Bayesian Analysis of Simple Step-stress Model under Weibull Lifetimes [1]

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Main Sections

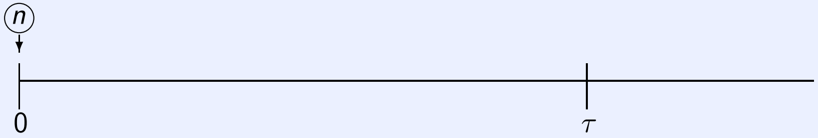
Censoring

Censoring

- Quite useful technique in reliability life testing.
- Possible termination of experiment before failing all the experimental units.
- Lower cost in terms of money and time than full experiment.
- Survival experimental units can be used for further experiments.

Type-I Censoring

- n **:** Number of items put on the test.
- τ **:** Pre-fixed time.
- $\tau^* = \tau$ **:** Experiment termination time.



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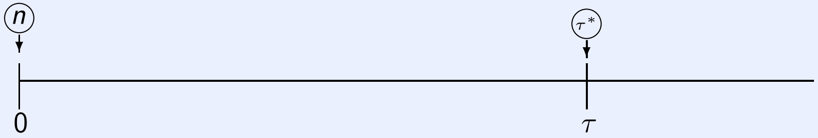

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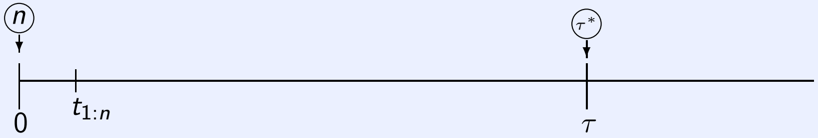

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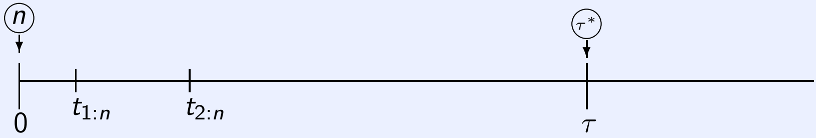

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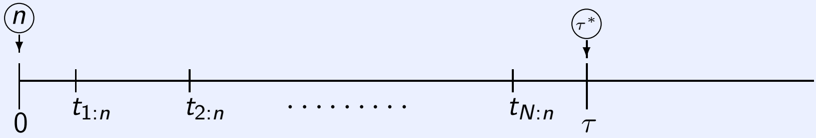

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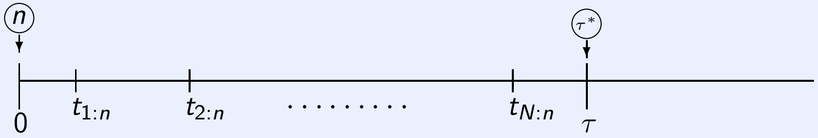

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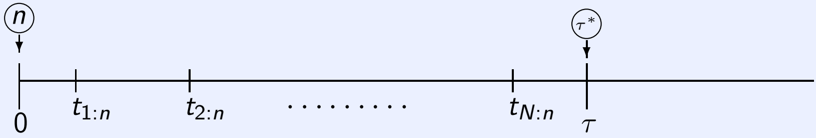
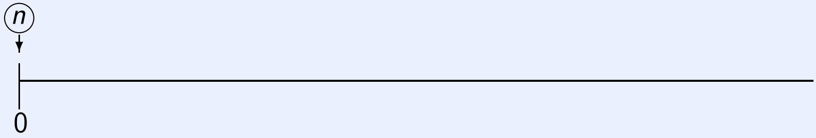


- Number of failures is a random variable.
- Advantage **:** Pre-fixed experiment termination time.
- Disadvantage **:** Very few failures, even no failure, before time τ.

Type-II Censoring

- n **:** Number of items put on the test.
- $r\,(\leq n)$ **:** Pre-fixed integer.
- $\tau^* = t_{r:n}$ **:** Experiment termination time.



