Studies in Quantized Fields and Particle Creation in the Early Universe

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CERTIFICATE

Certified that the work contained in the thesis entitled, **Studies in Quantized Fields and Particle Creation in the Early Universe** , is the bonafide work carried out by Mr.P.K.Suresh, under my supervision in the Department of Physics,Cochin University of Science and technology,Cochin-22 in partial fulfillment of the requirements for the award of the Degree of Doctor of Philosophy and the same has not been included in any other thesis submitted previously for the award of any degree or diploma of any University.

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PREFACE

The work presented in this thesis has been carried out by the author in the Department of Physics, Cochin University of Science and Technology during the period 1993-1998.

The Universe has been a wonder to the humanity and continues to be so. The search to understand the Universe, how it works and where it came from, can be considered to be the most persistent and greatest adventure in human history. Cosmology - the science of the Universe - based on Einstein's General Theory of Relativity has helped us very much to understand the evolution of the Universe and has opend new horizon in this field of study.

The thesis deals with the study of cosmological particle creation and related issues using squeezed state formalism. The thesis is organized in five chapters. Chapter I is an introductory one. It contains a brief description of different cosmological models, grand unified theories and the phenomena of structure formation and the objectives of the present study. Chapter II deals with the quantization of scalar field and basic mechanism of particle creation in curved spacetime. A brief review of the physical and mathematical properties of coherent states and squeezed states which are revlevant to the present study are also presented. Recent studies of cosmological problems using coherent states and squeezed states are also mentioned. In Chapter III, the quantization of scalar field coupled minimally with gravity in various quantum states, vis, coherent states, squeezed states and squeezed vacuum states are discussed. The expectation values of the energy-momentum tensor are calculated and the particle creation problem is examined in an anisotropic background cosmology. A brief discussion of standard cosmology and squeezing effect is also included. Chapter IV contains studies on the quantum fluctuations in different representations of the scalar field and the validity of the semiclassical theory is discussed. In Chater V we have examined the particle creation by black holes and the corresponding entropy generation and Hawking temprature for squeezed states and coherent states are studied. These studies showed that the change in entropy, the Hawking temperature and change in mass are related to the associated squeezing parameter.

A part of the present investigations has appeared in the form of the following publications:

(1) 'Squeezed states representation and vacuum fluctuations in the early universe', P.K.Suresh,V.C.Kuriakose and K.Babu Joseph,Int.J.Mod.Phys.D **6**,781(1995)

(2) 'Squeezed states , balck holes and entropy generation' P.K.Suresh and V.C.Kuriakose, Mod.Phys.Lett. **A 12** 1453 (1997)

(3) 'Squeezed states representation of quantum fluctuation and Semiclassical theory' P.K.Suresh and V.C.Kuriakode, Mod.Phys.Lett.**A 13** 165 (1998)

and also has been presented in the following symposia/conferences

(1)'Squeezed states and vacuum fluctuation in the early universe', Symposium on Early Universe held at IIT Madras (Dec 22-24,1994)

(2)'Squeezed states and black hole',XVIIIth IGARG Meeting held at IMSc Madras (Feb.16-19,1996)

SYNOPSIS

One of the great successes of classical cosmology is its ability to describe the main features of the observed physical Universe by using some specific initial conditions. To avoid postulating such initial conditions as well as the existence of particle horizons in isotropic models the study of inhomogeneous and anisotropic models of the Universe has been initiated. To bring about the observed isotropy in such models at a later time, it requires the introduction of a dynamical mechanism for damping the inhomogeneity and the anisotropy. One such mechanism is neutrino viscosity , which was investigated in Bianchi type I,V and IX cosmologies and found not to be rapid enough to bring about the observed isotropy later. Another mechanism coming into play at much earlier times (t$\sim$ Planck time) is the production of elementary particles by rapid expansions of the Universe. Zel'dovich suggested that this process could bring about isotropy near the Planck time(t_p). Quantum aspects of particle production and renormalization of the energy momentum tensor in Bianchi type I and IX Universe have been studied by many people. In an anisotropic expanding Universe we expect that when t$>t_p$ the process of anisotropic damping is dominated by the created particles. Therefore the investigation of particle creation in anisotropic cosmological models have much relevance near the singularity where the gravitational field is expected to be strong.

Particle creation in cosmological spacetime was first investigated by Parker, Sexl, Urbantke, Zel'dovich and Strobinsky in the late sixties. The basic mechanism can be understood as parametric amplification of vacuum fluctuations by the expanding Universe.

Recently, in order to probe quantum effects in cosmology, quantum optics concepts like coherent states and squeezed states are found to be suitable candidates. These states are important classes of quantum states well understood in the context of quantum optics and obey the Heisenberg minimum uncertanity principle. Creation of particles like gravitons and other primordial perturbations created from the zero point quantum fluctuations in the process of the cosmological evolution were studied by Grishchuk and Sidorov. Gaspernini and Giovananni have shown that the entropy growth in the cosmological process of pair creation can be completely determined by the associated squeezing parameter. Albreacht etal analysed inflationary cosmology in the light of squeezed states. Hu etal have given a systematic description of the dependence on the initial states in terms of squeezing parameter.

The present study emphasis that quantum phenomena are inevitable to understand the particle creation near the singularity. Squeezed vacuum states can be treated as possible quantum states in the early Universe epoch. In the context of the anisotropic background cosmology, the squeezing of vacuum lead to the production of the particle and hence the damping mechanism occur with the created particls. Squeezing of vacuum is achieved by means of the background gravitational field; the gravitational field plays the role of the paramertic medium. The validity of the semiclassical theory is based on the assumption that the fluctuations in the expectation value of the energy-momentum tensor is minimum. But the present study shows that particle creation near the singularity in this scenario is large and hence the semiclassical theory may break down near the singularity. The study of black hole problems shows that entropy generation and change of mass depend on squeezing paramater.

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Chapter 1

Introduction

Cosmology is the science of the Universe. It deals with the study of the formation of large scale structures of the Universe. The central issue in these studies is to understand the origin and the evolution of the Universe in a systematic way. Though many beautiful and fascinating models and theories have been proposed, only a few can even qualitatively explain some of the observed properties of the Universe. The standard cosmology is now considered to be the most promising candidate to describe the main features and evolution of the observed physical Universe. The main features of the standard cosmology are briefly described below.

1.1 Standard cosmology

General Theory of Relativity of Einstein [1, 2, 3] forms the corner stone of modern cosmology and there have been two important stages in the development of modern cosmology. The first stage began in 1920's when Friedmann [4, 5] developed a theoretical model of the Universe based on Einstein's equation and his model visualized a homogeneous, isotropic and expanding Universe. The discovery of redshift by Hubble [6] in 1929 has been interpreted as a consequence of the expansion of the Universe and is viewed as an experimental support to the theoretical model of Friedmann. The cause of this expansion is being speculated to be due to a big explosion in the beginning. This explosion is now called the big bang[7]. The second stage began with the discovery of cosmic microwave background radiation (CMBR) by Penzias and Wilson [8] in 1965. Though Friedmann envisaged an expanding Universe it was not clear then whether the early Universe was hot or cold. The discovery of CMBR led to the belief of a hot early Universe.

The most fundamental feature of the standard cosmology is the expansion of the Universe which is quantitatively expressed through the measurement of red shift which has a major role in the observational cosmology. The light we see today from the most distant objects might have been emitted when the Universe was only a few years old. The relationship between luminosity distance d_L and the redshift of a galaxy z can be written in a power series: [9]

$$z = H_0 d_L + \frac{1}{2}(q_0 - 1)(H_0 d_L)^2 + ... \tag{1.1}$$

where H_0 is the value of the Hubble constant which gives the present rate of expansion of the Universe and q_0 is called the deceleration parameter.

The assumption of an isotropic and homogeneous Universe dates back to the earliest work of Einstein and this assumption is called the cosmological principle. He used this principle to solve his field equations for gravitation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1.2}$$

where $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}$ is the energy-momentum tensor of all the fields present-matter and radiation and G is the universal gravitational constant. The observation of the uniformity in the temperature of the CMBR provides the best evidence for the isotropy of the Universe. If the expansion of the Universe were not isotropic, then the expansion would lead to a temperature anisotropy of small magnitude in the CMBR. Likewise, inhomogeneities in the density of the Universe on the last scattering surface would also lead to temperature anisotropies. The remarkable uniformity of CMBR indicates that at the epoch of last scattering of the CMBR (about 2×10^5 years after the big bang), the Universe was, to a high degree of precision isotropic and homogeneous.

1.1.1 Friedmann-Robertson-Walker metric

In Einstein's early work, however, he adopted what seemed to be like a perfectly natural assumption, that the Universe on the average was not only homogeneous and isotropic but also unchanging in time as well. The assumption that the Universe does not change in time is philosophically very pleasing, and when added to the cosmological principle, the resulting principle was called the perfect cosmological principle. Einstein found that physical meaningful static solutions to his equation could not be found except for the trivial case of an early Universe. To rectify this deficiency, Einstein modified his equations by incorporating a repulsive force of unknown origin, which is called the cosmological constant and the modified equation is given by:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}, \tag{1.3}$$

where Λ is known as cosmological constant. With the cosmological constant, the gravitational effects of a finite mass-energy density could be balanced; and it was possible to produce non-trivial static models of the Universe.

In 1922, Friedmann discovered three possible classes of nonempty cosmological models which did not require the cosmological constant; and they were also discovered independently later by Lematire. The Friedmann-Lemaitre models are for a homogeneous and isotropic, but not for a static Universe, i.e., they evolve in time, and therefore do not satisfy the perfect cosmological principle.

The assumption of homogeneity and isotropy restrict the geometry of spacetime. The metric for such a spacetime can be written as

$$ds^2 = c^2 dt^2 - S(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right] \tag{1.4}$$

This metric is now called Friedmann-Robertson-Walker-Lemaitre (FLRW) metric or simply Robertson-Walker (RW) metric [9]. The metric is characterized by two parameters, (i) the scale factor $S(t)$ and (ii) the spatial curvature parameter k which can take values + 1, - 1 or 0 and they refer respectively to closed, open and flat Universes.

The Einstein's field equation relates the behavior of $S(t)$ to the energy -momentum tensor. To be consistent with the symmetries of the metric, the total energy-mpmontum tensor must be diagonal and isotropy implies that the spatial components must be equal. The Universe is assumed to be filled with a perfect fluid. The energy-momentum tensor is assumed to take a form that of a perfect fluid and is given by [10]

$$T^{\mu\nu} = pg^{\mu\nu} + (\rho + p)u^{\mu}u^{\nu}. \qquad (1.5)$$

In a locally inertial comoving frame this expression reduces to $T^{\mu\nu} = \text{diag}\,[\,\rho, -p, -p, -p]$ so one can identify ρ as the energy density, p as the pressure, and u^{μ} as the four-velocity of the fluid. The field equations then imply that:

$$\frac{\ddot{S}}{S} = -\frac{4\pi G}{3}(\rho + 3p) \qquad (1.6)$$

and

$$\left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} = \frac{8\pi G}{3}\rho \qquad (1.7)$$

By combining (1.6) and (1.7), one can obtain a relation for conservation of energy which can be written as

$$\frac{d}{dS}(\rho S^3) + 3pS^2 = 0. \qquad (1.8)$$

The Hubble constant H is related to $S(t)$ through the relation:

$$\frac{\dot{S}}{S} = H \qquad (1.9)$$

All Friedmann models have a common feature of having $S = 0$ at a certain epoch (which can be chosen as at $t = 0$). As we approach the limit $S \to 0$, the Hubble constant increases rapidly, being infinity at $S = 0$. This epoch therefore indicates an epoch of violent activity and is given the name big bang. From a mathematical point of view, $S = 0$ describes a spacetime singularity, a region of infinite curvature and energy density at which the laws of physics break down. $S = 0$ also presents an insurmountable barrier to physicists. If we use the strong principle of equivalence to study how the physical processes operate in strong gravitational fields, our procedure will break down at $S = 0$.

Singularities are not artifacts of the models. Hawking and Penrose [11] have shown that singularity is unavoidable in standard cosmology and their famous results is now known as the 'singularity theorem'. The theorem does not imply, however, that a singularity will physically occur. Rather, the theory predicting it will break down at very high curvature. This can be suppressed by some better or more powerful theory. In such theories, near a singularity, spacetime becomes highly curved; its volume shrinks to small dimensions. Under such circumstances, one must appeal to quantum theory.

Our current understanding of the evolution of the Universe is based upon FRW cosmological model or the hot big bang model as it is usually called. This model is so successful that it has become now known as the standard cosmological model. It provides a reliable and tested account of the history of the Universe from at least as early as the time of synthesis of light elements ($t = 10^{-2}$ to 10^{2} sec after the big bang, $T \sim 10^{11} K$ to $10^{9} K$) until today ($t = 15$ Gyr, $T = 2.75\ K$) and provides a sensible frame work for describing the early history of the Universe.

1.2 Grand Unified Theory

The remarkable developments in particle physics during 70's and 80's, significantly the progress made towards unification of basic interactions in nature have their own impacts in cosmology also. There are four basic interactions in nature (i) gravitational interaction (ii) electromagnetic interaction (iii) weak interaction and (iv) strong nuclear interaction. Weinberg [12] and Salam [13] in 1964 independently showed that the strength of electromagnetic and weak interactions become comparable at energies of the order of 10^2 GeV(10^{15}K) and in 1980's the experimental verification came in. This gave a moral boost to particle physicists that at sufficiently high energy, three of the basic interactions - strong, weak and e.m-can be believed to be unified and the strength of the three interactions become comparable at energies of the order of 10^{15}GeV ($10^{28}K$), the energy at which unification is likely to take place [14]. This energy is not available now on earth and hence there is no way to verify this. Present calculations suggest that this might have occurred when the Universe was 10^{-37}sec old when the characteristic energy of a typical particle was as high as 10^{15} GeV. To particle physicists, these ideas are exciting because, the early universe could act as a high energy particle accelerator. By studying the particle interaction at energies $\geq$ 10^{15} GeV and looking for possible relics of those events today, one can place certain limits on the parameters of the theories meant to explain the physics of the early epochs. It is an indirect way to test a physical theory, but there is no alternative way to do this. This helps us to believe that at a still high energy all the four interactions might have been unified. Planck had shown that by suitable combinations of the three fundamental constants c, G, and h, one can obtain new scales for energy, time and length: Planck energy: $E_p = \sqrt{\frac{c^5\hbar}{G}} = 10^{19}$ GeV, Planck time: $t_p = \sqrt{\frac{G\hbar}{c^5}} = 10^{-44}$ sec, and Planck length: $L_p = \sqrt{\frac{G\hbar}{c^3}} \sim 10^{-33}$ km.

The era $0<t<t_p$ is usually called quantum gravity era and in this era the classical theory of gravity breaks down. The appearance of the three fundamental constants c, G, and h together suggest that we are reaching a synthesis of theories of relativity, gravitation and quantum theory. The concept of spacetime measurement so crucial to General Relativity breaks down at this level.

1.3 Inflationary cosmology

Though standard cosmology is an achievement, it is not free from shortcomings. The shortcomings of this model are flatness problem, horizon problem, entropy problem, the monopole problem, etc [15]. These shortcomings do not invalidate the standard cosmology in any way. They can be accommodated in the standard model, even if they lack proper explanations. To avoid these problems of standard cosmology, Guth [16] introduced the concept of inflationary cosmology. According to the inflationary model, the early universe underwent a brief period during which the system was in a metastable state called 'false vacuum' state, driving the evolution of the Universe into an exponential expansion. The successes of the original inflationary model depend on the assumption that the phase transition occurred quickly with rapid thermalization of the energy that was released. It is now known that this assumption is false. Also, found that the randomness of the bubble nucleation process leads to gross inhomogeneities, rendering the original inflationary model untenable. To overcome these problems new inflationary models have been proposed by Linde [17, 18], Albrecht and Steinhardt [19]. Even though the new inflationary model could find explanation to the problems raised in standard cosmology, the idea of inflation has difficulties of a different kind; there are no satisfactory explanations for the formation of structure, the exact value for the age of the Universe, lack of direct evidence for non-baryonic matter, etc.

1.4 Structure formation

The problem of galaxy formation is one of the greatest interests in modern cosmology and at present we do not have a satisfactory theory of structure formation. It has been argued that galaxy formation required the existence, in the early Universe, of some kind of initial perturbations slightly disturbing the homogeneous expanding background. Such perturbations grew under gravitational instability and later result in the formation of structures, the nature of the initial perturbations is not well understood even now. In the existing theories of galaxy formation, the spectrum of fluctuations is chosen by fitting the theoretical models to observational data. It seems that more natural and attractive theories are needed to obtain the spectrum of initial fluctuations [20, 21, 22] from fundamental physical principles. Several approximation methods have been developed to study the formation and growth of structures in the Universe [23, 24, 25]. The formation of these structures still remains to be an unsolved problem in cosmology (The real Universe contains inhomogeneous structures like galaxies, clusters of galaxies, superclustures etc.). The origin and growth of small inhomogeneities could be understood using linear perturbation theory. When deviation from a smooth Universe becomes large, the evolution becomes nonlinear and one has to use other techniques to study the nonlinear evolution. Recently, approximate analytical and N-body simulation techniques have been used in the study of nonlinear evolution [26].

1.5 Anisotropic background cosmology

The overall structure of the Universe is believed to be homogeneous and isotropic [27]. But it is inhomogeneous and anisotropic on the scale of galaxies and their clusters. The standard hot big bang model, now-a-days, generally incorporate the hypothesis of inflation and cold dark matter. After the Universe was rendered transparent by the decoupling of radiation and matter, the standard model asserts that all future clustering of matter is entirely due to the ponderous action of gravity working against the general Hubble expansion. Inflationary cosmology was prized as the only theory that offered a causal mechanism for the origin of perturbations large enough to account for the creation and clustering of galaxies. As the decade passed, the observing technology progressed, extensive redshift surveys accumulated, more and more evidences were obtained showing the clustering of structures much larger than anything the inflationary theories were expecting. Computer simulations of density fluctuations left behind by inflation, were able to generate structures much larger than 30 Mpc. The recently discovered Great wall and Great Attractor megastructures [28] appear to stretch over 170 Mpc. Attempts to understand the origin of such inordinate structures, theories have come up with imaginative elaborations of the standard model involving double inflation or very late phase transitions. Noninflationary theories employing concepts like supersymmetry, topological defects, cosmic strings,[29] etc have also been formulated. The observed uniformity of the cosmic micro wave background radiation provides more exotic theoretical speculations.

CMBR measurements support the assumption of a homogeneous and isotropic Universe. The conclusive evidence that the CMBR spectrum is indeed of blackbody type came from experiments conducted in Cosmic Background Explorer satellite (COBE) launched in 1989 [30]. The temperature deduced from the COBE experiments is $T = 2.726\ K$ with an accuracy of $0.01K$. When the measurements improved over these years, it was found that the CMBR exhibits anisotropy in temperature and the temperature varies minutely over the sky and it has been estimated that the temperature variation is of $\delta T = 30 \pm 5\mu K$ or $\frac{\delta T}{T} \sim 10^{-5}$. Half of this $\frac{\delta T}{T}$ could be due to quadruple anisotropy at 90^0 angular scale. Although some quadruple anisotropy is kinetic, the remainder is then of purely cosmological origin which could have arisen if the expansion had not been spherically symmetric. This would then contradict the cosmological principle and the FRW cosmology. The other half of $\frac{\delta T}{T}$, are intrinsic CMBR fluctuation on all scales, indicating the existence of 500 Mpc size which is much larger than the optically observed superstructure.

The recent COBE satellite experiments show large angular anisotropy in the temperature distribution of microwave background radiation [31]. The standard cosmological model that purports to describe the evolution of the Universe is being tested at both ends by spectacular astronomical observations. On the other hand the recently discovered Great wall and Great attractor suggest enormous agglomeration of galaxies, that attested to coherent structures, stretching over half a billion light years in the present epoch. The new measurements on the uniformity of the cosmic microwave background radiation tell us that the Universe was amazingly smooth. The question at issue is: how did the cosmos evolve from these almost wrinkle-free beginnings to a present structure of these manifest inhomogeneity.

A related question, but without any satisfactory explanation, is the creation of particles in the Universe especially in the vicinity of singularities, or almost singular regions. The most general solution of the problem of collapse turn out to be locally anisotropic near the singularity. Cosmological solutions are also known in which the expansion be anisotropic at first near the singularity, and later becomes isotropic [32]. Such models have received much attention during last decades.

