NUCLEOSYNTHESIS AND PHASE TRANSITIONS IN THE EARLY UNIVERSE

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CERTIFICATE

Certified that the work reported in the present thesis is based on the bonafide work done by Titus K Mathew, under my guidance in the Department of Physics, Cochin University of Science and Technology, and has not been included in any of the thesis submitted previously for the award of any degree.

Cochin-22 M Sabir

May 8, 1995 Supervising Teacher

DECLARATION

Certified that the work presented in this thesis is based on the original work done by me under the guidance of Prof. M Sabir in the Department of Physics, Cochin University of Science and Technology, and has not been included in any other thesis submitted previously for the award of any degree.

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May 8, 1995.

PREFACE

The work presented in this thesis has been carried out by the author as a research scholar under the supervision of Prof. M Sabir, in the Department of Physics, CUSAT. The thesis addresses some recent problems in the primordial synthesis of light elements in the early universe based on the standard hot big-bag theory and some related questions.

The recent advances in the observational arena, have led to a revision of the primordial abundances of light elements like H, ^{2}H, ^{3}H, ^{3}He, ^{4}He, ^{7}Li in the Universe. It is the aim of the standard big bang nucleosynthesis (SBBN) theory to predict these abundances thus to obtain the value of the baryon-to-photon ratio η of the present Universe. The other two parameters apart from η are number of light neutrinos and neutron life time, which are derived from experimental results.

There have been reports that the latest values of the abundances of deuterium (D), Tritium (^{3}H), helium-3 (^{3}He) and helium-4 (^{4}He) predicted by the theory do not agree with the observed values for a unique range of η. One of the aim of our work is to check this claim in the light of latest input parameters such as neutron life time, reaction rates etc. With a modified numerical cord, we find that discrepancy is there and it is shared by lithium-7 (^{7}Li) also. Even though the discrepancy is not very large, it is considerable and has the undesirable consequence that it can predict more than one values of η for our Universe. The removal of this discrepancy calls for some essential changes in the scenario of SBBN.

The inhomogeneous nucleosynthesis model based on first order quark-hadron phase transition in the early Universe, is an alternative scenario which has been extensively analysed in the last decade. Another class of models include neutrino degeneracy effects and neutrino masses. In this thesis we present our work in nucleosynthesis calculation and related aspects in these alternative models.

The contents of the thesis are organised as follows: In chapter 1, is given an introduction to the theory of standard big-bang nucleosynthesis, and a review of the method of inferring the values of the primordial abundance of various light elements like D, ^{3}H, ^{3}He, ^{4}He, ^{7}Li etc.

In chapter 2 we present the method of calculating the reaction rates of the various reactions of cosmological interest based on the method of astrophysical S-factor formalism. Here we report our work on the calculation of the rate of the reaction ^{7}Li(α, n)^{11}B, which is a very important reaction in the formation of the elements heavier than ^{7}Li. The calculation is based on the latest data obtained by Boyd et al. The reaction rate obtained by us leads to a small reduction in the primordial abundance of ^{7}Li.

In chapter 3 is presented our work about the modification of the Wagoner's numerical code for calculating the primordial abundance of the light elements. In Wagoners original code all thermodynamic functions are evaluated approximately. In order to increase the accuracy, we changed all these approximate evaluations with exact numerical calculation. We have updated the code by incorporating the latest results of reaction rates. By using our modified code and using the latest value of neutron life-time, we did recalculation of the primordial abundances of the light elements. By comparing our calculated results with the observed values of the abundances, we find that the abundances of light elements are not in agreement with the observed values for unique range of η values. This shows that the SBBN model is in trouble. Then we present our work regarding the removal of this discrepancy by including neutrino degeneracy. By including a small electron-neutrino degeneracy we find that the discrepancy can be removed. We also present here our investigations on the effects of massive neutrinos on primordial nucleosynthesis. Presence of massive neutrino can increase the neutron-proton ratio and thus the ^{4}He abundance. But this increase can be brought down by the neutrino degeneracy. The works done above are of extensive computational types, which do not reveal fully the physics of the process. In order to bring out the physics very clear, we also present an approximate analytical analysis of the neutrino degenerated nucleosynthesis.

In chapter 4 we present our work about the possibility of Mini-inflation prior to the quark-hadron transition in the early Universe. First order quark-hadron transition is considered as a candidate for introducing inhomogeneity in the early Universe prior to the nucleosynthesis. On studying the characteristic of this transition we find that there is a possibility of mini-inflation just before the transition, which may dilute the inhomogeneity in such a way that the proceeding nucleosynthesis will not be affected considerably. We also note the possibility of mini-inflation without supercooling.

CONTENTS

Chapter 1

Introduction to standard Big-bang theory and primordial nucleosynthesis

The standard cosmological model is the hot big bang model. The important points of observational supports for this model are:

(i) Hubble expansion of the Universe,

(ii) Existence of the cosmic microwave background radiation (CMBR),

(iii) Abundance of the light elements.

The distribution of matter in the Universe is assumed to be homogeneous and isotropic on a sufficiently large scale as indicated by the distribution of galaxies. This high degree of isotropy exhibited by CMBR provides further evidence for the spatial isotropy.

The standard model assumes a homogeneous and isotropic Universe which is described by the Friedmann-Robertson-Walker (FRW) metric [2,3,7,8,9]

$$ds^2 = c^2\, dt^2 \; - \; R^2(t) \left[\frac{dr^2}{1 - kr^2} \; + \; r^2 \left(d\theta^2 + sin^2\theta \; d\phi^2\right)\right] \tag{1.1}$$

where (t, r, θ, ϕ) are the comoving distance, $R(t)$ is the cosmic scale factor and the k is a curvature parameter. By an appropriate rescaling of the coordinates k can assume values +1, -1 or 0. For k = +1, the Universe is finite but unbounded, essentially a three sphere of radius R. The other two cases describe space of infinite volume, k=0 being flat and k=-1 being one of negative curvature.

For an expanding Universe, the Hubbles law, is the kinematical consequence of the FRW metric. Hubbles law [1] says that all galaxies are receding from each other, and the velocity of recession of a galaxy is proportional to the distance from the observer. The constant of proportionality known as the Hubbles constant gives the expansion rate of the Universe. The measured value of the Hubbles constant at the present epoch, H_0 with its uncertainties in measurements ranges between 50 and 100 km/s/Mpc. Because of its uncertainty, one usually denotes H_0 by h_0 in units of 100 km/s/Mpc, where $0.4 \leq h_0 \leq 1.0$. One of the implications of the Hubbles law is the finiteness of the age of the Universe $t_0 \approx H_0^{-1}$. This means that at a finite time in the past all the constituents of the Universe must have been concentrated at a point. In the big bang model it is assumed that Universe started expanding from such a singular state which was extremely hot and dense.

The dynamics of the Universe is determined by the Einsteins field equation [10]

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi\, G\, T_{\mu\nu} + \Lambda\, g_{\mu\nu} \tag{1.2}$$

where $R_{\mu\nu}$ is the Ricci tensor, R the Ricci scalar, $g_{\mu\nu}$ the metric, G the Newton's gravitational constant, $T_{\mu\nu}$ the energy-momentum tensor and Λ the cosmological constant. Since during the nucleosynthesis era the Λ is not significant, we will not consider it any more. The energy-momentum tensor $T_{\mu\nu}$ comprises of all the forms of energy and mass we can assume in the Universe. The solution for this equation for the dynamics of the Universe will depend on the choice of the energy-momentum tensor. For an istropic and homogeneous Universe with a FRW metric, the tensor $T_{\mu\nu}$ must be a diagonal one [2,3,4,5] and the nonvanishing space components equal to each other,

$$T_{\mu\nu} = diag\left(\rho c^2, -P, -P, -P\right) \tag{1.3}$$

where ρ is the energy density and P the pressure, which are functions of time. With this choice Einstein equation (1.2) yield the following two equations for the evolution of the scale factor [2,3].

$$3\ddot{R} = -\, 4\pi G\,(\,\rho + 3P\,)\,R \tag{1.4}$$

and

$$R\ddot{R} + 2\dot{R}^2 + 2k = 4\pi\, G\,(\,\rho - P\,)\,R^2 \tag{1.5}$$

If the equations (1.4) and (1.5) are combined to eliminate $\ddot{R}$, the result is a first order equation in R

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi\, G}{3}\,\rho \tag{1.6}$$

which is called as Friedmann equation. In addition to this the energy conservation yields the equation

$$\frac{d}{dt}(\rho R^3) + \frac{p}{c^2}\frac{dR^3}{dt} = 0$$

$$R^3 \frac{d\rho}{dt} = \frac{d}{dt}\left[R^3(\rho + P)\right] \tag{1.7}$$

Equations (1.6) and (1.7) can be solved by using suitable equations of states connecting pressure and energy density. Assuming that initially the radiation energy dominated over the matter energy, the equation of state is

$$P = \frac{\rho}{3} \tag{1.8}$$

Neglecting the curvature effect it follows from (1.7) that the energy density of the radiation dominated phase behaves as

$$\rho \propto R^{-4} \tag{1.9}$$

This leads to the connection between the scale factor and age of the radiation dominated Universe as,

$$R \propto t^{1/2}. \tag{1.10}$$

During the matter dominated phase pressure P = 0 and

$$\rho \propto R^{-3} \tag{1.11}$$

The corresponding equation which connects the scale factor to age of the Universe is

$$R \propto t^{2/3} \tag{1.12}$$

As Universe expands the energy density of the radiation decreases faster than that of the non-relativistic matter. But this difference in the decrease of the energy density will not manifest until the rate of thermalisation falls short of the expansion rate of the Universe. When this happens the radiation and matter will thermally decouple from each other and consequently the Universe will change from the radiation dominated phase to the matter dominated phase.

The expansion rate of the Universe is controlled by the total energy density of the Universe. This total energy density comprises the energy density due to the photons and other particle species present in the Universe, like electrons, positrons, neutrinos, antineutrinos, nucleons etc. For k=0, one can define a critical energy density ρ_c as,

$$\rho_c = \frac{3\,H^2}{8\,\pi\,G} \tag{1.13}$$

In terms of the present value of the Hubble constant, $\rho_c = 1.88\ h_0^2 \times 10^{-29}\ \ g\ cm^{-3}$. We also define a parameter Ω known as the density parameter as

$$\Omega = \frac{\rho}{\rho_c} \tag{1.14}$$

In terms of Ω the Friedmann equation can be written as

$$(\Omega - 1)\ H^2 = \frac{k}{R^2} \tag{1.15}$$

For $\Omega > 1$, $k = +1$ corresponds to a closed Universe. If $\Omega < 1$, k = -1 the Universe is open, which expands for ever and $\Omega = 1$ corresponds to a flat Universe with k = 0.

The age of the Universe is also determined by the total energy density through the Friedmann equation. For the radiation dominated phase of the Universe, the age can be written for $k = 0$ as

$$t = \left(\frac{3}{32\pi\, G\, \rho}\right)^{1/2} + constant \tag{1.16}$$

Similarly one can determine the age of the matter dominated phase also. For exact age determination it is needed account for both the radiation and matter energy densities. The present age of the Universe is thought to be in the range 10 to 20 Gyr.

1.1 Thermal Evolution of the Universe

The early Universe was to a good approximation in thermal equilibrium. The subsequent departures from the equilibrium causes the formation of the different structures in the early Universe. A particle species depart from the thermal equilibrium when its interaction with the other particles lag behind the expansion rate of the Universe. During thermal equilibrium the reaction rate is a function of several variables including temperature. The expansion rate is however a function of temperature alone.

According to the second law of thermodynamics, the entropy of the particles at temperature T in a volume V is [2,3]

$$dS = \frac{1}{T}\,\{d(\rho\, V) + P\, dV\} \tag{1.17}$$

where ρ and P are the equilibrium energy density and pressure. By the integrability condition,

$$\frac{\partial^2 S}{\partial T\,\partial V} = \frac{\partial^2 S}{\partial V\,\partial T} \tag{1.18}$$

The energy density and pressure then be related as

$$\frac{dP}{dT} = \frac{1}{T}\,\{\rho + P\} \tag{1.19}$$

Energy conservation relation (1.7) can then be written as

$$\frac{d}{dT}\left[\frac{V}{T}\,(\rho + P)\right] = 0. \tag{1.20}$$

The quantity $V(\rho + P)/T$ is nothing but the entropy per comoving volume S, as can be proved from equations (1.17) and (1.19). Equation(1.20 implies that the entropy per comoving volume is a constant under the thermal equilibrium condition. During the radiation dominated phase, the constituent particles are highly relativistic such that the energy density is

$$\rho \propto T^4 \tag{1.21}$$

By entropy conservation law the corresponding evolution of the scale factor R with temperature is

$$R \propto T^{-1} \tag{1.22}$$

This condition known as the adiabatic condition, holds true through out the history of the early Universe.

At very high temperature the Universe consisted of photons, leptons, nucleons and mesons and their antiparticles. As the temperature dropped below 10^{12}K the muons annihilate with their antiparticles. During the further expansion the main events that took place are the decoupling of neutrinos around $T \approx 2 \times 10^{11}$K, pair annihilation of $e^{\pm}$ pairs, the freezing out of the neutron-to-proton number density ratio and subsequently the synthesis of the light nuclei at about 10^9K. Due to further cooling the Universe changed over from the radiation dominated phase to matter dominated phase at about $T \approx 4000$K, followed by the decoupling of radiation and matter. The energy of the decoupled radiation was then red shifted due to expansion of the Universe, its present temperature being about 2.7 K.

The evolution of the thermodynamic functions like energy density, number density, pressure etc of the different particle species are as follows. In equilibrium condition the photons will obey the Planck distribution [1,2,11,15]

$$\bar{n}_\gamma(E_\gamma) = \frac{1}{\pi^2 c^2 \hbar^{\,3}} \frac{E_\gamma^2}{e^{E_\gamma/k_B T_\gamma} - 1} \tag{1.23}$$

where E_γ is the photon energy and T_γ is the photon temperature. The total energy density of the photons can then be written as [11],

$$\rho_\gamma = \int_0^\infty \bar{n}_\gamma(E_\gamma)\, E_\gamma\, dE_\gamma \;=\; a\,\frac{T_\gamma^4}{c^2} = 8.4182\, T_9^4\, gcm^{-3} \tag{1.24}$$

The total number density of the photons [11,16]

$$n_\gamma = \int_0^\infty \bar{n}_\gamma(E_\gamma)\, dE_\gamma = \frac{2}{\pi^2}\left(\frac{k_B T_\gamma}{c\hbar}\right)^3 \zeta(3) = 2.02872 \times 10^{28}\, T_9^3\, cm^{-3} \tag{1.25}$$

This corresponds to $3.99 \times 10^{29}\ (T_9/2.7)^3$ photons per cm^3, which is very large compared to the baryon density in a Universe with critical energy density equal to the baryon density. Hence the Universe is dominated by photons. The ratio of the baryonic number density to the photonic number density, η is related to the baryonic component of the density parameter Ω_b as,

$$\begin{aligned}\Omega_b &= 3.5569 \times 10^7 \, \eta \, h_0^2 \left(\frac{T_\gamma}{2.7}\right)^3 \\ &= 3.5569 \times 10^{34} \, \eta h_0^2 \left(\frac{T_9}{2.7}\right)^3\end{aligned} \tag{1.26}$$

where h_0 is the present value of the Hubble's constant in units of 100 km/sec/Mpc [2]. The pressure of the photon gas is

$$P_\gamma = \frac{1}{3} \rho_\gamma \, c^2 \; erg \; cm^{-3} \tag{1.27}$$

The thermodynamic functions of the other particles like electrons, positrons, neutrinos, anti neutrinos etc can be determined using the following distribution function [3,11]

$$\bar{n}_i(p) = \frac{p^2}{2\pi h^3} g_i \left(\frac{1}{e^{(E_i-\mu_i)/k_B T} \pm 1}\right) \tag{1.28}$$

where i refers to the particular species, p is the momentum, g_i is the spin multiplicity (which is 2 for electron, 2 for photons and 1 for neutrinos), $E_i = \sqrt{p_i^2 c^2 + m_i c^4}$, is the energy of the i^{th} particle of rest mass m_i, μ is the chemical potential and k_B is the Boltzman constant. The + sign is for fermions and - sign is for bosons. In equilibrium the number density of any species i is

$$n_i = \int_0^\infty \bar{n}_i(p) \, dp \tag{1.29}$$

the energy density is

$$c^2 \rho_i = \int_0^\infty \bar{n}_i(p) \; E_i(p) \; dp \tag{1.30}$$

and the pressure is given by the relation

$$P_i = \int_0^\infty \frac{p^2\, c^2 \bar{n}_i(p)}{3E_i(p)}\, dp \tag{1.31}$$

Using the above general prescription we obtain the thermodynamic functions for the constituent particles of the early Universe as follows. The total energy density of the electrons is

$$c^2(\rho_{e^-} + \rho_{e^+}) = \int_0^\infty p^2\; 2\pi^2 h^3\; g_e \left(\frac{1}{e^{(E_n-\mu_n)/k_BT}+1} + \frac{1}{e^{(E_i+\mu_n)/k_BT}+1}\right) E_e(p)\; dp \tag{1.32}$$

For zero chemical potential ($\mu = 0$) as assumed in the earlier works [15,16,21,22,], the above equation can be expanded in terms of the Bessel function as,

$$\rho_{e^-} + \rho_{e^+} = 15.56\; T_9^4 \left[M(z) - \frac{1}{16}M(2z) + ...\right]\; g\; cm^{-3} \tag{1.33}$$

where $z = m_ec^2/k_BT = 5.92986\ /T_9$, and

$$M(z) = \left[\bar{K}_3(z) + \frac{z^2}{24}\bar{K}_1(z)\right]$$

$$\bar{K}_n(z) = \frac{2}{(n-1)!}\left(\frac{Z}{2}\right)^n\; K_n(z)$$

and $K_n(z)$ is the modified function. In the ultrarelativistic limit, that is when T $> m_ec^2/k_B = 5.92986 \times 10^9$K, the $e^\pm$ pairs are in thermal equilibrium with the photons. During that stage the energy density of the electrons can be represented in terms of the photon density as

$$\rho_{e^\pm} = \frac{7}{8}\rho_\gamma \tag{1.34}$$

The pressure of the electron gas can be written as [16]

$$\frac{p_{e^+} + p_{e^-}}{c^2} = 5.19\; T_9^4\; \left[\bar{K}_2(z) - \frac{1}{16}\bar{K}_2(2z) + ...\right]\; g\; cm^{-3} \tag{1.35}$$

In the ultrarelativistic limit

$$\frac{p_{e^+} + p_{e^-}}{c^2} = \frac{1}{3} \left(\rho_{e^+} + \rho_{e^-}\right) \tag{1.36}$$

The number density of the electrons is

$$n_{e^-} + n_{e^+} = 3.38 \times 10^{28}\, T_9^3 \left[\bar{K}_2(x) - \frac{1}{8}\bar{K}_2(2x) + ...\right] \; cm^{-3} \tag{1.37}$$

The difference in the numbers of the electrons and positrons has got a special significance. It gives the net number of electrons over their antiparticles which will equal to the total negative charge in the Universe. Since the total charge of the Universe is assumed to be zero, there will be an equal number of positive charge also. So by assuming charge conservation, the net number of the electrons per unit volume can be related to the baryon number density as [11,16]

$$n_{e^-} - n_{e^+} = N_A \rho_b \sum_i X_i \frac{Z_i}{A_i} \tag{1.38}$$

where X_i, A_i and Z_i are the mass fraction, mass number and atomic number of the i^{th} species of the nuclei and N_A is the Avogadro number.

Next important constituent of the early Universe is the neutrino. It is a massles, weakly interacting particle, with spin equal to (1/2). There are two types of neutrinos, Majorana and Dirac types. Recent experiments on double beta decay shows that the neutrinos in the early Universe are of Majorana type. There are 3 types of neutrinos according to the recent experimental evidence [23,25,26,145] they are the electron-neutrino, the muon-neutrino, and taon-neutrino. These have corresponding antiparticles also. By assuming that the neutrinos are non-degenerate, the energy density of any one species, say the the electron type can be evaluated as,

$$\begin{aligned}\rho_{\nu_e} + \rho_{\bar{\nu}_e} &= \frac{7}{8}\, a\, c^{-2}\, T_\nu^4 \\ &= 7.36593\, T_{\nu_9}^4\, g\, cm^{-3}\end{aligned} \qquad (1.39)$$

where $T_{\nu 9}$ is the temperature of the neutrinos in units of 10^9K. The temperature of the neutrinos are different from that of photons after the pair annihilation of the $e^{\pm}$ pairs. This is because of the complete transference of the entropy of $e^{\pm}$ pairs to the photons due to their pair annihilation. Since the neutrinos will decouple form the thermodynamic equilibrium long before the $e^{\pm}$ annihilation, they will not able to share the entropy due to the pair annihilation. The neutrinos decouple from the thermodynamic equilibrium at a temperature T = 2.1 $\times 10^{10}$K [11] when the universal expansion rate overtakes the interaction rate of the neutrinos with the rest of the Universe. Even after the decoupling the neutrino temperature will evolve as T $\propto$ R^{-1}. After pair annihilation of electrons, neutrino temperature will be little less than that of the photons. The exact decrease of the neutrino temperature relative to photon temperature can be calculated from the law of the constancy of the entropy density. At high temperature almost all particles are relativistic in character. During such a stage the total energy density of the Universe can be expressed in terms of the photon density as,

$$\rho_{tot} = \rho_\gamma\, g_{eff} \qquad (1.40)$$

where g_{eff} is the effective spin multiplicity factor given as [11]

$$2\, g_{eff} = \sum g_b \left(\frac{T_b}{T}\right)^4 + \frac{7}{8}\, g_{eff} \left(\frac{T_f}{T}\right)^4 \qquad (1.41)$$

where the subscript b(f) denote bosons (fermions). Before electron-positron annihilation $T_b = T_f = T$, hence g_{eff} becomes

$$\begin{aligned} g_{eff} &= \frac{1}{2}\left(g_\gamma + \frac{7}{8}\left(g_{e^-} + g_{e^+} + g_{\nu_e} + g_{\bar{\nu}_e} + g_{\nu_\mu} + g_{\bar{\nu}_\mu} + g_{\nu_\tau} + g_{\bar{\nu}_\tau}\right)\right) \qquad (1.42) \\ &= 1 + \frac{35}{8} = \frac{43}{8} \end{aligned}$$

In this sum we include the photons, electrons, the three types of neutrinos and their antiparticles. Since the expansion of the Universe is adiabatic the entropy per comoving volume remains a constant. So if any species annihilate in to photons their entropy will be transferred to photons and hence the photons temperature will be increased. This increase in temperature is shared by the other particles also if there is thermodynamic equilibrium. Since the neutrinos are decoupled well before the pair annihilation of neutrinos electrons, the constancy of entropy implies that,

$$\left(g_\gamma + \frac{7}{8}(g_{e^-} + g_{e^+})\right)\, T_i^3 = g_\gamma\, T_f^3 \qquad (1.43)$$

where T_i is the temperature before the annihilation of the $e^\pm$ pairs and T_f is temperature after the annihilation process. After putting the required values in the above equation we get [2,3,15,16]

$$\frac{T_f}{T_i} = \left(\frac{11}{4}\right)^{1/3} = 1.401 \qquad (1.44)$$

The initial temperature will be identical to the temperature of the decoupled neutrinos, then the above equation shows that after the pair annihilation the neutrino temperature is related to the photon temperature as

$$T_\gamma = 1.401\, T_\nu \qquad (1.45)$$

which implies a 40% increase in the photon temperature over the neutrino temperature. If the present day photon temperature is assumed to be 2.75 K, then the decoupled primordial neutrinos have a present temperature 1.96 K.