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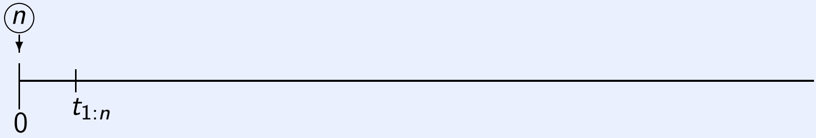

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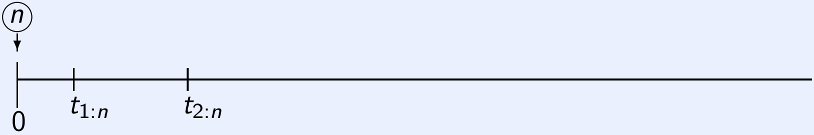

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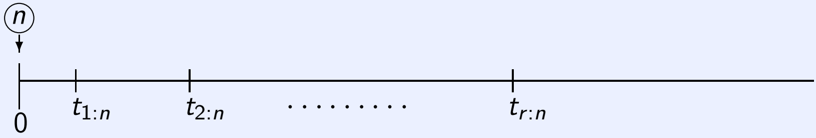

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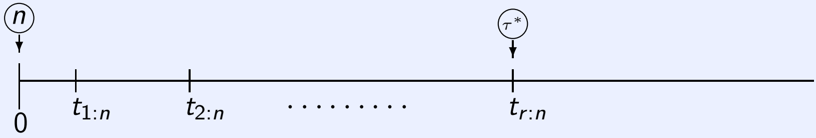

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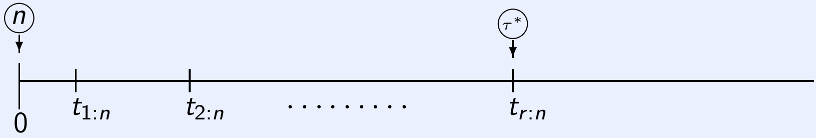

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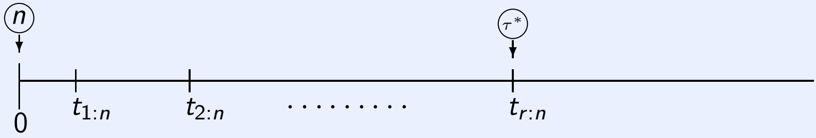

- Duration of experiment is a random variable.
- Advantage **:** Pre-fixed number of failures.
- Disadvantage **:** Long experimental duration.

Other Censoring Schemes

- Hybrid Censoring Schemes: Hybridization of Type-I and Type-II censoring.

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- Progressive Censoring Schemes: Allow to remove items from the test before completion of the experiment.
- Progressive Hybrid Censoring Schemes: Mixture of hybrid and progressive censoring schemes.
- All the censoring schemes suffer form the disadvantage of either Type-I or Type-II censoring scheme.

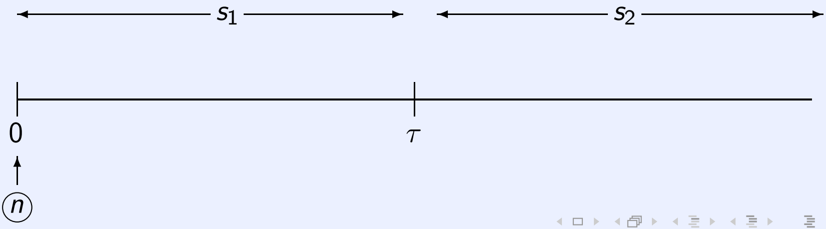
Step-stress Life Tests

Accelerated Life Tests

- Useful experimental technique to obtain data on the lifetime distribution of highly reliable products.
- Put a sample of products on the test in some extreme environmental conditions to get early failures.
- Need to extrapolate to estimate the lifetime distribution under the normal condition.

Step-stress Life Tests

- A particular type of accelerated life test.
- Allows the experimenter to change the stress levels during the life-testing experiments.
- n : Number of items put on the test.
- s_1, s_2 : Stress levels (Simple SSLT).
- τ : Stress changing time (Pre-fixed).



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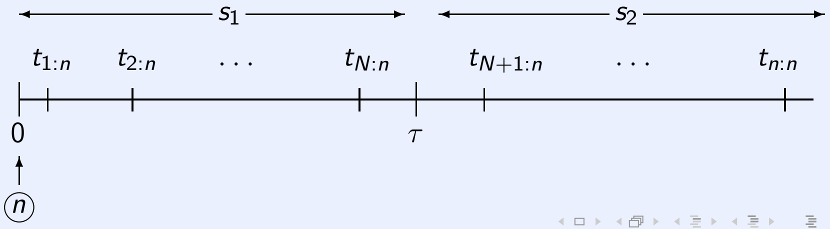
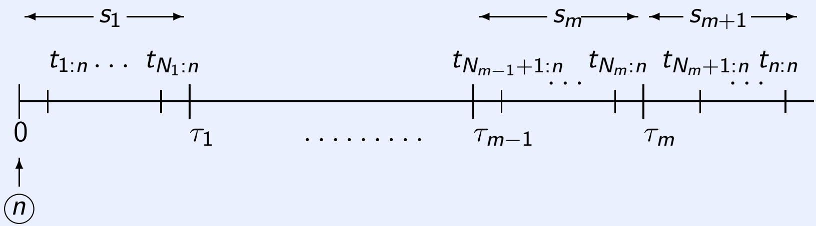