Particle creation resulting from the strong gravitational field near the cosmological singularity may have a profound influence on the evolution of the metric of very early times. The question is whether such a process could have brought about the isotropization of an initially anisotropic expansion at sufficiently early time. Therefore the investigations of the behavior of the Universe near a singularity are of great interest. Studies of quantum field theory in curved spacetime can throw more light on these problems. Therefore it is appopriate to discuss the formulation of quantum field theory in curved spacetime and the basic mechanism of particle creation in curved spacetime and are described in the next chapter.

Chapter 2

Quantum field theory in curved spacetime

2.1 Introduction

Einstein's General Theory of Relativity is revolutionary in the sense that a new concept on spacetime structure and gravitation has been put forth. But there is a major drawback for the theory as it is not based on the principles of quantum theory. There are two reasons which compell us to look for a quantum theory of gravity. Advances made in grand unified theories make us to believe that it may be possible to unify all the four forces of basic intractions. Interestingly, the natural length scale which arises in grand unified theory is only a few orders of magnitude $< L_p$. Thus it is possible that a quantum theory of gravity may even play an important role in the unification of the strong and electromagnetic interactions. A unified theoery of all forces might predict many new phenomena, and observations of them would justify the unification scheme [29]. The second reason arises directly from general theory of relativity. As mentioned earlier spacetime singularities occur in the solutions of classical general theory of relativity relevent to gravitational collapse and cosmology. In these singularities the classical description of spacetime structure breaks down. Thus it appears that the development of a quantum theory of gravitation will be an essential requirment for our understanding of the initial state of the Universe [10, 29]. The formulation of quantum gravity will be a great achievment as far as theoretical physics is concerned. All the attempts so far made to have a quantum theory of gravitation ran into difficulties. The lack of a satisfactory quantum theory of gravity does not mean that one can not perform any reliable calculations of quantum effects occuring in strong gravitational fields. A complete satisfactory theory exists for a free quantum matter field propagating in a fixed background spacetime. In this approach, the spacetime metric is treated classically and is coupled to the matter field which is treated quantum mechanically. Such a programme is known as semiclassical approximation [33] . Though this method is only an approximation to a full quantum theory of gravity this procedure can at least give a good indication of the types of quantum effects which might have occurred in strong gravitational fields. This programme is being used to study the effect of quantum gravity on other phenomena like creation of particles, black hole evaporation, etc.

The phenomena of particle creation can be understood by studying the matter field in a background metric. Therefore the essential ideas of formulation of filed theory in curved spacetime and particle creation mechanism are briefly discussed below.

2.2 Quantun fields in curved spacetime and particle creation

Advances made in Grand Unified Theories make us to believe that it may be possible to correlate observational data with quantum process in thè early Universe [34, 35, 36, 37]. This has caused increasing interests in the study of quantum theory in curved spacetime.

A great deal of the formalism of quantum field theory in Minkowski spacetime can be extended to curved spacetime with little modifications. In flat spacetime, Lorentz invariance plays an important role in each of the basic ingredients in the construction of a quantum field theory. In flat spacetime the Lorentz invariance allows us to identify a unique vacuum state for the theory. However in curved spacetime, we do not have Lorentz symmetry. The formulation of a classical field theory and its formal quantization may be carried through in an arbitrary spacetime. The real difference between flat space and curved space arises in the characterisation of the quantum states and the physical interpretations of the states [38]

In view of current quantum concepts, the physical vacuum (i.e. the state without real particles) is a quite complex entity. According to the formulation of quantum field theory virtual (short-lived) particles are constantly created, they interact with one another, and are annihilated in the vacuum. The vacuum is stable and real particles (long-lived) are not produced. But we can see that in the presence of external fields virtual particles may acquire sufficient energy for becoming real. The result is that quantum creation of particles from vacuum is possible in the presence of an external field [33, 38].

In general, there does not exit a unique vacuum state in a curved spacetime. As a result, the concept of particles becomes ambiguous, and the physical interpretations of particles become much more difficult. This issue can be realised by considering the formulation of quantum field theory in Minkowski spacetime and in curved spacetime. First we discuss quantum field theory in Minkowskai spacetime.

Consider a real scalar field in Minkowski spacetime satisfying the equation of motion:

$$\left(\eta^{\mu\nu}\nabla_\mu\nabla_\nu - m^2\right)\phi(x) = 0, \tag{2.1}$$

Let $\{u_k\}$ be a set of solutions of this equation, which are positive frequency modes with respect to some time-like Killing vector ζ_1, that is

$$L_{\zeta_1} u_k = -i\omega_1 u_k \tag{2.2}$$

where $\omega > 0$ and L denotes the Lie derivatives. Assuming that the u_k's are complete and orthonormal, we have:

$$\begin{aligned} (u_i, u_k) &= \delta_{ik} = -\left(u_i^*, u_k^*\right) \\ (u_i, u_k^*) &= 0 \end{aligned} \tag{2.3}$$

where

$$(\phi_1, \phi_2) = i\int_t \phi_1^* \partial_t \phi_2 d^{n-1}x \tag{2.4}$$

and t denotes a spacelike hyperplane of simultaneity at instant t. Now let us choose [38] a solution in the following form

$$u_k = \left[(2\pi)^{n-1}2\omega\right]^{\frac{-1}{2}} e^{i\mathbf{k}.\mathbf{x}-\omega \mathbf{t}} \tag{2.5}$$

where

$$\omega = \left(k^2 + m^2\right)^{\frac{1}{2}}. \tag{2.6}$$

Now the scalar field may be expanded in terms of these modes,

$$\phi = \sum_k \left(a_k u_k + a_k^\dagger u_k^*\right) \tag{2.7}$$

and the quantization of the theory is implemented by imposing canonical commutation relations:

$$[a_i, a_k^\dagger] = \delta_{ik}, \quad [a_i, a_k] = [a_i^\dagger, a_k^\dagger] = 0 \tag{2.8}$$

The vacuum state $\mid 0_I\rangle$ is defined as

$$a_k \mid 0_I\rangle = 0. \tag{2.9}$$

In Minkowski spacetime there is a natural set of modes, namely, as given by (2.5), that are closely associated wih the rectangular coordinate system (t,x,y,z). In turn, these coordinates are associated with the Poincare group, the action of which leaves the Minkowski line element invariant. Thus vacuum is invariant under the action of Poincare group. Therefore the soultions contain only posituve frequencies with respect to the Minkowski time coordinate. But the situation is quite different in curved spacetime.

Now consider a real scalar field in a curved manifold without horizons [38]. The field must satisfy the generally covariant Klein-Gordon equation:

$$\left(g^{\mu\nu}\nabla_\mu\nabla_\nu - m^2\right)\phi(x) = 0, \tag{2.10}$$

Let $\{u_k\}$ be a set of solutions of this equation, which are positive frequency modes with respect to some time-like Killing vector ζ_1, that is

$$L_{\zeta_1} u_k = -i\omega_1 u_k \tag{2.11}$$

where $\omega > 0$ and L denotes the Lie derivatives. Assume that the u_k's are complete and orthonormal then we have, as before the Klein-Gordon scalar product (generalized to curved space),

$$\begin{aligned} (u_i, u_k) &= \delta_{ik} = -\left(u_i^*, u_k^*\right) \\ (u_i, u_k^*) &= 0 \end{aligned} \tag{2.12}$$

where

$$(\phi_1, \phi_2) = i\int_\Sigma \phi_1^* \partial_\mu \phi_2 \sqrt{-g}\, d\Sigma^\mu \tag{2.13}$$

and Σ is a three-dimensional space like hypersurface. The scalar field may be expanded then in terms of these modes,

$$\phi = \sum_k \left(a_k u_k + a_k^\dagger u_k^*\right) \tag{2.14}$$

and the quantization of the theory is implemented as usual by imposing canonical commutation relations (2.8).

Defining a vacuum state $\mid 0_I\rangle$ such that

$$a_k \mid 0_I\rangle = 0, \tag{2.15}$$

then we can construct the Fock space by the action of the creation operators $a_k^\dagger$.

Suppose now to have a different family of solutions of the covariant wave equation (2.10), $\{v_k\}$, with only positive frequencies with respect to another Killing vector L_{ζ_2},

$$L_{\zeta_2} v_k = i\omega_2 v_k \tag{2.16}$$

($\omega_2 > 0$), and v_k form a complete orthnormal set :

$$\begin{aligned} (v_i, v_k) &= \delta_{ik} = -\left(v_i^*, v_k^*\right), \\ (v_i, v_k^*) &= 0. \end{aligned} \tag{2.17}$$

Then ϕ may be expanded in this set also:

$$\phi = \sum_k \left(b_k v_k + b_k^\dagger v_k^*\right) \tag{2.18}$$

and in this decomposition the canonical quantization commutuation relations are:

$$[b_i, b_k^\dagger] = \delta_{ik}, \quad [b_i, b_k] = [b_i^\dagger, b_k^\dagger] = 0. \tag{2.19}$$

This implies that a new vacuum state $\mid 0_{II}\rangle$ can be defined:

$$b_k \mid 0_{II}\rangle = 0 \tag{2.20}$$

which yields a new Fock space.

The annihilation and creation operators of the two quantization schemes, viz a and b can be related by the Bogolubov transformation. For this by considering the scalar product (u_i, ϕ), and using (2.14) and (2.18) we get

$$a_i = \sum_k \left(\alpha_{ik} b_k + \beta_{ik} b_k^\dagger \right) \tag{2.21}$$

where

$$\alpha_{ik} = (u_i, v_k)\,, \beta_{ik} = (u_i, v_k^*) \tag{2.22}$$

are known as Bogolubov transformation coefficients. In the same way, the scalar product (v_i, ϕ) gives the following inverse relation

$$b_i = \sum_k \left(\alpha_{ki}^* a_k - \beta_{ki} a_k^\dagger . \right) \tag{2.23}$$

Imposing the compatibility of (2.21) and (2.23) we find that the Bogoliubov coefficients satisfy the conditions

$$\sum_k \left(\alpha_{ik} \alpha_{jk}^* - \beta_{ik} \beta_{jk}^* \right) = \delta_{ij} \tag{2.24}$$

$$\sum_k \left(-\alpha_{ik} \beta_{jk}^* + \beta_{ik} \alpha_{jk}^* \right) = 0. \tag{2.25}$$

Using these coefficients we can expand u_k in terms of v_k and vice versa (as both set are complete). We find, e.g.,

$$v_i = \sum_k \left(-\alpha_{ki} u_k + \beta_{ki}^* u_k^* \right). \tag{2.26}$$

It is evident that, as long as $\beta_{ik} \neq 0$, the Bogolubov transformations can induce a mixing up of positive and negative frequency modes, and that v_i is not a positive frequency mode with respect to the Killing vectors ζ_1. This means, in other words, that the $| 0_I\rangle$ vacuum is not annihilated by b_k and the two vacua are not equivalent.

In particular, the state $| 0_I\rangle$ is not empty for an observer who defines positive frequencies with respect to ζ_2 : the state $| 0_I\rangle$ contains particles and in the mode v_k the expectation value of their number operator $b_k^\dagger b_k$ is, according to (2.23)

$$\langle 0_I \mid b_k^\dagger b_k \mid 0_I\rangle = \sum_{ij} \beta_{ik}\beta_{jk}^* \langle 0_I \mid a_i a_i^\dagger \mid 0_I\rangle \;\; = \;\; \sum_i \mid \beta_{ik} \mid^2. \tag{2.27}$$

Now we are in a position to describe the physical phenomenon of particle creation by time varying gravitational field. Let us assume that no particles were present before the gravitational field is turned on. If the Heisenberg picture is adopted to describe the quantum dynamics, then $| 0\rangle_{in}$ is the state of the system for all times. However, the physical number operator which counts particles in the out-region is $N_k = b_k^\dagger b_k$. Thus the mean number of particles created in the mode k is

$$\langle N_k\rangle =_{in} \langle 0\left|b_k^\dagger b_k\right|0\rangle_{in} = \sum_j |\beta_{jk}|^2. \tag{2.28}$$

If any of the β_{jk} coefficient are non-zero, i.e., if any mixing of positive and negative frequency solutions occur, then particles are created by gravitational field.

The formalism may be extended, with little complications, to describe particle creation in the presence of horizons. In that case the two Killing vectors ζ_1 and ζ_2, defining the in equivalent vacua, may correspond to observers using different coordinate systems to cover the same manifold: for example Minkowski and Rindler coordinates if we have a uniform accelerated observer in flat space or Kruskal or Schwarzschild coordinates in the so called eternal black hole model.

If we have a field in the vacuum state$\mid 0_I\rangle$, the probability amplitude to find the field in an excited state containing n particles in a given mode v_k, for an observer associated to the other vacuum $\mid 0_{II}\rangle$, is given by $\mathrm{A}_k(n) = \langle n_k \mid 0_I\rangle$, where

$$\mid n_k\rangle_{II} = \frac{1}{\sqrt{n!}}\,(b_k)^2 \mid 0_{II}\rangle \tag{2.29}$$

The probability distribution, $\mathrm{P}_k(n) = \mid A_k(n) \mid^2$, can be computed explicitly using the relation connecting the two vacua, i.e., expanding $\mid 0_I\rangle$ as a superposition of states belonging to the Fock space constructed from $\mid 0_{II}\rangle$. To simplify the formalism, let us consider spatial homogeneity. In this case the Bogolubov transformation is diagonal, because both set of modes u_k and v_k have the spatial dependence and the coefficient becomes

$$\alpha_{ik} = \alpha_i\delta_{ik}, \beta_{ik} = \beta_i\delta_{ik} \tag{2.30}$$

(no summation over i) From (2.21) we have then

$$a_k = a_k b_k + \beta_k b^\dagger_{-k} \tag{2.31}$$

and the condition (2.24) reduces to

$$\mid \alpha_i \mid^2 - \mid \beta_i \mid^2 = 1. \tag{2.32}$$

The special feature of Minkowski space is that the conventional vacuum state is the same for all initial measuring device throughout the spacetime. This is because the vacuum is invariant under the Poincare group and so are the set of inertial observers in Minkowski space.

In curved spacetime the definition of vacuum is associated with the quantum measurement processes used to detect the quanta present. The state of motion of the measuring device can affect whether particles are observed or not. For example, a free-falling detector will not always register the same particle density as a noninertial accelerating detector does. This means that particle concepts does not generally have a universal significance and is observer dependent.

In many problems of interest the spacetime can be treated as asymptotically Minkowskian in the remote past and or in the remote future and they are respectively refered to as 'in' and 'out' regions and in Minkowskian quantum field theory it is assumed that as t $\rightarrow \pm\infty$, all the field interactions approach zero. The analogue situation here is that the 'in' and 'out' regions admit natural particle states and a privileged quantum vacuum. If the state of the quantum field in the 'in' region is chosen to be the vacuum state, it will remain in that state during its subsequent evolution. However at later times, outside the 'in' region, freely falling particle detectors may still register particles in the vacuum state. If there is also an 'out' region then the 'in' vacuum may not coincide with the 'out' vacuum and observers in the 'out' region will detect presence of particles. This phenomenon is now refered to as 'particle creation by time-dependent gravitational field'[38].

Qunatun field theory in curved spacetime reveals that quantum concepts are essential to understand various problems in cosmology. Such studies naturally lead to quantum cosmology.

2.3 Quantum effects in cosmology

One of the greatest successes of classical cosmology is, its ability to describe the important features of the evolution of the Universe by using some specific initial conditions. The observed Universe could have arisen from a much larger class of initial conditions than in the hot big bang model, it is certainly not true that it could have arisen from any initial state - one could have chosen an initial quantum state for the matter which do not have correct density perturbation spectrum, and indeed could choose initial conditions for which inflation does not occur and so on. In order to have complete explanation of the presently observed state of the Universe, therefore, it is necessary to face up to the initial conditions. This is one reason compelling us for replacing classical cosmology by quantum cosmology.

Another fundamental problem facing the standard cosmology is the occurrence of singularities in spacetime, examples of which are the initial singularity of cosmological models and the curvature singularities on the behavior of black holes. General theorems have been proved which demonstrate that singularities are inevitable in standard cosmology, provided that certain conditions are imposed on the energy momentum-tensor [11]. These conditions are reasonable for classical matter, but are not expected to hold in general for energy-momentum tensor associated with quantized matter fields.

This holds out the hope that quantum effects associated with the matter fields can lead to the avoidance of singularities.

It has been assumed that quantum zero-point fluctuations got amplified during inflationary period and produced density perturbations, rotational perturbations and gravitational waves. Density perturbations seeded the observed inhomogeneity in the Universe. These perturbations which eventually give rise to galaxy clusters began as quantum fluctuations that were enormously stretched during the inflationary expansion phase in the first 10^{-35} sec after big bang. The inflationary scenario requires that the mean density of the Universe may be very close to its closure value. In the standard model, most of the remaining dark matter is presumed to consist of weakly interacting particles whose thermal velocities would have been negliable in the epoch when structures started to develop. However the origin of the inflationary stage still remain as an unsolved problem. Also what kind of evolution the Universe experienced before the inflationary stage and how did the Universe itself originate still remain as unanswered questions. A frequently made assumption is that initially the Universe was filled with radiations and the inflation era was preceded by an essentialy quantum gravitational phenomenon called the spontaneous birth of the Universe. Gravitational waves seem to be the only source of impartial information about the very early Universe and the quantum birth of the Universe. For this one need quantum theory of gravity [15, 29].

The second motivation for quantum cosmology comes from quantum gravity. At a deeper level, both the background geometry and matter fields are to be treated quantum mechanically, this is the realm of quantum cosmology. The main object in quantum cosmology is the introduction of a wavefunction of the Universe [15] which, in general, describes all degrees of freedom on an equal footing. But there is no unique wavefunction of the Universe. Presently, we do not know any guiding principle allowing one to prefer one cosmological wavefunction over others. We do not have a fully satisfactory and consistent quantum theory of gravity. But a viable programme is now accepted in which the quantum gravity effects can be ignored as they are likely to be small, but quantum mechanics plays a vital role in the behavior of matter fields. Thus we have a problem of defining a consistent scheme in which the spacetime metric is treated classically but is coupled to the matter fields which can be treated quantum mechanically except near the spacetime singularity. This formulation is now generally called the semiclassical theory of gravity. At the moment, classical general relativity still provides a most successful description of gravity and matter field is treated as quantum mechanically as the source of gravity. In semiclassical theory, Einstein equation takes the following form [33]:

$$G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle. \tag{2.33}$$

The right hand side of the equation is supposed to be the energy-momentum tensor of the matter field. This means that in semiclassical theory the source on the right hand side of Einstein equation is taken to be the expectation value of some suitably defined energy-momentum operator for the matter fields. Another satisfactory explanation is needed in cosmology for the mechanism of particle creation in the early Universe.

These observations emphasis the fact that quantum concépt and quantum effects are needed to understand the various stages of evolution of the Universe, especially the very early Universe.

Recently in order to probe quantum effects in cosmology, quantum optics concepts like coherent states [39] and squeezed states [40, 41] have been found to be very useful. It is now believed that relic gravitons and other primordial perturbations, created from zero-point quantum fluctuations in the course of cosmological evolution exist in specific quantum states known as squeezed states [42]. Creating particles like gravitons and other primordial perturbations from the zero-point quantum fluctuations in the process of the cosmological evolution were studied by Grishchuk and Sidorov [42] using squeezed state formalism. Gasperini and Giovannini [43] have shown that the entropy growth in the cosmological process of pair creation is completely determined by the associated squeezing parameter. Albreacht etal. [44] analyzed inflationary cosmology in the light of squeezed states. Hu etal.[45] have given a systematic description of the dependence on the initial states in terms of squeezing parameter. Using the squeezed state formalism Grishchuk [46] have studied generation of rotational cosmological peturbations. Novello etal [47] treated cosmological perturbations in the quantum frame work by using squeezed states. Caves [48].suggested that the concept of squeezed states might have a role in increasing the sensitivity of a gravitational wave detectors

At a first glance it seems that the two areas, viz, quantum optics and cosmology have no direct connections but the mathematical formalism and physical concepts are similar, the quantum optics concepts are found to be very useful to study many problems in cosmology. The basic properties of coherent states and squeezed states are discussed below.

2.4 Coherent states and Squeezed states

Coherent states and squeezed states are two important classes of quantum states well known in quantum optics [40]. A more appropriate basis for many optical fields are coherent states. The coherent states have an indefinite number of photons which allow them to have a more precisely defined phase than a number state where phase is completely random. The variances of quadrature components in a coherent state are equal having the minimum value allowed by the uncertainty principle. In this sense they are quantum mechanical state close to classical description of the field.