The pressure of the neutrinos can be written as

$$P_{\nu} = \frac{1}{3}\, \rho_{\nu}\, c^2 \tag{1.46}$$

To get the total energy density and pressure of all the neutrinos it is enough to multiply the above equations of energy density and pressure with a factor 3.

Baryon density ρ_b can be calculated by the relation, $\rho_b = hT_9^3$. Because of the nucleosynthesis the baryon energy density will be modified. An correct equation for baryon density by taking account of the nucleosynthesis process is given by Wagoner [16] as,

$$\rho_b = hT_9^3 \left[1 + \sum \left(\frac{\Delta M_i}{M_u} + \zeta T_9\right) Y_i\right] \tag{1.47}$$

where $M_u = 1.66043 \times 10^{-24}$ g is the atomic mass unit, $\zeta = 1.388 \times 10^{-4}$,$\Delta M_i$ is the mass excess of the species i produced during the primordial nucleosynthesis. and Y_i is the abundance of the i^{th} species. The last term in the above equation represents the kinetic energy contribution. The Baryon pressure is then written as

$$P_b = N_A\, \rho_b\, k_B\, T \sum Y_i \tag{1.48}$$

By knowing the form of the thermodynamic quantities, we can study the nature of the variation of the main variables in the theory, which will be useful later for calculating the primordial abundance of the light elements. They are T_9, h and ϕ_e, the chemical potential of the electron. The variation of T_9 can be calculated as,

$$\frac{dT_9}{dt} = \frac{dr/dt}{dr/dT_9} \tag{1.49}$$

where r= $\ln R^3$, and dr/dt = 3H. The quantity dr/dT_9 can be evaluated from the principle of conservation of energy. The law of conservation of energy is as given by the equation (1.7). This equation can be modified by taking account of the energy introduced due to the nucleeosynthesis process as [16]

$$\frac{d}{dt}\left(\rho R^3\right) + \frac{p}{c^2}\frac{dR^3}{dt} + R^3\frac{d\rho}{dt}\bigg|_{T=const.} = 0 \tag{1.50}$$

where ρ is the total energy density after the neutrinos were decoupled, that is

$$\rho = \rho_e + \rho_\gamma + \rho_b \tag{1.51}$$

and p is the corresponding pressure,

$$p = p_e + p_\gamma + p_b \tag{1.52}$$

Now dr/dT_9 can be obtained as [16]

$$\frac{dr}{dT_9} = -\frac{dr/dT_9 + d\rho_e/dT_9 + d\rho_b/dT_9}{\rho_\gamma + p_\gamma/c^2 + \rho_e + p_d/c^2 + p_b/c^2 + \left(\frac{1}{dr/dt}\right)\left(d\rho_b/dt|_{T_9} + d\rho_e/dt|_{T_9}\right)}. \tag{1.53}$$

The time evolution of the h parameter is given by

$$\frac{dh}{dt} = -3h\left[\frac{1}{R}\frac{dR}{dt} + \frac{1}{T_9}\frac{dT_9}{dt}\right]. \tag{1.54}$$

The time evolution of the chemical potential can be obtained as

$$\frac{d\phi_e}{dt} = \frac{\partial\phi_e}{\partial T_9}\frac{dT_9}{dt} + \frac{\partial\phi_e}{\partial r}\frac{dr}{dt} + \frac{\partial\phi_e}{\partial S}\frac{dS}{dt} \tag{1.55}$$

where $S = \sum_i Z_i\, Y_i$.

1.2 Primordial nucleosynthesis

It was Alpher, Bethe and Gamow [13] who were the first to consider the primordial production of the light elements in the Friedmann Universe. Two years later Fermi and Turkevich [14] did a similar work. In these earlier works it was assumed that essentially all the baryons are neutrons and baryon density of the form

$$\rho_b = hT_9^3 \tag{1.56}$$

where the parameter h is fixed by assuming the value of the baryon-to-photon number density ratio η, and T_9 is the temperature in units of 10^9K. Later Hayashi [17] proposed that initial baryon density would consists of mainly neutrons and protons, the nucleons, which are in thermal equilibrium with each other at high temperatures through the weak interaction. A set back to the earlier approaches was that due to the well known difficulty at mass numbers 5 and 8. Hayashi and Nishida [18] made an attempt to overcome this by proposing triple alpha reaction, $3^4\mathrm{He} \rightarrow \mathrm{C}^{12}$. But their solution had the drawback that, the reaction needs a high baryon density, which will in turn over produce other light elements and also predict higher abundance for elements heavier than carbon which was against the experimental evidence. Due to these difficulties the hope on primordial synthesis of elements was doomed for a short period in the fifties. The approach was resurrected with work of Fowler and Hoyle [19]. They showed that the observed helium abundance in the Universe can not be accounted for by the stellar nucleosynthesis alone and implies the necessary revival of the possibility of cosmological origin of light elements. By their primordial abundance theory Fowler and Hoyle [19] predicted an abundance of

^{4}He about 27%, which retrict the h-parameter to about 10^{-4}. In the mean time the discovery of the microwave background radiation by Penzias and Wilson [20], gave strong support to the standard big-bang theory based on the Friedmann Universe which in turn supported the theory of primordial nucleosynthesis.

One of the main process through which the primordial nucleosynthesis proceeds is the frrezing out of the neutron-to-proton ratio. Neutrons and protons are in thermal equilibrium at high temperature, T 10^{10}K. The equilibrium was kept between them through the following weak interactions [17]

$$\begin{aligned} n + \nu_e &\longleftrightarrow p + e^- \\ n + e^+ &\longleftrightarrow p + \bar{\nu}_e \\ n &\longleftrightarrow p + e^- + \bar{\nu}_e. \end{aligned} \tag{1.57}$$

During thermal equilibrium the number densities of neutrons and protons are slightly different due to difference in their masses. The relative number density is given by the relation [16]

$$\frac{n}{p} = e^{-\Delta mc^2/k_B T} \tag{1.58}$$

where $\Delta m = 1.293$ Mev/c^2, the rest mass difference between neutron and proton. During thermal equilibrium the total rates of the reactions which convert neutrons in to protons is almost equal to total rate of the reactions which convert protons into neutrons. The rate of the conversion of proton into neutron is about $\lambda(p \rightarrow n) = 4 \times 10^{-6}\ t_9^5\ sec^{-1}$. The reaction rate $\lambda(n \rightarrow p)$ and $\lambda(p \rightarrow n)$ are written as the sum of the rates of the individual reactions as,

$$\lambda(n \rightarrow p) = \lambda(n \rightarrow p + e^- + \nu_e) + \lambda(n + e^+ \rightarrow p + \bar{\nu}_e + \lambda(n + \nu_e \rightarrow p + e^-) \quad (1.59)$$

and

$$\lambda(p \rightarrow n) = \lambda(p + e^- + \bar{\nu}_e \rightarrow n) + \lambda(p + \bar{\nu}_e \rightarrow n + e^+) + \lambda(p + e^- \rightarrow n + \nu_e) \quad (1.60)$$

and the individual reaction rates can be calculated as [16,27]

$$\lambda(n \rightarrow e^- + \bar{\nu} + p) = (\tau\,\lambda_0)^{-1} \int_1^{\infty} \frac{\epsilon(\epsilon - q)^2\,\sqrt{\epsilon^2 - 1}}{[1 + exp(-\epsilon z)]\ \{1 + exp[(\epsilon - q)z_\nu]\}}\,d\epsilon \quad (1.61)$$

$$\lambda(n + e^+ \rightarrow \bar{\nu} + p) = (\tau\,\lambda_0)^{-1} \int_1^{\infty} \frac{\epsilon(\epsilon - q)^2\,\sqrt{\epsilon^2 - 1}}{[1 + exp(\epsilon z)]\ \{1 + exp[(-\epsilon + q)z_\nu]\}}\,d\epsilon \quad (1.62)$$

$$\lambda(n + \nu \rightarrow e^- + p) = (\tau\,\lambda_0)^{-1} \int_1^{\infty} \frac{\epsilon(\epsilon - q)^2\,\sqrt{\epsilon^2 - 1}}{[1 + exp(-\epsilon z)]\ \{1 + exp[(\epsilon - q)z_\nu]\}}\,d\epsilon \quad (1.63)$$

$$\lambda(p + e^- \rightarrow n + \bar{\nu}) = (\tau\,\lambda_0)^{-1} \int_1^{\infty} \frac{\epsilon(\epsilon - q)^2\,\sqrt{\epsilon^2 - 1}}{[1 + exp(\epsilon z)]\ \{1 + exp[(q - \epsilon)z_\nu]\}}\,d\epsilon \quad (1.64)$$

$$\lambda(p + \nu \rightarrow e^+ + n) = (\tau\,\lambda_0)^{-1} \int_1^{\infty} \frac{\epsilon(\epsilon - q)^2\,\sqrt{\epsilon^2 - 1}}{[1 + exp(-\epsilon z)]\ \{1 + exp[(\epsilon + q)z_\nu]\}}\,d\epsilon \quad (1.65)$$

$$\lambda(p + e^- + \nu \rightarrow n) = (\tau\,\lambda_0)^{-1} \int_1^{\infty} \frac{\epsilon(\epsilon - q)^2\,\sqrt{\epsilon^2 - 1}}{[1 + exp(\epsilon z)]\ \{1 + exp[(q - \epsilon)z_\nu]\}}\,d\epsilon \quad (1.66)$$

In the original evaluation due to Wagoner [15,16] these integarls are approximated in terms of the modified Bessel functions. In the above equations $q = \Delta m/m_e$, $z_\nu = m_e c^2/k_B T$, τ is the mean life time of the neutron under laboratory conditions and λ_0 is defined as

$$\lambda_0 = \int_1^{q} (q - \epsilon)^2\,(\epsilon^2 - 1)^{1/2}\,d\epsilon \quad (1.67)$$

which has a value 1.636 [16]. In Wogoner's original work these integral is also approximated in terms of the modified Bessel functions.

The abundance of ^{4}He is very much sensitive to the freezing out value n/p ratio compared to the other elements. The freezing value of n/p is in turn depend on the freezing out temperature of the weak interaction. Higher the freeze out temperature, higher will be the value of n/p ratio. Practically all the available neutrons will be processed in to ^{4}He.

The primordial nucleosynthesis process is begin with the formation of deuterium form neutron and proton through the reaction $n+p \rightarrow D+\gamma$. Due to the low binding (2.225 Mev) energy and large photo dissociation cross section, the deuterium (D) is photo dissociated as soon as it is formed because of the presence of large number of high energy photons at high temperature. This prevents the formation of the next heavy elements like tritium and helium. This is the well known deuterium bottleneck. As the Universe expands the temperature decrease hence more deuterium will be formed due to unavailability of the high energy photons. Hence deuterium starts building up at about $T_9 \sim 1$. This will set the platform for formation of next heavy elements. ^{3}H is mainly formed through the reactions D(n, γ)^{3}H and D(D,p)^{3}H. ^{3}He is formed mainly through the reactions ^{3}He(n,p)^{3}H and ^{3}He $\rightarrow$ ^{3}H + e^- + $\bar{\nu}_e$. The main reactions through which the element helium-4 is formed are ^{3}He(n, γ)^{4}He, ^{3}He(D,p)^{4}He and ^{3}He(^{3}He, 2p)^{4}He. As the Universe cools down to the temperature to the temperature of $T_9 \approx 0.1$, practically all the ^{3}H and ^{3}He are converted in to ^{4}He. Hence very soon the abundance of ^{4}He will exceeds that of all other elements except that of hydrogen. According to the periodic table of elements there are gaps at mass numbers 5 and 8. This corresponds to a pair of bottlenecks for the production of the elements heavier than ^{4}He like ^{7}Li, ^{7}Be, etc. However a trace of these elements were formed mainly through the reactions ^{4}He(^{3}H, γ)^{7}Li and ^{4}He(^{3}He, γ)^{7}Be. When temperature drops below $T < 4 \times 10^8$ K, the increase in the coulomb barrier will effectively stops the nucleosynthesis process.

The main part of the nuclear abundance calculation is to time evolve the abundances of various light elements and thus to predict the final abundance of all the light elements. One part of the calculation lies in the time evolution of the parameters T_9, h and ϕ_e, which can be done according to the prescriptions given above. Another part is the calculation of the reaction rates of the various reactions. The techniques of the calculation of the reaction rates will be discussed in chapter 2.

It was Robert Wagoner [16] who first developed the extensive numerical code for the calculation of the abundance of the light elements in the SBBN model. Later many workers modified the code [21,22,27,28,29] for various purposes. In the original code due to Wagoner more than 140 reactions were included to calculate the abundances. Both forward and backward rates of almost all the reactions are included in the code. The abundance of any species, i is given in terms of the mass fraction

$$X_i = \frac{A_i n_i}{\rho_b N_A} \tag{1.68}$$

where A_i is the mass number and and n_i is the number density of the i^{th} species. The overall abundance of the species is depends on the number of reactions in which it is participates, the reaction rates and the type of reactions.

If the i^{th} species is destroyed or created due to the interaction with a photon or a lepton and forming or destroying a species j, then the corresponding contribution to the rate of the abundance of i is $\pm\sum_j(X_j/A_j)\lambda_\omega(j)$. The + sign is chosen if process is a constructive and - sign if the process is destructive one. When the i^{th} species is destroyed or created due to the interaction between the species j and k, then contribution of such reactions to the abundance rate of i is $\pm\sum_{j\geq k}(X_j/A_j)(X_k/A_k)[jk]$.Here [jk] is the reaction rate given by [16]

$$[jk] = \rho_b N_A \langle\sigma v\rangle_{jk} \tag{1.69}$$

where

$$\langle\sigma v\rangle = \int_0^\infty f(v,T)\sigma(v)v dv$$

. and $f(v,T)$ is the Maxwell-Boltzman distribution function for the velocities of the reactants j and k, $\sigma(v)$ is the cross section of the reaction. Similarly if the i^{th} species is destroyed or created due to interaction of three species j, k and l, then the corresponding contribution to the rate of abundance of the species i is $\pm\sum_{j\geq k\geq l}(X_j/A_j)(X_k/A_k)(X_l/A_l)[jkl]$, where [jkl] is the three body reaction rate given by the relation [15]

$$[jkl] = \rho_b^2 N_A^2 \langle\sigma v\rangle_{jkl}\,. \tag{1.70}$$

Now the total rate of growth of any species i can be calculated as

$$\frac{1}{A_i}\frac{dX_i}{dt} = \pm\sum\frac{X_i}{A_j}\lambda_w(j) \pm \sum\frac{X_j}{A_j}\frac{X_k}{A_k}[jk] \pm \sum\frac{X_j}{A_j}\frac{X_k}{A_k}\frac{X_l}{A_l}[jkl] \qquad (1.71)$$

This equation is to be solved to get the final abundance subjected to the initial conditions.

In Wagoner's code the nuclear species were first numbered as i = 1,2,3,.....etc, their mass numbers (A_i), charge number (Z_i) and Q-value are listed. The reactions are then classified according to their type as photo nuclear, weak, two body type with proton, neutron or alpha as partners and three body type etc.

The rate equation (1.71) to be solved is a highly non-linear one, which can be written as matrix equation [16,11]

$$\dot{x}_i = a_{ij}x_j + a_{ijk}x_j x_k + a_{ijkl}x_j x_k x_l \qquad (1.72)$$

For practical calculation the above non-linear matrix equation is converted into a linear one by a proper choice of the time step Δt. The time step should be small as possible, but at the same time should be large enough to ensure the chemical equilibrium between the species. For chemical equilibrium $\dot{x}_i$=0, hence

$$\begin{aligned} 0 &= a_{ij}x_j + a_{ijk}x_j x_k^{(0)} + a_{ijkl}x_j x_k^{(0)} x_l^{(0)} \qquad (1.73) \\ &= \left[a_i j + a_i jk x_k^{(0)} + a_i jkl x_k^{(0)} x_l^{(0)}\right] x_j = b_i j x_j \end{aligned}$$

Here $x_k^{(0)}$ is the abundance obtained during the previous time step. A new set of x_i's are obtained by solving this linearised equation. The calculation is continued until the abundance values are saturated.

The abundance equation is solved subjected to the initial conditions. In Wagoners original code initial conditions are specified for a temperature T_9=60. The important quantities to which the initial values to be specified number densities, energy densities, h parameter etc. One of most important quantity whose initial values is to be specified is the initial abundance of the species. Wagoner used a relation for this in his original code as [15,16]

$$X(i) = 10^{-10} X(j) X(k) \rho_b T_9^{-3/2} e^{Q/k_B T} \tag{1.74}$$

According to this equation the initial abundance decreases with temperature. In later modified codes, however, many authors used a constant value for the initial abundance all for elements.

During the calculations of Wagoner *et. al.*, the observational abundance of ^{4}He was around 0.270 by mass fraction. Their calculation reproduced this value, and this gave a strong to support the idea of primordial nucleosynthesis. The corresponding value for the h parameter obtained by them was about 10^{-4}. But later determinations have altered the values of the primordial abundance of the light elements, in particular that of ^{4}He was somewhat reduced. These changes call for certain modifications of the theory. However there is no universal agreement on this point. Even in the early eighties it was argued that the original theory is compatible with the then observational data on abundances, without much modifications. But recent determinations on the primordial abundance shows that the ^{4}He abundance is still less [37] around 0.220. These latest results on abundances ofthe light elements necesitate some essential modifications in the theoryas proposed by many

[30,31,38,163]. Our aim in this thesis is to check the present status of such inconsistenciesin the SBBN model and suggest suitable modifications to SBBN accordingly. In the following we will try to infer the primordial abundance of the light elements from the latest observational data available.

1.3 Observational abundance of light elements

The main parameters in the SBBN model are number of neutrinos, neutron life time and baryon to photon number density. Once the first two parameters are obtained from direct laboratory results, the theory becomes a single parameter theory. The method of SBBN model is to match its results with the observational abundances so as to predict the value of the parameter η. For this one needs reliable observational results on the abundance of light elements. For this we have depend on the astronomical surveys. The main observational sights for inferring the primordial abundances of light elements are sun, other stars, galaxies and certain planets. There are many observational uncertainties to warrant a very cautious approach in setting the limits of the primordial abundance. The abundance we observe today are contaminated with the nuclear processes in the galaxies and stars. The metallicity of the stars can be taken as a measure of the contamination due to the stellar evolution. Metallicity means the presence of the heavy elements. Since heavy elements production was negligible during the SBBN period it is better to choose metal poor stars for inferring the primordial abundance of the elements. However we do not see a zero metallicity star to determine the correction due to finite age of the stars or due to galactic chemical evolution. The general method for inferring the value of the primordial abundance is to extrapolate the element-metallicity data to zero metallicity. In the following we will consider the observational constraints on each of the light elements separately.

1.3.1 Helium-4

Apart from hydrogen, helium is the most abundant element in the Universe which we are observing. The main sources of observations are sun, orion, galactic HII regions and some other high metallicity sources. In the past fifteen years there has been tremendous increase in the observational data on ^{4}He abundance. In order to infer the value of primordial ^{4}He, we have to subtract all the contributions made by the various astrophysical processes. During their chemical evolution stars will synthesis ^{4}He also along with the heavy elements. Let ΔY be the mass fraction of the astrophysical production of ^{4}He in a source. Since there was practically no production of heavy elements during the primordial synthesis, the whole content of the heavy elements present in the site is entirely due to the astrophysical production. Let ΔZ be the mass fraction of the heavy elements. If one succeed in finding a reliable relation between ΔY and ΔZ, that can be used to infer the primordial abundance of ^{4}He. But theories on the chemical evolution of stars and galaxies are manifold. It is found that the helium-to-metallicity ratio is a complicated function of stellar mass and composition. The relation between ΔY and ΔZ is not monotonic [149,150]. In spite of this situation what is usually done is to do a linear extrapolation to zero metallicity. Traditionally, ^{4}He is inferred by linear regression of ^{4}He with either oxygen or nitrogen metallicity [50,51]. However there is no compelling reason to believe that ^{4}He abundance always increase linearly with oxygen or nitrogen [37]. Carbon is also used as standard. By accepting a linear relationship between ΔY and ΔZ as [11]

$$\frac{\Delta Y}{\Delta Z} = \alpha \tag{1.75}$$

If we assume that the mass function of stars is a universal function then the average value of α is seems to be lie between 4 to 6 [11]. In order to get the primordial abundance of ^{4}He we have to subtract the the astrophysical contribution from the observed values as

$$Y_p = Y_{obs} - \Delta Y = Y_{obs} - \alpha \Delta Z \tag{1.76}$$

The reliability will be strong if the observational site is a very old one or one with less metallicity. Caution should be taken to make sure that star which we are identifying as the source must be a massive or of mass in the intermediate range. In massive stars the core where the synthesis is taking place is convective, while the envelope is radiative. Because the envelope is not convective no mixing will take place, hence the envelope of massive stars will retain its original composition. The same is the case with intermediate massive stars, where the core is radiative and the envelope is convective. But for low mass stars the core and the envelope will overlap each other hence they are not generally selected for primordial abundance determination.