Step-stress Life Tests

- Generalization
  - n : No of items placed on the test.
  - $s_1, s_2, s_3, \ldots, s_{m+1}$: Stress levels.
  - $\tau_1 < \tau_2 < \ldots < \tau_m$: Stress changing times (Pre-fixed).



Models

- Consider a simple SSLT, *i.e.*, only two stress levels, s_1 and s_2, present.
- $F_i(.)$: CDF of lifetime of an item under the stress level s_i, $i = 1, 2 \ldots, m + 1$.
- $F(.)$: CDF of life time of an item under the step-stress pattern.
- Model needed to relate $F(\cdot)$ to $F_i(\cdot)$, $i = 1, 2, \ldots, m + 1$.
- Popular models
    - Cumulative exposure model.
    - Tampered failure rate model.
    - Khamis-Higgins model.

Cumulative Exposure Model

- First proposed by Seydyakin (1966)[4] and later studied by Nelson(1980)[5].
- $F_i(\cdot)$ is the CDF of lifetime of an item under the stress level s_i, $i = 1, 2, \ldots, m + 1$.
- $F(\cdot)$ is the CDF of lifetime of an item under the step-stress pattern.

[4] Seydyakin, N. M. (1966) On one physical principle in reliability theory, *Technical Cybernatics*, 3:80-87.

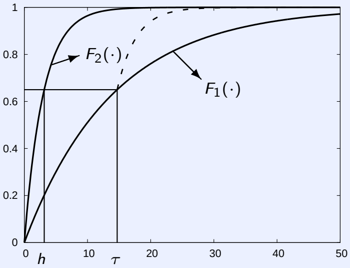
[5] Nelson (1980) Accelerated life testing: step-stress models and data analysis, *IEEE Transactions on Reliability*, 141:288-2838.

Cumulative Exposure Model

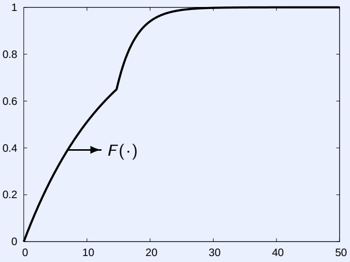
The CEM assumptions are:

- The remaining life of an item depends only on the current cumulative fraction accumulated, regardless how the fraction accumulated.
- If the stress level is fixed, the survivors will fail according to the distribution function of that stress level but starting at previous accumulated fraction failed.

Cumulative Exposure Model



(a) CDF under different stress level



(b) CDF under CEM

Figure: Example of CEM

Here $F_1(\cdot)$ and $F_2(\cdot)$ are CDF of $Exp(14)$ and $Exp(1)$ respectively.

Cumulative Exposure Model

Under the assumptions of CEM, the CDF of the lifetime is given by

$$F_{\text{CEM}}(t) = F_i(t - \tau_{i-1} + h_{i-1}) \quad \text{if } \tau_{i-1} \le t < \tau_i,\ i = 1, 2, \ldots, m+1,$$

where $\tau_0 = 0$, $\tau_{m+1} = \infty$, $h_0 = 0$ and h_i, $i = 1, 2, \ldots, m$, is the solution of

$$F_{i+1}(h_i) = F_i(\tau_i - \tau_{i-1} + h_{i-1}).$$

Tampered Failure Rate Model

- Proposed by Bhattacharyya and Soejoeti (1989)[1] for simple SSLT.
- Generalized by Madi (1993)[2] for multiple step SSLT.

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- Effect of switching the stress level is to multiply the failure rate of the first stress level by a positive constant.

$$\lambda_{\text{TFRM}}(t) = \left(\prod_{j=0}^{i-1} \alpha_j\right) \lambda(t) \text{ if } \tau_{i-1} \leq t < \tau_i,\ i = 1, 2, \ldots, m+1.$$

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[2]Xu, H. and Tang, Y. (2003), Commentary: The Khamis/Higgins model, *IEEE Transactions on Reliability*, 52:4–6.