A single mode coherent state (scs) [39] is defined as:

$$\mid \lambda\rangle = D(\lambda) \mid 0\rangle \tag{2.34}$$

where $D(\lambda)$ is the single mode displacement operator and is given by

$$D(\lambda) = \exp\left(\lambda a^{\dagger} - \lambda^{*} a\right). \tag{2.35}$$

The displacement operator have the following properties:

$$D^{\dagger}(\lambda) = D^{-1}(\lambda) \quad = \quad D(-\lambda) \tag{2.36}$$

$$
\begin{aligned}
D^\dagger(\lambda)\ a\ D(\lambda) &= a + \lambda \\
D^\dagger(\lambda)\ a^\dagger\ D(\lambda) &= a^\dagger + \lambda^*
\end{aligned}
$$

where $a^\dagger$ and a are creation and annihilation operators respectively and coherent states are eigen states of a:

$$
a \mid \lambda\rangle = \lambda \mid \lambda\rangle \tag{2.37}
$$

Similarly two mode coherent states (tcs) are defend as:

$$
\mid \lambda_+\lambda_-\rangle = D(\lambda_+\lambda_-) \mid 00\rangle \tag{2.38}
$$

where $D(\lambda_+\lambda_-)$ is the two mode displacement operator which is the product of two single mode displacement operators.

Now using these properties we can calculate the expectation values of the position and momentum operators for the harmonic oscillators in single mode coherent states.

$$
\begin{aligned}
\langle q\rangle_{scs} &= \sqrt{\frac{h}{2\omega}}(\lambda + \lambda^*) \qquad (2.39)\\
\langle p\rangle_{scs} &= \frac{1}{i}\sqrt{\frac{h}{2\omega}}(\lambda - \lambda^*), \\
\langle q^2\rangle_{scs} &= \frac{h}{2\omega}(\lambda^2 + \lambda^{*2} + 2|\ \lambda\ |^2 + 1), \\
\langle p^2\rangle_{scs} &= -\frac{h}{2\omega}(\lambda^2 + \lambda^{*2} - 2|\ \lambda\ |^2 + 1),
\end{aligned}
$$

where $q = \sqrt{\frac{h}{2\omega}}(a + a^\dagger)$ and $p = \frac{1}{i}\sqrt{\frac{h}{2\omega}}(a - a^\dagger)$

The coherent states form a two-dimensional continuum of states and are, in fact, overcomplete. The completeness relation is:

$$
\frac{1}{\pi}\int \mid \lambda\rangle\langle\lambda \mid d^2\lambda = 1 \tag{2.40}
$$

The coherent states have a physical significance that the field generated by a highly stabilized laser operating well above the threshold is a coherent state. They form a useful basis for expanding the optical field in laser physics and in nonlinear optics.

Another class of minimum uncertainty states is the squeezed states. Hollenhorst [49] first used the term squeezed states. These states are characterized by reduced quantum fluctuations in one quadrature component of the field at the expense of increased fluctuations in the other noncommuting component. This remarkable property of squeezed state field has no classical interpretation and makes sense only in models when the nonlinear medium and the radiation fields are treated quantum mechanically.

A single mode squeezed states (sss) (or displaced squeezed states) is defined as[40, 41]:

$$\mid \lambda, \xi\rangle = D(\lambda)S(r, \varphi) \mid 0\rangle \tag{2.41}$$

where $D(\lambda)$ is the single mode displacement operator given by (2.35) and $S(r, \varphi)$ is the single mode squeezing operator and is given as :

$$S(r, \varphi) = \exp \frac{r}{2}(e^{-i\varphi}a^2 - e^{i\varphi}{a^{\dagger}}^2) \tag{2.42}$$

where r is the squeezing parameter which determines the strength of squeezing and φ is the squeezing angle which determines the distribution between conjugate variables and $0 \leq r < \infty$ and $-\pi \leq \varphi \leq \pi$. While a is annihilation operator and $a^{\dagger}$ is the creation operator for the single mode states and they obey the following commutation relation.

$$[a, a^{\dagger}] = 1 \tag{2.43}$$

and all other commutation relations vanish.

When $\lambda = 0$ (2.41) reduces to single mode squeezed vacuum states (ssv).

$$\mid \xi\rangle = S(r, \varphi) \mid 0\rangle. \tag{2.44}$$

The squeezing operator obey the following relation

$$S^{\dagger}(r, \varphi) = S^{-}(r.\varphi) = S(-r, \varphi) \tag{2.45}$$

and has the following properties

$$\begin{aligned} S^{\dagger}aS &= a\cosh r - a^{\dagger}e^{i\varphi}\sinh r \\ S^{\dagger}a^{\dagger}S &= a^{\dagger}\cosh r - ae^{-i\varphi}\sinh r. \end{aligned} \tag{2.46}$$

Similarly two mode squeezed states (tss) are defined as

$$\mid \lambda_{+}, \lambda_{-}, \xi\rangle = D_{\div}(\lambda_{+}\lambda_{-})S_{t}(r, \varphi) \mid 0, 0\rangle \tag{2.47}$$

The two mode displacement operator is given by.

$$D(\lambda_{+}\lambda_{-}) = \exp\left(\lambda a^{\dagger}b^{\dagger} - \lambda^{*}ab\right) \tag{2.48}$$

and $S_t(r, \varphi)$ is the two mode squeezed vacuum operator and is given by:

$$S_{t}(r, \varphi) = \exp r(e^{-i\varphi}ab - e^{i\varphi}a^{\dagger}b^{\dagger}) \tag{2.49}$$

where λ_{+}, λ_{-} and ξ are complex numbers and a, b are annihilation operators for each mode and $a^{\dagger}, b^{\dagger}$ are creation operators and

$$[a, a^{\dagger}] = [b, b^{\dagger}] = 1 \tag{2.50}$$

and all other commutation relations vanish.

When $\lambda_+ = \lambda_- = 0$ (2.48) reduces to two mode squeezed vacuum states (tsv).

$$| \xi\rangle = S(r, \varphi) \mid 0, 0\rangle. \tag{2.51}$$

The most fundamental properties of two mode squeezed vacuum states are given below

$$\begin{aligned} S^\dagger a S &= a\cosh r - b^\dagger e^{i\varphi}\sinh r \\ S^\dagger a^\dagger S &= a^\dagger \cosh r - b e^{-i\varphi}\sinh r. \end{aligned} \tag{2.52}$$

The squeezed vacuum states under considerations are many particle states, hence the resulting field can be called classical but the statistical property is highly different from that of the coherent states. From that point of view, squeezed vacuum is purely a quantum phenomenon having no analogue in classical physics.

Basically single mode and two mode squeezed states are two photon problems. The displacement operator adds a constant to a, thus changing the mean values of the position and momentum variables. The single mode squeezed operator mixes a and $a^\dagger$. Consequently, it induces a correlation between the position and momentum variables that is independent of their mean values. The two mode squeezed operator mixes a with $b^\dagger$ and b with $a^\dagger$. Consequently, it induces correlation between the position and momentum of the different modes.

Theoretical predictions have shown that squeezing of quantum fluctuations can occur in a variety of nonlinear optical phenomena like ,four wave mixing [50, 51], parametric amplification [52, 53, 54], harmonic generation [55, 56, 57], multiphoton absorption process [58, 59], optical bistability [60], etc. Squeezed number states, squeezed coherent states and squeezed thermal states are some of the well known squeezed states. Over the past decade considerable efforts have been put into experiments aiming at the generation and detection of squeezed states of the field. Slusher et.al. [61] in 1985 first performed a four wave mixing experiment in sodium vapour to generate squeezing inside a resonant cavity.

In curved spacetime there is, in general, no unique choice of the $\{v\}$, and hence no unique vacuum state. This means that we cannot identify what constitutes a state without paricle content, and the notation of 'particle' becomes ambiguous. One possible resolution of this difficulty is to choose some quantities other than particle content to lable quantum states. Possible choice might include local expectation values of the field operator. In the particular case of asymptotically flat spacetime, we might use the particle content in an asympototic region. Even this characterisation is not unique. However, this non-uniqueness is an essential feature of the theory with physical consequences, namely, the phemomenon of particle creation.

The phenomenon particle creation in a nonstationary background mertic using squeezed state and coherent state formalisms are investigated in the next chapter.

Chapter 3

Quantization of the Scalar field and Particle Creation

3.1 Introduction

Although the present Universe in its over all structures seems to be spatially homogeneous and isotropic there are reasons to believe that it has not been so in all its evolution and that inhomogeneities and anisotropies might have played an important role in the early Universe [62, 63].

The isotropic model is adequate enough for the description of the later stages of evolution of the Universe but this does not mean that the model will be equally suited for the description of early stages of the evolution, especially near the time singularity [64].Also the most general solutions of the problem of gravitational collapse turns to be locally anisotropic near the singularity [65, 66, 67]. Cosmological solutions of Einstein's general relativity are also known in which the expansion be anisotropic at first near the singularity, and only later does the expansion became to be isotropic. Interests in such models have recently been increased [62, 67, 68, 69].

To avoid postulating specific initial conditions, as well as, the existence of particle horizons in isotropic models attempts have been made through the study of inhomogeneous anisotropic Universe [30, 31]. To bring about the observed isotropy in such models at sufficiently early times requires a dynamical mechanism for damping inhomogeneity and anisotropy. One such mechanism is neutrino viscosity, which was investigated in Bianchi type I, V and IX cosmologies, and was found not to be rapid enough to bring about isotropy at a sufficient early stage [70, 71]. Another mechanism coming into play at much earlier times ($t \sim t_p$) is the production of elementary particles by the expansion of the Universe. Zel'dovich [72] suggested that this process would bring about isotropy near the Planck time. In anisotropic expanding Universe we expect that for $t \geq t_P$ the process of anisotropic damping is dominated by the created particles. Therefore the investigations of particle creation mechanisms in anisotropic models, near singularity are of great physical interest.

Zel'dovich [72] has considered a cosmological mechanism for the production of particles near a Kasnar singularity. Zeldovich and Starobinskii [68] studied particle creation and vacuum polarization of a scalar field with arbitrary mass in a srtong anisotropic external gravitational field with a homogeneous spatially-flat nonstationary metric. In the presence of a strong gravitational field, it is natural to treat it in the classical approximation.On the other hand, the particles which are created necessarly must be described within the spirit of the theory of quantized fields.

The behavior of classical scalar field near the cosmological singularity can be best followed quantum mechanically by constructing an (over) complete set of coherent states for each mode of the scalar field [73]. The coherent states are parameterized by initial conditions for the scalar field. The states become the usual minimum uncertainty wave packet if the time scale for the evolution of the background spacetime is much greater than the periods of oscillation of the modes of the scalar field. Hawking [74] has proposed a way to avoid the requirement to specify the initial conditions for each modes. The quantum state of the scalar field near the initial singularity is inaccessible to an observer at the present time. Hawking [74] suggested that this ignorance of the actual state of the quantized field could be best expressed by taking a random superposition of all allowed states in the inaccessible region. Berger [73] has imposed this randomicity principle by superposing the coherent states and studied the expectation values of the energy-momentum tensor which gave rise to classical value except for the zero-point energy term. He also constructed a coherent state representation of the scalar field even valid near the singularity. But the problem of particle creation was not discussed in that study which gave us the motivation to study the saclar field in squeezed states and creation of particles in an anisotropic background metric. In this chapter we will consider a minimally coupled quantized massive scalar field in a spatially homogeneous and possibly anisotropic background spacetime and study the problem of particle creation in coherent states and squeezed states.

3.2 Representation of scalar fields in squeezed states and coherent states

The most important class of Friedmann's cosmological solutions pertain to conformally-flat metric. However, the more general singular solution and, in particular, the simplest of these, whose three scaling factors S_1, S_2 and S_3 along the three spatial axes depend on the time in different ways are not conformally-flat. The general form of such a background metric in which three-dimensional space is homogeneous and three-dimensionally flat and possibly anisotropic (with $\hbar$ = c = G = 1) is given by [68, 73]:

$$ds^2 = -dt^2 + \sum_{i=1}^{3} S_i^2(t)(dx^i)^2 \tag{3.1}$$

In this background a minimally coupled scalar field of mass m satisfing the Klein-Gordon equation:

$$(g^{\mu\nu}\nabla_\mu\nabla_\nu - m^2)\phi(x) = 0, \tag{3.2}$$

can be expanded in odd and even parity modes.$\phi(x)$ can be put as

$$\phi(x) = (2\pi)^{\frac{-3}{2}} \sum_k \left[q_{\mathbf{k}}(\tau)\cos\mathbf{k.x} + q_{-\mathbf{k}}(\tau)\sin\mathbf{k.x}\right] \tag{3.3}$$

wher $\sum_k$ represents a sum over both odd and even discrete modes in the three-torus. ∇_μ is the covariant derivative and μ= 0, 1,2,3. Transformation to a new time coordinate denoted by $g^{\frac{1}{2}}d\tau = dt$ for $g^{\frac{1}{2}} = S_1S_2S_3$ yields an equation for the mode amplitude q_k:

$$\frac{d^2q_k}{dt^2} + \omega^2(\tau)q_k = 0 \tag{3.4}$$

where

$$\omega_k^2 = g\left(\sum_{i=1}^{3}\frac{k_i^2}{S_i^2} + m^2\right) \tag{3.5}$$

where g = det $[g_{\mu\nu}]$. The action describing the scalar field is

$$I(\phi) = \frac{1}{2}\int d^3x\sqrt{-g}\left[\nabla\phi^2 - m^2\phi^2\right] \tag{3.6}$$

The energy-momentum tensor for the scalar fiels is then given by

$$\begin{aligned} T_{\mu\nu} &= \frac{2}{\sqrt{-g}}\frac{\delta I(\phi)}{\delta g^{\mu\nu}} \\ &= \partial_\mu\phi\ \partial_\nu\phi - \frac{1}{2}g_{\mu\ \nu}\left(g^{\delta\ \sigma}\partial_\delta\phi\partial_\sigma\phi + m^2\phi\right). \end{aligned} \tag{3.7}$$

where $\partial_\mu = \frac{\partial}{\partial x^\mu}$. This tensor is assumed to be symmetric and obeys the law of conservation of energy and this property can be expressed in the following mathematical forms:

$$\begin{aligned} T_{\mu\nu} &= T_{\nu\mu} \\ \nabla_\nu T^{\mu\nu} &= 0 \end{aligned} \tag{3.8}$$

The background mertic under consideration is assumed to be not quantized. Therefore for the metric (3.1), the diagonal components (τ as time coordinate) of energy-momentum tensor of (3.7) can be found as :

$$T_{00} = \frac{1}{2}\left\{\left(\frac{\partial\phi}{\partial\tau}\right)^2 + g\left[\sum_{i=1}^{3}\frac{1}{S_i{}^2}\left(\partial_i\phi\right)^2 + m^2\phi\right]\right\}, \tag{3.9}$$

and

$$T_{ii} = \left(\partial_i\phi\right)^2 + \frac{1}{2}S_i^2\left(\frac{\partial\phi}{\partial\tau}\right)^2\frac{1}{g} - \frac{1}{2}S_i^2\left[\sum_{j=1}^{3}\left(\partial_j\phi\right)^2 + m^2\phi^2\right] \tag{3.10}$$

where i = 1,2,3

Since the background metric is spatially homogeneous, we may require the quantum state of the system to be also spatially homogeneous. Thus we need consider only the spatially homogeneous modes of expressions (3.9) and (3.10) and $2\pi^{\frac{-3}{2}} \int d^3x$ applied to the result yields the spatially averaged components of (3.9) and (3.10) and they are given by

$$\bar{T}_{00} = \frac{1}{32\pi^3 g} \sum_k \left[\left(\frac{\partial q_k}{\partial \tau} \right)^2 + \omega_k^2(\tau) q_k^2 \right] \tag{3.11}$$

and

$$\bar{T}_{ii} = \frac{1}{32\pi^3 g} S_i^2 \sum_k \left[\left(\frac{\partial q_k}{\partial \tau} \right)^2 + \left[\frac{2k_i^2}{S_i^2} g - \omega_k^2(\tau) \right] q_k^2 \right], \tag{3.12}$$

The expectation values of the diagonal components of the energy-momentum tensor given by (3.11) and (3.12) respcetively represent the energy density and pressure of the scalar field under consideration. Since the background metric is taken to be anisotropic in the present study, these corresponding quantities can be conveniently termed as anisotropic density and anisotropic pressure which are to be distingushed from the corresponding quantities in an isotropic background metric case.

Consider the scalar field (3.3) and can be quantized mode by mode by defining

$$p_k = \frac{dq_k}{d\tau} \tag{3.13}$$

and imposing the usual canonical commutation relations. A complete set of orthonormal states $\mid n_k \rangle$ can be constructed to be eigenstates of a formal number operator

$$N_k = a_k^\dagger a_k \tag{3.14}$$

where

$$a_k = -i \frac{d\eta_k}{d\tau} q_k + i\eta_k p_k \tag{3.15}$$

$q_k, p_k = -i\frac{\partial}{\partial q_k}$ are now operators. The c number complex function η_k is a solution to the equation (3.4) such that the Wronskian is

$$\eta_k^* \frac{d\eta_k}{d\tau} - \eta_k \frac{d\eta_k^*}{d\tau} \;\; = \;\; i \qquad (3.16)$$

Here onwards we drop the suffix k for notational convenience. Now the scalar field can be represented by different states and the expectation values of energy-momentum tensor can be computed.

As an alternative to the number states representation we can have other states such as squeezed vacuum states ($\mid \xi\rangle$), squeezed states ($\mid \lambda, \xi\rangle$) and coherent states ($\mid \lambda\rangle$). These states are constructed in such way that they are complete and normalized sets. In the present study we use single mode as well as two modes of the aforementioned states.

To determine the expectation values of the energy-momentum tensor in these states we will first evaluate the expectation values of the q^2, p^2 in these states. The programme is as follows:

Single mode squeezed vacuum states

The single mode squeezed vacuum states and their properties have been described in Chapter 2.4. From the fundamental properties of squeezed vacuum states we can obtain the following expectation values using (2.47) and (3.15):

$$\begin{aligned} \langle a^2\rangle_{ssv} &= -e^{i\varphi}\cosh r \sin hr \qquad (3.17) \\ \langle a^{+2}\rangle_{ssv} &= -e^{-i\varphi}\cosh r \sinh r \end{aligned}$$

$$\langle a^{+}a\rangle_{ssv} = \sinh^2 r.$$

Now q and p are related to a and $a^\dagger$ through the equations (3.15) and (3.16) :

$$\begin{aligned} q &= \eta^* a + \eta a^\dagger \\ p &= \left(\frac{d\eta^*}{d\tau}\right) a + \left(\frac{d\eta}{d\tau}\right) a^\dagger \end{aligned} \tag{3.18}$$

Therefore the expectation values of q^2 and p^2 can be calculated and are given by:

$$\begin{aligned} \langle q^2\rangle_{ssv} = & -\eta^{*2} e^{i\varphi}\cosh r\sinh r - \eta^2 e^{-i\varphi}\cosh r\sinh r + \\ & \mid \eta\mid^2 \left(\cosh^2 r + \sinh^2 r\right) \\ \langle p^2\rangle_{ssv} = & -\left(\frac{d\eta^*}{d\tau}\right)^2 e^{i\varphi}\cosh r\sinh r + \left(\frac{d\eta}{d\tau}\right)^2 - e^{-i\varphi}\cosh r\sinh r + \\ & \mid \frac{d\eta}{d\tau}\mid^2 \left(\cosh^2 r + \sinh^2 r\right) \end{aligned} \tag{3.19}$$

Single mode squeezed states

The single mode squeezed states and their properties have been introduced in Chapter 2.4. As described above, we can evaualate $\langle a^2\rangle_{sss}$ and $\langle a^\dagger\rangle_{sss}$, $\langle a^\dagger a\rangle_{sss}$: followig the results given by (2.42), (2.37) and (2.42):

$$\begin{aligned} \langle a^2\rangle_{sss} &= \lambda^2 - e^{i\varphi}\cosh r \sin hr \\ \langle a^{+2}\rangle_{sss} &= \lambda^{*2} - e^{-i\varphi}\cosh r\sinh r \\ \langle a^{+}a\rangle_{sss} &= \sinh^2 r. \end{aligned} \tag{3.20}$$

The expectation values of q^2 and p^2 are computed using the above results :

$$\langle q^2\rangle_{sss} = \eta^{*2}\left(\lambda^2 - e^{i\varphi}\cosh r\sinh r\right) + \tag{3.21}$$

$$\eta^2\left(\lambda^{*2}-e^{-i\,\varphi}\cosh r\sinh r\right)+|\eta|^2\left(2|\lambda|^2+\cosh^2 r+\sinh^2 r\right)$$

$$\begin{aligned}\langle p^2\rangle_{sss} &= \left(\frac{d\eta^*}{d\tau}\right)^2\left(\lambda^2-e^{i\,\varphi}\cosh r\sinh r\right)+\\ &\left(\frac{d\eta}{d\tau}\right)^2\left(\lambda^{*2}-e^{-i\,\varphi}\cosh r\sinh r\right)+\left|\frac{d\eta}{d\tau}\right|^2\left(2|\lambda|^2+\cosh^2 r+\sinh^2 r\right)\end{aligned}$$