Let us briefly go through the various observationally inferred results due to various authors, with the aim of selecting the reasonable value for Y_p. A detailed list various values of ^{4}He abundance was compiled by Rana [11]. Pagel [106,107] carried out a linear regression analysis on all the available data on ^{4}He-metallicity including that from sun, orion and some other high metallicity sources and derived a value $Y_p = 0.24 \pm 0.01$. In the same year Pagel [55,107] showed that if one restrict to the data form extragalactic H II region with metallicity less than 0.25 of solar, then the result become $Y_p = 0.225 \pm 0.005$, where he did the extrapolation with oxygen. Hasenfrats *et. al.* [103] found no such correlation between ^{4}He and O in their data from the 12 metal poor galaxies of a gasious nebulae. They give a value based on their data as $Y_p = 0.245 \pm 0.003$. On the other hand the observational results due to Peimbert & Torres-Peimbert [98], shows that there is a strong correlation between ^{4}He and O, according to them $Y_p = 0.220$. Fuller et. al [37] argued that the no correlation between ^{4}He and O in the data obtained by Kunth & Sargent is due to extremely low ΔZ value of the object. apart from O, one can use N and C as standards for the extrapolation to find the primordial value of ^{4}He. ^{12}C and ^{16}O are processed in the first generation stars along with ^{4}He [149,150]. But N is formed gradually during the second generation. As a result ^{4}He abundance with respect to N will show a rapid increase initially, then slow down to lower rate due to the increase in N. So the ^{4}He verses N curve for low value of N can be used to predict the primordial value of ^{4}He at a reasonable level. The curve of ^{4}He with O, has a strong dependence on the initial mass function (IMF) [149,150], which is a very poorly known function, and also very much model dependend. So it is considerably more difficult to predict Y_p using the curve between ^{4}He and O. Fuller, Boyd and

Kalen [37] adopt the ^{4}He verses N curve method to predict the value of Y_p. They have used the data of Pagel [107]. They found that $Y_p = 0.233 \pm 0.009$ for all the 41 data points, $Y_p = 0.221 \pm 0.007$ for first 22 points of comparably low metallicity and $Y_p = 0.220 \pm 0.007$ for first 14 points of still low metallicity. They finally concluded by accepting $Y_p = 0.220$ as the upper limit for the primordial abundance of ^{4}He. Later Melnick et al [36] did an extensive analysis and proved that the primordial abundance of ^{4}He is still lower around 0.216 ± 0.006. It may be very difficult to judge between these to say what is the actual value of Y_p. In Table 1.1 we summarise some of these observational results. Our feeling is that we should rely on the value which is inferred from low metallicity objects. With that in mind we can very well set the primordial abundance of Helium between limits given below as

$$Y_p = 0.215 \quad to \quad 0.225. \tag{1.77}$$

by mass fraction. But some other authors [29,58,66] still consider values around 0.235 as primordial.

1.3.2 Deuterium and Helium-3

Here we will consider the observational limits on deuterium (D) and helium-3(^{3}He) together since most of the D present in early Universe will converted in to ^{3}He, through the astrophysical process. So their combined abundance should not change much with time. However first we will consider them separately.

Table 1.1: Observations on ^{4}He abundane

year	Authors	Y_p
1976	Peimbert and Torres-Peimbert	0.228
1979	Carney	0.19 ± 0.04
1980	French	0.216
1980	Talent	0.216
1980	Rayo et al	0.216
1983	Kunth and Sargent	0.245 ±0.003
1983	Peimbert	0.218
1983	Buzzoni et al	0.23 ± 0.02
1986	Pagel	0.236 ±0.005
1988	Pagel	0.230 ± 0.005
1989	Pagel	0.229 ± 0.004
1991	Fuller et al	0.220 ± 0.007
1992	Melnick et al	0.216 ± 0.006

The main observational sights of D are Solar system, UV absorption line studies in the local ISM, studies of the deuterated molecules (DCO, DHO) in the ISM. Because of its fragile nature D is destroyed (at temperatures greater than about 0.5 $\times 10^6$ K) during the stellar evolution. There is practically no astrophysical production of deuterium reported conclusively. Certain proposals are there for the astrophysical production D [155,156], but none are accepted widely [132]. So the present day abundance provide a lower limit to the abundance of D and in order to obtain the primordial value one should correct for the astrophysical destruction. One of important nature of D is its strong dependence on the baryon-to-photon ratio η. Its abundance can related directly to the η value. Detection of the exact lower limit or upper limit to D abundance is very difficult because of the absence of a well defined chemical evolution theory. In 1992 Rana and Basu [32] proposed a chemical evolution model, according to which the D abundance can be a factor 2 higher than

Table 1.2: Observations on D abundance

year	Authors	D/H
1976	York and Rogerson	1.6×10^{-5}
1978	Sarma and Mohanty	$< 6 \times 10^{-5}$
1979	Laurent et al	0.7 - 0.4×10^{-5}
1980	Ferlet et al	$< 1 \times 10^{-6}$
1983	York	0.6 - 1.0×10^{-5}
1984	Vidal-Madjar	$2.0 \pm 1.0 \times 10^{-5}$
1989	Smith et al	2.6 - 8.4×10^{-5}

the presently observed abundance. Certain other authors says that its abundance can be anywhere between 1 and 50 [49,50,51,155,158]. We will summarise some of the results about the observationally inferred values are listed in table 1.2

For ^3He there are only few determinations are there. The main observational sites are solar system, $^3He^+$ lines in the galactic H II region etc. The abundance of ^3He compared to ^4He in the oldest meteorites and carbonaceous chondrites is found to be $^3He/H = 1.4 \pm 0.4 \times 10^{-4}$ [113] similar to the case with deuterium the abundance of ^3He also difficult to infer because of the stellar processing, because low mass stars tend to produce ^3He and high mass stars tend to destroy ^3He. Results of these determinations are tabulated in table 1.3

The most reliable results on the primordial abundance of these elements is the combined abundance these elements in number fraction. The combined abundance

Table 1.3: Observations on ^3He abundance

year	Authors	($^3He/^4He$)
1970	Jeffrey and Anders	$1.43 \pm 0.4 \times 10^{-4}$
1977	Frick and Moniot	1.5 *pm*1.0×10^{-4}
1978	Eberhardt	$1.46 \pm 0.073 \times 10^{-4}$
1979	Rood et al	$< 5 \times 10^{-5}$ $^3He/H$

according to Pagel [54], Smith [28] and Walker [22] is,

$$\left(\frac{D+^3 He}{H}\right)_p < 5 \times 10^{-5}$$

by number fraction, where the subscript p denotes that the abundance shown is primordial. The abundance from the pre-solar nebulae is taken as [11]

$$\left(\frac{D+^3 He}{H}\right) = (3.6 \pm 0.60) \times 10^{-5}$$

by number fraction, Olive at al [29] have argued that the combined abundance should be less than 10^{-4}. However we will consider a consensus value.

$$\frac{D+^3 He}{H} = 7 \times 10^{-5} \;\; to \;\; 1.3 \times 10^{-4} \tag{1.78}$$

by number fraction.

1.3.3 Primordial abundance Lithium-7

The determination of the primordial abundance ^7Li is one of the most controversial part in inferring the primordial abundance of all the elements. There are two main sources for the determination of the ^{7}Li abundance, the Pop I stars and the Pop II stars. The observational limits from the Pop I star is that $^7\mathrm{Li/H} \approx 10^{-9}$ [28] and that from the Pop II stars is that $^7\mathrm{Li/H} \approx 10^{-10}$ [118]. The issue is which one is the primordial abundance. The consensus has been that the lower Pop II abundance represents the primordial abundance of ^{7}Li. The main reason for this is that the Pop II stars are older than the Pop I. The latest determination is due to Deliyannis et. al. [159], which tallies with the earlier determinations on the Pop II stars. For our purpose we will consider this latest value as the ^{7}Li abundance

$$\frac{^7Li}{H} = 1.2^{+0.30}_{-0.25} \times 10^{-10} \tag{1.79}$$

by number fraction. In this thesis we concentrate on the abundance of D, ^{3}He, ^{4}He and ^{7}Li only. So no datailed discussion will be given about the other possible elements formed during the primordial synthesis, for example lithium-6, Boron, Beryllium. However the beryllium-7 formed during the primordial synthesis will converted it ^{7}Li. So the abundance value of ^{7}Li as given above will actually represents the combined abundance of lithium-7 and beryllium-7.

Chapter 2

The effect of the enhanced ^{8}Li (α, n) ^{11} B reaction rate on primordial abundance of ^{7}Li

One of the essential inputs of the primordial nucleosynthesis calculation is the reaction rates of the various relevant reactions. The final abundance is strongly dependent on the accuracy of the reaction rates. One has to obtain these reaction rates from the laboratory measurements of the reaction cross-sections. Such calculations of reaction rates from the cross-sections are discussed in Wagoner *et. al.* [15], Fowler *et. al.* [19] and Mathew *et. al.*[35]. Usually the laboratory data are available at comparably large center of mass energy than that prevailed in the early Universe during the nucleosynthesis period. So one has to extrapolate the laboratory data available at high energies, to the appropriate lower energy region. This extrapolation is usually done through the so-called astrophysical S-factor formalism [161,16].

2.1 S-factor formalism for reaction rate

For a two body reaction where an i^{th} species is destroyed (or created) due to the interaction between the species j and k, the reaction rate per unit volume can be detrmined by equation (1.69). Since the Universe was in thermodynamic equilibrium with the components of the reaction, there exist a spectrum of relative velocities of the various particles. The temperature range during which the nucleosynthesis is taking place is $T_9 \sim 1$ to 0.1. In this temperature range all the matter particles are non-relativistic and non-degenerate. Hence the velocity distribution will be Maxwellian [28], which can be written as

$$f(v) = 4\pi^2 v^2 \left(\frac{\mu m_n}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{\mu m_n v^2}{2k_B T}\right) \tag{2.1}$$

which satisfies $\int f(v)dv = 1$, where $\mu = (1/m_1 + 1/m_2)^{-1}$ the reduced mass of the colliding system expressed in atomic mass unit ($1\mathrm{amu} = m_n = 1.6605 \times 10^{-27}\mathrm{Kg}$). Now the equation of $\langle \sigma v \rangle$ reduces to

$$\langle \sigma v \rangle = 4\pi \left(\frac{\mu m_n}{2\pi\, k_B T}\right)^{3/2} \int_0^\infty \sigma v^3 \left(-\frac{\mu m_n v^2}{2k_B T}\right) dv \tag{2.2}$$

which is the integral required for the calculation of the reaction rate. In principle the term $\langle \sigma v \rangle$ can include the contributions from a resonant part other than the prominent [16] non-resonant part. But the magnitude of the contributions from the resonant energy region depend up on the energy of the reactants. In the following however we will mainly concentrate on the most important part, the non-resonant part.

Apart from the knowledge about the velocity distribution, the quantity to be known for the calculation of the reaction rate is the cross-section of the reaction. While calculating the cross-section we have to take into account the Coulomb barrier penetration also because most of reactants posses charge during the primordial nucleosynthesis. Nuclear reactions are take place because the reacting nuclei are able to penetrate the coulomb repulsive barrier. The Coulomb energy between any species of atomic numbers Z_1 and Z_2 separated by a distance of R fermi is

$$\begin{aligned} E_c &= \frac{1}{4\pi\epsilon_0}\,\frac{Z_1\,Z_2\,e^2}{10^{-15}R(fermi)} \\ &= \frac{1.44\,Z_1\,Z_2}{R(fermi)}Mev. \end{aligned} \tag{2.3}$$

where ϵ_0 is the permitivity of vacuum. Classically the reactions will take place when the kinetic energy is greater than the coulomb repulsive potential. The kinetic energy of a reactant nuclei is determined by the Maxwell-Boltzmann distribution of velocities corresponding to the thermal energy,

$$\begin{aligned} k_B\,T &= 1.3807\times10^{-16}\,T \\ &= 8.62\times10^{-8}\,T\,Kev. \end{aligned} \tag{2.4}$$

From the above two relation it is clear that the Coulomb repulsive energy is many orders of magnitude greater than the average kinetic energy. The particles with the highest energy in the Maxwell-Boltzmann distribution, have a chance to overcome the coulomb barrier. Gamow showed that two particles of charges Z_1 and Z_2 moving with relative velocity v have a quantum mechanical probability for penetration approximately given by

$$\begin{aligned} P_r &\simeq \exp\left(-\frac{Z_1\, Z_2 e^2}{2\epsilon_0 \hbar v}\right) \\ &= \exp\left(31.29 Z_1 Z_2 \left[\frac{\mu}{E_{kev}}^{1/2}\right]\right) \end{aligned} \tag{2.5}$$

where E_{kev} is the center of mass energy in Kev. The cross-section will also be proportional to the same factor. Quantum mechanically the cross-section is proportional to $\pi\bar{\lambda}P_r$, where $\bar{\lambda}$ is the de Broglie wavelength of either nucleus in the center of mass frame. But $\pi\bar{\lambda} \;\propto (1/E)$, where E is the center of mass energy. So one can write the cross-section $\sigma(E)$ as promotional to

$$\sigma(E) \;\propto\; \frac{1}{E} \exp\left(-\frac{Z_1 Z_2 e^2}{2\epsilon \hbar v}\right) \tag{2.6}$$

or equivalently

$$\sigma(E) \;=\; \frac{S(E)}{E} \exp\left(\frac{Z_1 Z_2\, e^2}{2\epsilon \hbar v}\right) \tag{2.7}$$

where the factor $S(E)$ is by definition, the astrophysical S-factor, is a slowly varying function in general. The advantage of writing reaction rate in terms of S-factor is that the other two terms in the above equation are strongly varying function of energy, there factorisation leaving the situation in favour of the slowly varying function of energy, the S-factor. Now the reaction rate per particle can be written in terms of energy as

$$\langle \sigma v \rangle \;=\; \left(\frac{8}{\pi \mu m_n}\right)^{1/2} \frac{1}{(k_B T)^{3/2}} \int_0^\infty S(E)\, \exp\left(-\frac{E}{k_B T} - \frac{b}{\sqrt{E}}\right) dE \tag{2.8}$$

where

$$b \;=\; \sqrt{2\mu m_n}\, \frac{2\pi e^2 Z_1 Z_2}{4\epsilon \hbar} = 31.290 Z_1 Z_2 \mu^{1/2} \;\; keV. \tag{2.9}$$

The behaviour of the integrand is mainly determined by the exponential factor. Because of the opposite behaviour of the two terms in the exponential, they give rise to what is known as the Gamow peak of the reaction rate at energy E_0

$$E_0 = \left(\frac{bk_BT}{2}\right)^{2/3} = 1.220\,(Z_1 Z_2 \mu T_6)^{1/3}\, keV, \qquad (2.10)$$

and an effective width Δ given by

$$\Delta = 4 \left(\frac{E_0 k_B T}{3}\right)^{1/2} = 0.749 \left(Z_1^2 Z_2^2 T_6^5\right)^{1/6} \qquad (2.11)$$

where T_6 is the temperature in terms 10^6 K. The value of the integral in equation (2.8) has an approximated value

$$\langle \sigma v \rangle = \left(\frac{2}{\mu m_n}\right)^{1/2} \frac{\Delta}{(k_B T)^{3/2}}\, S_{eff}(E_0) \,\exp\left(-\frac{3E_0}{k_B T}\right). \qquad (2.12)$$

In the above equation the term $S_{eff}(E_0)$ the effective value of S-factor at the peak value E_0, is found to have a form

$$S_{eff}(E_0) = S(0)\left[1 + \frac{5}{36}\frac{k_B T}{E_0} + \frac{\dot{S}(0)}{S(0)}\left(E_0 + \frac{35}{36}k_B T\right) + ...\right] \qquad (2.13)$$

where S(0) is the astrophysical S-factor at $E_0 = 0$, provided S(E) is expanded in Taylors's series, $S(E) = S(0) + E\dot{S}(0) + (E^2/2)\ddot{S}(0) + ...$ So in order to calculate the reaction rate one has to calculate the $S(E_{eff})$.

If the energy of the reaction is so high that the Gamow peak is less than the nuclear resonance energy, then the resonance contribution towards the reaction rate will be comparably larger. In such a case the Taylor expansion for the S-factor will break down. The cross-section of such resonant reactions can be follows from the

Breit-Wigner formula as [16]

$$\sigma(E) \;=\; \pi\,\bar{\lambda}\omega\,\frac{\Gamma_a.\Gamma_b}{(E-E_R)^2 \;+\; (\Gamma/2)^2} \tag{2.14}$$

where $\Gamma = \Gamma_a + \Gamma_b + \ldots\ldots$ is the sum of partial energy widths of the resonances and ω characterising the spin multiplicities of nuclei taking part in the reaction. The corresponding reaction rate per particle is given as

$$\langle\sigma v\rangle_r \simeq \left(\frac{2\pi}{\mu k_B T}\right)^{3/2} \hbar^2 \left(\omega\frac{\Gamma_a\Gamma_b}{\Gamma}\right) \exp\left(-\frac{E_R}{k_B T}\right) \tag{2.15}$$

These equation must be added to the equation (2.14) in order to get the full reaction rate. If there are more than one resonances, an expression of the above form for each equation to be added to get final reaction rate.

2.2 Reaction rate of ^{8}Li(α, n)^{11}B.

The standard big-bang model of nucleosynthesis [15,16,22] is thought to be a successful model for predicting the primordial abundances of the light elements up ^{7}Li. But recent work [124] on ^{4}He has raised questions about the agreement between theory and experimental observations as mentioned in chapter 1. Even then the SBBN fits well with the observational results to some extent. The ultimate test of the model has to come from the prediction of the heavier elements like ^{11}B. Heavier elements are produced mainly through the cycle [163],

$$^{4}He\left(^{3}H,\gamma\right)^{7}Li\,(n,\gamma)^{8}\,Li\,(\alpha,n)^{11}\,B\,(n,\gamma)^{12}\,B\,(\beta)^{12}\,C\,(n,\gamma)^{14}\,C$$

The most important reaction in this series is ^{8}Li (α, n)^{11}B which determines the abundance of ^{11}B and through which other elements can subsequently form. To understand the detailed dynamics of the above series, one must know the reaction rate of each component reaction. For calculating the reaction rate we must know the cross-section of the component reactions. Since the half life of ^{8}Li is low as 840.3 ms, it is very difficult to produce the reaction, ^{8}Li (α, n) ^{11}B in the existing laboratory conditions. So what used to be done was measure the reaction rate of the inverse process and apply the principle detailed balance to infer the the rate of the forward reaction. The center of mass energy of the reaction in the laboratory was 1.5 Mev, so we have to extrapolate this data to the energy range prevailed in the early Universe during the primordial nucleosynthesis.

For the first time, Boyd *et. al.* [45] have been able to measure the direct reaction cross-section for ^{8}Li (α, n)^{11}B using radioactive beams of ^{8}Li of center of mass energy 1.5 MeV, which shows that the S-factor derived from the direct reaction is about 5-8 times larger than those obtained by Paradellis *et. al.* [67] from the study of its usual reverse reaction ^{11}B(n, α)^{8}Li. The strong depedance on energy and existence of several resonances are noted and therefore the assumption of existence of no resonance structure in the low energy region leading to the concept of S(0) factor is basically invalid. But since the big bang nucleosynthesis took place in energy range of 0.1 to 1 MeV, one has to extrapolate the data to the correct value of the astrophysical S-factor for the reaction.

We calculate the value of $S_{eff}(E_0)$ and hence $S_{eff}(T)$, by extrapolating the data of Boyd et al. The data of Boyd et al is given in Table 2.1. Since the data of Boyd *et al* for ^{8}Li(α, n)^{11}B reaction is available only up to the lowest energy of 1.5 MeV, we consider the data of Paradellis et al for the same reaction at energies less than 1.5 MeV, but we modified the data of Paradellis et al in view of the direct data of Boyd et al by multiplying it by an average factor that is derived from the comparison between the two data sets in the overlapping domain of the center of mass energy. In the low energy range where there is no data available, we took S(E) as a constant. We have evaluated the integral of the equation (2.8) for different temperatures ranging from 0.2×10^8K to 22×10^8K. A sample of our calculated data are presented in Table 2.2. With data, we have integrated the equation graphically. The value of the integral at different temperatures are given in Table 2.3.