Khamis-Higgins Model

- Proposed by Khamis and Higgins (1998)[1] for Weibull lifetimes.
- Under KHM, the CDF is given by

$$F_{\text{KHM}}(t) = 1 - e^{-\lambda_i (t^\beta - \tau_{i-1}^\beta) - \sum_{j=1}^{i-1} \lambda_j (\tau_j^\beta - \tau_{j-1}^\beta)} \quad \text{if} \quad \tau_{i-1} \leq t < \tau_i.$$

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- Xu and Tang (2003)[2] showed that KHM is a particular case of TFRM.

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Advantages

- By increasing the stress level, reasonable number of failure can be obtained.
- Experimental time is reduced.

Disadvantages

- Exact relationship between the stress level and lifetime of the product is needed.
- Model must take into account the effect of stress accumulated.
- Model becomes more complicated.

A Brief Literature Review

Literature Review

- Balakrishnan et al. (2007)[1].
  - Simple step-stress life test.
  - Type-II censoring.
  - Exponentially distributed failure times.
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[1] Balakrishnan, N., Kundu, D., Ng, H. K. T., and Kannan, N. Point and interval estimation for a simple step-stress model with Type-II censoring. *Journal of Quality Technology*, 9:35–47, 2007.

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  - c_{jk} involves $(-1)^k$, $\binom{n}{j}$, $\binom{j}{k}$, and $e^{-\frac{\tau}{\theta_1}(n-j+k)}$.
  - $f_{\mathrm{G}}(\cdot)$ is the PDF of Gamma distribution.

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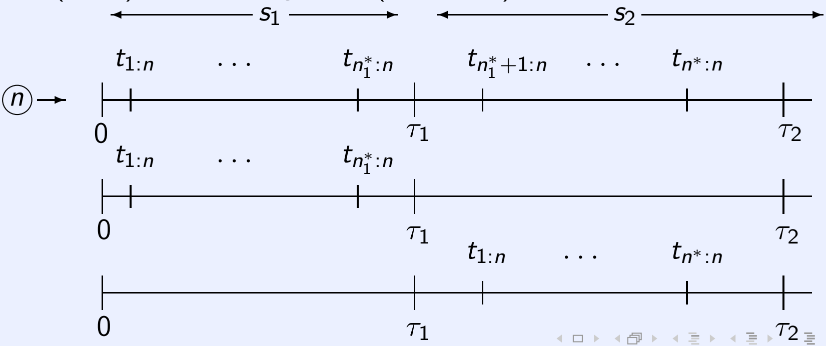
Model Description and Prior Assumptions

Model Description

- n : Number of item put on the test.
- s_1, s_2 : Stress levels.
- τ_1 : Stress changing time (Pre-fixed).
- Type-I censored data.
- $\tau_2 (> \tau_1)$: Censoring time (Pre-fixed).



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- τ_1 : Stress changing time (Pre-fixed).
- Type-I censored data.
- $\tau_2(> \tau_1)$: Censoring time (Pre-fixed).
- Life time at stress level s_i, $i = 1, 2$, has a Weibull(β, λ_i) distribution, *i.e.*, its CDF is given by

$$F_i(t) = \begin{cases} 1 - e^{-\lambda_i t^{\beta}} & \text{if } t > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Model Description

- Under CEM, the CDF is given by

$$F_{\text{CEM}}(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 - e^{-\lambda_1 t^{\beta}} & \text{if } 0 \leq t < \tau_1 \\ 1 - e^{-\lambda_2 \left(t - \tau_1 + \frac{\lambda_1}{\lambda_2}\tau_1\right)^{\beta}} & \text{if } t \geq \tau_1. \end{cases}$$

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- KHM is mathematically tractable than CEM.
- It is difficult to distinguish between CEM and KHM.

Prior Assumptions I

- $\lambda_1 \sim \text{Gamma}(a_1, b_1)$.
- $\lambda_2 \sim \text{Gamma}(a_2, b_2)$.
- $\beta \sim \text{Gamma}(a_3, b_3)$.
- λ_1, λ_2, and β are independently distributed.

Prior Assumptions II

- Main aim of SSLT is to get rapid failure by imposing extreme environmental condition.
- Plausible to assume that the mean life time at stress level s_2 is smaller than that at stress level s_1.
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- Weibull distribution is quite flexible and fits a large range of lifetime data.

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- MLEs of the model parameters do not have explicit form and all inferences rely on asymptotic distributions.

