Faculty of Science

(Mathematics and Statistics)

S.m.1. VELUKKUTTY, K.K.–Some Problems of Discrete Function Theory–1982–Dr. Wazir Hasan Abdi.

This thesis is a study of discrete analytic functions defined on the lattice $\{(q^m x_0, q^n y_0) / m, n \in z\}$ when (x_0, y_0) is a fixed point in C and q is a fixed number in (0,1).

In the discrete function theory, the differential operator of the classical complex analysis is replaced by a suitable difference operator. The theory of discrete functions had its start from R.P. Isaac's (1941) work who introduced two types of difference operators to describe analyticity namely monodiffricity of the first and second kinds. Ferrand (1944), Duffin (1956), Abdullaev are some of the names who developed discrete function theory. The theory developed by these people has been mainly in the Gaussian lattice. q-difference theory was developed by Jackson, Hahn and Abdi and using this theory C. Harman (1972) developed a discrete function theory on the lattice $\{(q^m x_0, q^n y_0)\}$

This thesis starts with the investigation of functions which are both p-and q-analytic in certain domain in the discrete geometric space. The solution is named bianalytic function. The continuation of such a function from two adjacent rays is examined. Then the problem is generalised as investigation of functions having p-and q-residues equal. It is found that such functions satisfy the notion of monodiffricity of second kind in the geometric lattice. Such functions are now named q-monodiffric functions.

Monodiffricity of second kind was not studied earlier in detail. Duffin and Harman had mistaken preholomorphicity as equivalent to this–which has been disproved in this thesis.

In the second chapter of thesis, q-monodiffric differentiation is discussed in detail, q-monodiffric constant which is the general solution of the derivative equation: first derivative is equated to zero, is studied.

In discrete function theory, the concept of construction of an entire discrete analytic function from its discrete analyticity in a known domain, using the difference operator defined to describe the discrete analyticity is important. In this thesis, the construction of bianalytic function and q-monodiffric function is explained. Bifunctions and q-monodiffric constants are well-studied. They stand to replace the concepts of functions and complex numbers respectively of the classical complex function theory. The condition that the usual product of two q-monodiffric functions in a given domain is also a q-monodiffric function there is also obtained and analysed.

Among the three approaches to analytic function theory, the second, namely through the Cauchy integral is considered in the third chapter, whereas the third, namely through power series is dealt with in the fourth chapter. Here two types of integrals are defined either of them will not stand as a counterpart to the classical integral. But both of them taken together represent the theory of integration in q-monodiffric theory and plays the same role of classical integration. Fundamental concepts of integration like Cauchy's integral formula and theorem are developed in the q-monodiffric sense. Meromorphic functions along with pole and polar residue is studied. The relation between these intregrals is also obtained.

The second difficulty in the formulation of the theory is solved by introducing discrete powers in the q-monodiffric sense which leads to the third approach namely representation of discrete analytic functions in the form of an infinite series in terms of discrete powers. Unlike the previous theories, results like nth discrete power of z has exactly n zeros hold in this theory. Some estimates of discrete powers are evolved. Using these estimates convergence of infinite series is discussed. Also a comparison test to decide the convergence of infinite series is found.

The last few sections of the fourth chapter deals with polynomials and zeros of them. Mainly three types of polynomials are studied: polynomials defined over complex numbers, biconstants and q-monodiffric constants. Quadratic-polynomials of each type are exercised in detail with roots of unity in the q-monodiffric sense (ie. the zeros of $z^{m}-1$).

Lastly, some special polynomials are discussed. A theory to classify the discrete polynomials is obtained and some special polynomials are classified in this line. Description of a set of discrete polynomials using generating functions is completely discussed in the final chapter.