Next we calculate the right hand side of equation (2.12) without the factor $S_{eff}(E_0)$, that is $\langle \sigma v \rangle / S_{eff}(E_0)$ at different temperatures and the results are tabulated in Table 2.4. We compare Table 2.4 and Table 2.3 at corresponding temperatures and thus find the value of S_{eff} at different temperatures. The newly found values of $S_{eff}(T)$ is plotted with temperatures as shown in figure 1. On extrapolating the curve to lower temperatures, we found that the value of $S_{eff}(0)$ is 2.0($\pm$0.05) $\times 10^4$ MeV barn [35]. The value of $S_{eff}(T)$ are then calculated for different temperatures and are given in Table 2.5

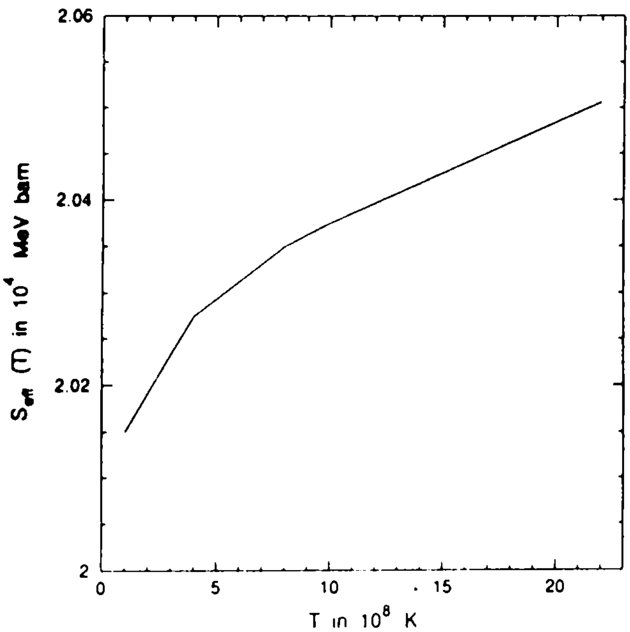

Fig 1. Plot of $\log(S_{eff}(T))$ and temperature in units of 10^8 K.

Table 2.1: Data of Boyd et al

E (MeV)	σ (mb)	S(E) (MeV barn)
1.62	381	1258
2.00	509	968
2.19	519	796
2.38	545	695
2.76	462	436
2.95	419	349
3.24	472	335
3.52	381	235
3.81	276	151
4.09	424	209
4.38	414	186
4.66	271	113
4.86	333	132
5.24	305	113
5.34	133	46
5.71	195	64
5.90	104	33
6.19	186	57
6.47	162	47

Table 2.2: Values of the integrand of eq. (5) at different values of temperature

E (MeV)	S(E) (MeV barn	at T=4 $\times 10^8$	at T=8 $\times 10^8$	at T=10 $\times 10^8$
0.2000(-01)	0.5433(05)	0.2168(-03)	0.1644(-03)	0.6527(-04)
0.2500(-01)	0.5433(05)	0.2817(00)	0.2167(00)	0.8859(-01)
0.5000(-01)	0.5433(05)	0.1252(08)	0.1035(08)	0.4889(07)
0.1000(00)	0.5433(05)	0.2000(13)	0.1913(13)	0.1208(13)
0.1500(00)	0.5433(05)	0.2698(15)	0.2983(15)	0.2518(15)
0.2000(00)	0.5433(05)	0.3746(16)	0.4788(16)	0.5401(16)
0.2500(00)	0.5433(05)	0.1792(17)	0.2647(17)	0.3992(17)
0.5971(00)	0.5433(05)	0.1099(18)	0.4444(18)	0.5020(19)
0.6119(00)	0.4688(05)	0.8915(17)	0.3763(18)	0.5020(19)
0.6545(00)	0.3228(05)	0.4989(17)	0.2383(18)	0.3755(19)
0.7158(000	0.2070(05)	0.2223(17)	0.1268(18)	0.2852(19)
0.7652(00)	0.9863(04)	0.7536(16)	0.4963(17)	0.1486(19)
0.7771(00)	0.2588(05)	0.1812(17)	0.1235(18)	0.3964(19)
0.8747(00)	0.6310(040	0.2118(16)	0.1826(17)	0.1032(19)
0.9352(00)	0.4246(04)	0.7945(15)	0.8567(16)	0.6879(18)
0.9563(00)	0.3622(04)	0.5578(15)	0.6395(16)	0.5804(18)
0.1000(01)	0.3083(04)	0.3136(15)	0.4082(16)	0.4773(18)
0.1109(01)	0.2075(04)	0.7055(14)	0.1261(16)	0.2782(18)
0.1231(01)	0.1770(04)	0.1644(14)	0.4181(15)	0.1869(18)
0.1269(01)	0.2851(04)	0.1752(14)	0.4969(15)	0.2759(18)
0.1366(01)	0.1919(04)	0.3930(13)	0.1478(15)	0.1443(18)
0.1494(01)	0.1396(04)	0.6421(12)	0.3501(14)	0.7187(17)
0.1618(01)	0.1258(04)	0.1298(12)	0.1016(14)	0.4294(17)
0.1999(01)	0.9678(03)	0.8569(09)	0.2023(12)	0.7790(16)
0.2190(01)	0.7660(03)	0.6043(08)	0.2479(11)	0.2880(16)
0.2380(01)	0.6949(03)	0.4358(07)	0.3105(10)	0.1089(16)
0.2761(01)	0.4361(03)	0.1713(05)	0.3683(08)	0.1176(15)
0.2951(01)	0.3493(03)	0.1050(04)	0.3922(07)	0.3781(14)
0.3237(01)	0.3347(03)	0.2062(02)	0.1764(06)	0.8917(13)
0.3522(01)	0.2350(03)	0.2976(00)	0.5634(04)	0.1497(13)
0.3808(01)	0.1512(03)	0.3582(-02)	0.1606(03)	0.2232(12)
0.4094(01)	0.2091(03)	0.9389(-04)	0.9643(01)	0.7024(11)
0.4379)(01)	0.1865(03)	0.1560(-05)	0.3669(00)	0.1401(11)
0.4665(01)	0.1127(03)	0.1730(-07)	0.9315(-02)	0.1865(10)
0.4855(01)	0.1318(03)	0.1397(-08)	0.1307(-02)	0.7899(09)
0.5141(01)	0.1127(03)	0/2149(-10)	0.4604(-04)	0.1458(09)
0.5427(01)	0.4645(03)	0.1575(-12)	0.7729(-06)	0.1285(08)

Table 2.3: Values of the integral at different temperatures.

T $\times 10^6$ K	Integral
0.6	2.01×10^{-35}
4	5.34×10^{-27}
8	5.35×10^{-25}
10	16.36×10^{-25}
20	25.24×10^{-24}
22	34.92×10^{-24}

Table 2.4: Values of $(\langle \sigma v \rangle / S_{eff}(E_0)$ at different temperatures.

T $\times 10^6$	$\langle \sigma v \rangle / S_{eff}(E_0)$
4	7393×10^6
8	1056×10^9
10	4064×10^9
14	2536×10^{10}
18	8626×10^{10}
20	1393×10^{11}
22	2115×10^{11}

Table 2.5: Values of $S_{eff}(T)$ at different temperatures.

T $\times 10^6$ K	$S_{eff}(T)$
1	2.015×10^4
4	2.027×10^4
8	2.035×10^4
10	2.038×10^4
20	2.049×10^4
22	2.051×10^4

The new value of S-factor will affect the old value of reaction rate of ^{8}Li(α, n)^{11}B substantially. According to Malaney and Fowler [163], the old reaction rate of ^{8}Li(α, n)^{11}B in the required energy range is given by

$$N_A \ \langle \sigma v \rangle \ = \ 8.62 \times 10^{13} \, T_{9A}^{5/6} \, T^{-3/2} \, \exp\left(- \frac{19.461}{T_{9A}^{1/3}} \right) \quad cm^2 s^{-1} mole^{-1} \qquad (2.16)$$

where N_A is the Avogadro number, T_9 is the temperature in units of 10^9K and

$$T_{9A} \ = \ \frac{T_9}{1 + T_9/15.1} \qquad (2.17)$$

On comparing equations (2.16) and (2.12) one can find that the $S_{eff}(E_0)$ used by Malaney and Fowler is 8.40 $\times 10^3$MeV barn. One can note that the difference between our value and their value of S_{eff}. By incorporating our value of $S_{eff}(E_0)$ in to the reaction rate equation instead of Malaney-Fowler's value the reaction rate become,

$$N_A \ \langle \sigma v \rangle \ = \ 2.05 \times 10^{14} \, T_{9A}^{5/6} \, T^{-3/2} \, \exp\left(- \frac{19.461}{T_{9A}^{1/3}} \right) \quad cm^2 s^{1} mole^{-1} \qquad (2.18)$$

Probably due to the above reaction rate the abundance of ^{11}B and ^{8}Li should change. Since this reaction is coming after the element ^{7}Li in the cycle this modified reaction rate will affect the abundance of ^{7}Li also. We modified the Wagoners code by incorporating our reaction rate for the above reaction and also some new reactions which are important for the synthesis of ^{7}Li, but are not incorporated in a recent work by Smith et al [28] and found that the abundance of ^{7} Li is reduced by a factor of 1.2.

Chapter 3

Neutrino degenerate big bang nucleosynthesis

The standard big-bang nucleosynthesis model is successful in predicting the microwave back ground radiation [13]. It has been claimed that its prediction about the abundances of the light elements is also perfect. But the abundances values of the various light elements inferred from the latest observational data are not in agreement with the the theory for a unique range of the baryon-to-photon ratio. The prediction of the present day baryon-to-photon ratio η is the main aim of the SBBN.

In the SBBN model η is treated as one of the parameters along with the other two, the number of light neutrinos and neutron life time. The parameters, number

Table 3.1: Number of light neutrinos, N_ν

Authors	N_ν
Adeva at al	3.29 ± 0.17
De Camp et al	3.27 ± 0.30
Akrawy et al	3.12 ± 0.42
Arnio et al	2.40 ± 0.40
Abrams et al	2.80 ± 0.60
Schramm et al	3
Shvartsman	3
Malaney et al	3

of light neutrinos and neutron life time are derived from the laboratory experiments. The number of light neutrinos has been fixed as 3 these beeing the electron neutrino, muon-neutrino and taon-neutrino. There are lot of experimental evidence for fixing the neutrino number as 3. We have summarised a set of results in Table 3.1.

There are various experimental determinations and theoretical inference on neutron life time. Since the freeze out value of the n/p ratio depends strongly on the life time of neutron [16], it has a crucial role in determining the abundance of ^{4}He, because ^{4}He abundance depends very much on the freeze out value of the n/p ratio. The abundance shows a sharp increase with the neutron life time. In the Table 3.2 is given the summary of some of the lifetime estimations of the life time of neutron. The latest measurements indicates a lower value for neutron life time, implying a lower abundance for ^{4}He. In our calculation we take the weighted mean of the values since 1986, as done by Smith et al [28], that is $\tau_n = 888.5 \pm 3.8$ sec. Once the number of neutrino and neutron life time are fixed the SBBN theory becomes a

Table 3.2: Neutron life time τ

Authors	Years	neutron life time
Sosnovskii et al	1959	1013 ± 26
Christensen et al	1972	919 ± 14
Krohn and Ringo	1975	907 ± 18
Bondarenko et al	1978	877 ± 16
Strataura et al	1978	902 ± 20
Erozolimskii et al	1979	905 ± 14
Byrne et al	1980	936 ± 17
Bopp et al	1984	889 ± 11
Byrne et	1984	914 ± 6
Kosvintsev et al	1986	903 ± 13
Last et al	1988	876 ± 22
Mampe et al	1989	887 ± 3
Olive et al	1990	889 ± 4.4
Walker et al	1991	889 ± 2.9

theory becomes a one parameter problem, the parameter being η. The expectation is that the SBBN model should predict a single value for η. But recent reports are against this expectation. It has been reported by many [30,31,38,164] that the theory predicts more than one value for η. This should not be, because a single Universe cannot have more than one value for η.

3.1 The discrepancy - earlier results

In order to review the earlier situation we mainly follow the work of Rana [30,31]. For comparing the calculated results, the following limits inferred form the observational data for the primordial abundance were used by him,

$$Y_p = 0.230 \pm 0.006$$

by mass fraction

$$\frac{D +^3 He}{H} = (3.6 \pm 0.60) \times 10^{-5}$$

by number fraction and

$$\frac{^7Li}{H} = (1.12 \pm 0.38) \times 10^{-5}$$

by number fraction. The number of neutrinos is 3 and the neutron life time is 891.6 sec. For the calculation Wagoners's code was used. The results due to Rana [11,30] shows that the range of η corresponding to Y_p and $(D +^3 He)/H$ mutually exclude each other, but the η corresponds to ^{7}Li/H is overlapping with both regions. So the discrepancy here is mainly in the case of Y_p and $(D +^3 He)/H$. Similar cases of inconsistencies in the SBBN model were reported by many as mentioned above. Even though the discrepancy is small, it is not negligible. It shows that the standard model predictions are not absolutely correct, but at the same time are not very far from truth. So SBBN model should be modified. Two alternative solutions have been proposed. One is the inhomogeneous nucleosynthesis model [37,68] and the second is the nucleosynthesis with neutrino degeneracy [30,31]. At the same time several works argued that standard model is sufficient [21,22,144]. We shall try to asses the status of the above reported discrepancy in the light of the refined values of the abundances, reaction rates and other parameters. First we propose to modify the Wagoners code to take account the latest results on reaction rates, neutron life times and include some other corrections which are described in the following sections.

3.2 Modifications of the Wagoners code

In the Wagoners code all the thermodynamic functions are evaluated approximately in terms of the modified Bessel function [15,16] as has been elaborated in chapter 1. Since the reported discrepancy regarding the non-uniqueness of η is small a precise evaluation is needed to check the absolute existential status of the inconsistency. We did an exact evaluation by changing all the approximately evaluated functions with accurate numerical calculations. The photon energy density is put in to the code as $\rho = 8.4182 T_9^4 \ gcm^{-3}$, and its number density as, $n_\gamma = 2.0282719 \times 10^{28}\ T_9^3 \ cm^{-3}$. Another important constituent is the electron, which determine the baryon density through the charge conservation law. The difference in the electrons and its antiparticle positrons can be evaluated using the exact relation,

$$n_{e^-} - n_{e^+} = \left(\frac{k_B}{c\hbar}\right)^3 \frac{10^{27}}{\pi^2} T_9^3 \, sinh\phi_e \int_1^\infty \frac{\epsilon\sqrt{\epsilon^2-1}}{cosh(z\epsilon) + cosh\phi_e} z^3 \, d\epsilon \tag{3.1}$$

This difference in number can be related to the baryon mass density by assuming charge coservation. The total number density of the electrons and positrons can be evaluated as,

$$n_{e^-} + n_{e^+} = \left(\frac{k_B}{c\hbar}\right)^3 \frac{10^{27}}{\pi^2} T_9^3 \int_1^\infty \frac{cosh\phi_e + e^{-z\epsilon}}{cosh(z\epsilon) + cosh\phi_e} \epsilon\sqrt{\epsilon^2-1} \, z^3 \, d\epsilon \tag{3.2}$$

and the total energy of the electrons and positrons is calculated as,

$$\rho_{e^-} + \rho_{e^+} = \left(\frac{k_B^4}{\pi^2 c^5 \hbar^3}\right) 10^{36} \, T_9^4 \int_1^\infty \frac{cosh\phi_e + e^{-z\epsilon}}{cosh(z\epsilon) + cosh\phi_e} \epsilon^2\sqrt{\epsilon^2-1} \, z^4 \, d\epsilon. \tag{3.3}$$

Electrons and positrons will annihilate each other during the pair annihilation period which starts at a temperature about 6×10^8K. But a small amount of electrons

Table 3.3: Evolution of electron density during pair annihilation.

T_9	$\rho_{e^\pm}/\rho_\gamma$
100	1.751
34.5	1.746
3.3	1.335
1.35	0.181
0.41	0.0002

ived and these are responsible for the electron degeneracy $\phi_e = \mu_e/k_B T$, where the chemical potential of the electrons. During the pair annihilation the mass ity of the electrons evolve with respect to that of photons as given in the Table The total pressure due to the electrons and the positrons is calculated exactly he relation,

$$(P_{e^-} + P_{e^+}) = \left(\frac{k_B^4}{c^5 \hbar^3}\right) \frac{10^{36}}{3\pi^2} T_9^4 \int_1^\infty \frac{cosh\phi_e + e^{-z\epsilon}}{cosh(z\epsilon) + cosh\phi_e} \sqrt{\epsilon^2 - 1}\, z^4\, d\epsilon. \quad (3.4)$$

calculating the evolution of the temperature with respect to time, we need derivatives of the above quantities with respect to temperature and electron nical potential. The temperature derivative of net electron number can be tly calculated using the relation,

$$\frac{\partial(n_{e^-} - n_{e^+})}{\partial T_9} = \left(\frac{k_B}{c\hbar}\right)^3 \frac{10^{27}}{\pi^2} T_9^2\, sinh\phi_e \times \int_1^\infty \frac{sinh(z\epsilon)}{(cosh(z\epsilon) + cosh\phi_e)^2} \epsilon^2 \sqrt{\epsilon^2 - 1}\, z^4\, d\epsilon. \quad (3.5)$$

nperature derivative of total electron energy density is,

$$\frac{\partial(\rho_{e^-} + \rho_{e^+})}{\partial T_9} = \left(\frac{k_B}{c^5 \hbar^3}\right) \frac{10^{36}}{\pi^2} T_9^3 \times \int_1^\infty \frac{cosh(z\epsilon)\, cosh\phi_e + 1}{(cosh(z\epsilon) + cosh\phi_e)^2} \epsilon^2 \sqrt{\epsilon^2 - 1}\, z^5\, d\epsilon \quad (3.6)$$

and the temperature derivative of the total pressure due to electrons and positrons is calculated as,

$$\frac{\partial(P_{e^-}+P_{e^+})}{\partial T_9} = \left(\frac{k_B^4}{c^5\hbar^3}\right)\frac{10^{36}}{3\pi^2}T_9^3 \times \int_1^\infty \frac{cosh(z\epsilon)cosh\phi_e+1}{(cosh(z\epsilon)+cosh\phi)}\epsilon^2\sqrt{\epsilon^2-1}\ z^5\ d\epsilon \tag{3.7}$$

The corresponding chemical potential derivatives are

$$\frac{\partial(n_{e^-}-n_{e^+})}{\partial\phi_e} = \frac{n_{e^-}-n_{e^+}}{sinh\phi_e}+\left(\frac{k_B}{c\hbar}\right)^3\frac{10^{27}}{\pi^2}T_9^3\ (cosh\phi_e-1) \times \int_1^\infty \frac{cosh(z\epsilon)-1}{(cosh(z\epsilon)+cosh\phi_e)^2}\ \epsilon\sqrt{\epsilon^2-1}\ d\epsilon \tag{3.8}$$

$$\frac{\partial(\rho_{e^-}+\rho_{e^+})}{\partial\phi_e} = \left(\frac{k_B^4}{c^5\hbar^3}\right)\frac{10^{36}}{\pi^2}T_9^4\ sinh\phi_e \times \int_1^\infty \frac{sinh(z\epsilon)}{(cosh(z\epsilon)+cosh\phi_e)^2}\epsilon^2\sqrt{\epsilon^2-1}\ z^4\ d\epsilon \tag{3.9}$$

$$\frac{1}{c^2}\frac{\partial(P_{e^-}+P_{e^+})}{\partial\phi_e} = \left(\frac{k_B}{c^5\hbar^3}\right)\frac{10^{36}}{3\pi^2}T_9^4\ sinh\phi_e \times \int_1^\infty \frac{sinh(z\epsilon)}{(cosh(z\epsilon)+cosh\phi_e)^2}\left(\epsilon^2-1\right)^{3/2}\ z^4\ d\epsilon \tag{3.10}$$

where the symbols have their usual meaning as given in the introduction. In the above equations we treat all particles as relativistic. In the case of baryons, they are treated as non-relativistic. But they show a relativistic character at temperatures above 10^{11}K. Their mass density can be evaluated using the equation (1.47). The temperature derivative of the baryon density can be written as,

$$\frac{d\rho_b}{dT_9} = hT_9^3\,\zeta \sum Y_i + hT_9^3 \left(\frac{\Delta M}{M_u} + \zeta\, T_9\right) \frac{dY_i}{dT_9} \tag{3.11}$$

The total energy density of the 3 species of the neutrinos and their antiparticles can be written as,

$$\rho_\nu + \rho_{\bar\nu} = 3 \times 7.365935 T_9^4 \tag{3.12}$$

Neutrons are decoupled from the thermal equilibrium at about a temperature 2.1 $\times 10^{10}$K. After that the temperature of the neutrinos decrease with the expansion of the Universe as, $T_\nu \propto R^{-1}$.

3.2.1 Overheating of neutrinos

Neutrinos decouple from the thermodynamic equilibrium (at about a temperature T $\sim 2 \times 10^{10}$K) well before the annihilation of electrons and positrons in to photons (at temperature of about T $\sim 5.93 \times 10^9$). Hence the energy associated with the $e^\pm$ pairs is transferred completely to the photonic sector. As a result the the neutrino temperature is reduced by 1.401 times that of the photon. This was the sequence assumed in the standard model of the hot big-bang theory. But Dicus *et. al.* [27] have shown that during the pair annihilation of the $e^\pm$ pairs a fraction of the energy associated with the $e^\pm$ pairs is added to the decoupled neutrino sector through the scattering interactions, $\nu + e^{\pm} \leftrightarrow \nu + e^{\pm}$, and also through the annihilation process $\nu + \bar{\nu} \leftrightarrow e^{-} + e^{+}$. Since the neutrinos are decoupled species this transference of energy to them, is an effective loss for further Universal processes like primordial nucleosynthesis. Dicus et al have shown that due this overheating the neutrino temperature can be increased by 0.3% [27]. This will affect the primordial nucleosynthesis process mainly in two ways. One is the modification in the weak interaction rate which controls the freeze out value of the n/p ration. The second is the effect on the number of the neutron decays after the freeze out of n/p ratio. Both the effects will be reflected in the abundance value of primordial ^{4}He. Dodelson and Smith have argued that the first effect will lead to a decrease in ^{4}He abundance by an order of 10^{-5}, and the second effect will lead to a decrease about $1 - 2 \times 10^{-4}$.

