Chaotic Dynamics of Semiconductor Laser with Current Modulation and optoelectronic feedback

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Abstract: Chaotic dynamics of directly modulated semiconductor lasers have been studied extensively over the last two decades because of their application in secure optical communication. However, chaos is generally suppressed in such systems when the nonlinear gain reduction factor is above 0.01 which is very much smaller than the reported values in semiconductors like InGaAsP. In this paper we show that by giving an optoelectronic feedback with appropriate delay one can increase the range of the values of the gain reduction factor for which chaos can be observed. Numerical studies show that negative feedback is more efficient in producing chaotic dynamics.

1 Introduction

Semiconductor lasers are widely used as coherent sources of light for several technological applications because of their compactness, low cost and convenience of operation. Consequently they have been studied exhaustively over the last several years. The dynamics of single mode semiconductor lasers such as InGaAsP are described by a simple set of coupled rate equations for the electron and photon densities [1]. They are pumped directly with an injection current. Semiconductor lasers when operated with a d.c bias current are generally stable and do not show chaotic behavior. One of the ways by which an additional degree of freedom can be introduced is by periodically modulating the current with a frequency in the GHz range. It was shown that under direct current modulation the system can exhibit a period doubling route to chaos [2,3]. The existence of chaotic behavior in these lasers is important since they are routinely employed for optical communications [4,5]. However Agrawal has shown [6] that chaos disappears altogether in such systems when the rate equations are modified to include power-dependent nonlinear gain reduction factor (ϵ) due to the phenomenon such as spectral hole burning. It was also shown that chaos can exist only when ϵ takes a low value very much less than their normal values in lasers like InGaAsP. A bidirectional coupling between two such lasers is also found to suppress the chaotic dynamics [7] and can also lead to chaotic synchronization [8]. For these results to be extended to realistic cases, it is essential to look for alternate ways by which chaotic behavior can be observed at comparatively higher values of the nonlinear gain reduction parameter.

In this paper we report the results of the numerical studies on the effect of a delayed optoelectronic feedback on the dynamics of semiconductor lasers. We consider both positive and negative feedbacks. The results show that chaotic behavior persists for larger values of ϵ that lie in the normal range of values for semiconductor lasers used for communication purposes. Delayed optical feedback has been employed earlier [9]. A delayed optoelectronic feedback with current modulation is found to suppress bistability in semiconductor lasers [10, 11] when appropriate values are chosen for delay time and the modulation strength. The paper is organized as follows. The rate equations are presented in section 2. In section 3. The numerical results are given with both positive and negative optoelectronic feedback. Section 4 contains conclusions.

2 Rate equations

Directly modulated semiconductor lasers are modeled by the following rate equations for the normalized carrier density (N)and the normalized photon density (P) [1].

$$\frac{dN}{dt} = (\frac{1}{\tau_e})[(\frac{I}{I_{th}}) - N - \{\frac{(N-\delta)}{(1-\delta)}\}P] \quad (1a)$$

$$\frac{dP}{dt} = (\frac{1}{\tau_p})[\frac{N-\delta}{1-\delta}(1-\epsilon P)P - P + \beta N] \quad (1b)$$

$$I(t) = I_b + I_m \sin(2\pi f_m t). \quad (1c)$$

Here τ_e and τ_p are the electron and photon lifetimes and $I(t)$ is the injection current. $\delta = \frac{n_0}{n_{th}}$ where n_0 is the carrier density for transparency and $n_{th} = \tau_e \frac{I_{th}}{eV}$ is the threshold carrier density, ϵ is the nonlinear gain reduction factor, I_{th} is the threshold current and V is the active volume. $I_b = bI_{th}$ is the bias current. $I_m = mI_{th}$ is the modulation current, m is the modulation strength and f_m is the modulation frequency in the GHz range. β is the spontaneous emission factor. The parameter values used for computation are given in table 2.

