculation)}$$

If the basic additivity assumption of the quark model be true and the very good agreement between theory and experiment in the case of $\mu(P)/\mu(N)$ [Eq.(7.1)] not accidental, then as Fritzsch [195] has pointed out

$$|\mu(\Xi^-)| < |\mu(\Lambda)| \tag{7.31}$$

This inequality sets a bound on $\mu(\Xi^-)$. The result of the recent high precision measurement by Schachinger et al. [203] has yielded

$$\mu(\Lambda) = -0.6138 \pm 0.0047 \text{ n.m.} \tag{7.32}$$

Hence (7.31) implies

$$|\mu(\Xi^-)| < 0.614 \tag{7.33}$$

All theoretical predictions [Eq.(7.30)] are compatible with (7.33), but the present experimental value badly violates this bound. We should therefore agree with the observation made by Fritzsch [195] that the experimental value of $\mu(\Xi^-)$ (as quoted in Table 7.2) may be incorrect and should emphasise the need for a more precise measurement of this quantity.

We conclude by noting that the general agreement of the present results with the experimental values is suggestive of the fact that the basic additivity assumption of the quark model works very well and that the modifications incorporated in the bag model have definitely improved its phenomenological potential in describing static properties of hadrons.

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APPENDIX I

NOTATION AND CONVENTIONS

Natural units are used throughout so that $c = \hbar = 1$.

The space-time coordinates are denoted by the 4-vector:

$$x^\mu \equiv (x^0, x^1, x^2, x^3) = (t, x, y, z) = (t, \vec{x})$$

Also $\quad x_\mu \equiv (x_0, x_1, x_2, x_3) = (t, -x, -y, -z) = g_{\mu\nu} x^\nu$

where

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Summation over repeated indices is implied unless otherwise specified. The inner product is

$$x^2 = x_\mu x^\mu = t^2 - \vec{x}^2$$

Momenta are defined

$$p^\mu = (E, p_x, p_y, p_z)$$

with the inner product

$$p_1 \cdot p_2 = p_1^\mu \, p_{2\mu} = E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2$$

Also $x \cdot p = tE - \vec{x} \cdot \vec{p}$

The momentum operator in the coordinate representation is

$$p^\mu = i \frac{\partial}{\partial x_\mu} = i \partial_\mu = (i\partial_t, -i \vec{\nabla})$$

with $p^\mu p_\mu = -\partial_\mu \partial^\mu = -\Box$

Dirac matrices:

The following representation is chosen for the Dirac γ-matrices.

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}, \quad k = 1, 2, 3$$

where

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

are the 2x2 Pauli matrices, and $1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the 2x2 unit matrix. One of the useful combinations is

$$\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3 = -\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Also, we have

$$(\gamma^0)^+ = \gamma^0, \; (\gamma^k)^+ = -\gamma^k, \; \gamma^\mu\gamma^5 = -\gamma^5\gamma^\mu.$$

APPENDIX II

WEAK INTERACTION HAMILTONIAN

In the GIM scheme the weak hadron current is given by

$$J^h_\mu = \bar{q}\ C_H\ \gamma_\mu\ (1 + \gamma_5) q$$

where $q = \begin{pmatrix} c \\ u \\ d \\ s \end{pmatrix}$

$$C_H = \left(\begin{array}{c|c} 0 & U \\ \hline 0 & 0 \end{array}\right)$$

$$U = \begin{pmatrix} -\sin\theta_c & \cos\theta_c \\ \cos\theta_c & \sin\theta_c \end{pmatrix}$$

θ_c being the Cabibbo angle.

Thus

$$J^h_\mu = -\sin\theta_c\ \bar{c}\gamma_\mu(1 + \gamma_5)d + \cos\theta_c\ \bar{u}\gamma_\mu(1 + \gamma_5)d$$

$$+ \cos\theta_c\ \bar{c}\gamma_\mu(1 + \gamma_5)s + \sin\theta_c\ \bar{u}\gamma_\mu(1 + \gamma_5)s$$

$$(J_\mu^h)^+ = -\sin\theta_c \bar{d}\,\gamma_\mu(1+\gamma_5)c + \cos\theta_c \bar{d}\,\gamma_\mu(1+\gamma_5)u$$
$$+ \cos\theta_c\,\bar{s}\,\gamma_\mu(1+\gamma_5)c + \sin\theta_c\,\bar{s}\,\gamma_\mu(1+\gamma_5)u$$

