S.m.6. VIJAYAKUMAR–A. Studies in the Geometry of the Discrete Plane–1986–Dr. Wazir Hasan Abdi and Dr. T. Thrivikraman.

In this thesis, an attempt is made to study some geometric properties of the discrete plane $H = \{(q^m x_0, q^n y_0) : m,n \in Z$, the set of intigers$\}$ where (x_0,y_0) is a fixed point in the first quadrant of the complex plane, $x_0 = 0$, $y_0 = 0$, and $q \in (0,1)$ is fixed. This discrete plane was first considered by C. Harman (1972) to develop the theory of q-analytic functions. The theory was a consequence of attempt made by Isaacs, Duffin, Abdullaev etc. since 1941, to evolve a discrete analytic function theory analogous to the classical complex analytic function

theory. These theories are free from the classical notion of continuity. Recently, concepts like discrete bianalytic functions, q-monodiffric functions (Velukutty, K.K., 1982) and discrete pseudo-analytic functions (Mercy K. Jacob, 1983) have been introduced and studied in detail. All such theories are function theoretic in nature.

In this thesis, the notion of a metric on H, discrete analogues of some classical geometric concepts, transformations on H, characterization of certain special types of transformations, group theoretic and discrete analytic properties of these transformations, discreate analogue of convexity and related concepts are introduced and investigated. This study, hence will initiate the development of discrete geometry of H.

In chapter 1, the basic principle of discretization, a sketch of the development of discrete analytic function theory, a brief descrption of geometry of a space and also the summary of results established in this thesis are given.

In chapter 2, using the concept of discrete curve given by Harman, the distance between any two points of H is defined. The distance functiond assumes non negative integral values and (H,d) is called the discrete holometric space. The notion of domain in H is defined and a metric characterization of it is obtained. Also, bounds for the diameter of any domain are obtained.

Not considered in this chapter are the discrete analogues of segments and circles which are termed, D-linear sets and *r*-sets respectively. It is proved that the intersection of two D-linear sets is also D-linear, but not the union. A necessary and sufficient condition for a subset of H to be D-linear is obtained. For *r*-sets, formulae for the number of points on it and in its interior are found. After defining the notions of contact set, intersecton, discrete annulus etc. for two *r*-sets, some results are established. Some contrasts with the Euclidean case are brought out. The intersection of discrete Pythagorean type in analogy with the orthogonal intersection of circles are then considered and some properties are obtained.

One of the most important concepts in the development of any geometry is that of a transformation. In chapter 3, bijective mappings of H onto itself called D-transformations are introduced. Special transformations like D-translation and D-isometry are defined and studied. Some results obtained seem to be interesting, to mention one, D-isometries map domains onto domains. D-linear transformations are defined and characterized.

The D-transformations that take *r*-sets onto *r*-sets have also been studied in detail. In this case, the transformations between *r*-sets of equal radii only need be considered in order to maintain the bijective nature of the D-transformation. It is found that these special type of transformations form a finite, non abelian, solvable, nilpotent group. In the last section of this chapter, discrete analyticity properties of these transformations have been investigated. The geometry developed here could be used in the analysis done by earlier authors like Harman. The guidelines are provided in this section.

The notion of convexity outside the framework of linear spaces, has been extensively studied. In the first two sections of chapter 4, we define D-convexity for subsets of H and obtain a sufficient condition for a domain to be not D-convex. Also, concepts like D-kernel and D-convex hull are considered and some characterization theorems are obtained.

In the next section, some results obtained in the course of the investigation, which are interesting, although not directly along the main line of thought in the thesis, are presented. These include, a matrix representation of domains, the

entries of the matrix being the distance between points of it, and the notion of metric content for subsets of H, which is the sum of elements in the upper (lower) triangular part of the distance matrix associated with the subset. The notion of E-set analogous to the ellipse is also considered.