In our model we replace equation 1c by adding a delayed feedback to the current term and write

$$I(t) = I_b + I_m \sin(2\pi f_m t) \pm rP(t-\tau) \quad (2)$$

In equation 2, r is the feedback fraction and τ is the delay time. The feedback is introduced as follows: the light output from the laser is converted to an electronic signal using a photodetector (PD) and then amplified. time delay is produced by allowing the output light to travel through a specific distance in space before reaching the PD. A delay of the order of nanoseconds is achieved for GHz modulation. The output from the PD is added to the input current of the laser. Equations 1a 1b and 2 are solved numerically using Fourth order Runge-Kutta algorithm.

3 Results and discussions

Table 1: Calculated values of maximal Lyapunov exponent using TISEAN (with sampling frequency 10^{11}) for positive and negative feedback.

ϵ	r	$\lambda_{max}(+ve\ fb)$	$\lambda_{max}(-ve\ fb)$
0.0001	0	$+3.758031e-2$	$+3.758031e-2$
0.0001	0.002	$+5.353506e-2$	$+5.345553e-2$
0.0001	0.005	$+6.377417e-2$	$+4.825806e-2$
0.05	0.02	$-4.592604e-5$	$+7.055892e-2$

The chaotic dynamics of directly modulated semiconductor lasers has been studied in detail because of its possible applications in secure communications. Agrawal [6] has derived the condition for the occurrence of chaos in such systems. For the parameter values that we have chosen for computation (2) the system exhibits chaotic behavior (in the absence of feedback) when $\epsilon < 0.01$ and for a small window of modulation depth. In fig 1a we plot the output photon density for $\epsilon = 0.0001$ (without feedback) against the modulation depth m. It shows a period doubling route to chaos for values of m between 0.45 and 0.6. for further increase of modulation depth it goes to a stable behavior through a reverse period doubling route. Fig 1b gives the bifurcation diagram corresponding to an increase in the value of ϵ while all other control parameters are held at the chaotic operating condition. given in 2. It is evident from the figure that the output becomes stable at $\epsilon = 0.01$. The results are in agrement with the conditions for stability of periodic solutions given in [6]. Therefore, one can expect chaotic dynamics in directly modulated semiconductor lasers only when $\epsilon < 0.01$. However, for practical semiconductor lasers like InGaAsP used in communication technology ϵ lies in the range $0.03 \leq \epsilon \leq 0.05$. In order to check whether chaotic behavior can be observed in this enhanced range we investigated the effect of both positive and negative delayed feedback for systems with higher values of ϵ. In figures 2a and 2b we have plotted the bifurcation diagram for positive and negative feedback, as a function of increasing ϵ. The feedback fraction r was kept at the value 0.005. The time delay was chosen to be $3.78ns$ which is the optimum value for which the dynamics is chaotic for both positive and negative feedback at $\epsilon = 0.0001$. It is evident from the diagram that both the types of feedbacks are effective in producing chaos for $\epsilon < 0.02$. Specifically, both configurations can give rise to chaotic behavior at $\epsilon = 0.01$ where the output is stable in the absence of feedback. To confirm whether the method is efficient at $\epsilon = 0.05$ we increased r to 0.01 and the bifurcation diagram was drawn (fig 3 a and 3 b). Positive feedback does not induce chaos above $\epsilon = 0.02$ even when r is increased to 0.01. However, a negative feedback of the same strength and delay τ can produce chaotic output at higher values of $\epsilon \approx 0.05$. Similar results can also be seen from figs 4a and 4b. To confirm these results we have calculated the maximal Lyapunov exponent from the timeseries for the output photon density P. The Lyapunov exponents were calculated for various values of ϵ and for various values of the feedback fraction r using the software TISEAN 7.0 for both positive and negative feedbacks. these values are given in table 1. These results show that delayed negative feedback is an effective method

Table 2: The Parameter values used for simulation.