Two mode squeezed vacuum states

In Chapter 2.4 we have introduced two mode squeezed vacuum states and their properties. By using the properties of these states the expectation values of, q^2 and p^2 are obtained from (2.53) :

$$\begin{aligned}\langle q^2\rangle_{tsv} &= 2\eta^{*2}e^{i\,\varphi}\cosh r\sinh r+2\eta^2e^{-i\,\varphi}\cosh r\sinh r+\\ &2|\,\eta\,|^2\left(\cosh^2 r+\sinh^2 r\right)\\ \langle p^2\rangle_{tsv} &= 2\left(\frac{d\eta^*}{d\tau}\right)^2e^{i\,\varphi}(\cosh r\sinh r)+2\left(\frac{d\eta}{d\tau}\right)^2e^{-i\,\varphi}(\cosh r\sinh r)+\\ &2|\,\frac{d\eta}{d\tau}\,|^2\left(\cosh^2 r+\sinh^2 r\right)\end{aligned} \tag{3.22}$$

Two mode squeezed states

The two mode displaced squeezed states or simply two mode squeezed states have been introduced in Chapter 2.4. The expectation values of q^2 and p^2 in these states can be computed using (2.48) and (2.53) and are obtained as:

$$\begin{aligned}\langle q^2\rangle_{tss} &= \eta^{*2}\left(\lambda_+^2+\lambda_-^2+2\lambda_-\lambda_+-2e^{i\varphi}\cosh r\sinh r\right)+\\ &\eta^2(\lambda_+^{*\,2}+\lambda_-^{*\,2}+2\lambda_-^*\lambda_+^*)-2e^{-i\varphi}\cosh r\sinh r)+\\ &|\eta|^2(2|\lambda_+|^2+2|\,\lambda_-\,|^2+2\lambda_+\lambda_-^*+2\lambda_+^*\lambda_-+2(\cosh^2 r+\sinh^2 r))\\ \langle p^2\rangle_{tss} &= \left(\frac{d\eta^*}{d\tau}\right)^2\left(\lambda_+^2+\lambda_-^2+2\lambda_-\lambda_+-e^{i\varphi}\cosh r\sinh r\right)+\end{aligned} \tag{3.23}$$

$$\left(\frac{d\eta}{d\tau}\right)^2 \left(\lambda_+^{*} + \lambda_-^{*} + 2\lambda_+^{*}\lambda_-^{*} - e^{-i\varphi}\cosh r \sinh r\right) +$$
$$\left|\frac{d\eta}{d\tau}\right|^2 \left(2|\lambda_+|^2 + 2|\lambda_-|^2 + 2\lambda_+^{*}\lambda_- + 2\lambda_+\lambda_-^{*} + 2(\cosh^2 r + \sinh^2 r)\right)$$

Two mode coherent states

The two mode coherent states have been introduced in Chapter 2.4 and their properties are different from single mode coherent states. By using the same procedure adopted in the above and using the proerties of two mode coherent states the expectation values of q^2 and p^2 are obtained:

$$\begin{aligned} \langle q^2\rangle_{tcs} &= \eta^{*2}(\lambda_+^2 + \lambda_-^2 + \lambda_-\lambda_+) + \eta^2(\lambda_+^{*2} + \lambda_-^{*2} + \lambda_-^{*}\lambda_+^{*}) + \\ &\quad |\eta|^2(2|\lambda_+|^2 + 1 + 2|\lambda_-|^2 + 1 + 2\lambda_+\lambda_-^{*} + 2\lambda_+^{*}\lambda_-) \\ \langle p^2\rangle_{tcs} &= \left(\frac{d\eta^*}{d\tau}\right)^2 \left(\lambda_+^2 + \lambda_-^2 + \lambda_-\lambda_+\right) + \left(\frac{d\eta}{d\tau}\right)^2 (\lambda_+^{*} + \lambda_-^{*} + \lambda_+^{*}\lambda_-^{*}) + \\ &\quad \left|\frac{d\eta}{d\tau}\right|^2 \left(2|\lambda_+|^2 + 1 + 2|\lambda_-|^2 + 1 + 2\lambda_+^{*}\lambda_- + 2\lambda_+\lambda_-^{*}\right) \end{aligned} \tag{3.24}$$

Now we are in a position to compute the expectation values of the energy-momentum tensor in different states under considerations.

3.3 Squeezed vacuum and Particle creation

The energy-momentum tensor and its expectation values have a siginificant role in understanding the particle creation phenomena and related problems. Since the choice of the quantum states is not unique the expectation values of the energy-momentum tensor depend on the selection of quantum states chosen. In this section the expectation values of the energy-momentum tensor are calculated in various quantum states and then the possiblity of particle creations in an anisotropic background cosmology is discussed. The form of such an anisotropic background metric is given by (3.1).

Investigations on the effects of gravity on quantum fields dated back at least since the work of Schrodinger [75]. Particle creation in strong gravitational fields, in particular near the cosmological singularity has been considered by a number of people[76, 77]. Particle creation in cosmological spacetime was first introduced by Parker [78], Sexl and Urbantke [79], Zel'dovich and Starobinskii [68] in the late sixties. The basic mechanism can be understood as parametric amplification of vacuum fluctuations in an expanding Universe.

Quantum aspects of particle production and renormalization of the energy-momentum tensor in Bianchi type I and IX Universe were studied by various people. The reaction back on the metric of the created particles has been studied by Lukash and Starobanskii. They assumed that the particles created at a time t_0 large with respect to t_p, was so great that the evolution of the metric at times near t_0 could be treated independently of the created particles.

Parker [78] has studied spin 0 field of arbitrary mass and quantised this field in an expanding universe by cannonical procedure and shown that particle number is an adiabatic invariant, but not a static constant of motion and has obtained an expression for the average particle density as a function of the states and shown that particle creation occurs in pairs.

Particle creation mechanism can be investigated by studying the expectation values of energy-momentum tensor of the scalar field for various quantum states. Therefore the expectation values of the energy-memenutm tensor will be now calculated for various states under considerations using the results of the previous section. For calculational simplicity we consider only the $\vec{k}^{th}$ mode pieces of $\bar{T}_{00}$ and $\bar{T}_{ii}$:

Single mode squeezed vacuum

The expectation values of the energy-momentum tensor in single mode squeezed vacuum states are obtaind by using (3.11) and (3.19). The temporal component and spatial components of $T_{\mu\nu}$ are given by:

$$
\begin{aligned}
\langle T_{00}^{\vec{k}}\rangle_{ssv} &= \frac{1}{32\pi^3 g}\{\left[\left(\frac{d\eta^*}{d\tau}\right)^2 e^{i\varphi} + \left(\frac{d\eta}{d\tau}\right)^2 e^{-i\varphi}\right] \\
&\times(-\cosh r\sinh r) + \left|\frac{d\eta}{d\tau}\right|^2\left[\cosh^2 r + \sinh^2 r\right] + \\
&\omega^2\left[\eta^{*2}e^{i\varphi} + \eta^2 e^{-i\varphi}\right](-\cosh r\sinh r) + \omega^2|\eta|^2\left[\cosh^2 r + \sinh^2 r\right]\} \\
&= \frac{1}{32\pi^3 g}\{\left[\left(\frac{d\eta^*}{d\tau}\right)^2 + \omega^2\eta^{*2}\right]\left(-e^{i\varphi}\cosh r\sinh r\right) + \\
&\left[\left(\frac{d\eta}{d\tau}\right)^2 + \omega^2\eta^2\right]\left(-e^{-i\varphi}\cosh r\sinh r\right) + \left[\left|\frac{d\eta}{d\tau}\right|^2 + \omega^2|\eta|^2\right]\left(1 + 2\sinh^2 r\right)\}
\end{aligned}
\tag{3.25}
$$

and

$$
\begin{aligned}
\langle T_{ii}^{\vec{k}}\rangle_{ssv} &= \frac{1}{32\pi^3 g}S_i^2\{\left[\left(\frac{d\eta^*}{d\tau}\right)^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)\eta^{*2}\right]\left(-e^{i\varphi}\cosh r\sinh r\right) + \\
&\left[\left(\frac{d\eta}{d\tau}\right)^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)\eta^2\right]\left(-e^{-i\varphi}\cosh r\sinh r\right) + \\
&\left[\left|\frac{d\eta}{d\tau}\right|^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)|\eta|^2\right]\left(1 + 2\sinh^2 r\right)\}
\end{aligned}
\tag{3.26}
$$

Single mode squeezed state

The expectation values of the energy-momentum tensor can be computed using (3.11) and (3.21) in single mode squeezed states. The expressions for the temporal component and spatial components are obtained :

$$\langle T^{\vec{k}}_{00}\rangle_{ssv} = \frac{1}{32\pi^3 g}\{\left[\left(\frac{d\eta^*}{d\tau}\right)^2 + \omega^2\eta^{*2}\right]\left(\lambda^2 - e^{i\varphi}\cosh r\sinh r\right) + \left[\left(\frac{d\eta}{d\tau}\right)^2 + \omega^2\eta^2\right]\left(\lambda^{*2} - e^{-i\varphi}\cosh r\sinh r\right) + \left[\left|\frac{d\eta}{d\tau}\right|^2 + \omega^2|\eta|^2\right]\left(2|\lambda|^2 + 1 + 2\sinh^2 r\right)\} \tag{3.27}$$

and

$$\langle T^{\vec{k}}_{ii}\rangle_{ssv} = \frac{1}{32\pi^3 g}S_i^2\{\left[\left(\frac{d\eta^*}{d\tau}\right)^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)\eta^{*2}\right]\left(\lambda^2 - e^{i\varphi}\cosh r\sinh r\right) + \left[\left(\frac{d\eta}{d\tau}\right)^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)\eta^2\right]\left(\lambda^{*2} - e^{-i\varphi}\cosh r\sinh r\right) + \left[\left|\frac{d\eta}{d\tau}\right|^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)|\eta|^2\right]\left(2|\lambda|^2 + 1 + 2\sinh^2 r\right)\} \tag{3.28}$$

Similarly using the properties of two mode squeezed vacuum states, squeezed states and coherent states we can compute the expectation values of energy-momnetunm tensor.

Two mode squeezed vacuum

The expectation values of the energy-momentum tensor in two mode squeezed vacuum states are computed using (3.11) and (3.22) and are given by :

$$\langle T^{\vec{k}}_{00}\rangle_{tsv} = \frac{1}{32\pi^3 g}\{\left[\left(\frac{d\eta^*}{d\tau}\right)^2 + \omega^2\eta^{*2}\right]\left(-2e^{i\varphi}\cosh r\sinh r\right) + \left[\left(\frac{d\eta}{d\tau}\right)^2 + \omega^2\eta^2\right]\left(-2e^{-i\varphi}\cosh r\sinh r\right) + \left[\left|\frac{d\eta}{d\tau}\right|^2 + \omega^2|\eta|^2\right] 2\left(1+2\sinh^2 r\right)\} \tag{3.29}$$

and

$$\langle T^{\vec{k}}_{ii}\rangle_{tsv} = \frac{1}{32\pi^3 g}S_i^2\{\left[\left(\frac{d\eta^*}{d\tau}\right)^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)\eta^{*2}\right]\left(-2e^{i\varphi}\cosh r\sinh r\right) + \left[\left(\frac{d\eta}{d\tau}\right)^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)\eta^2\right]\left(-2e^{-i\varphi}\cosh r\sinh r\right) + \left[\left|\frac{d\eta}{d\tau}\right|^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)|\eta|^2\right] 2\left(1+2\sinh^2 r\right)\} \tag{3.30}$$

The expectation values of the energy-momentum tensor in two mode squeezed staes are computed by using (3.11) and (3.23) and lead to the following results:

$$\langle T^{\vec{k}}_{00}\rangle_{tss} = \frac{1}{32\pi^3 g}\{\left[\left(\frac{d\eta^*}{d\tau}\right)^2 + \omega^2\eta^{*2}\right] \times\left(\lambda_+^2 + \lambda_-^2 + 2\lambda_+\lambda_- - 2e^{i\varphi}\cosh r\sinh r\right) + \left[\left(\frac{d\eta}{d\tau}\right)^2 + \omega^2\eta^2\right] \times\left(\lambda_+^{*2} + \lambda_-^{*2} + 2\lambda_+^*\lambda_-^* - 2e^{-i\varphi}\cosh r\sinh r\right) + + \left[\left|\frac{d\eta}{d\tau}\right|^2 + \omega^2|\eta|^2\right] \left(2|\lambda_+|^2 + 2|\lambda_-|^2 + 2\lambda_+^*\lambda_- + 2\lambda_+\lambda_-^* + 2\left(1+2\sinh^2 r\right)\right)\} \tag{3.31}$$

and

$$\langle T^{\vec{k}}_{ii}\rangle_{tss} = \frac{1}{32\pi^3 g}S_i^2\{\left[\left(\frac{d\eta^*}{d\tau}\right)^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)\eta^{*2}\right]$$
$$\times\left(\lambda_+^2 + \lambda_-^2 + 2\lambda_+\lambda_- - 2e^{i\varphi}\cosh r\sinh r\right) +$$
$$\left[\left(\frac{d\eta}{d\tau}\right)^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)\eta^2\right]$$
$$\times\left(\lambda_+^{*2} + \lambda_-^{*2} + 2\lambda_+^*\lambda_-^* - 2e^{-i\varphi}\cosh r\sinh r\right) +$$
$$\left[\left|\frac{d\eta}{d\tau}\right|^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)|\eta|^2\right]$$
$$\times\left(2|\lambda_+|^2 + 2|\lambda_-|^2 + 2\lambda_+^*\lambda_- + 2\lambda_+\lambda_-^* + 2\left(1 + 2\sinh^2 r\right)\right)\} \quad (3.32)$$

The expectation values of the energy-momentum tensor in two mode coherent states are also computed using (3.11) and (3.24). The diagonal componets of the energy-momentum tensor for temporal and spatial components are given by :

$$\langle T^{\vec{k}}_{00}\rangle_{tcs} = \frac{1}{32\pi^3 g}\{\left[\left(\frac{d\eta^*}{d\tau}\right)^2 + \omega^2\eta^{*2}\right]\left(\lambda_+^2 + \lambda_-^2 + 2\lambda_+\lambda_-\right) +$$
$$\left[\left(\frac{d\eta}{d\tau}\right)^2 + \omega^2\eta^2\right]\left(\lambda_+^{*2} + \lambda_-^{*2} + 2\lambda_+^*\lambda_-^*\right) +$$
$$\left[\left|\frac{d\eta}{d\tau}\right|^2 + \omega^2|\eta|^2\right]\left(2|\lambda_+|^2 + 2|\lambda_-|^2 + 2\lambda_+^*\lambda_- + 2\lambda_+\lambda_-^* + 2\right)\} \quad (3.33)$$

and

$$\langle T^{\vec{k}}_{ii}\rangle_{tcs} = \frac{1}{32\pi^3 g}S_i^2\{\left[\left(\frac{d\eta^*}{d\tau}\right)^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)\eta^{*2}\right]\left(\lambda_+^2 + \lambda_-^2 + 2\lambda_+\lambda_-\right) + \quad (3.34)$$
$$\left[\left(\frac{d\eta}{d\tau}\right)^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)\eta^2\right]\left(\lambda_+^{*2} + \lambda_-^{*2} + 2\lambda_+^*\lambda_-^*\right) +$$
$$\left[\left|\frac{d\eta}{d\tau}\right|^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)|\eta|^2\right]\left(2|\lambda_+|^2 + 2|\lambda_-|^2 + 2\lambda_+^*\lambda_- + 2\lambda_+\lambda_-^* + 2\right)\}$$

$\langle T_{0\ 0}\rangle = \rho$, gives the energy density and $\langle T_{i\ i}\rangle = P$, gives the pressure of the source field. Since the background metric under our consideration is an anisotroic metric, we call the energy density and pressure as anisotropic energy density and anisptropic pressure in order to distingush them from the isotropic case.

Now, from the expectation values of the energy-momentum tensor obtained for various states, expression for the corrsponding anisotropic energy density and pressure can be obtained.

The expectation values of the temporal component of the energy-momentum tensor gives the anisotropic energy density and in single mode squeezed vacuum states it can be written as:

$$\langle T_{0\ 0}\rangle_{ssv} = \rho_0 + \rho_{ssv} \tag{3.35}$$

where

$$\rho_0 = \frac{1}{32\pi^3 g}\left[\left|\frac{d\eta}{d\tau}\right|^2 + \omega^2|\eta|^2\right] \tag{3.36}$$

is called vacuum energy density.

The expectation values of spatial components of the energy-momentum tensor give the anisotropic pressure and in single mode squeezed vacuum states it can be written as:

$$\langle T_{i\ i}\rangle_{ssv} = p_0 + p_{ssv} \tag{3.37}$$

where

$$p_0 = \frac{1}{32\pi^3 g}S_i^2\left[\left|\frac{d\eta}{d\tau}\right|^2 + \left(2\frac{k_i^2}{S_i^2}g - \omega^2\right)|\eta|^2\right] \tag{3.38}$$

is called **vacuum** pressure.

The expectation value of energy-momentum tensor in single mode coherent states considered by Berger [73] shows that the energy density can be written as zero-point term plus an additional term The additional term corresponds to the classical value. But the present study shows that the similar situation occurs for single mode squeezed states calculations. But we are primarly interested in squeezed vacuum states. The vacuum expectation values of energy-momentum tensor include the energy density and the pressure of the ordinary zero-point term plus extra term, which can contribute to the zero-point fluctuations and can provide quantum results. The vacuum value of the energy-momentum tensor defined prior to any dynamics in the background field gives us all the information about the particle creation and vacuum polarization. Therefore one can expect particle creation in squeezed vacuum state formalism. Fig3.1 shows that when $r \neq 0$ and $\varphi \neq 0$, there is maximum value for ρ_{ssv}, the scale factor is chosen such that $\rho_0 = 0$. The same arguments can be extented to two mode squeezed vacuum states with the only exception that mode-mode correleation terms are also present in the expression for $\langle T_{00} \rangle$ and $\langle T_{ii} \rangle$.

In curved spacetime the choice of vacuum is not unique and therefore one needs to correleate the evolution of one vacuum state at a time t_0 to another vacuum state at a later time and can be done by Bogolubov transformation method. The transformation coefficients satisfy the following condition.

$$| \alpha |^2 - | \beta |^2 = 1 \tag{3.39}$$

Let us consider the special case of a metric whose evolution is such that $S_1,S_2,S_3 \mid_{\tau=-\infty}$ and $S_1,S_2,S_3 \mid_{\tau=+\infty} = 1$. Now consider a single mode $\vec{k}$. As $\tau \to -\infty$,let the function $\eta(\tau)$ corresponding to this mode have the form $\eta(\tau) = e^{-i\Omega\tau}$,where $\Omega = \sqrt{m^2+\vec{k}^2}$. Then, as $\tau \to +\infty$ this same function η has the asymptotic form

$$\eta = \alpha e^{-i\Omega\tau} + \beta e^{i\Omega\tau}, \tag{3.40}$$

where α and β safisfy the condition given by (3.47) .$\beta \neq 0$ in the general case. Thus when the amplification of the waves occures,its energy increases by $1+2|\beta|^2$ times. The same thing also pertains to the second elementary wave: if $\eta = e^{i\Omega\tau}$ as $\eta \to -\infty$, then for $\eta \to +\infty$ one has

$$\eta = \alpha^* e^{i\Omega\tau} + \beta^* e^{-i\Omega\tau}, \tag{3.41}$$

The energy of this wave also increases by $1+2|\beta|^2$ times. An arbitrary linear combinations of both waves with different signs of the frequency for $\tau \to -\infty$ obviously can be both intensified and weakened.

From the quantum point of view, the energy increase associated with this process implies the creation of new quanta of the field. In the classical theory the increase of energy is proportional to its initial magnitude by virtue of the linearity of the field equations. The quantum theory of Bose particles is equivalent to a classical theory with a nonvanishing energy $\hbar\Omega/2$ of the state without any particles, and therefore gives a non-zero value for the production of particles from this state.