In order to incorporate the effect of overheating in the Wagoners code we adopt the formulation by Rana and Mitra [33]. They showed by a more careful method that there can be a 0.36% increase in the neutrino temperature compared to the photon temperature, which will lead to reduction about 0.003 in the ^{4}He abundance. The lose in the primordial soup due to the neutrino overheating can represented as,

$$\frac{du}{dt} = \frac{du_\nu}{dt} = \sum_i I_i \,(T - T_\nu) \tag{3.13}$$

where I_i is the energy loss integral (its exact form is given in of Rana and Mitra [33]). It is assumed here that $T - T_\nu << T$. The summation in the equation extends over all types of neutrinos. The above equation can be recast in to more suitable form by using the following equations,

$$I_i = \frac{C_T}{\tau_i}\frac{du_\nu}{dt} \tag{3.14}$$

$$= 4C_\nu \frac{dT_\nu}{dt} \tag{3.15}$$

as

$$\frac{dT_\nu}{dt} = \frac{C_T}{C_\nu}\frac{T - T_\nu}{\tau} - H\,\tau_\nu \tag{3.16}$$

where

$$C_T^{-1} = C_e^{-1} + C_\nu^{-1}$$

C_e and C_ν are the specific heat capacities of electron and neutrino respectively. τ_i is the total relaxation time for a given type neutrinos. The relaxation time of electron neutrino is slightly different from that of mu/tau neutrinos and as a result ν_e will decouple at about 1.5 $\times 10^{10}$K, but ν_μ and ν_τ decouple at a slightly higher temperature 2.5 $\times 10^{10}$K. The second term in the above equation arises due to the expansion of the Universe, where H is the Hubble constant. When we incorporates this effect in the our modified code it is found that the ^{4}He abundance is reduced by 0.001 only.

3.2.2 Effect of plasma on the electron mass

Due to the interaction of the electron with the rest of the plasma in the early Universe, its propagation will be modified. This interaction will effect a net increase in the rest mass of the electron at finite temperature [27]. Increase in the mass of the electron can be calculated using the finite temperature propagator of the electron and photon , by assuming fermi distribution for electron. Dicus et al [27] have noted that the finite temperature increase in the electron mass is, if the chemical potential is negligibly small,

$$\delta m_T = \frac{B\,\alpha\,T^2}{m_e} \tag{3.17}$$

where m_e is the zero-temperature rest mass of the electron, B is a slowly varying function of temperature having value between 1 - 2 and α is the coupling constant which arises due to the addition of the gauge fixing term in the finite temperature propagator of electron. This correction cab be added to the zero temperature rest mass of the electron, which is 0.511 MeV, to get correct mass of the electron, at given temperature. The corrected mass mainly affect the weak interaction rate, which will in turn affect the ^{4}He abundance through the freeze out value of the n/p ratio. Due to the inclusion of this correction in our modified code there can be a slight decrease in the weak interaction rate, about -0.0013, which would result in a slight increase in the ^{4}He abundance about 0.0002.

3.2.3 Coulomb and radiative correction

In Wagoner's original code [16] the Coulomb correction is included by simply increasing the term λ_0 appearing in the weak interaction rate by 2%. This has the drawback that at low temperature $\lambda(n \to pe\nu)$ approaches 0.98 rather than unity. The correct treatment of the Coulomb and radiative interactions was worked out by Dicus *et. al* [27]. We will adopt their results to Wagoners code accordingly. They proposed that, the correction is to multiply all the weak interaction rate by $(1+\frac{\alpha}{2\pi}C(\beta,y)$, where α is the coupling constant, β is the velocity of the electron in the rest frame of the positron and y is the neutrino energy divided by the electron mass m_e. The function C is given by,

$$C(\beta,y) \simeq 40+4(R-1)\left(\frac{y}{3\epsilon}-\frac{3}{2}+ln2y\right)+R\left(2(1+\beta^2)+\frac{y^2}{6\epsilon^2}\right.$$
$$\left.-4\beta R\right)-4\left(2+11\beta+25\beta^2+30\beta^3+20\beta^4+8\beta^5\right)/(1+\beta)^6 \quad (3.18)$$

where R is defined to be

$$R \simeq \beta^{-1}\tanh^{-1}\beta$$

In the above correction the largest part is that due to the radiative correction. The overall effect is about a 7% increase in λ_0, out of that 3.4% comes from the radiative part and the remaining part from the Coulomb part. Due to these correction λ_0 increase from its old value 1.53515 to the new value 1.75321. Consequently ^{4}He abundance is reduced by a 0.0005.

3.2.4 Updating the reaction rates

We have included about 250 reactions in the code. In the original code of Wagoner around 180 reactions were included. In a recent work of Smith *et. al* [28] only

81 reactions were included. They neglect other reactions arguing that the remaining ones are unimportant. We think that however small be the contribution of a particular reaction, it should be included. Of course the computation time will become large due to the inclusion of more reaction. But if one looking for accuracy in the prediction all the possible reactions must be included. In that sense we have more reactions which are not included in the work of Smith *et. al.*. A list of some important reactions and their rates are given in the appendix.

We have updated the reaction rates according to the latest results. The updating of the reaction is mostly from the paper due to Caughlan *et. al.* [46]. For the rates of some of the reaction we adopt the results from Smith *et. al.* [28]. In the case of the reaction ^{8}Li (α, n)^{11}B we included the new rate determined by us in chapter 2.

3.2.5 Correcting the values of the nuclear weight

For calculating the primordial abundance we need the weight of all the nuclear species in atomic mass units. In the original code the weight of the corresponding atomic species are included, which include the weight of the nuclei plus the corresponding number of the electrons also. Strictly speaking only the weights of nuclei are neede. So we have modified the code by replacing all these atomic weights with the corresponding nuclear weights.

3.3 Comparison of the theory and observation

We perform the computation with our modified code. For the calculation we chooses the following initial condition. The initial temperature is chosen as $T_9 = 100$. This temperature is high enough to have all the particles in statistical thermal equilibrium with each other. The initial value of all other variables are set for the temperature $T_9 = 100$. For example the initial value of the baryon density can be specified as follows. We know that the baryon-to-photon number density ratio is,

$$\eta = \frac{n_b}{n_\gamma} \tag{3.19}$$

Let i refer to the initial value of the corresponding quantity, then

$$(n_b)_i = (n_\gamma)_i \ \eta_i \tag{3.20}$$

where

$$(n_\gamma)_i = 2.028719 \times 10^{34} \tag{3.21}$$

Since the temperature T_9=100 is well above the neutrino decoupling temperature (2.1×10^{10}K) one should be cautious in calculating η_i. The value of the baryon-to-photon ratio will change due to the addition of the photons during the $e^\pm$ annihilation. Let η_f be the value of the baryon-to-photon ration after the $e^\pm$ annihilation, then it can be written as,

$$\begin{aligned} \eta_f &= \left(\frac{T_\nu}{T_\gamma}\right) \eta_i \\ &= 0.366\eta_i \end{aligned} \tag{3.22}$$

Using the relation (3.20) and (3.21) the initial value of n_b, the baryon number density turns out to be

$$(n_b) \;=\; 5.54298 \times 10^{34}\, \eta_f \qquad (3.23)$$

Now we can easily calculate the initial value of h parameter by using the following relation,

$$n_b \;=\; N_A\, h\, T_9^{\,3}. \qquad (3.24)$$

Thus the initial value of h is turns out to be $9.20434 \times \eta_f$. Initial value of the abundance of all the elements is fixed as 10^{-25} by number fraction in order to avoid underflow in the numerical calculation. Basically the initial values of all relevant variables are fixed by the initial value of the temperature and baryon-to-photon ratio.

The ultimate test of the SBBN model is that whether the predicted abundances are all matching with those inferred from the observations for a unique range of η values. The results of the calculation are as shown in the figure 2a and 2b, where we plotted the abundance, mass fraction of ^{4}He, number fractions of $(D +^3 He)/H$ and ^{7}Li/H. Figure 2a is that result which obtained using our modified code. After this calculation, there was a paper by Smith et. al. [28], who proposed modified reaction rates for some of the important reactions. In constructing figure 2b, we include these modified reactions also. The shaded regions are the allowed ranges of η obtained from the constraints due to the observationally estimated primordial abundances of the elements as described in chapter 1. The shaded zones are not found to overlap each other, thus bringing out the inconsistency of the SBBN model.

According to figure 2a, the observational abundance of ^4He by mass fraction corresponds to a range $(0.7 - 1.1)\times10^{-10}$ of η, and that of (D + ^3He)/H corresponds to the range $(4 - 5.6)\times10^{-10}$. The observational limits of $^7Li/H$ is corresponds to $(1.8 - 3.5)\times10^{-10}$. These three ranges of η are mutually excluding each other.

According to figure 2b, the range of η corresponds to the ^4He abundance is $(0.5 - 1)\times10^{-10}$ that for (D + ^3He)/H abundance corresponds to $(4.8 - 6.9)\times10^{-10}$ and that for $^7Li/H$ abundance corresponds to $(1.7 - 2.6)\times10^{-10}$. These results also shows that three ranges of η are mutually excluding each other. Earlier it was reported [30] that the discrepancy is there for ^4He and (D + ^3He)/H only. Our calculations using a more realistically modified code shows that the discrepancy is shared by $^7Li/H$ also. Even though the inconsistency is small, it is not negligible. A small modification of the model must be sufficient to cure the theory.

Two alternative solutions have been proposed. One of them as proposed by Rana [30,31] and Sherrer [38], involves the introduction of one more free parameter, $\phi_{\nu_e} = \mu_{\nu_e}$, called the degeneracy of electron neutrinos, characterizing the ratio of the excess number density of neutrino of electron type over their antiparticles to the photon number density. Another alternative solution [6,68], is the inhomogeneous primordial nucleosynthesis, where the inhomogeneity is introduced in to the early Universe, prior to the nucleosynthesis by a possible first order transition from the quark-gluon to hadronic state of matter. The dynamics of this transition is the least understood one so far as the exact quantifications are concerned, but it

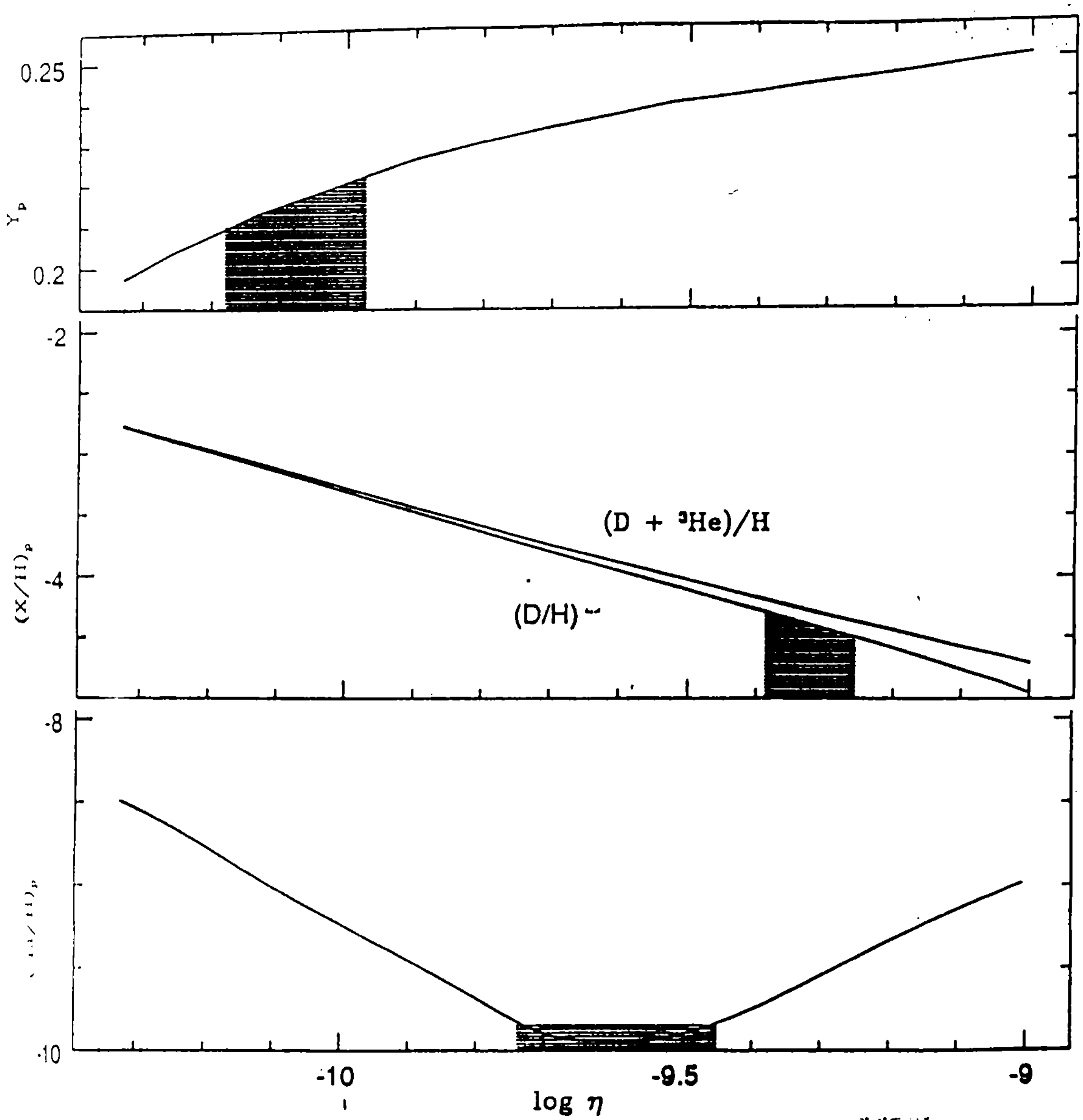

Fig. 2a Primordial abundances ^{4}He, D/H and ^{7}Li/H in the SBBN model with the shaded regions representing the allowed ranges of the baryon to photon number density ratio η. Non-overlapping of these shaded ranges of η implies inconsistency of the model predictions and the observational estimates.

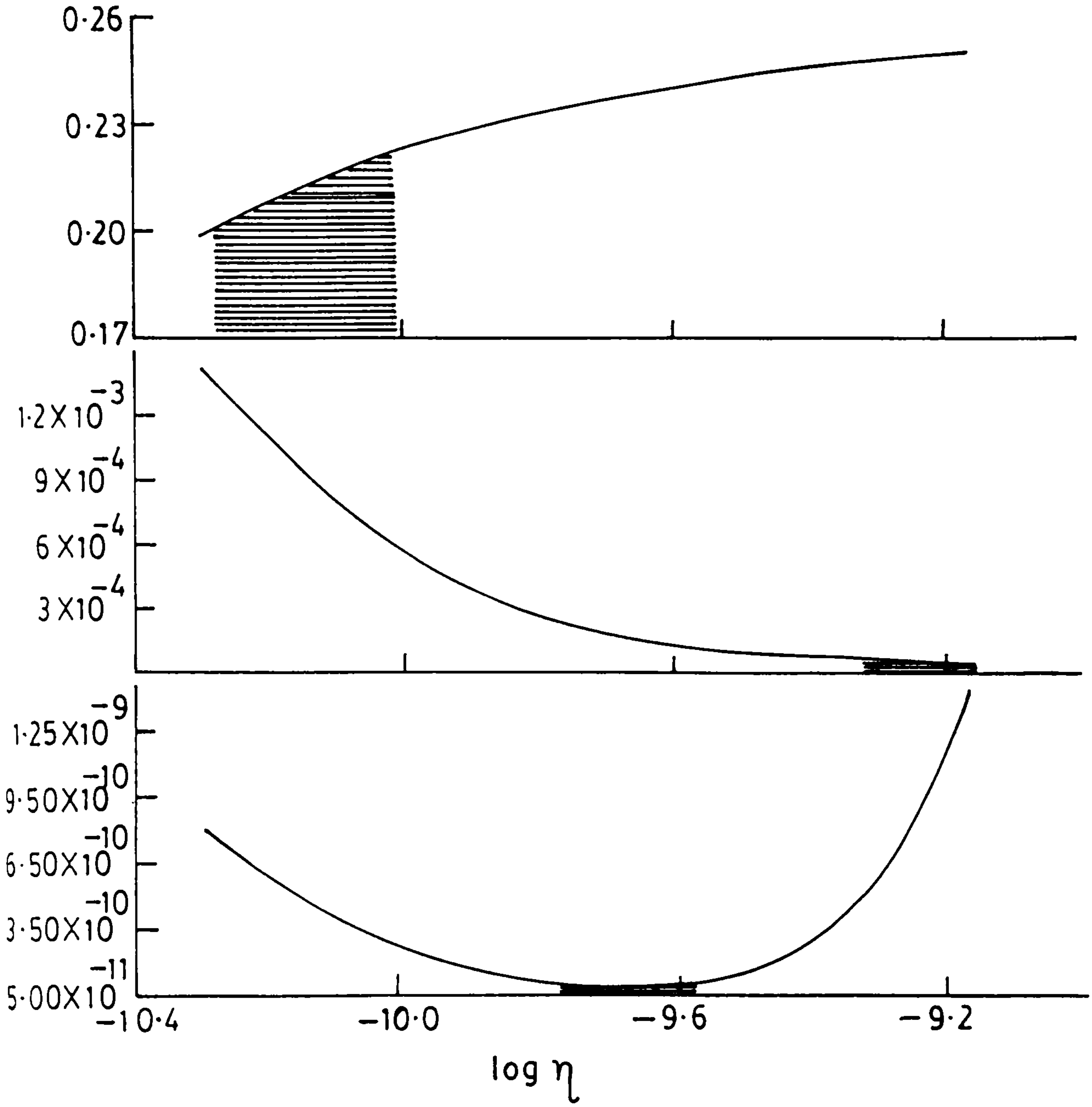

Fig. 2b. Same as fig. 2a, but with the inclusion of modified reaction rates due to Smith *et. al.*.

allowed the introduction of three more parameters to the model at a time when it had already three parameters in it. When the inhomogeneities were taken in to account the abundance ^{4}He and ^{7}Li shoot up considerably, in particular the later. We will discuss some aspects of inhomogeneous nucleosynthesis as solution to the above mentioned inconsistency in the next chapter. Here we will concentrate on the neutrino degenerate case.

3.4 Degenerate big-bang nucleosynthesis

In the work of Wagoner it was assumed that the neutrinos are non-degenerate. There is no firm experimental basis for such an assumption. A small neutrino degeneracy is natural according to many grand unified theories [57,29]. The meaning of neutrino degeneracy in the present context is that the chemical potential of the neutrino is non-zero, which implies an excess of neutrinos over their antiparticle [30]. We consider a degeneracy in the electron type neutrinos and there is no degeneracy in the other two types, the muon type and the taon type. The small neutrino degeneracy can affect the nucleosynthesis process in two ways. First the excess density of the neutrinos due to the degeneracy can increase the expansion factor causing an earlier freeze out of the n/p ratio. Because of the freeze out the value of n/p ration will be higher than its canonical value, which in turn increase the ^{4}He abundance. Second effect is that, because of the excess electron neutrino over their antiparticles, the rate of the forward reaction $n + \nu_e \rightarrow e^- + p$ will dominate compared to the corresponding rate of the backward reaction. This causes a substantial decrease in the number density of neutrons. This in turn will reduce the ^{4}He abundance. So the first and second effects are mutually opposing effects. Since we are assuming a small degeneracy, the increase in the expansion rate will be comparably negligible, as a result the second effect will dominate the first one. We analyse the situation below with relevant calculations.

If there is no neutrino degeneracy, the energy density of electron neutrinos will be given by the relation,

$$\begin{aligned}
\rho_{\nu_e} &= \rho_{\nu_e} + \rho_{\bar{\nu}_e} \\
&= \frac{7}{8}\,\frac{\pi^2}{15(c\hbar)^3}\,T_\nu^4
\end{aligned} \tag{3.25}$$

Because of the electron neutrino degeneracy the electron neutrino energy density become,

$$\rho_{\nu_e} = \frac{\pi^2}{15(c\hbar)^3}\,T_\nu^4\,\left[\frac{7}{8} + \frac{15}{4\pi^2}\,\phi_{\nu_e}^2 + \frac{15}{8\pi^2}\,\phi_{\nu_e}^4\right] \tag{3.26}$$

provided the rest mass of the neutrino is negligible. With respect to the photon energy density, the neutrino energy density become

$$\frac{\rho_{\nu_e}}{\rho_\gamma} = \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)\,(1 + \Delta N_{\nu_e}) \tag{3.27}$$

where

$$\Delta N_{\nu_e} = \frac{30}{7\pi^2}\,\phi_{\nu_e}^2 + \frac{15}{7\pi^2}\,\phi_{\nu_e}^4. \tag{3.28}$$

Here we assumed that the electron neutrino degeneracy will affect much the photon-neutrino temperature difference, which is realistic assumption in the case of small degeneracy. However if the degeneracy is higher [165] one should apply the correction of neutrino overheating due to the degeneracy also. Now the total energy density would become,

$$\tilde{\rho}_{tot} = \tilde{g}_{eff}\,\rho_\gamma \tag{3.29}$$

by assuming that all the relevant constituents are relativistic. Here $\tilde{g}_{eff}$ is the effective spin multiplicity with neutrino degeneracy, which can be written as,

$$\tilde{g}_{eff} = \frac{11}{4} + \frac{7}{8}\,(\Delta N_{\nu_e}) \tag{3.30}$$

before $e^{\pm}$ annihilation,and

$$\tilde{g}_{eff} = 1 + \frac{7}{8}\,\left(\frac{4}{11}\right)^{4/3}(1 + \Delta N_{\nu_e}) \tag{3.31}$$

after $e^{\pm}$ annihilation. According to the above equations the increase in energy density compared to photons due to the degeneracy of electron neutrinos is

$$(7/8)T_\nu/T_\gamma)^4\Delta N_{\nu_e}.$$

Due to this there is a speed up in the expansion of the Universe, which can be characterized by a speed up factor S as,

$$\begin{aligned} S &= \frac{\tilde{H}}{H} = \left(\frac{\tilde{\rho}_{tot}}{\rho_{tot}}\right)^{1/2} \\ &= \left(\frac{\tilde{g}_{eff}}{g_{eff}}\right)^{1/2} \end{aligned} \tag{3.32}$$

where H' is the Hubble constant of the Universe with electron neutrino degeneracy. Due to a finite degeneracy S is always greater than 1. Because of this the n/p ratio freezes out at a higher temperature due to the early overtaking of the weak interaction rate by expansion rate.