Parameter	*Value*
τ_e	$3ns$
τ_e	$6ps$
ϵ	1×10^{-4}
δ	692×10^{-3}
β	5×10^{-5}
f_m	$0.8GHz$
I_{th}	$26mA$
m	0.55
b	1.5

for inducing chaos in semiconductor lasers at reasonably high values of the nonlinear gain reduction factor.

We have also studied the effect of increasing the value of the feedback fraction on the output dynamics. The results are shown in figs 5a 5b for $\tau = 3.78ns$ and $\epsilon = 0.05$. One can see that for negative feedback there is an optimum window in r lying between 0.01 and 0.03 where the output is chaotic.

4 Conclusion

We have numerically investigated the dynamics of semiconductor laser of the type InGaAsP under current modulation and delayed optoelectronic feedback for experimentally reported values of the nonlinear gain reduction factor ϵ. For low values of ϵ, both types of feedback are effective in producing chaotic outputs. However, where ϵ is increased the output dynamics of the systems stable for positive feedback while negative of the same strength and delay time can induce chaos. Therefore, it is possible to bypass the restriction $\epsilon < 0.01$ in [6] by incorporating a delayed negative optoelectronic feedback. The mechanism responsible for this is currently being investigated.

Acknowledgement

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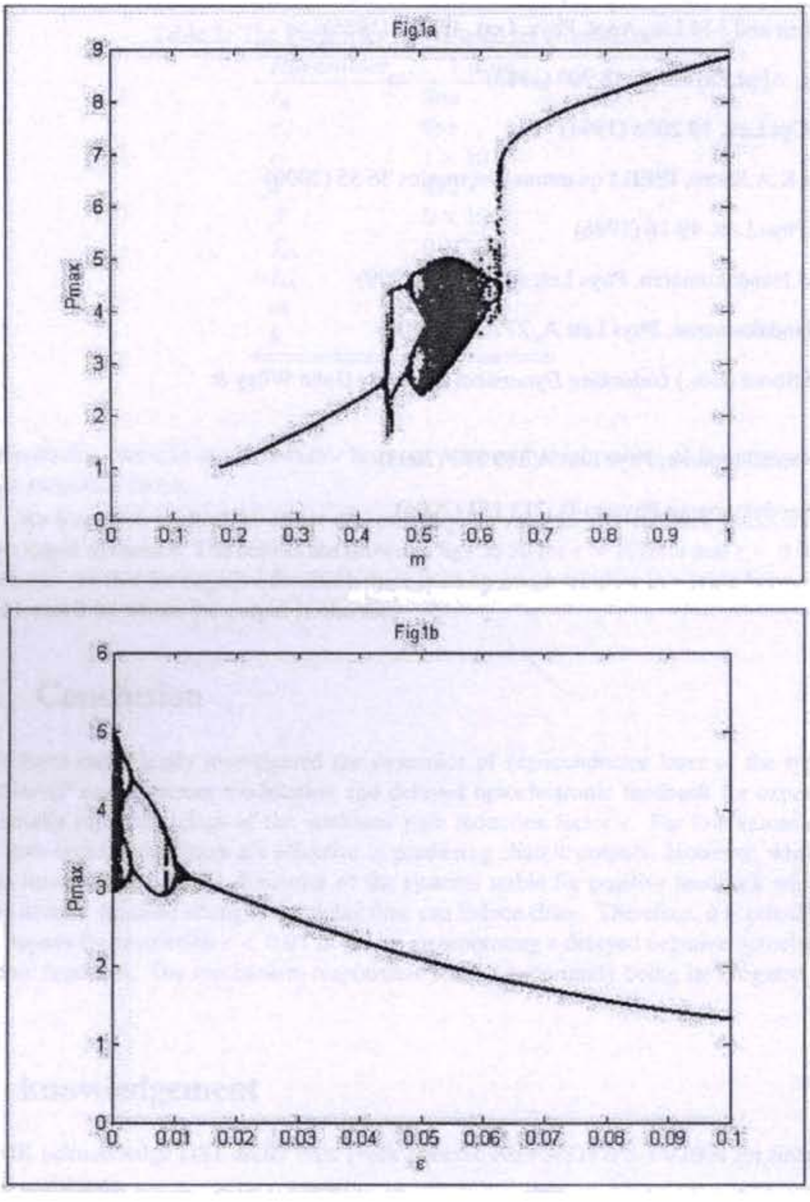

