Modern Physics Letters B Vol. 4, No. 1 (1990) 47–51

TWO DIMENSIONAL LARGE AMPLITUDE QUASI SOLITONS IN THIN HELIUM FILMS

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Received 28 August 1989

Large amplitude local density fluctuations in a thin superfluid ^{4}He film is considered. It is shown that these large amplitude fluctuations travel and behave like "quasi-solitons" under collision, even when the full nonlinearity arising from the Van der Waals potential is taken into account.

Recently, the propagation of solitons on thin helium films has attracted considerable attention of both experimentalists and theorists. It has provided a wealth of information on the microscopic properties of superfluid helium. Huberman[1] was the first to point out the possibility that the nonlinear local density fluctuations in very thin ^{4}He films may travel unattenuated for large times. Later Nakajima *et al.*[2] derived the Korteweg de Vries (KdV) equation for two dimensional helium films by considering one dimensional solitary waves propagating along one direction. Starting from the phenomenological equation of motion as proposed by Rutledge *et al.*,[3] a quite different result was obtained by Biswas and Warke,[4] wherein the coefficients of the nonlinear term in the KdV equation was chosen differently from that considered by Huberman.

Later Biswas and Warke[5] generalized their earlier result to quasi two dimensional wave propagation (where essentially the direction of the propagation was chosen to be along the x-axis and the y dependence was assumed to be weak) and obtained the Kadomtsev-Petviashvili (K-P) equation. They established that in two dimensional superfluid ^{4}He films, the one dimensional solitons do not represent stable states in general. Using these results, we have[6] studied the phenomenon of two soliton resonance of the K-P equation for the superfluid surface density fluctuations and obtained the velocity of the resonant soliton.

In all the work sited above, only the lowest nonlinearity was taken into account. Kurihara[7] was the first to study the dynamics of wave propagation when the complete nonlinear form of the Van der Waals potential is retained. The analysis was

PACS Nos.: 67.40. 03.40K.

done numerically and he obtained the result that there exists soliton-like localized excitations, "quasi-solitons", which behave like quasi-particles and have a high degree of stability under collisions. These solitons had profiles which are localized along only one of the directions.

In this letter, we report the study of wave propagation on a two-dimensional ^{4}He film, taking into account the full nonlinearity of the Van der Waals potential. The analysis is done numerically as an initial value problem where an arbitrary gaussian initial profile is used. This is essentially an extension of the work of Kurihara to a situation where the initial profile decayed in both x and y directions. These are hence localized in two dimensions.

We start with the equation of motion for the superfluid density fluctuation.[3]

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - \frac{A\psi}{(a+|\psi|^2)^3} - \mu\psi - B\psi\nabla^2|\psi|^2, \tag{1}$$

where m = mass of ^{4}He atom, A and a are constants of Van der Waals interaction, μ = chemical potential and B is the surface tension. This equation has recently been obtained by Balakrishnan *et al.*[8] by starting from a microscopic theory of nonlinear dynamics in superfluid ^{4}He, formulated using a model in which a system of bosons with hard cores plus attractive nearest neighbor interactions is described by a pseudospin hamiltonian on a lattice. For the monolayer films, we are going to consider, $B = 0$.[3] If we search for a solution of the form $\psi(x, y, t) = \rho_s^{1/2}(x, y, t)\, e^{i\phi(x,y,t)}$, where $\rho_s(x, y, t)$ is the superfluid density, one would get the two dimensional continuity equation[3]

$$\frac{\partial\rho_s}{\partial t} + \nabla\cdot\mathbf{j}_s = 0, \tag{2}$$

where $\mathbf{j}_s(x, y, t) = \mathrm{Re}\,[(\hbar/im)\psi^*\nabla\psi]$ is the quantum-mechanical current density.

For the purpose of numerical analysis, we transform Eq. (1) to a dimensionless form. We assume that ψ depends only on the time coordinate t and space coordinates x and y. The scale for ψ is its equilibrium value ψ_0, obtained from the relation

$$\mu + \frac{A}{(a+|\psi_0|^2)^3} = 0. \tag{3}$$

We can fix the equilibrium value of the superfluid thickness as

$$d_0 = \psi_0^2/a. \tag{4}$$

Scales for the space coordinates and time coordinate are fixed by the characteristic wave vector k and frequency ω

$$k = \left[\frac{2mW}{\hbar^2}\right]^{1/2}, \quad \omega = W/\hbar, \tag{5}$$

where $W = A/a^3(1+d_0)^3$ is the Van der Waals energy. Now we can rewrite Eq. (1) in the normalized form

$$i\frac{\partial\chi}{\partial\tau} = -\frac{\partial^2\chi}{\partial\xi^2} - \frac{\partial^2\chi}{\partial\eta^2} - \left(\left[\frac{1+d_0}{1+d_0|\chi|^2}\right]^3 - 1\right)\chi, \tag{6}$$

where $\chi = \psi/\psi_0$, $\xi = kx$, $\eta = ky$, and $\tau = \omega t$.