Now in the present context as $\tau \to +\infty$ the function η has the asymptotic form

$$\eta = \cosh r e^{i\Omega\tau} + \sinh r e^{i\varphi} e^{i\omega\tau} \tag{3.42}$$

where

$$|\cosh r|^2 - |\sinh r|^2 = 1 \tag{3.43}$$

Thus, when the amplification for the wave occures, its energy is increased by $1{+}2|\sinh r|^2$ times. The same thing also pertains to the second elementary wave, then as $\eta \to +\infty$

$$\eta = \cosh r e^{i\Omega\tau} + \sinh r e^{-i\varphi} e^{i\omega\tau} \tag{3.44}$$

The energy of this wave is also increased by $1{+}2|\sinh r|^2$. The wave equation is invariant with regard to the replacement of τ by $-\tau$

Parker [78] has shown that the probability of observing particles at time t is

$$P_\tau = \frac{|\beta|^2}{1+|\beta|^2} = \frac{|\beta|^2}{|\alpha|^2} \tag{3.45}$$

where α and β are Bogolubov transformation coefficients and in the present context, one can identify $|\alpha|^2$ by $\cosh^2 r$ and $|\beta|^2$ by $\sinh^2 r$ and they satisfy the condition (2.47). Therefore, the probability of observing particles in a squeezed vacuum can be written as

$$P_{sqv} = \left|\frac{\sinh r}{\cosh r}\right|^2 \tag{3.46}$$

Also, the probability of observing n_k pair of particles at t in the state $|0\rangle$ where one particle is in the mode k and the other in the mode -k for some set of occupied modes $\{k\}$ is

$$p_{n_k} = \prod_k \left[\left(\left|\frac{\beta}{\alpha}\right|^2\right)^{n_k} \frac{1}{|\alpha|^2}\right] \tag{3.47}$$

The probability of observing the n_k pair in the squeezed vacuum is then given by

$$p_{n_k sqv} = \prod_k \left[\left(\left| \frac{\sinh r}{\cosh r} \right|^2 \right)^{n_k} \frac{1}{\mid \cosh r \mid^2} \right] \tag{3.48}$$

where $n_k = 1,2,...$ Fig 3.3 shows behaviour of probability of observing pair of parricles in squeezed vacuum states, which reaches a maximum and then decreases.

Now in terms of barrier reflection coefficient R and tunneling coefficient D, the particle creation condition is

$$R = 1 - D \tag{3.49}$$

Grishchuk and Sidorov [42] showed that $D = \frac{1}{\cosh^2 r}$. Therefore the particle creation condition is fully satisfied in the squeezed vacuum state formalism. The increase in energy is proportional to $1 + 2\mid \sinh r \mid^2$.

The essential idea of creation of squeezed states of light is that a nonlinear medium acts like a meterial with a time-dependent dielectric function when a strong , time-varing classical electromagnetic field is applied. Photon modes propagating through a time dependent dielectric will undergo a mixing up of positive and negative frequencies, and photons will be quantum mechanically created into a squeezed vacuum state.

The basic idea of the present work is that a similar programme is performed for particle creation in the early Universe. Time dependent background gravitational field acts as an nonlinear medium (parametric amplification) which squeezes the scalar field vacuum and hence squeezed vacuum states can be taken as the initial states for calculationg the expectation values od the energy-momentum tensor.

The gravitationally induced pair creation is analogous to excitation of an oscillator or pendulum when the length l of the string is changed. For the pendulum, $\left|\frac{\dot{l}}{l}\right|$ is larger than the charactristic frequency ω. Similarly one can argue that the probability of creation of pair of paricles of energy ω_i will be significant when $\left|\frac{\dot{a}_i}{a_i}\right|$ is larger than ω_i. Therefore in this case the probability of paricle creation in different directions will be different. So the produced particles may move in different directions and this gives rise to anisortiopic pressure.

3.4 Standard cosmology and squeezing effect

In this section we study the semiclassical quantum gravity on quantum FRW cosmological model by using the squeezed states and coherent states formalisms. Since the quantum field preserves the unitarity throughout evolution, the application of squeezing to the FRW model is expected to be particuarly useful in probing the quantum effect of matter field on gravity.

Let us consider the FRW Universe in which the classical Einstein's equation is

$$\left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} = \frac{8\pi}{3m_p}\left(\frac{\dot{\varphi}^2}{2} + \frac{dV(\varphi)}{d\varphi}\right) \tag{3.50}$$

and the classical field equation is

$$\ddot{\varphi} + 3\frac{\dot{S}}{S} + \frac{dV(\varphi)}{d\varphi} = 0. \tag{3.51}$$

In semiclasical quantum gravity Einstein's equation can be written [9, 15, 27]:

$$\left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} = \frac{8\pi}{3m_p}S^3\langle\hat{H}\rangle. \tag{3.52}$$

and

$$i\hbar\frac{\partial}{\partial t}\Phi(\varphi,t) = \hat{H}\Phi(\varphi,t) \tag{3.53}$$

where

$$\hat{H} = \frac{1}{2S^3}\hat{\pi}^2 + S^3V(\hat{\varphi}) \tag{3.54}$$

Now we can represent the semiclassical Einstein's equation in squeezed vacuum states, squeezed states and coherent states. Let us construct the the Fock space by introducing creation and annihilation operators:

$$\begin{aligned} a^\dagger &= -i\left[\dot{\eta}(t)\hat{\pi}^2 - S^3\dot{\eta}(t)\hat{\varphi}^2\right] \\ a &= i\left[\dot{\eta}^*(t)\hat{\pi}^2 - S^3\dot{\eta}^*(t)\hat{\varphi}^2\right] \end{aligned} \tag{3.55}$$

We require that $a^\dagger$ and a are to be invariant operators:

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}a^\dagger + \left[a^\dagger,\hat{H}\right] &= 0 \\ i\hbar\frac{\partial}{\partial t}a + \left[a,\hat{H}\right] &= 0 \end{aligned} \tag{3.56}$$

then η satisfies the equation

$$\left(S^3\dot{\eta}\right)\dot{\varphi} + S^3\eta\frac{dV(\varphi)}{d\varphi}(\hat{\varphi}) = \dot{0}. \tag{3.57}$$

From the usual commutation relations it follows that

$$\hbar S^3\left(\eta_k^*\frac{d\eta_k}{d\tau} - \eta_k\frac{d\eta_k^*}{d\tau}\right) = i \tag{3.58}$$

In this context the position and the momentum operators are given by

$$\begin{aligned} \hat{\varphi} &= \hbar\left(\eta\hat{a} + \eta^{*}\hat{a}^{\dagger}\right) \\ \hat{\pi} &= \hbar S^{3}\left(\dot{\eta}\hat{a} + \dot{\eta}^{*}\hat{a}^{\dagger}\right). \end{aligned} \tag{3.59}$$

Now we can calculate the expectation value of the Hamiltonian in squeezed states and coherent states.In the present study we choose the potential to be of the form:

$$V(\varphi) = \frac{m^2\hat{\varphi}^2}{2} \tag{3.60}$$

In the single mode squeezed vacuum states the expectation values of the Hamiltonian is obtained:

$$\begin{aligned} \langle\hat{H}\rangle_{ssv} &= \frac{\hbar}{2}S^{3}\{\left[\dot{\eta}^{2} + m^{2}\eta^{2}\right]\left(-e^{i\varphi}\cosh r\sinh r\right) + \\ &\left[\dot{\eta^{*}}^{2} + m^{2}\eta^{*2}\right]\left(-e^{-i\varphi}\cosh r\sinh r\right) + \\ &\left[|\dot{\eta}|^{2} + m^{2}|\eta|\,\eta^{2}\right]\left(1 + 2\sinh^{2} r\right)\} \end{aligned} \tag{3.61}$$

The expectation values of the Hamiltonian in squeezed states can be calculated :

$$\begin{aligned} \langle\hat{H}\rangle_{sss} &= \frac{\hbar}{2}S^{3}\{\left[\dot{\eta}^{2} + m^{2}\eta^{2}\right]\left(\lambda^{2} - e^{i\varphi}\cosh r\sinh r\right) + \\ &\left[\dot{\eta^{*}}^{2} + m^{2}\eta^{*2}\right]\left(\lambda^{*2} - e^{-i\varphi}\cosh r\sinh r\right) + \\ &\left[|\dot{\eta}|^{2} + m^{2}|\eta|\,\eta^{2}\right]\left(2|\lambda|^{2} + 1 + 2\sinh^{2} r\right)\} \end{aligned} \tag{3.62}$$

Similarly in single mode coherent state the expectation value of the Hamiltonian is obtained :

$$\langle\hat{H}\rangle_{scs} = \frac{\hbar}{2}S^{3}\{\left[\dot{\eta}^{2} + m^{2}\eta^{2}\right]\lambda^{2} + \tag{3.63}$$

$$\left[\dot{\eta}^{*2} + m^2\eta^{*2}\right]\lambda^{*2} +$$
$$\left[|\dot{\eta}|^2 + m^2|\eta|\,\eta^2\right]|\lambda|^2\}$$

Now we will consider the semiclassical Einstein's in squeezed vacuum.

$$\begin{aligned} \left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} &= \frac{8\pi}{3m_p}\langle\hat{H}\rangle_{ssv} \\ &= \frac{8\pi}{3m_p}\left(H_0 + H_{ssv}\right) \end{aligned} \tag{3.64}$$

where

$$\begin{aligned} H_{ssv} &= \frac{\hbar}{2}S^3\{\left[\dot{\eta}^2 + m^2\eta^2\right]\left(-e^{i\varphi}\cosh r\sinh r\right) + \\ &\left[\dot{\eta}^{*2} + m^2\eta^{*2}\right]\left(-e^{-i\varphi}\cosh r\sinh r\right) + \\ &\left[|\dot{\eta}|^2 + m^2|\eta|\,\eta^2\right]\left(2\sinh^2 r\right)\} \end{aligned} \tag{3.65}$$

and

$$H_0 = \frac{\hbar}{2}S^3\left[|\dot{\eta}|^2 + m^2|\eta|^2\right] \tag{3.66}$$

is the zero-point energy.

In squeezed state representation the semiclassical Einstein's equation takes the following form:

$$\begin{aligned} \left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} &= \frac{8\pi}{3m_p}\langle\hat{H}\rangle_{ssv} \\ &= \frac{8\pi}{3m_p}H_0 + \frac{8\pi}{3m_p}H_{ssc} + \frac{8\pi}{3m_p}H_{ssv} \end{aligned} \tag{3.67}$$

where H_{ssv} is given by (3.65) and

$$H_{scs} = \frac{\hbar}{2}S^3\{\left[\dot{\eta}^2 + m^2\eta^2\right]\lambda^2 + \left[\dot{\eta}^{*2} + m^2\eta^{*2}\right]\lambda^{*2} + \tag{3.68}$$

$$\left[|\dot{\eta}|^2 + m^2|\eta|\,\eta^2\right]\left(2|\lambda|^2\right)\}$$

Similarly in coherent state representation the semiclassical Einstein's equation is obtained as:

$$\begin{aligned} \left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} &= \frac{8\pi}{3m_p}\langle \hat{H} \rangle_{ssv} \\ &= \frac{8\pi}{3m_p} H_0 + \frac{8\pi}{3m_p} H_{scs} \end{aligned} \tag{3.69}$$

where H_0 is given (3.66) and H_{scs} is (3.68)

3.5 Conclusions

We have examined the problem of particle creation near the cosmological singularity by means of single mode squeezed vacuum state formalism. The particle production probability is found to be fully dependent upon the associated squeezing parameter. The squeezed vacuum gives rise to fluctuations in anisotropic energy density and anisotropic pressure. Particle creation near the singularity in this scenario is very high and hence it might account for the initial anisotropic damping. The calculations are done in single mode as well as two mode representations Plots for energy density, probability of observing particles with squeezing parameter are also drawn. When the squeezing parameter becomes zero all results are consistent with that of coherent states formalism. From these results we can conculde that for creating a large number of particles near the singularity quantum phenomena are inevitable and it seems that squeezing phenomenon can play a major role in accounting for the particle creation in the early Universe. Therefore the gravitationally induced particle creation in squeezed vacuum states may lead to fundamental new insights in cosmology.

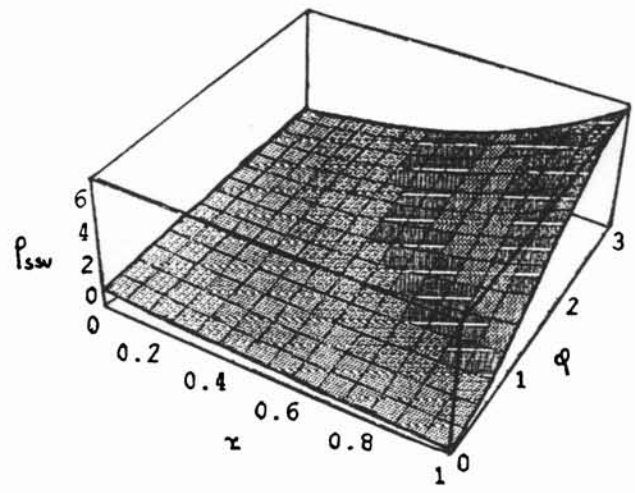


Fig 3.1 Plot for squeezed vacuum energy density ρ_{ssv} with squeezing parameter r and squeezing angle φ for single mode case.

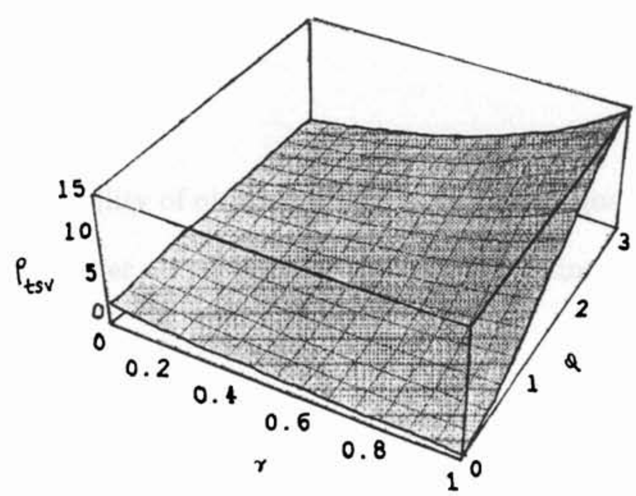


Fig 3.2 Plot for squeezed vacuum energy density ρ_{tsv} with squeezing parameter r and squeezing angle φ for two mode case.

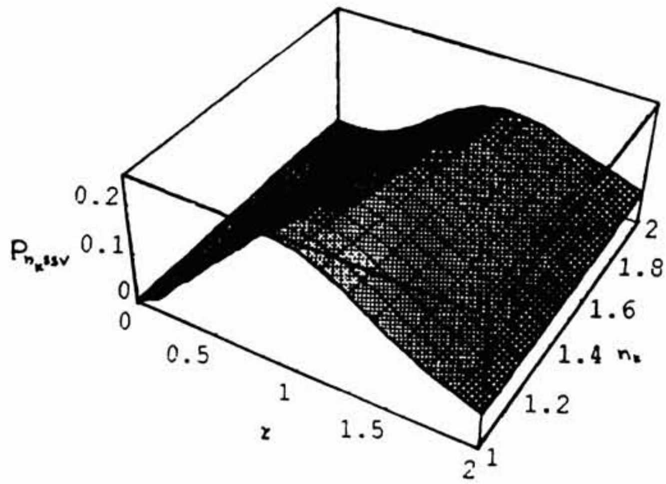


Fig 3.3 Plot for probability of observing pair of particles squeezed vacuum states $P_{n_{ksqv}}$ with squeezing parameter r and squeezing angle n_k for single mode case.

Chapter 4

Squeezed states and Semiclassical theory

4.1 Introduction

At present there does not exist a complete quantum theory of gravity and hence the gravitational field of quantum system is being described by a semiclassical theory based on Einstein's equation and in this theory Einstein's equation takes following form (G=1)

$$G_{\mu\nu} = 8\pi\langle T_{\mu\nu}\rangle, \tag{4.1}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, is the Einstein tensor and $\langle T_{\mu\nu}\rangle$ denotes the expectation value of the energy-momentum tensor of the matter field under consideration. This theory is almost certain to fail at the Planck scale where quantum nature of gravity becomes important. It can also fail far away from Planck scale if fluctuations in the energy-momentum tensor become relevent. The semiclassical theory gives reliable results when the fluctuations in the energy-momentum tensor are not too large, ie when [80]

$$\langle T_{\delta\sigma}(x)T_{\mu\nu}(y)\rangle \approx \langle T_{\delta\sigma}(x)\rangle\langle T_{\mu\nu}(y)\rangle. \tag{4.2}$$

For quantum states in which the energy density fluctuations are large, the semiclassical theory based upon (4.1) may not be valid.

In this chapter we will explore the issue of the limit of validity of the semiclassical theory by evaluating the expectation value of the energy-momentum tensor using the formalisms of squeezed states and coherent states. The study of the validity of the semiclassical theory will be probed in a spatially homogeneous and possibly anisotropic background metric. The form of such a metric is given in (3.1) and is minimally coupled to the scalar field which has been discussed in Chapter 3.

4.2 Limits on the semiclassical theory

To discuss a criterion for the validity of semiclassical gravity theory we have to consider the energy flux of gravitational radiation in linearized gravity produced by matter field in both the semiclassical theory and in linear quantum theory. In the semiclassical theory based upon (4.1), the flux depends on products of expectation values of stress tensor operators, whereas in a theory in which the metric perturbations are quantized, it depends upon the corresponding products of expectation values [80].

The semiclassical theory of gravity holds good only if the fluctuations in the energy-momentum density of the quantum field are not large [80]. This means that we have to evaluate $(\Delta T)^2 = \langle T^2_{\mu\nu}\rangle - \langle T_{\mu\nu}\rangle^2$ and it must be small compared to $\langle T^2_{\mu\nu}\rangle$. The evaluation of $(\Delta T)^2$ will be extremely cumbersome. For the sake of calculational simplicity we will evaluate temporal and spatial components of $(\Delta T)^2$ separately. The temporal component leads to fluctuations in energy density while the spatial components lead to fluctuations in anisotropic pressure. Thus we find first expressions for $\bar{T^k}^2_{00}$ and $\bar{T^k}^2_{ii}$ and are respectively obtined from (3.9) and (3.10):

$$\bar{T^k}^2_{00} = \left(\frac{1}{32\pi^3 g}\right)^2 \left[\left(\frac{\partial q}{\partial \tau}\right)^4 + \omega^2 \left[\left(\frac{\partial q}{\partial \tau}\right)^2 q^2 + q^2 \left(\frac{\partial q}{\partial \tau}\right)^2\right] + \omega^4 q^4\right] \tag{4.3}$$

and

$$\bar{T^k}^2_{ii} = \left(\frac{1}{32\pi^3 g}\right)^2 S_i^4 \left[\left(\frac{\partial q}{\partial \tau}\right)^4 + \left[\frac{2k_i^2}{S_i^2} g - \omega^2\right] \left[\left(\frac{\partial q}{\partial \tau}\right)^2 q^2 + q^2 \left(\frac{\partial q}{\partial \tau}\right)^2\right] + \omega^4 q^4\right] \tag{4.4}$$

where

$$\omega^4 = g^2 \left[\sum_{i=1}^{3} \frac{k_i^4}{q_i 4} + 2\sum_{i=1}^{3} \frac{k_i^2}{S_i^2} m^2 + m^4\right]. \tag{4.5}$$

The expectation values of $\bar{T^k}^2_{00}$ and $\bar{T^k}^2_{ii}$ can be calculated in squeezed states and coherent states and then the fluctuations in energy density ρ and pressure p can be studied. Thus the validity of semiclassical theory can be understood by evaluating the dimensionless quantity:

$$\frac{\delta\rho}{\langle\rho^2\rangle} = \left|\frac{\langle\rho^2\rangle - \langle\rho\rangle^2}{\langle\rho^2\rangle}\right| \tag{4.6}$$

for energy density fluctuations and

$$\frac{\delta p}{\langle p^2\rangle} = \left|\frac{\langle p^2\rangle - \langle p\rangle^2}{\langle p^2\rangle}\right|. \tag{4.7}$$

for pressure fluctuations.

If the values of $\frac{\delta \rho}{\langle \rho^2\rangle}$ and $\frac{\delta p}{\langle p^2\rangle}$ are very small compared to unity then it means that fluctuations are small and semiclassical theory holds good.

4.3 Squeezed states formalism and validity of semiclassical theory

In order to study the validity of semiclassical theory of gravity for various representations of the scalar field we have to compute the above mentioned dimension less quantities. To obtain these quantites we have to compute the square of the expectation values of the energy-momentum tensor and the expectation values of squared energy-momentum tensor for each states under consideration. The expectation values of $\bar{T^k}^2_{00}$ and $\bar{T^k}^2_{ii}$ can be calculated in squeezed vacuum states, squeezed states and coherent states by using their properties which have been introduced in Chapter 2.4. In addition to the earlier calculations we have to find the expectation values of q^4, p^4, q^2p^2 and p^2q^2 in each states and they are as given below.