Figure 1: (a) Bifurcation diagram of output (maxima of the normalized photon density, P_{max}) of the modulated semiconductor laser for increasing values of modulation depth 'm'. The nonlinear gain reduction factor is kept at a low value of $\epsilon = 0.0001$. (b) Bifurcation diagram of output P_{max} of modulated semiconductor laser without feedback for increasing values of nonlinear gain reduction factor ϵ. The modulation depth is kept at $m = 0.55$.

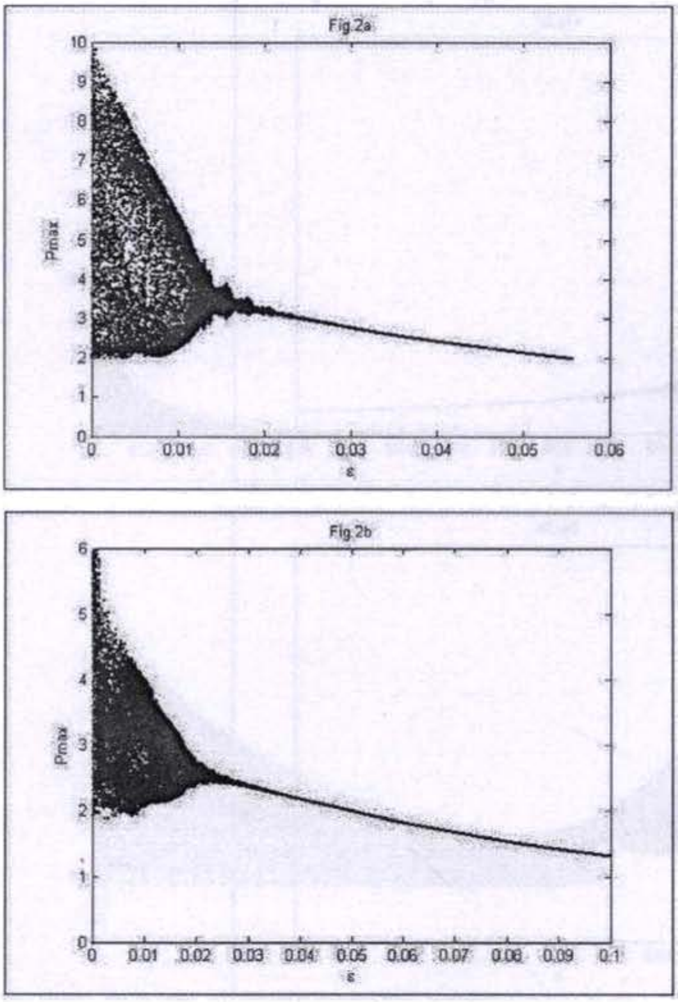

Figure 2: (a) Bifurcation diagram of output P_{max} of modulated semiconductor laser with positive delayed optoelectronic feedback of strength is $r = 0.005$ and delay time $\tau = 3.87ns$, for increasing values of nonlinear gain reduction factor. (b) Bifurcation diagram of output P_{max}, of modulated semiconductor laser with negative delayed optoelectronic feedback of strength $r = 0.005$ and delay time $\tau = 3.78ns$, for increasing values of nonlinear gain reduction factor.