Posterior Analysis

Posterior Distribution under Prior Assumptions I

- For $\beta > 0$, $\lambda_1 > 0$, and $\lambda_2 > 0$

$$l_1(\beta,\, \lambda_1,\, \lambda_2 \,|\, \text{Data}) \propto \beta^{n^*+a_3-1} \lambda_1^{n_1^*+a_1-1} \lambda_2^{n_2^*+a_2-1} \times e^{-(b_3-c_1)\beta - \lambda_1 A_1(\beta) - \lambda_2 A_2(\beta)},$$

$$n^* = n_1^* + n_2^*,\; c_1 = \sum_{j=1}^{n^*} \ln t_{j:n},$$

$$A_1(\beta) = b_1 + \sum_{j=1}^{n_1^*} t_{j:n}^{\beta} + (n - n_1^*)\tau_1^{\beta},$$

$$A_2(\beta) = b_2 + \sum_{j=n_1^*+1}^{n^*} (t_{j:n}^{\beta} - \tau_1^{\beta}) + (n - n^*)(\tau_2^{\beta} - \tau_1^{\beta}).$$

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- $l_1(\beta,\, \lambda_1,\, \lambda_2 \,|\, \text{Data})$ is integrable if proper priors are assumed on all the unknown parameters.

Posterior Distribution under Prior Assumptions II

- For $0 < \alpha < 1$, $\beta > 0$, and $\lambda_2 > 0$

$$l_2(\alpha,\, \beta,\, \lambda_2 \,|\, \text{Data}) \propto \alpha^{n_1^* + a_4 - 1}(1-\alpha)^{b_4 - 1}\beta^{n^* + a_3 - 1}\lambda_2^{n^* + a_2 - 1} \times e^{-(b_3 - c_1)\beta - \lambda_2(\alpha D_1(\beta) + D_2(\beta) + b_2)},$$

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- Bayes estimate of $g(\beta, \lambda_1, \lambda_2)$ cannot be obtained explicitly in general.
- An algorithm based on importance sampling is proposed to compute $\widehat{g}_B(\beta, \lambda_1, \lambda_2)$ and to construct CRI for $g(\beta, \lambda_1, \lambda_2)$ in both the cases.

Bayes Estimate and Credible Interval

$$l_1(\beta,\, \lambda_1,\, \lambda_2 \,|\, \mathrm{Data}) = l_3(\lambda_1,\, |\, \beta,\, \mathrm{Data}) \times l_4(\lambda_2,\, |\, \beta,\, \mathrm{Data}) \times l_5(\beta \,|\, \mathrm{Data}),$$

where

$$l_3(\lambda_1,\, |\, \beta,\, \mathrm{Data}) = \frac{\{A_1(\beta)\}^{n_1^* + a_1}}{\Gamma(n_1^* + a_1)}\, \lambda_1^{n_1^* + a_1 - 1}\, e^{-\lambda_1 A_1(\beta)} \quad \text{if} \quad \lambda_1 > 0,$$

$$l_4(\lambda_2,\, |\, \beta,\, \mathrm{Data}) = \frac{\{A_2(\beta)\}^{n_2^* + a_2}}{\Gamma(n_2^* + a_2)}\, \lambda_2^{n_2^* + a_2 - 1}\, e^{-\lambda_2 A_2(\beta)} \quad \text{if} \quad \lambda_2 > 0,$$

$$l_5(\beta \,|\, \mathrm{Data}) = c_2 \frac{\beta^{n^* + a_3 - 1} e^{-(b_3 - c_1)\beta}}{\{A_1(\beta)\}^{n_1^* + a_1}\{A_2(\beta)\}^{n_2^* + a_2}} \quad \text{if} \quad \beta > 0.$$

Data Analysis

Illustrative Example

- An artificial data is generated from KHM with $n = 40$, $\beta = 2$, $\lambda_1 = 1/1.2 \backsimeq 0.833$, $\lambda_2 = 1/4.5 \backsimeq 2.222$, and $\tau_1 = 0.6$.

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- Prior I : $\widehat{\beta} = 2.35$, $\widehat{\lambda}_1 = 0.93$, $\widehat{\lambda}_2 = 2.61$.
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- Prior I **:** 95% symmetric CRI for β is (1.12, 4.04).
- Prior II **:** 95% symmetric CRI for β is (0.45, 2.44).

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- MSEs of BE of all unknown parameters are smaller in case of Prior II than those in case of Prior I.
- Other loss functions and other censoring schemes can be handled in a very similar fashion.

Future Works

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- Optimality of SSLT under Bayesian framework.
- Prior elicitation is becoming a popular topic among Bayesian. It will be a challenging task to find a subjective prior for step-stress life testing models.

Thank You