Single mode squeezed vacuum

The expectation values of q^4, p^4, q^2p^2 and p^2q^2 in single mode squeezed vacuum states are obtained by using (2.44), (2.46) :

$$\begin{aligned}
\langle q^4\rangle_{ssv} &= \eta^{*4}\left(e^{2i\varphi}\cosh^2 r\sinh^2 r\right)+\eta^4\left(e^{-2i\varphi}\cosh^2 r\sinh^2 r\right) \qquad (4.8)\\
&\mid\eta\mid^4\left(\cosh^4 r+\sinh^4 r+2\cosh^2 r\sinh^2 r\right)\\
\langle p^4\rangle_{ssv} &= \left(\frac{d\eta^*}{d\tau}\right)^4\left(e^{2i\varphi}\cosh^2 r\sinh^2 r\right)+\left(\frac{d\eta}{d\tau}\right)^4\left(e^{-2i\varphi}\cosh r\sinh r\right)+\\
&\left|\frac{d\eta}{d\tau}\right|^4\left(\cosh^4 r+\sinh^4 r+2\cosh^2 r\sinh^2 r\right)\\
\langle p^2q^2\rangle_{ssv} &= \left(\frac{d\eta^*}{d\tau}\right)^2\eta^{*2}\left(e^{2i\varphi}\cosh^2 r\sinh^2 r\right)+\left(\frac{d\eta}{d\tau}\right)^2\eta^2\left(e^{-2i\varphi}\cosh^2 r\sinh^2 r\right)+\\
&\left|\frac{d\eta}{d\tau}\right|^2|\eta|^2\left(\cosh^4 r+\sinh^4 r+2\cosh^2\sinh^2 r\right)\\
\langle q^2p^2\rangle_{ssv} &= \eta^{*2}\left(\frac{d\eta^*}{d\tau}\right)^2\left(e^{2i\varphi}\cosh^2 r\sinh^2 r\right)+\eta^2\left(\frac{d\eta}{d\tau}\right)^2\left(e^{-2i\varphi}\cosh^2 r\sinh^2 r\right)+\\
&|\eta|^2\left|\frac{d\eta}{d\tau}\right|^2\left(\cosh^4 r+\sinh^4 r+2\cosh^2 r\sinh^2 r\right)
\end{aligned}$$

Now the expectation value of $T^{\vec{k}^2}{}_{00}$ is obtained by using (4.3) and (4.8): Thus

$$\begin{aligned}
\langle T^{\vec{k}^2}{}_{00}\rangle_{ssv} &= \left(\frac{1}{32\pi^3 g}\right)^2\Bigg\{\left[\left(\frac{d\eta^*}{d\tau}\right)^4+\omega^4\eta^{*4}+\omega^2\left(\frac{d\eta^*}{d\tau}\right)^2\eta^{*2}+\omega^2\eta^{*2}\left(\frac{d\eta^*}{d\tau}\right)^2\right] \qquad (4.9)\\
&\times\left(e^{2i\varphi}\cosh^2 r\sinh^2 r\right)+\\
&\left[\left(\frac{d\eta}{d\tau}\right)^4+\omega^4\eta^4+\omega^2\left(\frac{d\eta}{d\tau}\right)^2\eta^2+\omega^2\eta^2\left(\frac{d\eta}{d\tau}\right)^2\right]\\
&\times\left(e^{-2i\varphi}\cosh^2 r\sinh^2 r\right)+\\
&\left[\left|\frac{d\eta}{d\tau}\right|^4+\omega^4|\eta|^4+\omega^2\left|\frac{d\eta}{d\tau}\right|^2|\eta|^2+\omega^2|\eta|^2\left|\frac{d\eta}{d\tau}\right|^2\right]\\
&\times\left(\cosh^4 r+\sinh^4 r+2\cosh^2 r\sinh^2 r\right)\Bigg\}
\end{aligned}$$

The square of the expectation value of the $T^{\vec{k}}_{00}$ is computed from (3.25):

$$
\begin{aligned}
\langle T^{\vec{k}}_{00}\rangle^2_{ssv} = & \left(\frac{1}{32\pi^3 g}\right)^2 \{ \left[\left(\frac{d\eta^*}{d\tau}\right)^4 + \omega^4\eta^{*4} + \omega^2\left(\frac{d\eta^*}{d\tau}\right)^2\eta^{*2} + \omega^2\eta^{*2}\left(\frac{d\eta^*}{d\tau}\right)^2\right] \\
& \times \left(e^{2i\varphi}\cosh^2 r\sinh^2 r\right) + \\
& \left[\left(\frac{d\eta}{d\tau}\right)^4 + \omega^4\eta^4 + \omega^2\left(\frac{d\eta}{d\tau}\right)^2\eta^2 + \omega^2\eta^2\left(\frac{d\eta}{d\tau}\right)^2\right] \\
& \times \left(e^{-2i\varphi}\cosh^2 r\sinh^2 r\right) + \\
& \left[\left|\frac{d\eta}{d\tau}\right|^4 + \omega^4|\eta|^4 + \omega^2\left|\frac{d\eta}{d\tau}\right|^2|\eta|^2 + \omega^2|\eta|^2\left|\frac{d\eta}{d\tau}\right|^2\right] \\
& \times \left(\cosh^4 r + \sinh^4 r + 2\cosh^2 r\sinh^2 r\right) + \\
& \left[\left(\frac{d\eta^*}{d\tau}\right)^2\left(\frac{d\eta}{d\tau}\right)^2 + \omega^4\eta^{*2}\eta^2 + \omega^2\left(\frac{d\eta^*}{d\tau}\right)^2\eta^2 + \omega^2\eta^{*2}\left(\frac{d\eta}{d\tau}\right)^2\right] \\
& \times \left(\cosh^2 r\sinh^2\right) \\
& \left[\left(\frac{d\eta^*}{d\tau}\right)^2\left|\frac{d\eta}{d\tau}\right|^2 + \omega^4\eta^{*2}|\eta|^2 + \omega^2\left(\frac{d\eta^*}{d\tau}\right)^2|\eta|^2 + \omega^2|\eta|^2\eta^*2\right] \\
& \times \left(cosh^2 r + \sinh^2 r\right)\left(e^{i\varphi}\cosh r\sinh r\right) \\
& \left[\left(\frac{d\eta}{d\tau}\right)^2\left(\frac{d\eta^*}{d\tau}\right)^2 + \omega^4\eta^2\eta^{*2} + \omega^2\left(\frac{d\eta}{d\tau}\right)^2\eta^{*2} + \omega^2\eta^2\left(\frac{d\eta^*}{d\tau}\right)^2\right] \\
& \times \left(\cosh^2 r\sinh^2\right) \\
& \left[\left(\frac{d\eta}{d\tau}\right)^2\left|\frac{d\eta}{d\tau}\right|^2 + \omega^4\eta^2|\eta|^2 + \omega^2\left(\frac{d\eta}{d\tau}\right)^2|\eta|^2 + \omega^2|\eta|^2\eta^*2\right] \\
& \times \left(\cosh^2 r + \sinh^2 r\right)\left(e^{-i\varphi}\cosh r\sinh r\right) \\
& \left[\left|\frac{d\eta}{d\tau}\right|^2\left(\frac{d\eta^*}{d\tau}\right)^2 + \omega^4|\eta|^2\eta^{*2} + \omega^2|\eta|^2\left(\frac{d\eta^*}{d\tau}\right)^2 + \omega^2\left|\frac{d\eta}{d\tau}\right|^2\eta^{*2}\right] \\
& \times \left(\cosh^2 r + \sinh^2 r\right)e^{i\varphi}(\cosh r\sinh r) \\
& \left[\left|\frac{d\eta}{d\tau}\right|^2\left(\frac{d\eta}{d\tau}\right)^2 + \omega^4|\eta|^2\eta^2 + \omega^2|\eta|^2\left(\frac{d\eta}{d\tau}\right)^2 + \omega^2\left|\frac{d\eta}{d\tau}\right|^2\eta^2\right] \\
& \times \left(\cosh^2 r + \sinh^2 r\right)\left(e^{-i\varphi}\cosh r\sinh r\right)\}
\end{aligned}
\tag{4.10}
$$

Therefore the dimensionaless quantity for the density fluctuations can be computed in squeezed vacuum by applying (4.9) and (4.10) in (4.6):

$$\frac{\delta\rho}{\langle\rho^2\rangle_{ssv}} = \left|\frac{4\cos\varphi\cosh r\sinh r\left(1+2\sinh^2 r\right) - 2\cos 2\varphi\cosh^2\sinh^2 r}{2\cosh^2 r\sinh^2 r\left(1+\cos 2\varphi\right) + 4\sinh^2\left(1+\sinh^2 r\right)+1}\right| \tag{4.11}$$

The expressions for $\langle\bar{T^k}_{ii}^2\rangle_{ssv}$ and $\langle\bar{T^k}_{ii}\rangle_{ssv}^2$ can be calculated by a similar procedure and the dimensioness quantity for pressure fluctuation in squeezed vacuum states is obtained using (4.7) :

$$\frac{\delta p}{\langle p^2\rangle_{svs}} = \left|\frac{4\cos\varphi\cosh r\sinh r\left(1+2\sinh^2 r\right) - 2\cos 2\varphi\cosh^2\sinh^2 r}{2\cosh^2 r\sinh^2 r\left(1+\cos 2\varphi\right) + 4\sinh^2\left(1+\sinh^2 r\right)+1}\right| \tag{4.12}$$

Lengthy but straight forward algebra that we employed in the above case can be extented to other states also and we find

Single mode squeezed states

The dimensionless quantity for density fluctuation is obtained using (4.6)

$$\frac{\delta\rho}{\langle\rho^2\rangle_{ss}} = \left|\frac{-2\lambda^{*2} - 2\lambda^2 - 4\sinh^2 r\left(\lambda^2+\lambda^{*2}+|\lambda|^2\right)+\psi_1}{\lambda^4+\lambda^{*4}+6|\lambda|^4+4|\lambda|^2+4\lambda^2|\lambda|^2+4\lambda^{*2}|\lambda|^2+\psi_2+1}\right| \tag{4.13}$$

where ψ_1 and ψ_2 are given by

$$\begin{aligned}\psi_1 &= -2\cos 2\varphi\cosh^2\sinh^2 r \\ &\quad -4\cos\varphi\cosh r\sinh r\left(\lambda^2+\lambda^{*2}-1+2\sinh^2 r\right)\end{aligned} \tag{4.14}$$

$$+4\cos\varphi\cosh r\sinh r\left(1+2\sinh^2 r\right)$$

$$\begin{aligned}\psi_2 &= -4\cos\varphi\sinh r\cosh r\left(\lambda^2+\lambda^{*2}+2|\lambda|^2+\lambda+\lambda^*\right)\\ &+4\sinh^2 r\left(2\lambda^2+2\lambda^{*2}+|\lambda|^2\right)+2\cosh^2 r\sinh^2 r\,(1+\cos 2\varphi)\\ &+4\sinh^2 r\left(1+\sinh^2 r\right)\end{aligned}$$

The dimensionless quantity for pressure fluctuations in single mode squeezed states is obtained :

$$\frac{\delta p}{\langle p^2\rangle_{sss}}=\left|\frac{-2\lambda^{*2}-2\lambda^2-4\sinh^2 r\left(\lambda^2+\lambda^{*2}+|\lambda|^2\right)+\psi_1}{\lambda^4+\lambda^{*4}+6|\lambda|^4+4|\lambda|^2+4\lambda^2|\lambda|^2+4\lambda^{*2}|\lambda|^2+1+\psi_2}\right| \tag{4.15}$$

Single mode coherent states

Now we will calculate the dimensionless quantities given by (4.6) and (4.7) in single mode coherent staes. Thus we find:

$$\frac{\delta\rho}{\langle\rho^2\rangle_{scs}}=\left|\frac{-2\lambda^{*2}-2\lambda^2}{\lambda^4+\lambda^{*4}+6|\lambda|^4+4|\lambda|^2+4\lambda^2|\lambda|^2+4\lambda^{*2}|\lambda|^2+1}\right| \tag{4.16}$$

For the pressure fluctuation study the dimesionless quantities in single mode coherent states is obtained by using (4.7) :

$$\frac{\delta p}{\langle p^2\rangle_{scs}}=\left|\frac{-2\lambda^{*2}-2\lambda^2}{\lambda^4+\lambda^{*4}+6|\lambda|^4+4|\lambda|^2+4\lambda^2|\lambda|^2+4\lambda^{*2}|\lambda|^2+1}\right| \tag{4.17}$$

The calculations performed in single mode states can be extented to two mode states.

Two mode squeezed vacuum

In this case the dimensionless quantity for density fluctuation is obtained :

$$\frac{\delta\rho}{\langle\rho^2\rangle_{tsv}} = \left|\frac{8\cos\varphi\cosh r\sinh r(\cosh^2 r+\sinh^2 r) - 4\cos 2\varphi\cosh^2 r\sinh^2 r}{4\cosh^2 r\sinh^2 r(1+\cos 2\varphi)+8\sinh^2 r(\cosh^2 r+\sinh^2 r)+2}\right| \tag{4.18}$$

The dimensionless quantity for anisotropic pressure fluctuation is

$$\frac{\delta p}{\langle p^2\rangle_{tsv}} = \left|\frac{8\cos\varphi\cosh r\sinh r(\cosh^2 r+\sinh^2 r) - 4\cos 2\varphi\cosh^2 r\sinh^2 r}{4\cosh^2 r\sinh^2 r(1+\cos 2\varphi)+8\sinh^2 r(\cosh^2 r+\sinh^2 r)+2}\right| \tag{4.19}$$

Two mode squeezed states

The dimensionless quantity in two mode squeezed states is obtained :

$$\frac{\delta\rho}{\langle\rho^2\rangle_{tss}} = \left|\frac{-4\cos\varphi\cosh r\sinh r(\lambda_+^2+\lambda_+^{*2}+\lambda_-^2+\lambda_-^{*2}+4\sinh^2 r-2)+C}{A+B+D}\right| \tag{4.20}$$

where

$$\begin{aligned}
A &= 12(\lambda_+\lambda_- + \lambda_+^*\lambda_-^* + \lambda_+^*\lambda_- + \lambda_+\lambda_-^*) + 24|\lambda_+|^2|\lambda_-|^2 + \\
&\quad 4\lambda_+^3\lambda_- + \lambda_+\lambda_-^3 + \lambda_-^{*3}\lambda_+ + 4\lambda_-^*\lambda_+^{*3} + 4\lambda_+^3\lambda_-^* + 4\lambda_-^3\lambda_- + \\
&\quad 4\lambda_+^{*2}|\lambda_+|^2 + 4\lambda_-^{*2}|\lambda_-|^2 + 2 + 6\lambda_-^2\lambda_-^2 + 6\lambda_+^*\lambda_-^* + 6\lambda_+^{*2}\lambda_-^2 + 6\lambda_+^2\lambda_-^{*2} \\
B &= 4\cosh^2 r\sinh^2 r(1+\cos 2\varphi) + 8\sinh^2 r(\cosh^2 r+\sinh^2 r) + 2 \\
C &= -4\sinh^2 r(\lambda_+^2+\lambda_+^{*2}+|\lambda_+|^2) - 4\sinh^2 r(\lambda_-^2+\lambda_-^{*2}+|\lambda_-|^2) \\
&\quad -2\lambda_+^2 - 2\lambda_-^2 - 2\lambda_+^* - 2\lambda_-^* - 4\lambda_-\lambda_+ - 4\lambda_-^*\lambda_-^* - 4\lambda_+^*\lambda_- - 4\lambda_+\lambda_-^* + \\
&\quad 8\cos\varphi\cosh r\sinh r(\cosh^2 r+\sinh^2 r) - 4\cos 2\varphi\cosh^2 r\sinh^2 r \\
D &= (8\sinh^2 r + 4\cos\varphi\cosh r\sinh r)(\lambda_+^2+\lambda_+^{*2}+2|\lambda_+|^2+\lambda_-^2+\lambda_-^{*2}+2|\lambda_-|^2) + \\
&\quad (8\sinh^2 + 4\cos\varphi\cosh r\sinh r)(\lambda_+\lambda_- + \lambda_+^*\lambda_-^* + \lambda_+^*\lambda_- + \lambda_+\lambda_-^*) +
\end{aligned} \tag{4.21}$$

The dimensionless quantity for anisotropic pressure fluctuation in two mode squeezed states can be obtined by using (4.7):

$$\frac{\delta p}{\langle p^2 \rangle_{tss}} = \left| \frac{-4\cos\varphi \cosh r \sinh r(\lambda_+^2 + \lambda_+^{*\,2} + \lambda_-^2 + \lambda_-^{*\,2} + 4\sinh^2 r - 2) + C}{A + B + D} \right| \tag{4.22}$$

where A,B,C and D are given by (4.20).

Two mode coherent states

Similarly the dimensionless quantity for density can be also computed in two mode coherent states :

$$\frac{\delta \rho}{\langle \rho^2 \rangle_{tcs}} = \left| \frac{-2\lambda_+^2 - 2\lambda_-^2 - 2\lambda_+^* - 2\lambda_-^* - 4\lambda_+\lambda_+ - 4\lambda_-^* 4\lambda_-^* - 4\lambda_+^*\lambda_- - 4\lambda_+\lambda_-^*}{\lambda_+^4 + \lambda_-^4 + 6|\lambda_+|^4 + 6|\lambda_-|^4 + \lambda_+^{*\,4} + \lambda_-^{*\,4} + 4\lambda_+^2|\lambda_+|^2 + A} \right| \tag{4.23}$$

where A is given by (4.20)

The dimensionless quantity for anisotropic pressure fluctuation :

$$\frac{\delta p}{\langle p^2 \rangle_{tcs}} = \left| \frac{-2\lambda_+^2 - 2\lambda_-^2 - 2\lambda_+^* - 2\lambda_-^* - 4\lambda_+\lambda_+ - 4\lambda_-^* 4\lambda_-^* - 4\lambda_+^*\lambda_- - 4\lambda_+\lambda_-^*}{\lambda_+^4 + \lambda_-^4 + 6|\lambda_+|^4 + 6|\lambda_-|^4 + \lambda_+^{*\,4} + \lambda_-^{*\,4} + 4\lambda_+^2|\lambda_+|^2 + A} \right| \tag{4.24}$$

where A is given by (4.20)

Fig 4.1 shows the variation of energy density ρ_{ssv} with squeezing parameter r and squeezing angle φ. Fig 4.2 represnts the variation of $\frac{\delta\rho}{\langle\rho^2\rangle}_{ssv}$ with r and φ. From this graph we can see that $\frac{\delta\rho}{\langle\rho^2\rangle} = 0$ for $r = 0, \varphi = 0$ and increases with r and φ. Plot for $\frac{\delta\rho}{\langle\rho^2\rangle}$ for coherent states shows that the fluctuation is less compared to unity (fig.4.3). The same arguments can be extented to the pressure fluctuation studies also. Hence the semiclassical theory may break down in squeezed states formalisms.

4.4 Disscussions and conclusions

In this chapter we have examined the validity of semiclassical theory using of squeezed vacuum states, squeezed states and coherent states. The semiclassical theory is valid only when the fluctuations in the energy-momentum tensor become very small compared to unity. If the cosmological perturbations were generated quantum mechanically, they can be represented by the squeezed vacuum states. When vacuum states are squeezed an increase in the variance of phase which means a decrease in the variances of amplitude.

In the case of two mode squeezed vacuum states the calculations show that the result is twice the result of using a single mode squeezed vacuum state, while this is not so for the two mode coherent states and two mode squeezed states.

In the case of squeezed vacuum state, fluctuations can be expected to be large because of the quantum nature of the system involved. This implies a break down of the semiclassical theory near the initial singularity. The initial anisotropic damping can be interpreted as due to the particle production. If the cosmological perturbations were generated quantum mechanically we believe that squeezed vacuum states are good candidates as non-vacuum initial states for understanding the particle production phenomena. Squeezing of the vacuum of the scalar field is achieved by means of the background gravitational field and squeezed vacuum states evolve to squeezed states and later to coherent states in the process of cosmological evolution.

From the present study we can conclude that the coherent states and squeezed states representations of the scalar field are consistent with the semiclassical theory. Therefore we can say that the squeezed vacuum state is a possible quantum states to describe the early Universe. and during the evolution of the Universe quantum effects are deminishing and squeezed vacuum evolve to squeezed states and then to coherent staes which resembles the classical states.