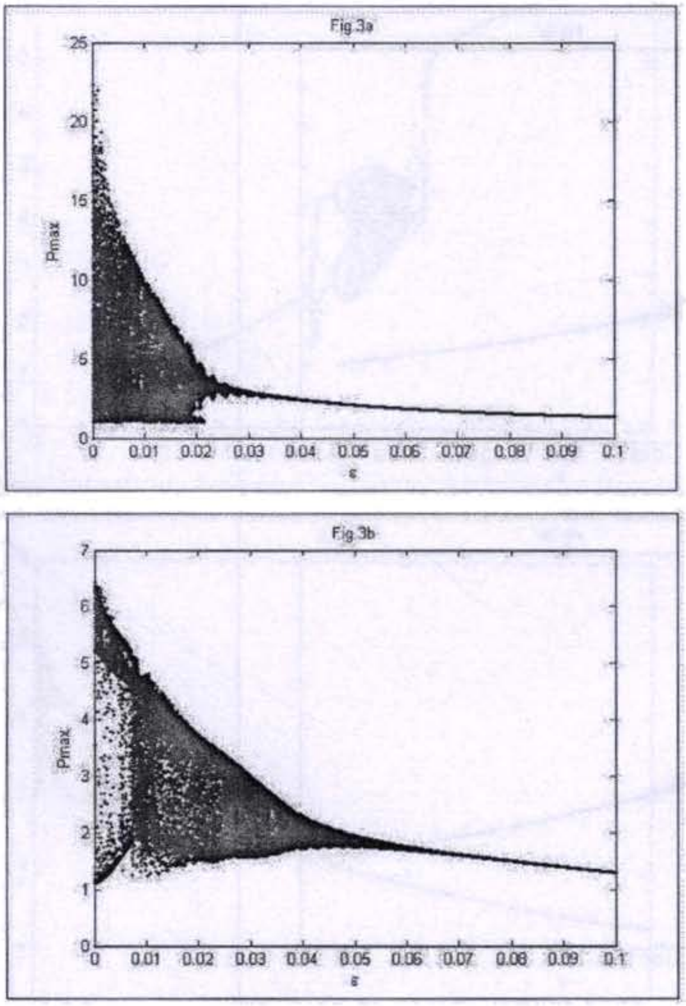

Figure 3: (a) Bifurcation diagram of output P_{max} of modulated semiconductor laser with positive delayed optoelectronic feedback of strength is $r = 0.01$ and delay time $\tau = 3.87ns$ for increasing values of nonlinear gain reduction factor. (b) Bifurcation diagram of output P_{max} of modulated semiconductor laser with negarive delayed optoelectronic feedback of strength is $r = 0.005$ and delay time $\tau = 3.87ns$, for increasing values of nonlinear gain reduction factor.

