S.m.4. JACOB K. DANIEL–Queueing and Inventory Models with Rest Periods–1985–Dr. A. Krishnamoorthy

In this thesis some queueing and inventory problems with vacation to the server are considered. This is the first work which introduces vacation in an inventory.

In the queueing problems considered stationary solution is obtained using the matrix geometric method. The waiting time distribution is also discussed. The following are the models discussed:

(i) GI/M/1 Queue with rest periods

The interarrival times are i.i.d.r.vs. having a general distribution and service times have eponential distribution. The server goes on vacation either after serving k units without any break in between services or when the system becomes empty, whichever occurs first. The vacation times are i.i.d. exponential variates and are independent of the arrival and service processes. After a vacation if the system is found empty the server remains idle. Observing the system at epochs just prior to arrival points, one can get a Markov chain (two dimensional; one component indicating the servers position (idle or vacation) the other indicating the number of units in the system. If the server is serving then the first component stands for the number in the system and the second indicates the number of units served after the previous vacation. This Markov chain is analysed and its stationary distribution obtained. Also the waiting time distribution is computed. The particular case of an M/M/1 queue with vacation is discussed in detail and an optimal control problem solved.

(ii) $M/M^{a,b}/1$ queue with vacation

Here the arrival process is a Poisson process. Service times are in batches of size taking values a, a+1, ... b and this depends on the number in the system. ie., if more than b units are in the system the first b are taken for service. Otherwise all are served simultaneously. The service times have exponential

distribution with parameter depending on the number of units in the batch undergoing service. If the number waiting for service is less than a, at a service completion point the server goes for vacation having an exponential distribution. On return again if the system size is found to be less than a then he remains idle. Here again a two dimensional Markov chain is identified and its limiting stationary distribution obtained.

(iii) $M/E_k/1$ queueing system with feedback and vacation:

The service times are Erlang of order k. After serving m units consecutively the server goes on vacation of an exponentially distributed duration. The served unit feedback to the system with probability θ, $(0 < \theta < 1)$ immediately after service. A three dimensional Markov chain is identified and its steady state probability determined.

Also considered is two queues in series with an intermediate waiting room having finite capacity. The service times with the first and second servers have exponential distributions. The first server goes on vacation when the intermediate waiting room is full or whenever there is none wating to be served by him. He remains idle on return from vacation if either the intermediate waiting room is full or there is none in the first queue. A four-dimensional Markov chain is identified and its limiting behaviour studied. The condition for the existence of steady state is also derived. Finally some numerical examples are given.

In Inventory three problems are investigated. Their time dependent behaviour alone is looked at. It is difficult (or rather impossible to get the stationary behaviour via the present approach).

The arrival of demands constitute renewal process. The lead times are i.i.d.r.vs. having a general distribution. The policy adopted is of the (s,S) type. The models discussed are:

(i) no queue of demands is permitted when the server is on vacation.

(ii) a queue of demand is permitted when the server takes rest, but only after the replacement of stock.

(iii) Two servers S_1 and S_2 with rest periods alternately to the servers.

The vacation times are i.i.d. random variables having a general distribution. The vacation is not extended and the server remains idle if the replenishment has not taken place on return from vacation. The system size probabilities are obtained.