The second effect is just the opposite of the first as we mentioned above, that is to decrease n/p ratio by enhancing the rate of conversion neutrons into protons. Due to this effect the ^{4}He abundance decrease, without affecting the abundance of the other elements. Our aim is to find a unique range of η for which theory is satisfied with the observational abundance of the ^{4}He, D, ^{3}He, and ^{7}Li. We run the modified code for various values η and ϕ_{ν_e}. The value of η used by us is in the range, 0.5×10^{-10} to 1.0×10^{-8} and ϕ_{ν_e} is in the range 0.05 to 1. We span a 2-dimensional parameter space of η and ϕ. Results are as shown figures 3a and 3b. In figure 3a, is that figure which corresponds to figure 2a, where the most latest results of some important reaction rates as reported by Smith et al [28] have not been used. Those results are incorporated to obtain the figure 3b. Figure 3a, which gives the isoyield curves shows that the the abundance of elements, that is Y_p, (D + ^{3}He)/H and ^{7}Li/H can be fitted for a unique range of baryon-to-photon ratio, $\eta = 4(\pm 1) \times 10^{-10}$, where the required value of the degeneracy parameter is $\phi_{\nu_e} \simeq 0.11 \pm 0.04$ [34]., However 3b, is more reliable in that it include all the reaction rates in its most uptodate form. According to figure 3b, the value of baryon-to-photon ratio is $\eta = 3(\pm 1) \times 10^{-10}$ and the corresponding value of degeneracy is $\phi_{\nu_e} = 0.3 \pm 0.05$. If the Universe is not strictly obey the conservation of (B - L) (the difference between baryons and lepton numbers), it will then be possible to accommodate such a large value of ϕ_{ν_e} compared

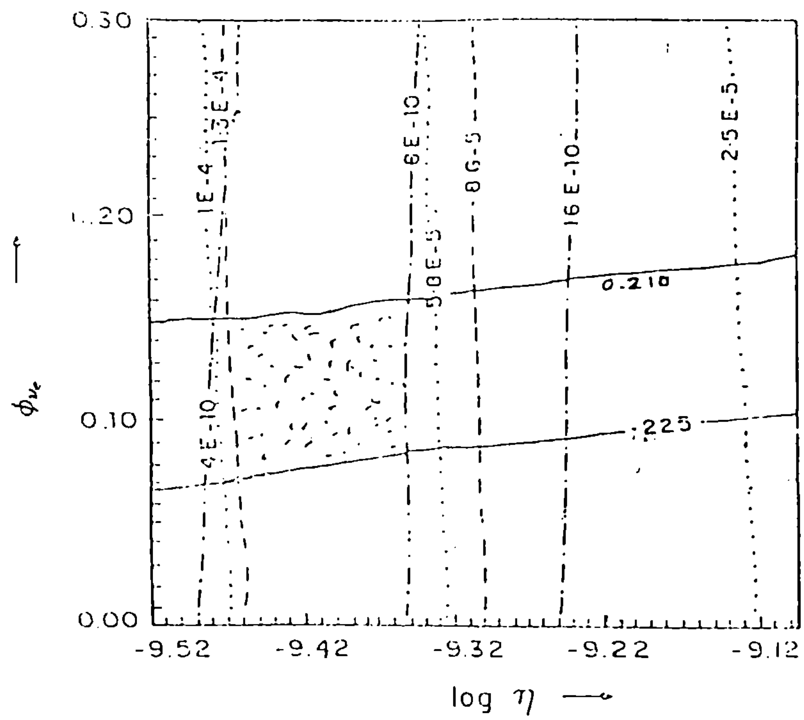

Fig. 3a. The plot for spanning the parameter space for the primordial abundance (by mass fraction) of ^{4}He, D and ^{7}Li (solid, doted and dot-dashed lines respectively) for ϕ_{ν_e} versus η. The shaded zone corresponds to the acceptable solution for $\phi_{\nu_e} = 0.11 \pm 0.01$ and $\eta = 4\ (\pm) \times 10^{-10}$.

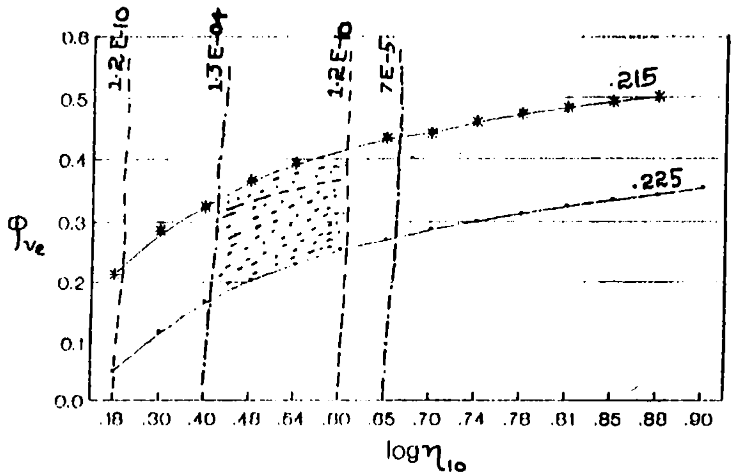

Fig. 3b. Same as figure 3a, but with the inclusion of the modified reaction rates of Smith *et. al*. Here the shaded zone corresponds to the acceptable solution for $\phi_{\nu_e} = 0.30 \pm 0.05$ and $\eta = 3\ (\pm 1) \times 10^{-10}$.

to the smallness of η. That the Majorana type of neutrinos with little or zero rest mass can under the circumstance of the early Universe develop such a large values of ϕ_{ν_e} has been shown by Langacker et al [63]. The above reported value of degeneracy parameter (*ie* .3) would correspond to an excess neutrino number density of

$$L = \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_\gamma} \simeq 0.06 \tag{3.33}$$

The corresponding speed up factor is about 0.4%, which justifies our assumption that for smaller degeneracy parameter (of course the degeneracy is higher compared to the photon number density) the speed up in the Universal expansion is negligibly small.

Another point to be noted is that, given the error bars, possibly the shaded regions for (D + ^{3}He)/H and ^{7}Li/H can merge for a value around 4.0 $\times 10^{-10}$ with no neutrino degeneracy. But if the value of $Y_p \leq 0.220$ we need a mechanism to reduce the SBBN value of ^{4}He abundance. This can be achieve by introducing significant degeneracy of electron neutrino ($\phi_{\nu_e} = \mu_{\nu_e}/k_B T \simeq 0.30$) is the main result of our work. However this suggestion must be confirmed by further tests. If one extend the degeneracy to the other types of neutrinos also, there appears to be a possibility of constructing a baryon dominated Universe, even with $\Omega_b = 1$.

3.5 An analytical analysis of the degenerate big-bang nucleosynthesis

Most of works, reported in the literature in the field of primordial nucleosynthesis icludes, the one we have given above, are of computational in nature. The over numerical reliance and computational nature gives the theory a kind of black box character, hiding the real physics from view most of the time. Hence analytical approaches which can throw light on the physical processes in the primordial nucleosynthesis are considered a welcome addition. This sorts of analytical treatment can devoloped only in an approximate way. For exact quantifications one canot by pass the exact numerical calculations. Bernstein *et. al.* [44] did an approximate analysis about the ^{4}He formation, where they mainly concentrated on case with non-degenerated neutrinos. Here our aim is to do such an analysis with some suitable assumptions for simplification of the calculation, for the ^{4}He for formation in Universe with electron neutrino degeneracy.

3.5.1 Neutron abundance with small electron neutrino degeneracy

Let $X_n(t)$ be the neutron abundance. During the evolution of the Universe, rate of neutron abundance change compared to the total mass density of the Universe is governed by the equation,

$$\frac{dX_n(t)}{dt} = \lambda_{np}\, X_n - \lambda_{pn}\, (1 - X_n(t)) \tag{3.34}$$

where λ_{np} is the total rate of the weak interactions that convert neutrons in to protons and λ_{pn} is the rate of conversion of protons in to neutrons. The relevant weak interaction are as given in the first chapter. At temperature of about 1 MeV, the weak interactions fall out of equilibrium and after that only an occasional neutron will remain active. If we neglect the neutron decay and synthesis of elements, then the neutron abundance will reach a constant finite value $X_n^{(0)}$ as $t \to \infty$ (or as $T \to 0$). The effect of the neutron decay is to multiply $X_n^{(0)}$ with an exponential factor as

$$X_n = X_n^{(0)} \exp\left(- \frac{t_c}{\tau} \right) \tag{3.35}$$

where t_c is the neutron capture time in the ^{4}He nuclei and τ is the neutron life time. Once X_n is known, the ^{4}He abundance by mass fraction is equal to $2X_n$. So the quantities one has to evaluate are $X_n^{(0)}$ and t_c. In the case of neutrino degeneracy our assumption is that the electron neutrino degeneracy is small but the mu/tau degeneracy need not be small. If there is a small electron neutrino degeneracy, the equilibrium abundance of neutron can be written as

$$X_{eq} = \frac{1}{1 + \exp(y + \phi_{\nu_e})} \tag{3.36}$$

where the variable is defined as, y = $\Delta m/T$, $\Delta m = 1.293$ the neutron-proton mass difference. By neglecting the neutron decay, the solution of the equation can be written as

$$X_n^{(0)} = X_{eq} - \int_0^{y} I(y, y^{'})\, \frac{d}{dy^{'}} \left[\frac{\lambda_{pn}(y^{'})}{\Lambda(y^{'})} \right] dy^{'} \tag{3.37}$$

where

$$I(y, y^{\sim}) = \exp\left[- \int_{y^{\sim}}^{y} dy^{''} \frac{dt^{''}}{dy^{''}} \Lambda(y^{''}) \right] \tag{3.38}$$

is the integrating factor. The derivative the time with respect to the variable y can be written as,

$$\begin{aligned} \frac{dt}{dy} &= \frac{dt}{dT}\frac{dT}{dy} \\ &= \left(\frac{45}{4\pi^3 g_{eff}}\right)^{1/2} \frac{M_{pl}}{\Delta m^2}\, y \end{aligned} \tag{3.39}$$

and

$$\begin{aligned} \Lambda(y) &= \lambda_{pn} + \lambda_{np} \\ &= (1 + \exp(-y - \phi_{\nu_e}))\ \lambda_{np} \end{aligned} \tag{3.40}$$

is the total reaction rate of neutrons and protons except for the free neutron decay, since we have ignored the neutron decay for calculating $X_n^{(0)}$. Hence the rate λ_{np} is

$$\lambda_{np} = \lambda(n + \nu_e \to p + e^-) + \lambda(n + e^+ \to p + \bar{\nu}_e)$$

The corresponding individual rates are

$$\begin{aligned} \lambda(n + \nu_e \to p + e^-) &= A \int_0^\infty dp_{\nu_e}\, p_{\nu_e}^2 p_e\, E_e\ (1 - f_e)\ f_{\nu_e} \\ \lambda(n + e^+ \to p + \bar{\nu}_e &= A \int_0^\infty dp_e\, p_e^2 p_{\nu_e}\, E_{\nu_e}\ (1 - f_{\nu_e})\ f_e \end{aligned} \tag{3.41}$$

where A is a constant whose value can be determined by calculating the neutron decay rate or equivalently the neutron life time. The rate of the neutron decay can be written as,

$$\lambda(n \to p + e^- + \bar{\nu}_e) = A \int_0^\infty dp_e\, p_e^2 p_{\nu_e}(1 - f_{\nu_e})\,(1 - f_e) \tag{3.42}$$

In the above equations p_{ν_e} (p_e) denote the momentum of electron-neutrino (electron) and $E_{\nu_e}(E_e)$ is the energy of the electron-neutrino (electron). They are related as

$$E_e = E_{\nu_e} + \Delta m \quad for \;\; n + \nu_e \leftrightarrow p + e^- ,$$

$$E_{\nu_e} = E_e + \Delta m \quad for \;\; n + e^+ \leftrightarrow p + \bar{\nu}_e$$

$$E_{\nu_e} = \Delta m - E_e \quad for \;\; n \leftrightarrow p + e^+ + \nu_e$$

The last relation gives the upper limit of the integration in the rate of $\lambda(n \rightarrow p + e^+ + \bar{\nu}_e)$. In this case we neglect the kinetic energy of the nucleus because the recoil of the nucleus is negligibly small in the temperature of interest. The integration limit p_0, can then written as,

$$p_0 = \left(\Delta m^2 - m_e^2\right)^{1/2}$$

The distribution functions have the form

$$f_{\nu_e} = \frac{1}{\exp\left(E_{\nu_e}/T_{\nu_e} - \phi_{\nu_e}\right) + 1} \tag{3.43}$$

$$f_e = \frac{1}{\exp\left(E_e/T_e\right) + 1} \tag{3.44}$$

In order to simplify the analysis we make the following assumptions

1. $$T_{\nu_e} \simeq T_\gamma \simeq T_e = T \tag{3.45}$$

2. $$f_e \simeq \exp(E_e/T) \;\; and \;\; f_{\nu_e} \simeq \exp(-E_{\nu_e}/T + \phi_{\nu_e}) \tag{3.46}$$

3. $$(1 - f_e) \simeq 1 \;\; and \;\; (1 - f_{\nu_e}) \simeq 1 \tag{3.47}$$

4. $$electron \;\; mass \;\; m_e \simeq 0 \tag{3.48}$$

Actually the neutrino temperature T_{ν_e} is less than the photon temperature T_γ by about 10% after the $e^\pm$ annihilation. In the second assumption we approximate the Fermi distribution by the Boltzmann distribution. Since during the nucleosynthesis the temperature is comparably less than Δm, this assumption will not do much harm to the final assumption. Under these assumptions the weak rates are become

$$\lambda(n + \nu_e \rightarrow p + e^-) = 2A\Delta m^5 \, (1 + \phi_{\nu_e}) \left\{ \frac{12}{y^5} + \frac{6}{y^4} + \frac{1}{y^3} \right\} \quad (3.49)$$

$$\lambda(n + e^+ \rightarrow p + \bar{\nu}_e) = 2A\Delta m^5 \left\{ \frac{12}{y^5} + \frac{6}{y^4} + \frac{1}{y^3} \right\} \quad (3.50)$$

The rate of the reaction $n \rightarrow p + e^- + \bar{\nu}_e$ is

$$\lambda(n \rightarrow p + e^- \bar{\nu}_e) = 0.0157 \, A \, \Delta m^5 \quad (3.51)$$

So A can be written as

$$A = \frac{255}{4\tau \Delta m^5}$$

where $\tau = 1/\lambda$. Total rate for the conversion of neutrons in to protons λ_{np} can the written as,

$$\lambda_{np} = \left(1 + \frac{\phi_{\nu_e}}{2}\right) \left(\frac{255}{\tau y^5}\right) \left(12 + 6y + y^2\right) \quad (3.52)$$

Substituting dt/dy and $\Lambda(y)$ in equation (3.38), the equation for $I(y, \tilde{y})$ can be expressed as,

$$I(y, \tilde{y}) = \exp\left[K(y) - K(\tilde{y})\right] \quad (3.53)$$

where

$$K(y) = b \left\{ \left(1 + \frac{\phi_{\nu_e}}{2}\right) \left(\frac{4}{y^3} + \frac{3}{y^2} + \frac{1}{y}\right) + \left(1 - \frac{\phi_{\nu_e}}{2}\right) \left(\frac{4}{y^3} + \frac{1}{y^2}\right) e^{-y} \right\} \quad (3.54)$$

with

$$b = 255 \left(\frac{45}{4\pi^3 g_{eff}}\right) \frac{M_{pl}}{\tau \Delta m^2}.$$

Table 3.4: Variation of helium abundance with ν_e degeneracy.

ϕ_{ν_e}	freeze out n/p ratio	Y_p for $\eta = 3 \times 10^{-10}$	Y_p for $\eta = 4 \times 10^{-10}$
0.00	0.154	0.245	0.248
0.05	0.146	0.233	0.236
0.10	0.139	0.222	0.224
0.12	0.136	0.217	0.219
0.16	0.131	0.208	0.211
0.20	0.126	0.200	0.202

The solution to the neutron abundance equation now becomes

$$X(y) = X_{eq} + \int_0^y dy^- e^{y^-} \phi_{\nu_e} X_{eq}(y^-)^2 I(y, y^-). \tag{3.55}$$

The integral can be easily evaluated for different values of ϕ_{ν_e}. The consistency of this type of simplified calculation for $\phi_{\nu_e} = 0$, was verified by Benstein *et al* [44], where they have calculated the integral for $\tau = 896 sec$. In our calculation we used the most recent value for the neutron life time, $\tau = 888.5$ [28]. The results of our calculation are tabulated in Table 3.4. We have calculated $X_n^{(0)}$ also as shown in the table. Peebles [64] calculated $X_n^{(0)}$ for $\phi_{\nu_e} = 0$ as 0.155, where he used the neutron life time as $\tau = 1013 sec$. In order to calculate Y_p one should know the capture t_c also. We calculate the capture in the presence of electron neutrino degeneracy. The capture time is that time when all the neutrons in the early Universe are captured into the ^{4}He nuclei. Since the ^{4}He synthesis is taking place at comparably low temperature than the electron rest mass, the temperature difference between electron and neutrinos must be taken in to account. The capture time can be written as [44].

$$t_s = \left(\frac{45}{16\pi^3 g_{eff}}\right)^{1/2} \left(\frac{11}{4}\right)^{2/3} \frac{M_{pl}}{T_{\gamma c}} + t_0 \tag{3.56}$$

where $t_0 = 2$ see [44] and $T_{\gamma c}$ is the capture temperature. By calculating the capture temperature, the calculation of the capture time is straight forward. In the following we will calculate the capture temperature.

The following are the main reactions through which ^{4}He is formed in the early Universe,

$$n + p \longrightarrow D + \gamma \tag{3.57}$$

$$D + D \longrightarrow T + P$$

$$D + T \longrightarrow {}^4He + n \tag{3.58}$$

During capture temperature most of the neutrons will be incorporated in to ^{4}He. ^{4}He is formed due third reaction in the above sequence, due to the reaction between D and T. For most of the neutrons capture in to ^{4}He, it is required that the destruction rate of D shoul be maximum, since the destruction of D will finally end up as helium. So we can calculate the capture temperature $T_{\gamma c}$ by the condition,

$$\left.\frac{dX_D}{dT}\right|_{T_{\gamma c}} = 0 \tag{3.59}$$

The calculations are done in equilibrium condition. The destruction rate of D will depend on the rate of formation D through the first reaction, and the rate of formation T through the second reaction. In equilibrium state the following equations are valid

$$G_{np} = \frac{X_n X_p}{X_D} = \frac{\lambda_\gamma(D)}{[pn]} \; for \; n + p \rightarrow D + \gamma \tag{3.60}$$

$$G_{DD} = \frac{X_D^2}{X_T X_P} = \frac{[TP]}{[DD]} \; for \; D + D \rightarrow T + P \tag{3.61}$$

where G is the Saha factor [44] of the corresponding reaction. The number fraction X_A of any element of mass number A and atomic number Z can be written as [3]

$$X_A = g_A \left[\zeta(3)^{A-1} \pi^{(1-A)/2} 2^{(3A-5)/2}\right] A^{3/2} \left(\frac{T}{m_N}\right)^{3(A-1)/2} \eta^{A-1} X_P^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right) \tag{3.62}$$

Now the Saha factors can be rewritten as,

$$G_{np} = \frac{\pi^{1/2}}{g_D \zeta(3) \, \eta} \left(\frac{m_N}{T_\gamma}\right) \exp\left(-\frac{B_D}{T}\right) \tag{3.63}$$

$$G_{DD} = \frac{g_D^2}{g_T} \left(\frac{4}{3^{3/2}}\right) \exp\left(-\frac{B}{T}\right) \tag{3.64}$$

where $B = 2m_D - m_P - m_T \approx 4.02 MEV$, $g_D = 3$, $g_T = 2$. The deuterium destruction can now be written approximately as

$$\frac{dX_D}{dZ} = R_{np} \left(X_p X_n - G_{np} X_D\right) - R_{DD} \left(2X_D^2 - G_{DD} X_T X_p\right) \tag{3.65}$$

where $Z = B/T_\gamma$. The equation is takes into account the first two reactions in the sequence given equation (3.59). However the destruction of deuterium through second process will enhance the production of ^{4}He. The quantities R_{np} and R_{DD}, are given by the relations,

$$R_{np} = \frac{dt}{dZ} \langle \sigma V \rangle \, n_B \tag{3.66}$$

$$= 4 \left(\frac{45}{16\pi^7 g_{eff}}\right)^{1/2} \zeta(3) B_D M_{pl} \frac{\eta}{Z^2} \langle \sigma V \rangle$$

$$= \frac{1.55 \times 10^{13}}{\sqrt{g_{eff}}} \left(\frac{\eta}{Z^2}\right) \tag{3.67}$$

Table 3.5: Variation of capture time with ν_e degeneracy.

ϕ_{ν_e}	Value of t_c for $\eta_{10} = 3$	Value of t_c for $\eta_{10} = 4$
0.00	202.82	192.98
0.10	204.25	194.38
0.12	204.55	194.67
0.16	205.14	195.25
0.20	205.75	195.84
0.30	207.29	197.36

where we take $\langle \sigma V \rangle = 4.55 \times 10^{-20}$

$$R_{DD} = \frac{1.55 \times 10^{17}}{\sqrt{g_{eff}}} \frac{\eta}{\epsilon^{2/3}} Z^{-4/3} \exp\left(- 1.44 Z^{1/3} \right) \tag{3.68}$$

Using the maximizing condition as given equation in 3.5.1 the condition for capture temperature is become

$$X_D^1 \, R_D D = 1 \tag{3.69}$$

After substitution of the relevant quantities, the above condition become

$$Z^{-17/6} \exp\left(Z_c - 1.44 Z_c^{1/3} \right) = 6.124 \times 10^5 \sqrt{g_{eff}} \left(\frac{1}{X_p^{(0)} \, X_n^{(0)} \eta_{10}} \right). \tag{3.70}$$

where η_{10} is the baryon-to-photon ratio in units of 10^{-10}. We can now calculate t_c for various values of ϕ_{ν_e}. The results are shown in table 3.5. We find that a slight increase in capture time with increasing ϕ_{ν_e}. Using the capture time for different ϕ_{ν_e} values the abundance ^{4}He (Y_p) are calculated and are given Table 3.4. Comparing the results obtained here with our earlier numerical calculations we find that becauuse of the various approximations used the accuracy of the analytical calculations are rather poor. In spirits of this approach this approach helps to understand the physics behind the primordial synthesis of ^{4}He. Similar works can be carried for other elements also, but will need a diifferent method which take account of the various reaction rates of the important reactions involved. Such an analysis is beyond the scope of the present work.