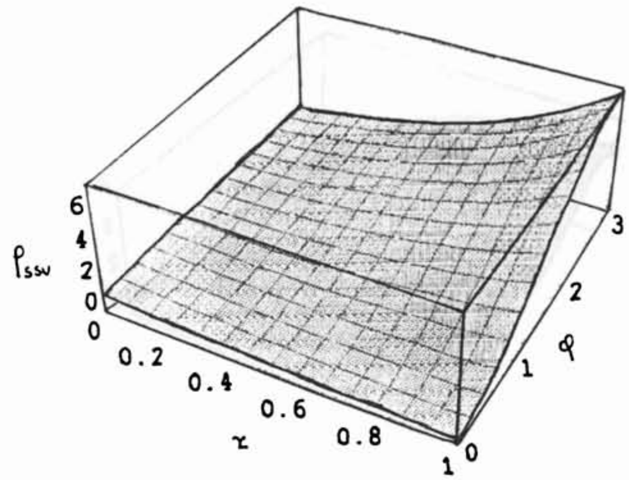
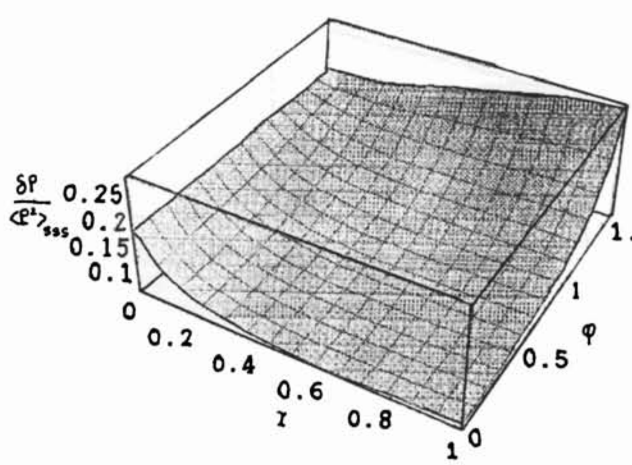


Fig 4.1 Plot for squeezed vacuum energy density ρ_{ssv} with squeezing parameter r and squeezing angle φ



. Fig 4.2 Variation of density fluctuation parameter $\frac{\delta\rho}{\langle\rho^2\rangle_{sss}}$ with squeezing parameter r and squeezing angle φ for real λ.

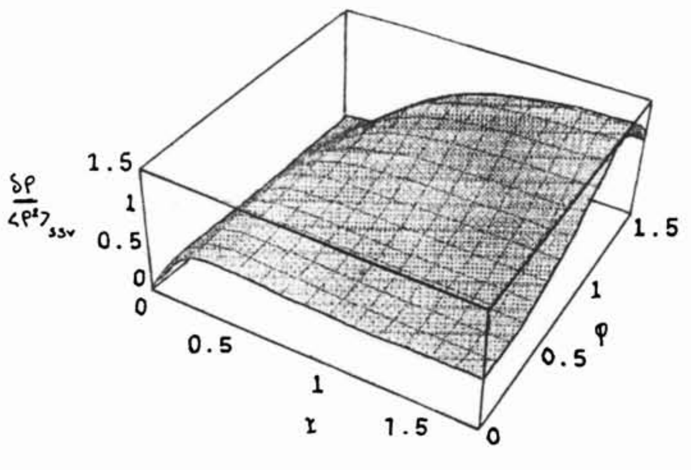


Fig 4.3 Variation of density fluctuation parameter $\frac{\delta\rho}{\langle\rho^2\rangle}_{ssv}$ with squeezing parameter r and squeezing angle φ .

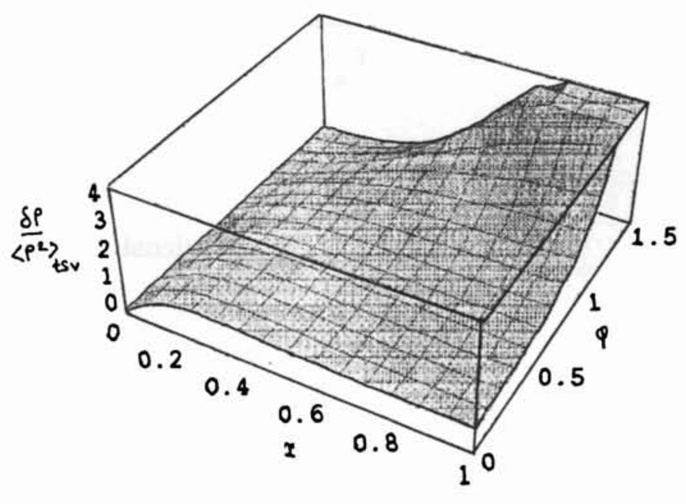


Fig 4.4 Variation of density fluctuation parameter $\frac{\delta\rho}{\langle\rho^2\rangle}_{tsv}$ with squeezing parameter r and squeezing angle φ.

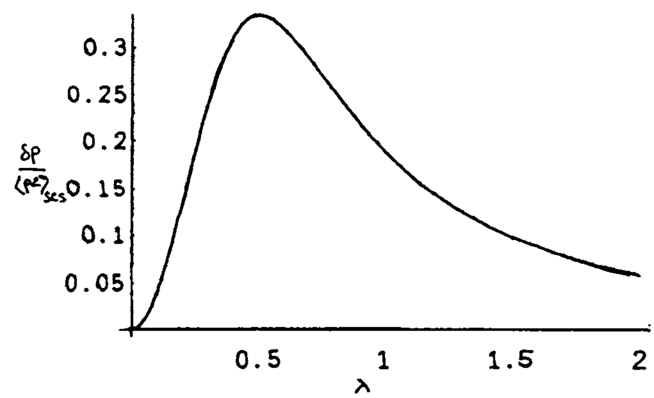


Fig 4.5 Variation of density fluctuation parameter $\frac{\delta\rho}{\langle\rho^2\rangle_{scs}}$ with real λ.

Chapter 5

Quantum effects in Black hole radiation

5.1 Introduction

Black holes are objects whose gravitational fields are so strong that no physical bodies and or signals can escape to infinity from them. The concept of black holes arose in Einstein's General Relativity, after Schwarzschild obtained first the exact solutions of Einstein's equations in vacuum. A black hole is formed when a body of mass M contracts to a size less than the so called gravitational radius $r_g = \frac{2G}{c^2}$, where G is Newton's graviational constant and c is the speed of light. Schwarzschild's solution possesses singularity at $r = 0$ and also on the gravitational radius surface ie at $r = r_g$.

Theoretical predictions made by Hawking in mid 1970's drew great attention of physicists on black holes. He found that as a result of the instability of the vacuum in the strong gravitational field of a black hole, black holes can act as sources of radiation.

He found that if the black hole mass is less than $10^{15}g$ it would decay over a time shorter than the age of the universe. Such black holes, now called primordial black holes, might have been formed at a very early stage of the Universe's evolution. He also found that quantum creation of particles takes place in neutral nonrotating black holes and that black holes create and emit particles as if it were a black body heated to a temprature $T_H = \frac{\hbar}{8\pi MG}$. In the process of Hawking radiation, a black hole loses mass so that its surface area decreases. In the general case of a black hole having charge and angular momemtum, the radiation accompanies the processes which remove angular momentum and electric charge. The classical theory of black holes is based on Einstein's General Relativity. The Schwarzschild black holes and Kerr black holes are exact classical solutions of Einstein's field equations. Classical theory can not account for the predictions of Hawking. The effect of black hole emission is the consequence of the quantum theory of black holes and leads to black hole theomodynamics. The quantum theory of black hole is based on the quantum field theory in curved space-time. Hawking has showed that the black hole emission is thermal and is black body radiation in nature.

In any classical process, the area A of the black hole and hence its entropy do not decrease, [81]i.e.,

$$\delta S^{BH} \geq 0. \tag{5.1}$$

But the quantum evaporation reduces the area of black holes and hence the equation (5.1) is violated. Since the black hole evaporation is thermal in nature, the entropy under consideration need not be the entropy of the black hole alone but it comprises the entropy of the black hole and the entropy of the matter out side the black hole :

$$S = S^{BH} + S^{m} \tag{5.2}$$

In any physical process involving black holes, the generalised entropy S do not decrease, i.e., $\delta S \geq 0$ and this law is known as generalised second law of black hole physics. In Hawking radiation, black hole absorbs negative energy and its area decreases and so does the mass. But the temperature and the luminosity rise and hence the black hole has a negative specific heat.

We will now apply the concept of squeezed states develpoed earlier to black hole radation process. We first consider Schwarzschild black holes and then extent the calculations to rotating black holes. Before considering the black radiation in squeezed states we briefly discuss the Hawking effect first.

5.2 Hawking effect

Hawking's discovery [82, 83] of the thermal radiation of black holes indicates of a profound relationship between black hole physics and thermodynamic laws. It further shows that quantum phenomena are also essential to understand the radiation processes of black holes.

Consider a black hole formed at some time in the past by gravitational collapse. Following Hawking [83], let us assume that no scalar particles were present before the collapse began. In this case, the quantum state is the in-vacuum: $| \psi\rangle = | 0\rangle_{in}$. The in-modes,$f_{\omega lm}$ are pure positive frequency on I^-, so $f_{\omega lm} \sim e^{-i\omega v}$ as $v \to -\infty$, where $v = t + r^*$is advanced time coordinate, Similarly, the out-modes, $F_{\omega lm}$, are pure positive frequncy on I^+, so $F_{\omega lm} \sim e^{-i\omega u}$ as $u \to \infty$, where $u = t - r^*$ is the retarded time coordinate (Fig 5.1). Now as discussed in Chapter 2 we have to find the relation between thse two sets of modes in order to compute the Bogolubov coefficients and to explain the particle creation processes. Our main concern is in particle emissiom at later times. Sincee these modes had an extremely high frequency during the passage through the body, we may describe their propagation by use of geometrical optics [83, 84].

A $u = constant$ ingoing ray passes through the body emerges as a $v = constant$ out going ray, where $u = g(v)$ or equivalently, $v = g^{-1}(u) = G(u)$. The geometrical optics approximation leads to the following aymptotic forms for the modes

$$\begin{aligned} f_{\omega lm} &\sim e^{-i\omega v}, \quad on \;\; I^- \\ f_{\omega lm} &\sim e^{-i\omega G(u)}, \quad on \;\; I^+ \end{aligned} \tag{5.3}$$

and

$$\begin{aligned} F_{\omega lm} &\sim e^{-i\omega v}, \quad on \;\; I^+ \\ F_{\omega lm} &\sim e^{-i\omega g(v)}, \quad on \;\; I^- \end{aligned} \tag{5.4}$$

Hawking gave a general ray-tracing argument which led to the result that

$$\begin{aligned} u = g(v) &= -4M\ln\left(\frac{v_o - v}{C}\right), \quad or \\ v = G(u) &= v_0 - Ce^{\frac{u}{4M}}, \end{aligned} \tag{5.5}$$

where M is the mass of the black hole, C is a constant, and v_0 is the limiting value of u for rays which pass through the body before the horizon forms.

From (5.3), the out modes, when traced back to I^-, have the form

$$F_{\omega lm} \sim e^{\Theta}, \quad v < v_0 \tag{5.6}$$

$$F_{\omega lm} \sim 0, \quad v > v_0$$

wher $\Theta = 4Mi\omega \ln[(v_0 - v)/C]$.

Now the Bogolubov coefficients are obtained by taking the Fourier transformation of the function:

$$F_{\omega lm} = \int_0^\infty d\omega' \left(\alpha^*_{\omega'\omega lm} f_{\omega' lm} - \beta_{\omega'\omega lm} f^*_{\omega' lm}\right) \tag{5.7}$$

The angular coordinates is same for each term in the above equation. Thus:

$$\alpha^*_{\omega'\omega lm} = \frac{1}{2\pi}\sqrt{\frac{\omega'}{\omega}} \int_{-\infty}^{v_0} dv e^{i\omega' v} e^{\Theta}, \tag{5.8}$$

and

$$\beta_{\omega'\omega lm} = -\frac{1}{2\pi}\sqrt{\frac{\omega'}{\omega}} \int_{-\infty}^{v_0} dv e^{-i\omega' v} e^{\Theta}, \tag{5.9}$$

or, equivalently

$$\alpha^*_{\omega'\omega lm} = \frac{1}{2\pi}\sqrt{\frac{\omega'}{\omega}} \int_0^\infty dv' e^{i\omega' v} e^{\Theta}, \tag{5.10}$$

$$\beta_{\omega'\omega lm} = -\frac{1}{2\pi}\sqrt{\frac{\omega'}{\omega}} \int_0^\infty dv e^{-i\omega' v} e^{\Theta}. \tag{5.11}$$

Now using the following result

$$\oint_C dv' e^{-i\omega' v} e^{\Theta} = 0, \tag{5.12}$$

we may write

$$\begin{aligned} \int_0^\infty dv' e^{i\omega' v} e^{\Theta} &= \int_0^\infty dv' e^{-i\omega' v} e^{\Theta'} \\ &= e^{4\pi M\omega} \int_0^\infty dv' e^{-i\omega' v} e^{\Theta_1} \end{aligned} \tag{5.13}$$

where $\Theta' = 4Mi\omega \ln[-(v_0 - v)/C]$ and $\Theta_1 = 4Mi\omega \ln[-\pi i + \ln[-(v_0 - v)/C]$. Also using $v' \to -v'$ and $\ln[-(v_0 - v)/C - i\epsilon] = -\pi i + \ln[-(v_0 - v)/C]$.

Now comparision of (5.13) with (5.10) and (5.11) leads to:

$$|\alpha_{\omega'\omega lm}| = e^{4\pi M\omega} |\beta_{\omega'\omega lm}|. \tag{5.14}$$

Then using the condition (2.24) in the present context, the Bogolubov coefficients can be obtained as:

$$\sum_{\omega'} \left(|\alpha_{\omega'\omega lm}|^2 - e^{4\pi M\omega} |\beta_{\omega'\omega lm}|^2 \right) = \sum_{\omega'} \left(e^{4\pi M\omega} - 1 \right) |\beta_{\omega'\omega lm}|^2 = 1 \tag{5.15}$$

Therefore the mean number of particles created into mode (ωlm) is:

$$N_{\omega lm} = \sum_{\omega'} |\beta_{\omega'\omega lm}|^2 = \frac{1}{e^{4\pi M\omega} - 1}. \tag{5.16}$$

This exhibits the nature of the Planck spectrum with a temperature

$$T_H = \frac{1}{8\pi GM} \tag{5.17}$$

which is known as Hawking temperature.

5.3 Black hole radiation and entropy generation

Hawking's calculations [82, 83]showed that if quantum effects were taken into account, the radiation from a black hole has the characteristic nature of a black boby spectrum. Let a system occupy the vacuum state before the black hole was formed. After the formation, the black hole becomes a source of radiation. The mean number of particles radiated in the mode k can be written by considering $\mid \Omega\rangle$ to be the vacuum state, then $\langle \mid N \mid \rangle$ is [81, 85]

$$\langle \Omega \mid N \mid \Omega \rangle = \frac{1}{e^{\frac{\omega}{T_H}} - 1} \tag{5.18}$$

where $N = a^{\dagger}a$ and T_H is the Hawking temprature.

Hawking temperature and the entropy of black hole are related by the equation [86]

$$T_H = G^2 \left[\left(\frac{\partial S}{\partial M} \right)_{J,Q} \right]^{-1} \tag{5.19}$$

where S is the entropy associated with it, M is the mass parameter, J is the angular momentum and Q is the charge.

Now the radiation emitted by a black hole can be considered to be in single mode squeezed vacuum states, squeezed states and coherent states.

Single mode squeezed vacuum

First we consider that the radiation coming from a black hole is in single mode squeezed vacuum state. The functional form of the radiation spectrum in squeezed vacuum follows from (2.45) and (5.19) and it takes the following form

$$\langle \xi \mid N \mid \xi \rangle = \frac{1}{e^{\frac{\omega}{T_H}} - 1}. \tag{5.20}$$

From the properties of squeezed vacumm states introduced in Chapter 2.4, we can write

$$\langle \xi \mid N \mid \xi \rangle = \sinh^2 r. \tag{5.21}$$

which accounts to spontaneous creation of particles. Using (5.20) and (5.21) we can write

$$\sinh^2 r = \frac{1}{e^{\frac{\omega}{T_H}} - 1}. \tag{5.22}$$

Thus we find :

$$T_H = -\frac{\omega}{\ln \tanh^2 r}. \tag{5.23}$$

A plot of T_H with r is given in fig.5.2.

From (5.15), (5.17) and (5.23) we find

$$\tanh^2 r = e^{-8\pi GM\omega} \tag{5.24}$$

This result shows that the Hawking temperature depends on squeezing parameter through (5.23). Thus we find that (5.24) gives Hawking temperature in terms of the squeezing parameter.

Now our programme is to calculate the change in entropy produced due to black hole evaporation in squeezed vacuum state formalism. Following the definition of entropy given by Shannon's information theory [87] we can write:

$$S = -Tr\rho ln\rho \tag{5.25}$$

where ρ is the density matix.

Since the actual values of ξ forming the squeezed state $\mid \xi\rangle$ are not known we may assume a Gaussian distribution for ξ values. We may also assume that the state of the system is

$$\mid \rangle_{ssv} = \int d^2\xi\sigma(\xi) \mid \xi\rangle. \tag{5.26}$$

Superposition of the squeezed vacuum state in a random manner may lose the phase information of the amplitude and hence only $\mid \sigma(\xi) \mid^2$ may be specified. This allows construction of a density matrix in the following way [73]

$$\rho_{ssv} = \mid \rangle_{ssv\,ssv}\langle \mid = \int d^2\xi \mid \sigma(\xi) \mid^2 \mid \xi\rangle\langle\xi \mid \tag{5.27}$$

where we have assumed that the random phase approximation replaces

$$\int d^2\xi \int d^2\xi' \sigma(\xi)^* \sigma(\xi) \mid \xi\rangle\langle\xi' \mid \tag{5.28}$$

with

$$\int d^2\xi \int d^2\xi' \delta(\xi' - \xi)\sigma(\xi) \mid \xi\rangle\langle\xi' \mid \tag{5.29}$$

The diagonal matrix elements of the density matrix are $\mid \sigma(\xi) \mid^2 \mid \xi\rangle\langle\xi' \mid$ for each value of ξ. The required Gaussian distribution is given by.

$$\begin{aligned} P_\xi &= \mid \sigma(\xi) \mid^2 \\ &= \frac{1}{\pi\langle N\rangle} \exp\left[\frac{-\mid \xi \mid^2}{\langle N\rangle}\right] \end{aligned} \tag{5.30}$$

Since

$$\frac{1}{\pi}\int | \xi\rangle\langle\xi^{'} | \, d^2\xi = 1 \tag{5.31}$$

we find

$$Tr\rho_{ssv} = 2\pi \int d^2\xi | \, \sigma(\xi) \, |^2 \, | \, \xi\rangle\langle\xi' \, || \, \xi\rangle\langle\xi' \, |= 1 \tag{5.32}$$

In squeezed vacuum representation (5.25) can be written as

$$\begin{aligned} S &= -Tr\rho_{ssv}\ln\rho_{ssv} \qquad (5.33)\\ &= \int d^2\xi P_\xi \ln P_\xi. \\ &= -\int d^2\xi \frac{1}{\pi\langle N\rangle}\exp\left[\frac{-| \, \xi \, |^2}{\langle N\rangle}\right]\ln\left(\frac{1}{\pi\langle N\rangle}\exp\left[\frac{-| \, \xi \, |^2}{\langle N\rangle}\right]\right) \\ &= -\ln\left(\frac{1}{\pi\langle N\rangle}\right)\left(\frac{1}{\pi\langle N\rangle}\right)\pi\int_0^\infty 2rdre^{-\frac{r^2}{\langle N\rangle}} + \left(\frac{1}{\pi\langle N\rangle}\right)\left(\frac{1}{\pi\langle N\rangle}\right)\pi\int_0^\infty 2rdre^{-\frac{r^2}{\langle N\rangle}} \end{aligned}$$

Now using the properties of Γ function and (5.21) we find

$$S = \ln\sinh^2 r + 1 + \pi \tag{5.34}$$

Now $\Delta S = S - S_0$, where S_0 is the initial entropy which is defined as [88]

$$S_0 = -\int_0^\infty dx d\bar{x}\left(\frac{1}{\pi}\right)e^{-x^2+\bar{x}^2}\ln\left(\frac{e^{-x^2+\bar{x}^2}}{\pi}\right) \tag{5.35}$$

and using the properties of Γ function we obtaind

$$S_0 = 1 + \pi \tag{5.36}$$

Hence the change in entropy produced due to black hole radation is given by

$$\Delta S = \ln\sinh^2 r. \tag{5.37}$$

The variation of ΔS with r is shown in fig.5.3. It is found that ΔS is positive only when $r \geq 1$.

From (5.19), (5.23) and (5.37) we obtain the following relation

$$\Delta M \sim \ln \cosh^2 r. \tag{5.38}$$

(5.38) gives the change of mass of the balck hole in trems of squeezing parameter . The dependence of ΔM on r is as shown in fig.5.4. ΔM is positive for $0 \leq r < \infty$ and vanishes when $r = 0$.