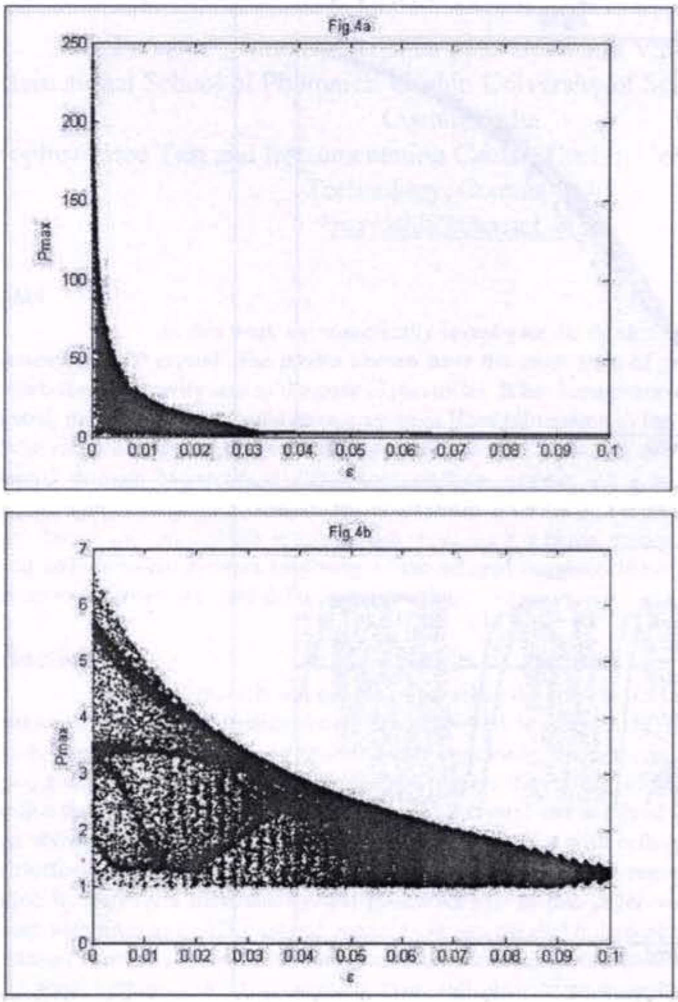

Figure 4: (a) Bifurcation diagram of output P_{max} of modulated semiconductor laser with positive delayed optoelectronic feedback of strength is $r = 0.02$ and delay time $\tau = 3.87ns$ for increasing values of nonlinear gain reduction factor. (b) Bifurcation diagram of output P_{max} of modulated semiconductor laser with negarive delayed optoelectronic feedback of strength is $r = 0.02$ and delay time $\tau = 3.87ns$, for increasing values of nonlinear gain reduction factor.

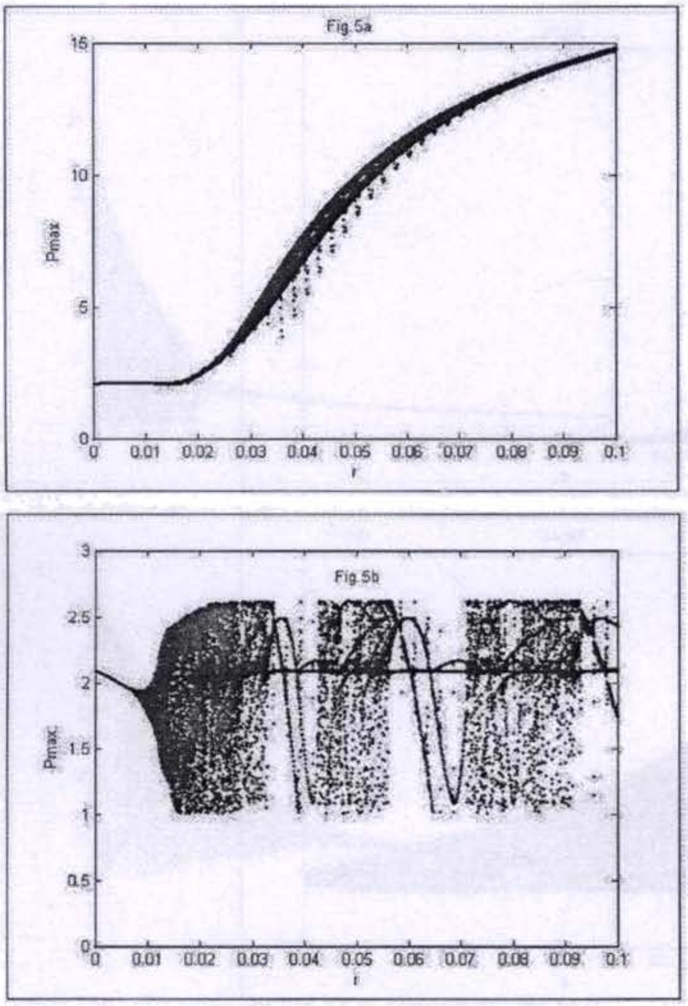

Figure 5: (a) Bifurcation diagram of output P_{max} of modulated semiconductor laser at nonlinear gain reduction factor value of $\epsilon = 0.05$ with positive optoelectronic feedback, for increasing values of feedback strength r. The delay time is $\tau = 3.87ns$. (b) Bifurcation diagram of output P_{max} of modulated semiconductor laser at nonlinear gain reduction factor value of $\epsilon = 0.05$ with negative optoelectronic feedback, for increasing values of feedback strength r. The delay time is $\tau = 3.87ns$.