3.6 Massive neutrinos and nucleosynthesis

In SBBN and IBBN, the neutrinos are assumed to be massless species [15,21,22]. The SBBN theory restricts the number of neutrinos to 3 [53,60]. The condition $\Omega h^2 \leq 1$, restrict the mass of the neutrinos to less than about 92 h^2 eV [143]. The laboratory limits for the masses of the μ and τ neutrinos are around 250 keV and 35 MeV respectively.

In the following we consider the effect of massive unstable neutrinos, with the presupposition that, these neutrinos decay only after the primordial nucleosynthesis process. The crucial effect of the massive neutrinos is through their contribution to the total mass density of the Universe, there by increasing the expansion rate of the Universe. The first attempt to include the effect of the massive neutrinos to primordial nucleosynthesis was done by Dicus *et. al.* [27]. Here we report a work in which neutrino mass and the electron-neutrino degeneracy are included. Our aim is to limit the degeneracy of electron-neutrino in the presence of the massive neutrinos using the limit on the ^{4}He abundance.

The time evolution of the number density n_ν of the massive neutrinos of mass m_ν can be evaluated by using the Boltzmann relation [3],

$$\frac{dn_\nu}{dt} + 3Hn_\nu = -\langle\sigma v\rangle \left(n_\nu^2 - n_\nu^{eq2}\right) \tag{3.71}$$

where n_ν^{eq} is the equilibrium number density, σ is the neutrino-antineutrino annihilation cross-section. The term 3Hn_ν represents the dilution in the neutrino density due to the expansion of the Universe. The above relation can rewritten using the variables $Y = n_\nu/n_\gamma$ and $x = m_\nu/T$ as

$$\frac{dY}{dx} = -\frac{x\langle\sigma v\rangle s}{H(m)} \left(Y^2 - Y^{eq2}\right) \tag{3.72}$$

where $H(m)$ is the Hubble constant in the presence of the massive neutrinos which has the form

$$H(m) = 1.66\, g^{1/2} \frac{m_\nu}{m_{pl}}$$

$g^{1/2}$ is the spin multiplicity factor of the constituents of the Universe and s is the entropy density. By substituting for H(m) and S the equation (3.72) can be brought to the form,

$$\frac{dy}{dx} = \frac{2m_\nu m_{pl}}{\pi^2} \left(\frac{90}{8\pi^2 g}\right) \zeta(3) \frac{\langle\sigma v\rangle}{x^2} \left(Y^2 - y^{eq2}\right). \tag{3.73}$$

We consider $\langle\sigma v\rangle = N_A G_f^2 m_\nu^2/2\pi^2$ [4], which is a constant for given m_ν, but in real case $\langle\sigma v\rangle$ is dependent on the momentum of the neutrinos. The contribution to the total energy density by the massive neutrinos can be calculated by the relation [3]

$$(\rho_\nu)_{massive} \simeq 2n_\gamma Y \left[(3.151T_\nu)^2 + m_\nu^2\right]^{1/2} \tag{3.74}$$

The factor 2 will account for neutrino-antineutrino pairs. To obtain $(\rho_\nu)_{massive}$ we have to solve equation (3.73). We numerically solved that differential equation taking $m_\nu = 5$MeV. The speed up factor then be calculated as

$$S = \left(\frac{\tilde{\rho}}{\rho_0}\right)^{1/2} \tag{3.75}$$

where

$$\rho_0 = \rho_\gamma + \rho_{e^\pm} + 2\rho_\nu$$

and

$$\tilde{\rho} = \rho_0 + (\rho_\nu)_{massive}$$

In figure 4 we plotted the ratio of densities that is S^2 versus the temperature, which shows a considerable increase in the expansion factor.

The increase in the speed up factor will affect the ^{4}He abundance in the following way. The speed up in the expansion will cause an earlier freeze out of the weak interaction and thus produce a high n/p ratio. This increase in the n/p ratio will increase the ^{4}He abundance. The effect of the electron-neutrino degeneracy is to bring down the value of ^{4}He abundance. By using the modified Wagoners code we found that for $m_\nu = 5$ MeV, and electron-neutrino degeneracy is $\phi_{\nu_e} = 0.5$, for Y_p $= 0.225$ is obtained.

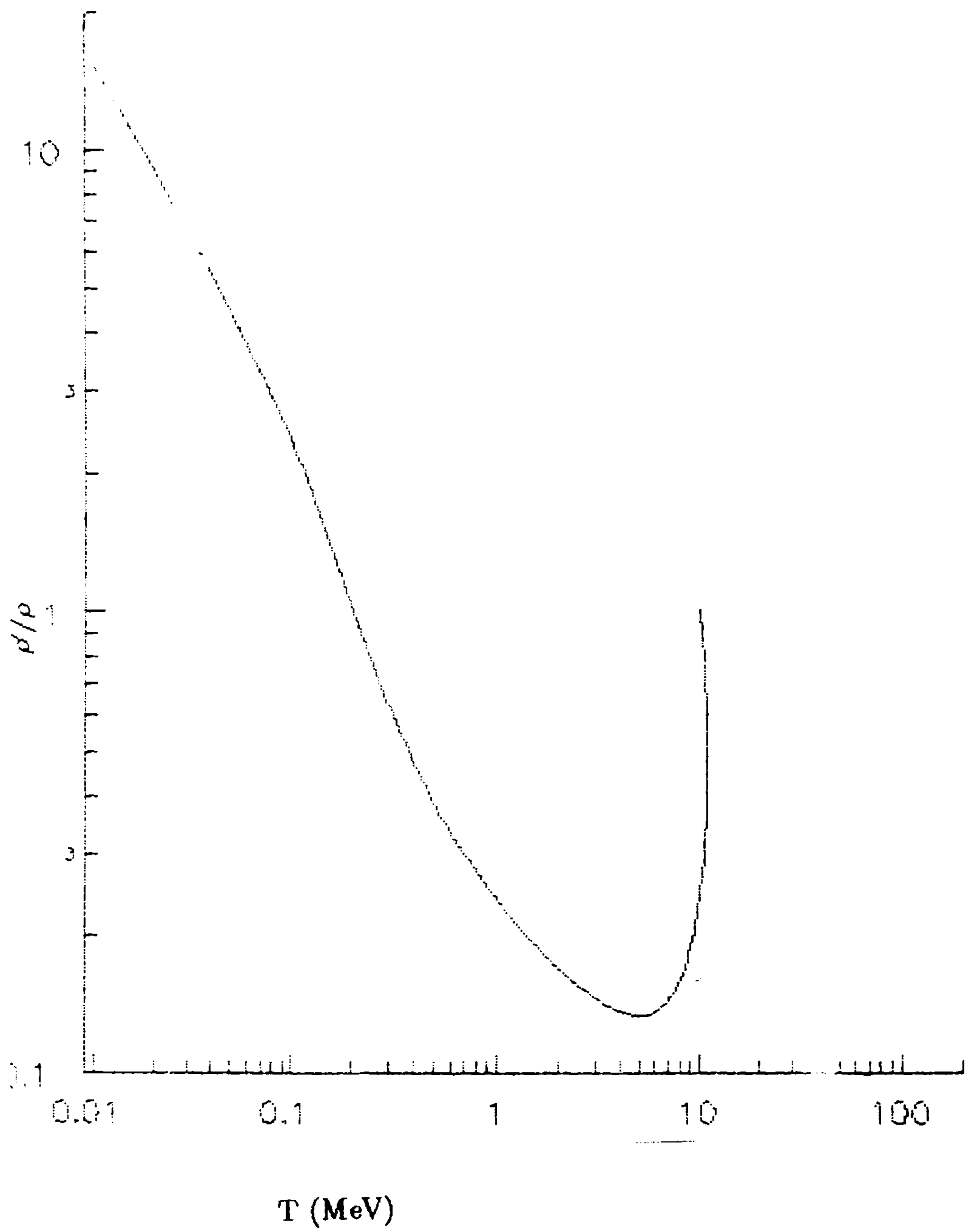

Fig. 4. Plot of the ratio of density with massive neutrinos to that with massless neutrinos (*ie* the square of the speed up factor) and temperature in Mev.

Chapter 4

Mini-Inflation before the QCD phase transition

One of the alternative solutions proposed for the removal of the discrepancy in the SBBN due to the appearance of the multivalues of the baryon-to-photon ratio is the inhomogeneous big-bang nucleosynthesis (IBBN) [38,6]. Quark-hadron transition is found to be the best agent to produce density inhomogeneity in the early Universe prior to the nucleosynthesis. The quark-hadron transitio were happened around the temperature of about 200 MeV. Theoretical analysis and numerical lattice studies suggest that the quark-hadron transition in the early Universe may be a first order transition. Several studies have been made about the nature and dynamics of this transition [81,121,122,123,149,83,167,162,126]. The inhomogeneous nucleosynthesis after the quark-hadron transition has been worked out in detail by coexist together at the critical temperature T_c. This phase seperation between the quark and hadronic state will cause density fluctuations. This isothermal density fluctuation will be modified later due to the diffusive separation of neutrons and protons [89], resulting in low density neutron-rich regions and high density proton-rich regions. This density fluctuation will affect the primordial nucleosynthesis which happens just after this transition. One of the attractive feature of this model is that, this opens the possibility of accounting for the primordial abundances for Ω = 1 in baryons. A Universe with Ω_b = 1, has the merit that, it is in accord with the inflationary scenario (since inflationary scenario predicts that the geometry of the Universe should be flat), and at the same time gives an answer for the dark matter problem.

The essential ingredient to study the phase transition is the equation of state of the two phases participating in the transition. If the quark-gluon plasma is treated as an ideal ralativistic gas then its pressure can be written in the bag model approach as [148]

$$P_{qp} = \frac{7}{180} N_c \, N_f \, T^4 \left[1 + \frac{30}{7\,\pi^2} \left(\frac{\mu_q}{T}\right)^2 + \frac{15}{7\pi^4} \left(\frac{\mu_q}{T}\right)^4 \right] + \frac{\pi^2}{45} \, N_g T^4 \; - \; B \quad (4.1)$$

where N_c is the number of colours (which is 3), N_f is the number of quark flavours, N_g=8, the number of gluons, μ_q is the chemical potential of quarks, which equal to one-third of the baryonic chemical potential μ_b, in the early Universe $(\mu/T) \sim 10^{-8}$. The term B appearing in the above equation is called the bag constant, which characterizes the vacuum energy of the quark-gluon plasma. The exact value of B for the early Universe is uncertain. Muller [125] have argued that the accepted range of B is in between 60 MeV fm^{-3} and 400 MeV fm^{-3}, but higher values are used in literature, for example, Kajantie and Suonio [126] have shown that if the transition temperature is 200 MeV, then B $\sim$ 780 MeV fm^{-3}. Spectroscopic studies [128] shows that the value of B is around (250 Mev)^4.If there is interaction in the quark-gluon plasma, then the pressure will be given as [78,127],

$$\begin{aligned} P_{qgp} &= \left(\frac{8\pi^2 T^4}{45}\right)\left(1-\frac{15\alpha_s}{4\pi}\right) + N_f\left[\left(\frac{7\pi^2 T^4}{60}\right)\left(1-\frac{50\alpha_s}{21\pi}\right) + \right. \\ & \left.\left(\frac{\mu_f^2 T^2}{2}+\frac{\mu_f^4}{4\pi^2}\right)\left(1-\frac{2\alpha_s}{\pi}\right)\right] - B \end{aligned} \tag{4.2}$$

where α_s is the coupling constant. We have assumed $\hbar = C = 1$ and μ_f is the quark chemical potential. The coupling constant α_s can be written as [127]

$$\alpha_s(\mu_q, T) = \left[\frac{12\pi}{(33-2N_f)}\right]\left(ln\left[\left(0.8\mu_q^2 + 15.622T^2\right)/\Lambda^2\right]\right)^{-1} \tag{4.3}$$

where Λ parameterizes the absolute strength of the interaction, whose value is in between 100 and 400 MeV. The other thermodynamic variables are then calculated by using the relations,

$$n = \frac{dP}{d\mu} \tag{4.4}$$

$$S = \frac{dP}{dT} \tag{4.5}$$

$$E = -PV + ST + \mu n V \tag{4.6}$$

where n, S and E are the number density, entropy and energy respectively and V is the volume of the quark-gluon plasma. For non-interacting massles quarks with zero chemical potential the equations will take the form [148]

$$P_{qgp} = \frac{1}{3} g_q a T^4 - B \tag{4.7}$$

$$E_{qgp} = g_q a T^4 + B \tag{4.8}$$

$$s_{qgp} = \frac{4}{3} g_q a T^3 \tag{4.9}$$

where $g_q = 51.25$, is the statistical factor of quark phase which the back ground leptons and s_q is the entropy density.

The equations of state for hadronic state, by considering it as a massless ideal gas of zero chemical potential are [148]

$$P_h = \frac{1}{3} g_h a T^4 \tag{4.10}$$

$$E_h = g_h a T^4 \tag{4.11}$$

$$s_h = \frac{3}{4} g_h a T^3 \tag{4.12}$$

where $g_h = 17.25$, which also includes the lepton background particles contribution. The hadronic constituents are mainly consists of nucleons. These equations are simplest to analyse the nature and dynamics of the first order QCD transition. Even though the T_c is considered as the starting temperature of the phase transition, in real case there will be a supercooling below the critical temperature, in order to nucleate the hadronic phase in the quark-gluon plasma. The coexistence temperature can be obtained by equating the quark-gluon plasma pressure and hadron pressure since it is assumed to be a first order transitions. So the critical temperature become,

$$T_c = \left(\frac{a(g_q - g_h)}{3} \right)^{-1/4} B^{1/4} \sim 0.72 B^{1/4}. \tag{4.13}$$

This will lead to a coexistence temperature of $T_c < 250$ MeV, when $B < (300 Mev)^4$.

The latent heat liberating during the transition can be calculated as,

$$L = T_c \frac{\partial (P_{qgp} - P_h)}{\partial T} = T_c (s_q - s_h) \tag{4.14}$$

where the derivative is evaluated at $T = T_c$. By using the ideal equation of state for both quark-gluon plasma and hadronic phase it can be shown that the latent heat $L \sim 4B$. The latent heat by including 2 and 3 flavours of quarks are studied by Fuller *et al* [148]. Their studies by treating both the phases as ideal gases shows that the latent heat will be slightly higher for 3 flavours of quarks than for 2 flavours. Other thermodynamic quantities for example, transition temperature T_c also show a slight increase with the number of quark flavours. The effect of including interaction in the quark-gluon plasma has been studied by many [78,80,86,87,88]. It was found that the effect of interaction is to increase the thermodynamic quantities like T_c, L etc.

The important consequence of this first order phase transition is the generation of isothermal baryon number fluctuation which will alter the preceding primordial synthesis of light nuclei [148] in the early Universe. The baryon number fluctuation is characterized by the ratio of the baryon number density in the quark phase to that in the hadron phase at the coexistence temperature, as represented below,

$$R = \frac{n_b^q}{n_b^h} \tag{4.15}$$

Fuller *et al* [148] calculated the value of R by assuming ideal gas equation of states for both quark-gluon and hadronic phase with chemical and thermal equilibrium between the two phases as,

$$R \approx \frac{2}{9}\left(\frac{\pi^3}{8}\right)^{11/2}\left(\frac{T_c}{m}\right)^{3/2} e^{m/T_c} \tag{4.16}$$

where m is the mass of the baryons in the hadronic phase. They have shown that the inhomogeneity may have significant effect on primordial nucleosynthesis when $R \geq 20$ for which the coexistence temperature should be less than $T_c < 125 MeV$. This result was later confirmed by Murugesan et al [80] by including interaction in the quark-gluon phase and Hagedorns pressure ensemble correction [79,129,130] to the hadronic phase. The exact value of the transition temperature is still uncertain due to the lack of understanding of QCD, however lattice field calculations [134,141,166] shows that, $T_c = 235 \pm 42$MeV. If so the inhomogeneity due to this transition will not affect the primordial nucleosynthesis. But due to the uncertainty in the exact quantification of the QCD parameters, a firm conclusion cannot be drawn yet, which motivate lot of works in the inhomogeneous nucleosynthesis.

The two main factors to be taken in to account while computing the abundances of the light elements in IBBN are the different values of the baryon-to-photon ratio prevailing in the different parts of the same Universe due to inhomogeneity in baryon density and the different diffusion probability of the neutrons and protons of the hadronic phase. Compared to the charged proton, the diffusion length of neutron is large because the protons diffusion will be hindered by the proton-electron scattering unlike in the case of neutrons whose diffusion length will be affected by the nucleons collision only due to its dipole moment. This difference in diffusion length will cause the generation of low density neutron rich region and high density proton rich region [162]. The two regions will participate in the nucleosynthesis process. Several works on IBBN are there in which the authors assumed that [71,89,131,148] the neutron diffusion is over before the nucleosynthesis. On the other hand examples are there [68,85,163] which include the possibility of neutron diffusion during the inhomogeneous nucleosynthesis. Fuller at al have [148] shown, by assuming that the neutron diffusion is over before the nucleosynthesis, that the inhomogeneous nucleosynthesis with $\Omega = 1$, will overproduce ^{7}Li. The same authors later extended their work for $\Omega \neq 1$ case also and proved that the deuterium also will be overproduced. Terasawa and Sato [69,70] have considered the case where the neutron diffusion is continued during the nucleosynthesis also, and showed that the IBBN prediction for the light elements will be compatible with the observational abundance only if the density fluctuation parameter $R \geq 300$ and the other parameters were tuned accordingly. Various values for R are considered in the literature, varying from 1 to 10^6 [71,124,167]. A consensus value is still not derived due to the enormous number of possibilities and the lack of understanding of the exact dynamics of the quark-hadron transition.

4.2 Mini-inflation

The Universe may supercools below the transition temperature T_c to facilitate the nucleation of the hadronic phase. If this supercooling is large enough it might be possible that the QCD vacuum energy contribute to the energy density and pressure of the quark-gluon phase will come to dominate and as a result the Universe can undergo a mini-inflation [72,148]. This mini-inflation may be present even during the transition time [73]. If there is sufficient supercooling, such an inflation can affect the scale factor, and baryon density fluctuation. However it is possible that the supercooling may be quite small as has been pointed out by Banerjee [40]. In the following we study the possibility of mini-inflation by including interaction in the quark-gluon plasma and Hagedorn's pressure ensemble correction to the hadronic phase. For including interaction we make use of the formula for the temperature dependence of the coupling constant, suggested by Kapusta [127] as given equation (4.3). For the temperature dependence of the coupling constant we make use of another formula also, derived by Nakkagawa-Niegawa [76], which seems to be more realistic one. An interesting result we obtained is possibility of mini-inflation above the transition temperature, which does not need a supercooling. The mini-inflation without supercooling is possible for a reasonable value of the vacuum energy constant (the bag constant) when we use the Nakkagawa-Niegawa equation for the temperature dependence of the QCD coupling constant.

We use the following equation of state for the quark-gluon plasma by considering it as unconfined gas of relativistic particles with overall vacuum energy and the interaction between the quarks and gluons is included in the lowest order of the perturbation through the running coupling constant α_s, as

$$\rho_{qgp} = \frac{51.25\pi^2}{30} T^4 \left(1 - \frac{110}{51.25\pi}\alpha_s\right) + B \tag{4.17}$$

where we included the degrees freedom of quarks and the background particles like electrons, photons and neutrinos, ρ_{eq} is the energy density of plasma. The pressure of the quark-gluon plasma can be calculated as,

$$P_{qgp} = \frac{(\rho - 4B)}{3} \tag{4.18}$$

. In our analysis we consider a range of values for the bag constant form 50 MeV$/fm^{-3}$ to 400 MeV$/fm^{-3}$. Even though the use of high values of B parameter may not be physically meaningful our final results are in agreement with reasonable values of B.

In the absence of interaction between hadrons the equation of state for hadronic state is [78],

$$P_{had} = \frac{P^{pt}_{had}}{\left(1 + \rho^{pt}_{had}/4B\right)} \tag{4.19}$$

$$\rho_{had} = \frac{\rho^{pt}_{had}}{\left(1 + \rho^{pt}_{had}/4B\right)} \tag{4.20}$$

where P_{had} is the pressure of the hadronic phase and ρ_{had} is the energy density. Denominators in equations 4.19 and 4.20 are due to the Hagedorn's pressure ensemble correction for the finite size of the hadrons. P^{pt}_{had} and ρ^{pt}_{had} are given by

$$P^{pt}_{had} = \sum_i P_i \tag{4.21}$$

where

$$P_i = \int_{m_i}^{\infty} dE \left(E^2 - m_i^2\right)^{3/2} \left(\exp\left[\beta(E - \mu_i)\right] \pm \theta_i\right)^{-1} \tag{4.22}$$

and

$$\rho^{pt}_{had} = \sum_i \rho_i \tag{4.23}$$

where

$$\rho_i = \frac{d_i}{6\pi^2} \int_{m_i}^{\infty} dE\, E^2 \left(E^2 - m_i^2\right)^{1/2} \left(\exp[\beta(E-\mu_i)] \pm \theta_i\right)^{-1}. \tag{4.24}$$

The subscript pt refers to point like hadrons and i denote a single hadron. We consider only nucleons in the hadronic phase. μ_i is the chemical potential for the i^{th} hadron of mass m_i, with spin-isospin degeneracy d_i and $\theta_i = +1$ for fermions and θ_i = -1 for bosons.