In the current x current form, the Hamiltonian is given by

$$H = \frac{G}{\sqrt{2}}(J_\mu^h)^+(J_\mu^h)$$

i.e.,

$$\begin{aligned}
H = \frac{G}{\sqrt{2}}[\sin^2\theta_c \{ & \bar{d}\,\gamma_\mu(1+\gamma_5)c\,\bar{c}\,\gamma_\mu(1+\gamma_5)d \\
& - \bar{d}\gamma_\mu\,(1+\gamma_5)c\,\bar{u}\,\gamma_\mu(1+\gamma_5)s \\
& - \bar{s}\,\gamma_\mu(1+\gamma_5)u\,\bar{c}\,\gamma_\mu(1+\gamma_5)d \\
& + \bar{s}\,\gamma_\mu(1+\gamma_5)u\,\bar{u}\,\gamma_\mu(1+\gamma_5)s\} \\
+ \cos^2\theta_c \{ & \bar{d}\,\gamma_\mu(1+\gamma_5)u\,\bar{u}\,\gamma_\mu(1+\gamma_5)d \\
& + \bar{d}\,\gamma_\mu(1+\gamma_5)u\,\bar{c}\,\gamma_\mu(1+\gamma_5)s \\
& + \bar{s}\,\gamma_\mu(1+\gamma_5)c\,\bar{u}\,\gamma_\mu(1+\gamma_5)d \\
& + \bar{s}\,\gamma_\mu(1+\gamma_5)c\,\bar{c}\,\gamma_\mu(1+\gamma_5)s\} \\
+ \sin\theta_c\,\cos\theta_c \{ & -\bar{d}\,\gamma_\mu(1+\gamma_5)c\,\bar{u}\,\gamma_\mu(1+\gamma_5)d \\
& -\bar{d}\,\gamma_\mu(1+\gamma_5)c\,\bar{c}\,\gamma_\mu(1+\gamma_5)s \\
& -\bar{d}\,\gamma_\mu(1+\gamma_5)u\,\bar{c}\,\gamma_\mu(1+\gamma_5)d \\
& +\bar{d}\,\gamma_\mu(1+\gamma_5)u\,\bar{u}\,\gamma_\mu(1+\gamma_5)s
\end{aligned}$$

$$+\bar{s}\,\gamma_\mu(1+\gamma_5)u\;\bar{u}\,\gamma_\mu(1+\gamma_5)d$$

$$+\bar{s}\,\gamma_\mu(1+\gamma_5)u\;\bar{c}\,\gamma_\mu(1+\gamma_5)s$$

$$+\bar{s}\,\gamma_\mu(1+\gamma_5)c\;\bar{u}\,\gamma_\mu(1+\gamma_5)s$$

$$-\bar{s}\,\gamma_\mu(1+\gamma_5)c\;\bar{c}\,\gamma_\mu(1+\gamma_5)d\Big\}\Big]$$

We are interested in the charm-changing and strangeness-changing parts of H only. Consequently terms involving $c\bar{c}$ may be left out. Also terms which do not contain c or $\bar{c}$ may be left out for the same reason. We are thus left with 8 terms. They correspond to $\Delta c = \pm 1$, and $\Delta s = \pm 1, 0$.

From these we pick out terms satisfying particular selection rules:

1. $\Delta c = -1$ and $\Delta s = -1$,

$\Delta c = +1$ and $\Delta s = +1$,

so that $\Delta s / \Delta c = +1$

$\Delta s/\Delta c = +1$

$$H_W = \frac{G}{\sqrt{2}}\ \cos^2\theta_c)\Big\{\bar{d}\,\gamma_\mu(1+\gamma_5)u\;\bar{c}\,\gamma_\mu(1+\gamma_5)s + \bar{s}\gamma_\mu(1+\gamma_5)c\;\bar{u}\,\gamma_\mu(1+\gamma_5)d\Big\}$$

2. $\Delta c = -1$, and $\Delta s = +1$,

$\Delta c = +1$ and $\Delta s = -1$,

so that $\Delta s/\Delta c = -1$.

$$H_W^{\Delta s/\Delta c=-1} = \frac{G}{\sqrt{2}}(-\sin^2\theta_c)\left\{\bar{d}\,\gamma_\mu(1+\gamma_5)c\;\bar{u}\,\gamma_\mu(1+\gamma_5)s + \bar{s}\,\gamma_\mu(1+\gamma_5)u\;\bar{c}\,\gamma_\mu(1+\gamma_5)d\right\}$$

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