Since for such a monolayer superfluid film we cannot have surface deformations, we will be studying the superfluid density fluctuations occuring in the two dimensional film. We assume that initially the superfluid density is locally altered. For example, this could be done by heating the film locally. After this is done Eq. (2) would hold.

For the sake of simplicity we look for solutions propagating along ξ-axis. The size of the superfluid film (is 100 along the ξ-direction) is chosen arbitrarily in such a way as to be larger than the characteristic size of the localized excitations. Equation (6) is treated as an initial value problem. It is assumed that at $t = 0$ the whole superfluid is at rest – that is we choose the initial value of the phase of the wave to be constant throughout the film. The dynamics of the system is independent of the actual value of this constant.

Our numerical results are shown in Figs. 1 and 2. Figure 1 shows the time evolution of the superfluid density fluctuations $a(\xi, \eta, \tau) = |\psi(\xi, \eta, \tau)|^2 - 1$ for an initial gaussian profile

$$a(\xi, \eta, \tau) = a_0 \exp\{-[(\xi - 25)/5]^2 - [(\eta - 50)/55]^2\}. \tag{7}$$

We impose the periodic boundary condition, $a(100, \eta, \tau) = a(0, \eta, \tau)$. Since no surface deformations occurs to the monolayer film, we do not have to take the kinematic boundary conditions. The parameters chosen are $d_0 = 1$ and $a_0 = 0.2$. The superfluid velocity corresponding to the density fluctuations are plotted in Fig. 2. Results for other values of the parameters will be reported elsewhere.[9]

Two solitons emerge from the single peak and travel in opposite directions. These solitons preserve their identity after interaction among each other and are quite stable. Under close examination these peaks are found to be asymmetric. These solitons, "quasi-solitons" are not completely stable as the large time behavior might suggest. The finite life time of the solitons as well as the asymmetry arise essentially from the higher order nonlinearity.

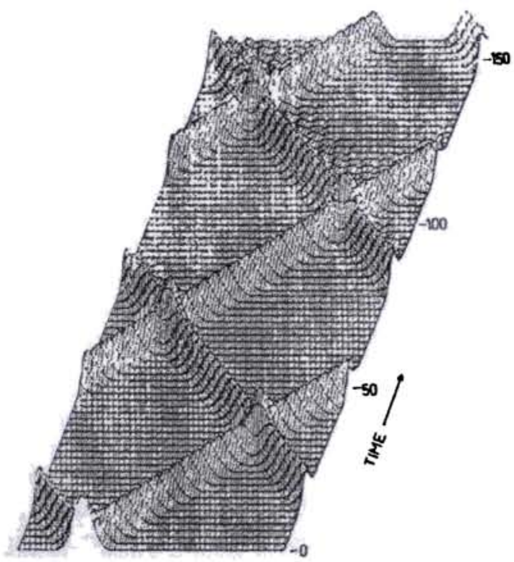

Fig. 1. Time evolution of the amplitude of the superfluid density fluctuations occuring along the x-direction, in arbitrary units.

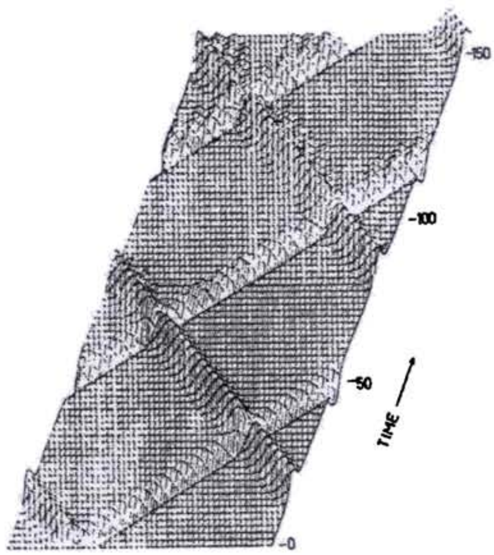

Fig. 2. The superfluid velocity, corresponding to the density fluctuations plotted in Fig. 1, in arbitrary units.

We have shown numerically that even under strong nonlinearity the two-dimensional ^{4}He films admit stable composite quasi-solitons of the superfluid density fluctuations and the superfluid velocity. These solitons are quite different from those obtained in the case of K-P equation or the two dimensional cubic nonlinear Schrödinger equation. In this letter we have not studied the resistance of the soliton to diffraction, which we plan to do under separate work.

Acknowledgments

We would like to thank Prof. J. C. Parikh, P. R. L. Ahmedabad for providing us with computational facilities and Dr. B. R. Sitharam, P. R. L. for helping with the computations. One of us (J. S.) would like to thank Council of Scientific and Industrial Research, New Delhi for financial support through the award of a Senior fellowship.

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