Earlier we have calculated the critical T_c as $0.72B^{1/4}$ by considering the both phases as massless ideal gases. When we includes the interaction in the quark-gluon phase and apply Hagedorn's correction [130] to the equation of state of hadrons T_c will change accordingly. Behaviour of T_c was studied by Murugasan *et. al.* [80] by using the equations of state given above. To calculate the T_c they made use of the temperature and chemical dependence of the coupling constant α_s as given in equation (4.3) suggested Kapusta [127] and in a later work by Heins *et. al.* [78]. In zero temperature QCD the scale parameter Λ appearing in α_s is lies between 100 and 400 MeV. [78]. We assume values for Λ between 0 and 400 Mev., and calculated the T_c for different values of B and corresponding values of α_s are computed and are given Table 4.2. Our calculation shows that T_c increase with Λ and also with B. The coupling constant also increases with Λ and T_c. It is clear from the table that for a given value of Λ, T_c increases with B, but α_s decrease with B.

Kapusta [127] obtained the coupling constant using the momentum space subtraction method. In this calculation it was assumed that the temperature dependence of the coupling constant can be taken in to account by naively choosing the normalisation scale to be the energy scale comparable to the temperature of the environment. But this is not generally true as has been pointed out by Nakkagawa and Niegawa [76]. One should treat the dependence of the coupling constant on the renormalisation scale and the temperature of the environment separately, since there is no compelling reason to take the the renormalisation scale to be equal to the environment temperature. Nakkagawa and Niegawa's analysis showed that the coupling constant α_s has a power like dependence on temperature in contrast to the logarithmic dependence as predicted by Kapusta, the form of the coupling constant obtained by Nakkagawa and Niegawa is shown below.

$$\alpha_s = (b\,[ln\,(\mu/\Lambda) - ln\,(\Lambda(\xi)/\Lambda)])^{-1}\,. \tag{4.25}$$

Here μ is the energy scale characterizing the process considered, $b = 29/6$ for two flavours of quarks and $\xi = T/\mu$ where T is the temperature of the surrounding. At large temperature the second term in the parenthesis of the right hand side of the equation (4.25) can be written as,

$$b\ln\left(\frac{\Lambda(\xi)}{\Lambda}\right) = \pi^2 \sum_{i=1}^{N} A_i\ \left(\delta_i\xi^2 + \epsilon_i\,\xi\right). \tag{4.26}$$

With N = 3, for two quark flavours Nakkagawa and Niegawa have obtained,

$$A_1 = 1 \quad A_2 = 1 \quad A_3 = 4/3 \tag{4.27}$$

$$\delta_1 = 0 \quad \delta_2 = 0 \quad \delta_3 = -2 \tag{4.28}$$

$$\epsilon_1 = 13/8 \quad \epsilon_2 = 0 \quad \epsilon_3 = -5/4 \tag{4.29}$$

The variations of α_s with Λ for $\mu = 0.5$, 1, 1.5 GeV. and $\xi = 1$ (T = μ) is given in table 4.1

Table 4.1:
The variations of α_s with Λ for ξ=1, according to Nakkagawa-Niegawa equation

μ (GeV.)	Λ (MeV.)	$b\ln(\mu/\Lambda)$	$b\ \ln(\Lambda(xi)/\Lambda)$	α_s
0.5	50	11.129	5.313	0.172
0.5	100	7.779	5.313	0.406
0.5	150	5.819	5.313	1.976
1	50	14.479	5.313	0.109
1	100	11.129	5.313	0.172
1	150	7.779	5.313	0.406
1	300	5.819	5.313	1.976
1.5	100	13.089	5.313	0.129
1.5	200	11.129	5.313	0.226
1.5	300	7.779	5.313	0.406
1.5	400	5.819	5.313	1.976

It is clear from that α_s increases with Λ, but the rate of increase of α_s here is very large compared to the rate increase as by Kapusta equation for the coupling constant.

The condition for inflation can be derived from the Friedmann equations satisfied in the early Universe. The condition for inflation is

$$\ddot{R} = -\frac{C^2 R}{2}(\rho_{qgp} + 3P_{qgp}) \tag{4.30}$$

The inflation possible when $\ddot{R} \geq 0$, and from the above equation it follows that the required condition is

$$\rho_{qgp} + 3P_{qgp} \leq 0 \tag{4.31}$$

The critical temperature T_i at which the above condition is satisfied can be deter-

Table 4.2: Variation of T_c with Λ & B and corresponding values of α_s & T_i according to Kapusta's equation for α_s

Λ (MeV.)	B (MeV./fm^3	T_c (MeV.)	α_s	T_i (MeV.)
0	60	103	0	30
0	250	148	0	123
0	400	166	0	197
100	60	117	0.42	32
100	250	163	0.35	132
100	400	183	0.33	210
250	60	145	0.78	36
250	250	184	0.61	141
250	400	202	0.56	223

mined by equating the left hand side of the above equation to zero:

$$\rho_{qgp} + 3P_{qgp} = 0. \tag{4.32}$$

Using equation (4.17) and (4.18) with α_s given by equation (4.3) we compute the values of T_i for different choices of Λ and B. Some of these are shown in Table 4.2 It is seen that both T_i and T_c increases with α_s, but the rate of increase of T_i is large compared to that of T_c. With B given a reasonable value (comparably high value if Kapusta's equation for coupling constant is used), T_i exceeds T_c. The consequences of this is that and inflationary stage can start above the coexistence temperature T_c, without supercooling. This possibility was not noted in the work of Fuller *et. al.* [148] and Boyko *et. al.* [72], where supercooling below T_c is considered as an essential condition for mini-inflation to happen. The value of B needed to realise this scenario appears to be quite high in comparison with the phenominologically permissible range of B if Kapusta's equation for the temperature dependence of the coupling constant is used. On the other hand when use the Nakkagawa-Niegawa

Table 4.3: Values of B according to Nakkagawa-Niegawa equation for α_s

Λ (MeV.)	T_c (MeV.)	α_s	B (MeV./fm^3
100	100	0.171	132
100	200	0.171	234
250	100	0.721	130
250	200	0.721	260

equation [76], the condition for mini-inflation without supercooling is found to be satisfied with lower values of α_s and hence with smaller values of B, the bag constant. This is clear from Table 4.3, which shows the variation of B with α_s. It is seen from our calculation that the mini-inflation without supercooling is possible for a value of B $\approx 250 MeV/fm^3$.

A solution describing the time evolution of the scale factor during the mini-inflation can be obtained from the Friedmann equations as

$$\dot{R} - CR\sqrt{\rho} = 0 \tag{4.33}$$

$$\dot{\rho} + 3\frac{\dot{R}}{R}\ (\rho + P)\ = 0 \tag{4.34}$$

where C = $(8\pi^2/3)^{1/2} M_{pl}$. On applying the equations of state of quark-gluon plasma phase to the above equation, it become,

$$\dot{\rho} + 4\,C\,\rho^{3/2} - 4\,CB\,\rho^{1/2}\ = 0. \tag{4.35}$$

This has solution solution

$$\rho\ =\ B\,Cth^2\left(2\,C\,\sqrt{B}\,(t - t_o) + ArCth\left(\sqrt{\rho_c/B}\right)\right). \tag{4.36}$$

This result is in agreement with obtained by Boyko *et. al.* [72]. The Friedmann now gives the scale factor as,

$$R = Sh^{1/2}\left(2\,C\,\sqrt{B}\,(t-t_c) + ArCth\left(\sqrt{\rho_c/B}\right)\right)\,Sh^{1/2}\left(ArCth\sqrt{\rho_c/B}\right) \quad (4.37)$$

where t_c is the zero point of time given as

$$t_c = \left(2\,C\,\sqrt{b}\right)^{-1}\,ArCth\left(\sqrt{\rho_c/B}\right) \quad (4.38)$$

and ρ_c is the integration constant which is equal to the energy density at critical temperature T_c. It is found that $\rho_c \approx 5.516$B for $\alpha_s = 0$ and decreases as α_s increases. Equation (4.37) shows that scale factor is increasing not as a pure exponential function, but approaches an exponential form when $t \geq \left(2\,C\sqrt{B}\right)^{-1}$.

During the expansion of the Universe RT is a constant. Using this we can calculate the increase in the scale factor due to mini-inflation without supercooling. In table 4.4 we give the percentage increase of the scale factor due to mini-inflation without supercooling. In this calculation we have made use of the Kapusta's formula for the coupling constant with a high value of B around 400 Mev/fm^{-3}. But if the Nakkagawa-Niegawa equation is used the required value of B is B $\sim$ 250 MeV/fm^{-3}. Table 4.4 also reveals that the increase of the scale factor due to mini-inflation without supercooling is reduced because of the inclusion of interaction in the quark-gluon plasma.

If there is a supercooling before the phase transition the mini-inflation will continue and the scale factor may increase rapidly. This will depend on the dynamics

Table 4.4: Percentage increase in R due to mini-inflation without supercooling

Λ	T_c	T_i	R_c/R_i	percentage increase in R
0	166	197	1.19	19%
100	183	210	1.15	15%
250	202	223	1.10	10%

of the phase transition and will also be probably influenced by the rapid expansion. Specific conclusions regarding this require a detailed study of the transition and the effect of mini-inflation.

In our analysis we consider hadron phase as an ideal gas. One can take in to account the interaction between the hadronic states also, for example, by including a density dependent interaction between the hadronic states as done by Heins *et. al.* [78]. If this is done the coexistence temperature shows an increase by a small factor which does not affect conclusion much. Another question that can be raised is regarding the effect of the inclusion of strange quarks in the quark-gluon phase. Murugesan *et. al.* [80] have shown that the inclusion of s quarks can lead to a slight decrease in the transition temperature. The effect of this slight decrease will only strengthen our conclusion.

The effect of mini-inflation on inhomogeneity may be a dilution of the density inhomogeneity. If the inflation is sufficient enough to smoothen the density inhomogeneity before nucleosynthesis, the primordial nucleosynthesis will become a homogeneous process. There also a number of other problems as well that the IBBN model faces. Only future works on the experimental side and theoretical side can clarify the exact status of the inhomogeneous nucleosynthesis model.

Appendix

Modified rates of some important reactions

No.	Reaction	Nuclear Reaction Rate ($cm^3 s^{-1} mole^{-1}$
1.	P (n , γ) d	4.472×10^4 $\left(1 - 0.850 T_9^{1/2} + 0.490 T_9 - 0.0962 T_9^{3/2}\right.$ $\left.+ 8.47 \times 10^{-3} T_9^2 - 2.80 \times 10^{-4} T_9^{5/2}\right)$
2.	^{7}Li (n , γ)^{8}Li	3.144×10^3 $+4.26 \times 10^3 T_9^{-3/2} \exp(-2.576/T_9)$
3.	^{7}Be (n , p) ^{7}Li	2.675×10^9 $\times \left(1 - 0.560 T_9^{1/2} + 0.179 T_9 - 0.0283 T_9^{3/2}\right.$ $\left.+2.21 \times 10^{-3} T_9^2 - 6.85 \times 10^{-5} T_9^{5/2}\right)$ $+9.391 \times 10^8 T_{9a}^{3/2} T_9^{-3/2}$ $+4.461 \times 10^7 T_9^{-3/2} \exp(-0.07486/T_9)$ $T_{9a} = [T_9/(1 + 13.08 T_9)]$
4.	^{3}He(n , p)T	$7.21\ 10^8 \left(1 - 0.508 T_9^{1/2} + 0.228 T_9\right)$
5.	^{11}B (n , γ) ^{12}B	7.29×10^2 $+2.40 \times 10^3 T_9^{-3/2} \exp(-0.223/T_9)$
6.	d (p , γ)^{3}He	2.65×10^3 $T_9^{-2/3} \exp(-3.720/T_9^{1/3})$ $\times \left(1 + 0.112 T_9^{1/3} + 1.99 T_9^{2/3}\right.$ $\left.+1.56 T_9 + 0.162 T_9^{4/3} + 0.324 T_9^{5/4}\right)$
7	^{10}Be (n , ^{4}He) ^{7}Li	5.07×10^8
8	^{6}Li (n , T) ^{4}He	$2.54 \times 10^9 + T_9^{-3/2} \exp(-2.39/T_9)$
9	d (d , n) ^{4}He	$3.95 \times 10^8 T_9^{-2/3}$ $\exp(-4.259/T_9^{1/3}) \times \left(1 + 0.098 T_9^{1/3}\right.$ $\left.+ 0.765 T_9^{2/3} + 0.525 T_9 + 9.61 \times 10^{-3} T_9^{4/3} + 0.0167 T_9^{5/3}\right)$
10.	d (d p) T	4.17×10^8 $T_9^{2/3} \exp(-4.258/T_9^{1/3})$ $\times(1 - 0.098 T_9^{1/3} + 0.518 T_9^{2/3} + 0.355 T_9$ $-0.010 T_9^{4/3} - 0.018 T_9^{5/3}$
11.	T (d , n) ^{4}He	1.063×10^{11} $T_9^{-2/3} \exp[-4.559/T_9^{1/3} - (T_9/0.0754)^2]$ $\times(1 - 0.092 T_9^{1/3} - 0.375 v T_9^{2/3} - 0.242 T_9$ $+33.82 T_9^{4/3} + 55.42 T_9^{5/3}) + 8.047 \times 10^8 T_9^{-2/3} \exp(-0.4857/T_9)$
12.	T (α , γ) ^{7}Li	3.032×10^5 $T_9^{-2/3} \exp(-8.090/T_9^{1/3}) \times (1 + 0.0516 T_9^{1/3} +$ $0.0229 T_9^{2/3} +$ $8.828 \times 10^{-3} T_9 - 3.28 \times 10^{-4} T_9^{4/3} - 3.01 \times 10^{-4}\ T_9^{5/3}) + 5.109 \times 10^5$ $T_{9a}^{5/6} T_9^{3/2} \exp(-8.068/T_{9a}^{1/3})$ $T_{9a} = T_9/(1 + 0.1378 T_9)$

13.	^{3}He (d , p) ^{4}He	5.021×10^{10}
		$T_9^{-2/3} \exp[-7.144/T_9^{1/3} - (T_9/0.270)^2]$
		$(1 + 0.058T_9^{1/3} + 0.603T_9^{2/3} + 0.245$
		$T_9 + 6.97T_9^{4/3} + 7.19T_9^{5/3})$
		$5.212 \times 10^8 T_9^{-1/2} \exp(-1.762/T_9)$
14.	^{3}He (α , γ) ^{7}Be	4.817×10^6
		$T_9^{-2/3} \exp(-14.964/T_9^{1/3})$
		$\times(1 + 0.0325T_9^{1/3} - 1.04 \times 10^{-3} T_9^{2/3} - 2.37 \times 10^{-4} T_9$
		$-8.11 \times 10^{-5} T_9^{4/3} - 4.69 \times 10^{-5} T_9^{5/3})$
		$+ 5.938 \times 10^6 T_{9a}^{5/6} T_9^{-3/2} \exp(-12.859/T_{9a}^{1/3})$
		$T_{9a} = T_9/(1 + 0.1071T_9)$
15	^{7}Li (p , α) ^{4}He	$1.096 \times 10^9 T_9^{-2/3} \exp(-8.472/T_9^{1/3})$
		$-4.8330 \times 10^8 T_{9a}^{5/6} T_9^{-3/2} \exp(-8.472/T_{9a}^{1/3})$
		$+1.06 \times 10^{10} T_9^{-3/2} \exp(-30.442/T_{9a}^{1/3})$
		$+ 1.56 \times 10^5 T_9^{-2/3} \exp[-8.472/T_9^{1/3} - (T_9/1.696)^2]$
		$\times(1 + 0.049T_9^{1/3} - 2.50T_9^{2/3} + 0.860T_9 + 3.52T_9^{4/3} + 3.08T_9^{5/3})$
		$1.55 \times 10^6 T_9^{-3/2} \exp(-4.478/T_9)$
		$T_{9a} = [T_9/(1 + 0.759T_9)]$
16	^{3}He (n , γ) ^{4}He	$6.62\ (1 + 0.905T_9)$
17	d (n , γ) T	$6.62 \times 10^1 (1 + 18.9T_9)$
18.	^{3}He (n , γ) ^{4}He	$6.62\ (1 + 905T_9)$
19.	^{7}Be (n , γ) ^{4}He	$2.05 \times 10^4 (1 + 3760T_9)$
20	^{6}Li (n , T) ^{4}He	$1.25 \times 10^8 + 3.65 \times 10^9 T_9^{-3/2} \exp(-2.53/T_9)$
21.	^{10}Be (n , ^{4}He) ^{7}Li	5.08×10^8
22.	^{4}He (d , γ) ^{6}Li	3.01×10^1
		$T_9^{-2/3} \exp(-7.42/T_9^{1/3}$
		$(1 + 0.056T_9^{1/3} - 4.85T_9^{2/3} + 8.85T_9 - 0.585T_9^{4/3} - 0.584T_9^{5/3})$
		$+ 8.55 \times 10^1 / T_9^{3/2} \exp(-8.228/T_9)$
23.	^{4}He (T , γ) ^{7}Li	$1.23 \times 10^6 / T_9^{2/3} \exp(-8.08/T_9^{1/3} \times$
		$(1 + 0.052T_9^{1/3} - 0.715T_9^{2/3} - 0.258T_9$
		$+ 0.272T_9^{4/3} + 0.250T_9^{5/3})$
24.	^{4}He (^{3}He , γ) ^{7}Be	5.79×10^6
		$T_{9a}^{5/6} / T_9^{3/2} \exp(-12.826/T_{9a}^{1/3}$
		$T_{9a} = T_9/(1 + 0.095T_9)$
25.	^{7}Li (^{4}He , γ) ^{11}B	3.55×10^7
		$T_9^{-2/3} \exp(-18.79/T_9^{1/3} - (T_9/1.326)^{1/2})$
		$(1 + 0.022T_9^{1/3} + 1.54T_9^{2/3} + 0.239T_9 + 2.20T_9^{4/3} + 0.869T_9^{5/3})$
		$1.91 \times 10^3 / T_9^{3/2} \exp(-3.484/T_9) + 1.01 \times 10^4 / T_9 \exp(-7.269/T_9)$

26	^{4}He (n , γ) ^{9}Be	$(2.59 \times 10^{-6}$ $/((1+0.344T_9)T_9^2))\exp(-1.062/T_9)$
27.	^{8}Li (p , n) ^{4}He	8.65×10^9 $T_9^{-2/3}\exp(-8.52/T_9^{1/3} - (T_9/2.53)^2) + 2.31$ $\times 10^9 T_9^{-3/2}\exp(-4.64/T_9)$
28.	^{8}Be (n , p) ^{4}He	4.02×10^8
29	^{9}Be (p , d) ^{4}He	2.11×10^{11} $T_9^{-2/3}\exp(-10.359/T_9^{1/3} - (T_9/0.520)^2)$ $(1+0.040T_9^{1/3} + 1.09T_9^{2/3} + 0.307T_9 + 3.21T_9^{4/3} + 2.30T_9^{5/3})$ $+ 5.79 \times 10^8/T_9\exp(-3.046/T_9) + 8.50 \times 10^8/T_9^{3/4}\exp(-5.80/T_9)$
30.	^{8}Li (α , n) ^{11}B	2.05×10^14 $T_{9a}^{5/6}T_9^{-3/2}\exp(-19.461/T_{9a})$ $T_{9a} = T_9/(1+T_9/15.1)$
31.	^{3}H (p , n) ^{3}He	7.07×10^8 $(1 - 0.15T_9^{1/2} + 0.098T_9)\exp(-8.863/T_9)$
32.	^{3}H (γ , n) ^{4}He	$1.67 \times 10^9/T_9^{2/3}$ $\exp(-4.872/T_9^{1/3})$ $(1+0.086T_9^{1/3} - 0.455T_9^{2/3} - 0.272T_9 + 0.148T_9^{4/3} + 0.225T_9^{5/3})$
33.	^{3}He (T , d) ^{4}He	5.46×10^9 $T_{9a}^{5/6}/T_9^{3/2}\exp(-7.733/T_{9a}^{1/3})$ $T_{9a} = T_9/(1+0.128T_9)$
34.	^{6}Li (d , n)^{7}Be	$1.48 \times 10^{12}/T_9^{2/3}$ $\exp(-10.135/T_9^{1/3}$
35.	^{6}Li (d , p)^{7}Li	$1.48 \times 10^{12}/T_9 2/3$ $\exp(-10.135/T_9^{1/3}$
36.	^{7}Li (d , p) ^{8}Li	$8.31 \times 10^8/T_9^{3/2}$ $\exp(-6.998/T_9)$
37.	^{7}Li (T , n) ^{9}Be	$1.4 \times 10^{11}/T_9^{2/3}\exp(-11.333/T_9^{1/3}$
38.	^{8}LI (d , n) ^{9}Be	$3.22 \times 10^{11}/T_9 2/3\exp(-10.357/T_9^{1/3}$
39.	^{7}Li (T , n) ^{4}He	$8.81 \times 10^{11}/T_9^{2/3}/T_9^{2/3}\exp(-11.333/T_9^{1/3}$
40.	^{7}Be (^{3}He , p) ^{4}He	$6.11 \times 10^{13}/T_9^{2/3}\exp(-21.793/T_9^{1/3}$
41.	^{7}Li (^{3}He , p) ^{4}He	$1.11 \times 10^{13}/T_9^{2/3}\exp(-17.989/T_9^{1/3}$
42.	^{7}Be (^{3}H , p) ^{4}He	$2.91 \times 10^{12}/T_9^{2/3}\exp(-13.719/T_9^{1/3}$
43.	^{7}Li (p , γ) ^{4}He	Rate of raction 15. $+ 1.56 \times 10^5/T_9^{2/3}\exp(-8.472/T_9^{1/3} - (T_9/1.696)^2)$ $(1+0.049T_9^{1/3} + 2.498T_9^{2/3} + 0.860T_9 + 3.518T_9^{4/3} + 0.308T_9^{5/3})$ $1.555 \times 10^6/T_9^{3/2}\exp(-10.70/T_9)$

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