The entropy generation of black holes may be interpreted due to spontaneous creation of particles by the black hole. The particles produced are from squeezed vacuum states of the radiation field.

Single mode squeezed state.

Related to Hawking effect, a similar phenomenon of creation of particles can take place in gravitational field of rotating black holes [89, 90, 91, 92]. This is known as Starobinskii-Unruh process [90, 92]. Due to vacuum instablity the ergosphere of the black hole leads to the classical phenomenon of wave amplification which is known as superradiance. This aspect manifests itself, in the independence of the enhancement of coefficient on Planck's constant. The superradiance can be described in quantum terms. A quantum analogue can be found for the classical phenomenon of superradiance : spontaneous creation of particles from the vacuum in the gravitational field of a roatating black hole. Since the radiation from a black hole contains an increased number of particles, we can say that the out going radiation contains stimulated emission as well as spontaneous emission of particles. Such a situation can be explained using the squeezed state representation of the radiation field.

Now the black hole radiation spectrum can be considered to be in squeezed states. The functional form of the spectrum in squeezed states become:

$$\langle \lambda, \xi \mid N \mid \lambda, \xi \rangle = \frac{1}{e^{\frac{\omega'}{T_H}} - 1}. \tag{5.39}$$

where $\omega' = \omega - m\Omega$, where m is the azimuthal quantum number and Ω is the angular speed of the event horizon. When the radiation field is in squeezed states, the quanta of the field is given by

$$\langle \lambda, \xi \mid N \mid \lambda, \xi \rangle = |\lambda|^2 + \sinh^2 r. \tag{5.40}$$

when the first term in the right hand side represents stimulated emission while the second term represents spontaneous emission of particles. Using (5.40) and (5.41) we find :

$$T_{H_{sss}} = \frac{\omega'}{\ln\left[\frac{|\lambda|^2}{|\lambda|^2 + \sinh^2 r} + \frac{1}{\frac{|\lambda|^2}{\cosh^2} + \tanh^2 r}\right]} \tag{5.41}$$

Plots of $T_{H_{sss}}$ with λ and r are given in fig 5.4

The entropy change can be evaluated in squeezed states also. Since the actual value of λ andξ forming the squeezed state $\mid \lambda\xi\rangle$ are not known we may assume a Gaussian distribution of $\lambda\xi$ values. We may also assume that the state of the system is

$$\mid \rangle_{sss} = \int d^2\lambda d^2\xi |\sigma(\lambda\xi)|^2 \mid \lambda\xi\rangle. \tag{5.42}$$

Superposition of the squeezed state in a random manner may lose the phase information of the amplitude and hence only $| \sigma(\lambda\xi) |^2$ may be specified. This allows construction of a density matrix in the following way [73]

$$\rho_{sss} = | \rangle_{ssssss} \langle \; | = \int d^2\lambda d^2\xi | \; \sigma(\lambda\xi) \; |^2 \; | \; \lambda\xi\rangle\langle\lambda\xi \; | \tag{5.43}$$

where we have assumed that the random phase approximation replaces

$$\int d^2\lambda d^2\xi \int d^2\lambda' d^2\xi' \sigma(\lambda\xi)^* \sigma(\lambda\xi) \; | \; \lambda\xi\rangle\langle(\lambda\xi)' \; | \tag{5.44}$$

with

$$\int d^2\lambda \int d^2(\lambda\xi)' \delta(\lambda'\xi' - \lambda\xi)\sigma(\lambda\xi) \; | \; \lambda\xi\rangle\langle\lambda'\xi' \; | \tag{5.45}$$

The diagonal matrix element of the density matrix are $| \; \sigma(\lambda) \; |^2 \; | \; \lambda\xi\rangle\langle\lambda'\xi' \; |$ for each value of $\lambda\xi$. Then the required Gaussian distribution is

$$\begin{aligned} P_{\lambda\xi} &= | \; \sigma(\lambda\xi) \; |^2 \\ &= \frac{1}{\pi\langle N\rangle} \exp\left[\frac{-| \; \lambda\xi \; |^2}{\langle N\rangle}\right] \end{aligned} \tag{5.46}$$

since we have a unit norm

$$\frac{1}{\pi}\int | \; \lambda\xi\rangle\langle\lambda'\xi' \; | \; d^2\xi d^2\lambda = 1 \tag{5.47}$$

we find

$$Tr\rho_{sss} = 2\pi \int d^2\lambda d^2\xi | \; \sigma(\lambda\xi) \; |^2 \; | \; \lambda\xi\rangle\langle\lambda'\xi' \; || \; \lambda\xi\rangle\langle\lambda'\xi' \; | = 1 \tag{5.48}$$

In squeezed state representation (5.26) can be written as

$$\begin{aligned} S &= -Tr\rho_{sss} \ln \rho_{sss} \\ &= \int d^2\lambda d^2\xi P_{\lambda\xi} \ln P_{\lambda\xi}. \end{aligned} \tag{5.49}$$

Thus we find

$$S = \ln|\lambda|^2 + \ln\sinh^2 r + 1 + \pi \tag{5.50}$$

Now $\Delta S = S - S_0$, where S_0 is the initial entropy which is given by (5.36)

Hence the change in entropy produced due to black hole radation is given by

$$\Delta S = \ln|\lambda|^2 + \ln\sinh^2 r \tag{5.51}$$

The variation of ΔS with λ and r is shown in fig.5.5.

The change in mass parameter in this representation is obtained by using (5.20) and (5.55) and is given by

$$\Delta M_{sss} \sim \frac{\ln|\lambda|^2 + \sinh^2 r}{\ln\left[\frac{|\lambda|^2}{|\lambda|^2+\sinh^2 r} + \frac{1}{\frac{|\lambda|^2}{\cosh^2}+\tanh^2 r}\right]} \tag{5.52}$$

If the' black hole radiation contains only stimulated emission of particles and no spontaneous emission particles', then the radiation field of the black hole may be represented by coherent states. In that case we can find the following results:

Hawking temperature:

$$T_{H_{ssc}} = \frac{\omega}{\ln(1 + \frac{1}{|\lambda|^2})} \tag{5.53}$$

the change in entropy :

$$\Delta S = \ln|\lambda|^2 \tag{5.54}$$

and the change in mass parameter :

$$\Delta M_{scs} \sim \frac{\ln \mid \lambda \mid^2}{\ln\left(1+\frac{1}{|\lambda|^2}\right)}. \tag{5.55}$$

A plot of T_{ssc} with λ is given in fig 5.7. Fig.5.8 shows the variation of ΔS_{scs} withλ while Fig 5.9 shows the variation of ΔM_{scs} with λ.

5.4 Conclusions

In this chapter we have studied the generation of entropy via squeezed vacuum state and found that it is complectly determined by the associated squeezing parameter and is caused by the particle creation which can be understood as a squeezed vacuum phenomenon. For large squeezing limit our result for entropy change is given by $\Delta S = \ln \sinh^2 r$. An expression for the Hawking temperature is also obtained in terms of the squeezing parameter and our result leads to the observation made by Hawking [83]. The change in the mass parameter of the black hole is also related to the squeezing parameter. We found that the change in mass parameter of the black hole is completely determined by the associated squeezing parameter. Therefore in order to understand change in entropy and change in mass due to particle production by the black hole squeezing can be a possible mechanism. We have also studied these parametres in squeezed states and coherent states representations of radiation field. We hope that these types of studies can throw more light on the problem of loss of information paradox [93, 94, 95]of black holes.

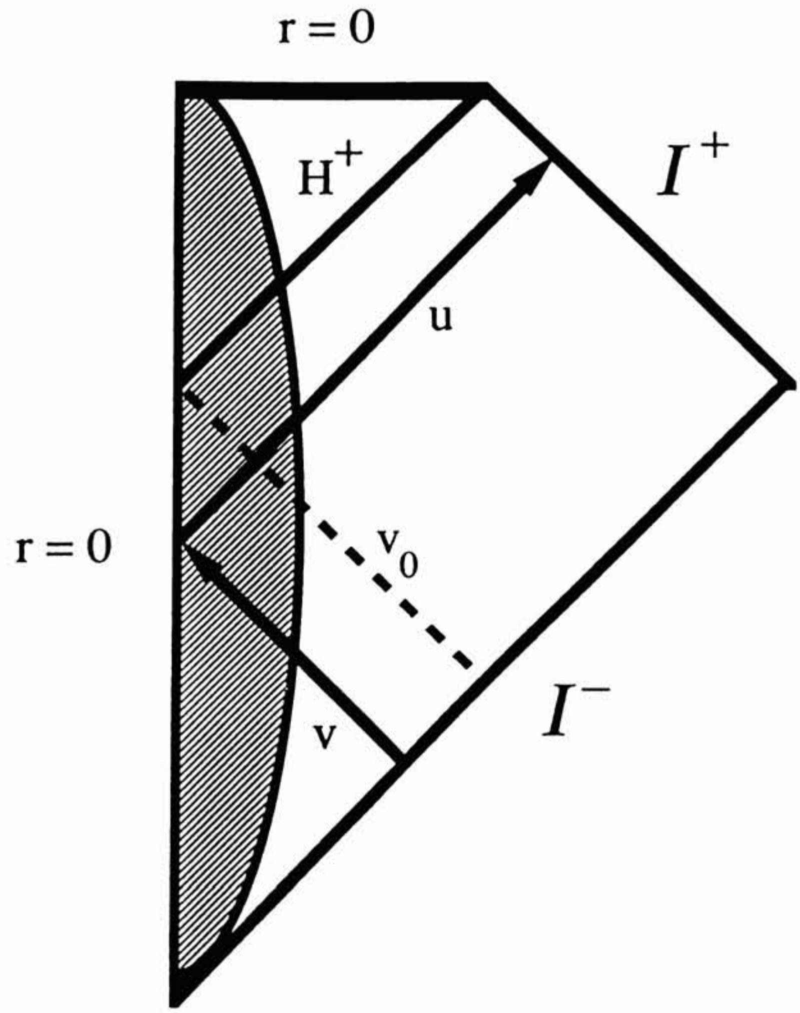


Fig.5.1. The penrose diagram for the spacetime of a black hole formed by gravitational collapse. The shaded region is the interior of the collapsing body, the $r = 0$ line on the left is worldline of the center of this body, the $r = 0$ line at the top of the diagram is the curvature singularity, and H^+ is the future event horizon. An ingoing light ray with $v < v_0$ from I^- passes through the body and escapes to I^+ as a $u = constant$ light ray. Ingoing raya with $v > v_0$ do not escape and eventually reach the singularity.

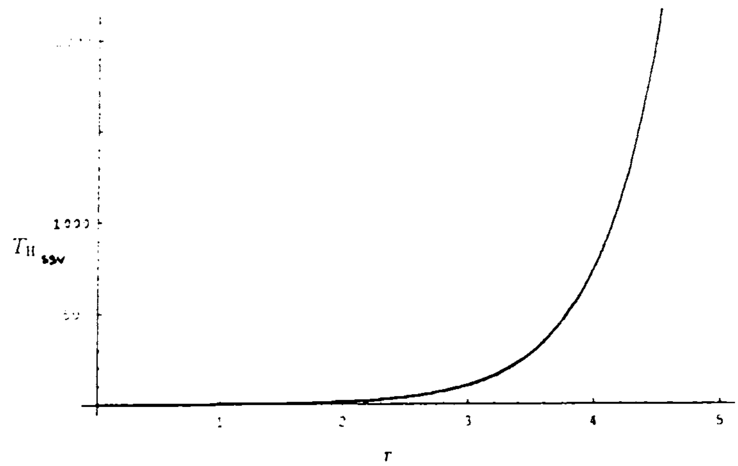


Fig 5.2. Plot for Hawking temprature T_{Hssv} with squeezing parameter r.

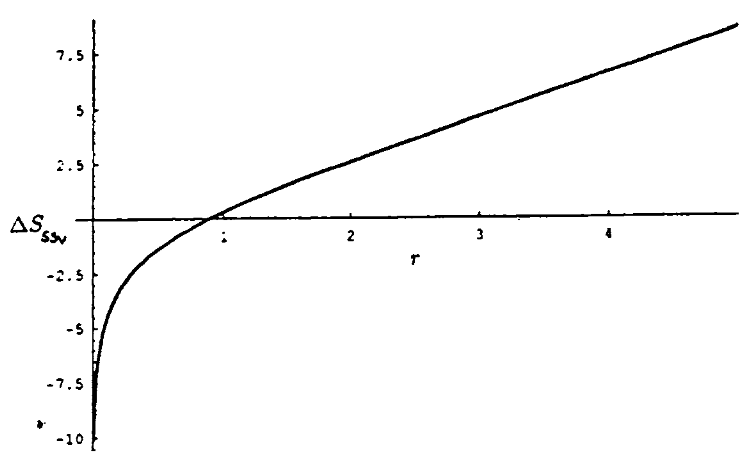
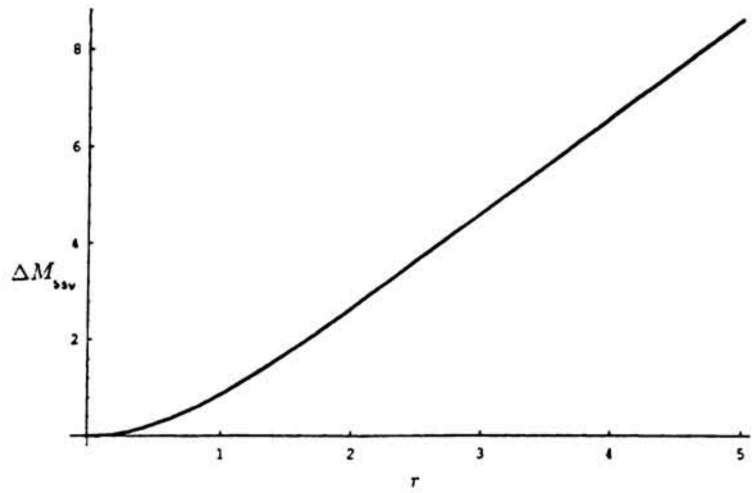
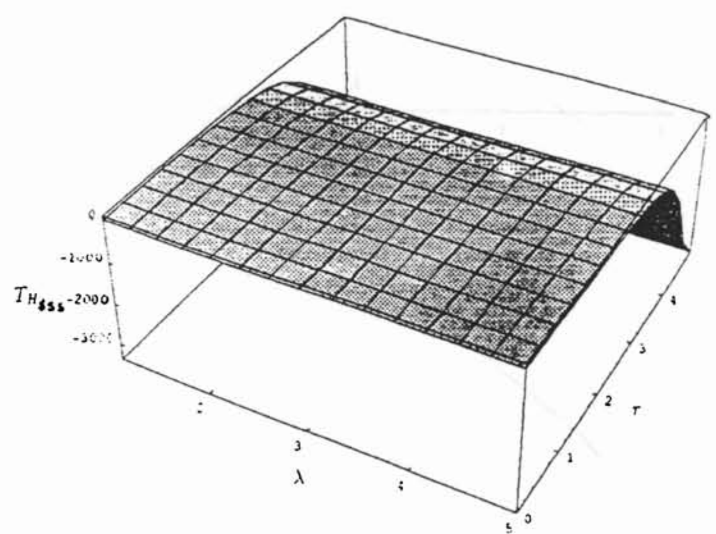


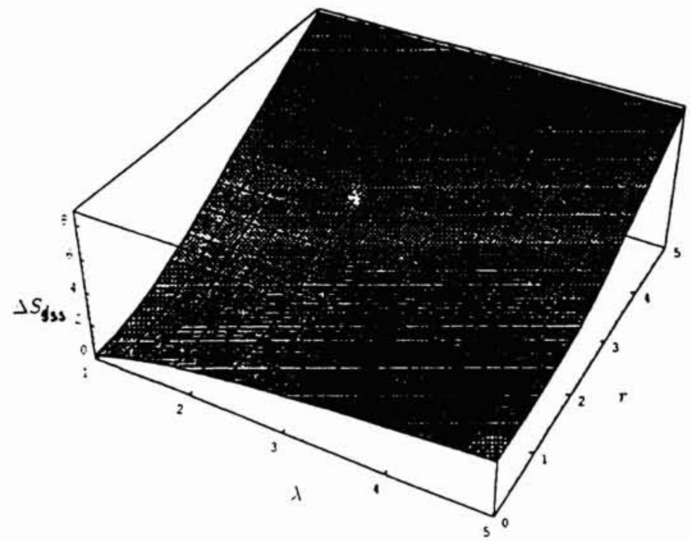
Fig 5.3 Plot for change in entropy ΔS_{ssv} with squeezing parameter r.



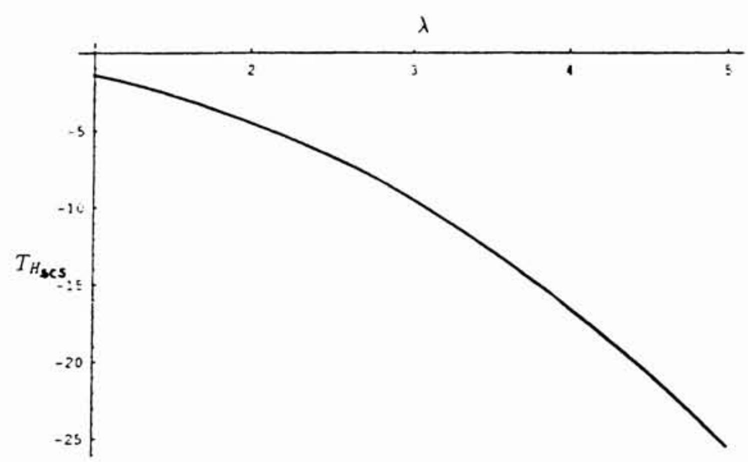
ig 5.4. Plot for massparameter ΔM_{ssv} with squeezing parameter r.



g 5.5 Plot for Hawking temprature T_{Hsss} with squeezing parameter r and λ.



g 5.6 Plot for change in entropy ΔS_{sss} with squeezing parameter r and λ



g 5.7 Plot for Hawking temprature T_{Hscs} with λ.

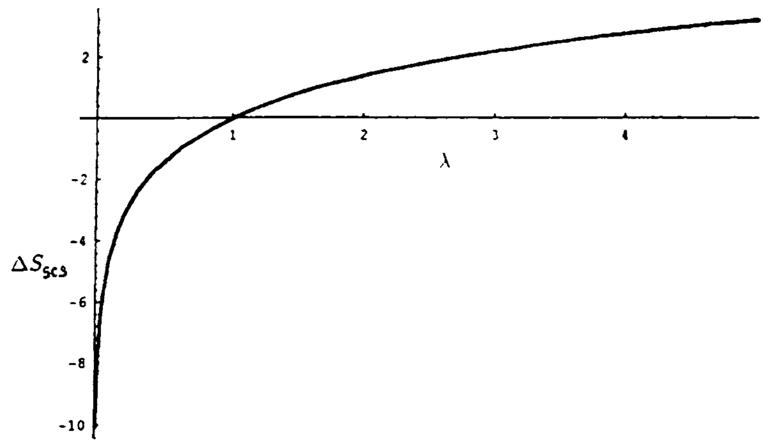


Fig 5.8 Plot for change in entropy ΔS_{scs} with λ

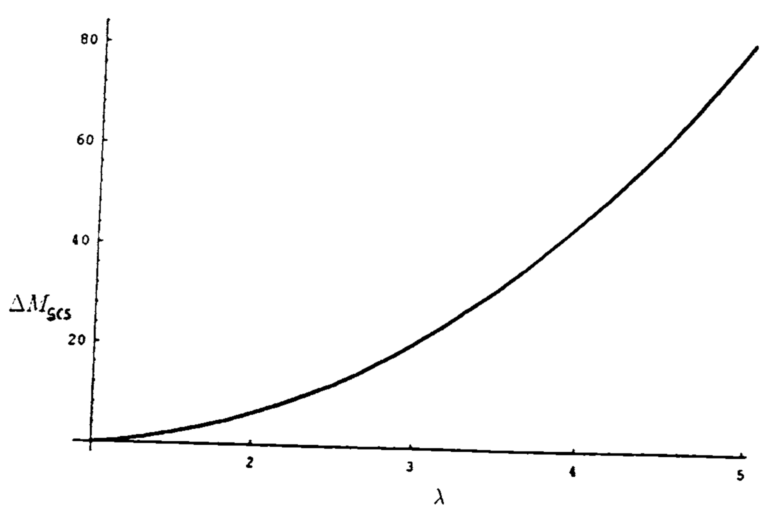


Fig 5.9. Plot for massparameter ΔM_{scs} with λ

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