## Squeezed Coherent State Representation of Scalar Field and Particle Production in the Early Universe

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The quantum field theory of a scalar field in curved space-time is studied using the squeezed coherent state representation. In this representation the expectation values of the stress-energy tensor of the scalar field is calculated. The present calculation can account for the production of particles in the early universe.

## 1. INTRODUCTION

Particle production is a quantum phenomenon which results from vacuum fluctuations in a strong gravitational field [1-3]. The angular variations in cosmological microwave background radiation observed recently in COBE experiments have been conjectured as a consequence of cosmological perturbations of the vacuum. Progress made in the grand unified theory make us believe that it may be possible to correlate observational data to quantum processes in the early universe. This has caused increasing interest in the study of quantum field theory in curved space-time. Recently Grishchuck and Sidorov [4] introduced the language of squeezed states in the realm of cosmology to explain the cosmological particle creation. Prokopec [5] used the formalism of two-mode squeezed states and the theory of gauge-invarient cosmological perturbations to calculate two-point correlation functions and the entropy of density perturbations. Using the language of squeezed states Hu et al. [6] addressed the dependence of particle creation on the initial state and the relation of spontaneous and stimulated particle creation and their dependence on the initial state.

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The present work is an attempt to explain particle production in the early univese. We argue that nonzero values of the stress-energy tensor evaluated in squeezed vacuum state can be due to particle production and this supports the concept of particle production from zero-point quantum fluctuations. In the present calculation we use the squeezed coherent state introduced by Fan and Xiao [7]. The vacuum expectation values of stressenergy tensor defined prior to any dynamics in the background gravitational field give all information about particle production. Squeezing of the vacuum is achieved by means of the background gravitational field, which plays the role of a parametric amplifier [8]. The present calculation shows that the vacuum expectation value of the energy density and pressure contain terms in addition to the classical zero-point energy terms. The calculation of the particle production probability shows that the probability increases as the squeezing parameter increases, reaches a maximum value, and then decreases.

#### 2. SCALAR FIELD AND STRESS-ENERGY TENSOR

The study of quantum field theory in curved space-time is important, as it is an essential key to understanding the scenario in the early universe. Many authors [9–11] have shown that quantum effects may play a significant role in the history of the early universe. The behavior of the classical scalar field near the initial singularity is best approximated quantum mechanically by constructing a complete set of coherent states for each mode of the scalar field. The quantum state of the scalar field near the initial singularity is inaccessible to an observer at the present time; Hawking [12] proposed that this inaccessible nature can be expressed by taking a random superposition of all allowed states in the inaccessible region. Berger [13] realized this by superposing coherent states in a random manner. Christenson [14] showed that the vacuum expectation values of the stress-energy tensor defined prior to any dynamics in the background gravitational field give all information about particle production. Parker [2] studied the particle creation in an expanding universe, with gravitational metric treated as an unquantized external field.

Here we consider a scalar field and calculate the expectation values of different components of the stress-energy tensor by assuming that the field is in a special type of squeezed coherent state. In this calculation a general form of a background cosmological metric which is spatially homogeneous, possibly anisotropic, and topologically a three-torus is taken (here  $\hbar = c = 1$ ):

$$ds^{2} = -dt^{2} + \sum_{i=1}^{3} a_{i}^{2}(t)(dx^{i})^{2}$$

In this background, a minimally coupled scalar field of mass m satisfies the equation

$$(g^{\mu\nu} \nabla_{\mu} \nabla_{\nu} - m^2) \phi(x) = 0$$

The scalar field is expanded as

$$\phi(x) = (2\pi)^{-3/2} \sum_{k} [q_k(\tau) \cos k x + q_{-k}(\tau) \sin k x]$$

where  $\Sigma_k$  represents a sum over both odd and even discrete modes. The stress-energy tensor for a scalar field is given by

$$T_{\mu\nu} = \partial_{\mu} \phi \ \partial_{\nu} \phi - \frac{1}{2} g_{\mu\nu} (g^{\delta\sigma} \ \partial_{\delta} \phi \ \partial_{\sigma} \phi + m^2 \phi^2)$$

Thus, we can write

$$T_{00} = \frac{1}{2g} \left( \frac{\partial \Phi}{\partial \tau} \right)^2 + g \left( \sum_{i=1}^3 \frac{1}{a_i^2} (\partial_i \Phi)^2 + m^2 \Phi^2 \right)$$
$$T_{ii} = (\partial_i \Phi)^2 + \frac{a_i^2}{2g} \left( \frac{\partial \Phi}{\partial \tau} \right)^2 - \frac{a_i^2}{2} \left( \sum_{j=1}^3 \frac{1}{a_j^2} (\partial_j \Phi)^2 + m^2 \Phi^2 \right)$$

The spatially averaged components are

$$\overline{T}_{00} = \frac{1}{32\pi^3 g} \sum_{k} \left( \left( \frac{\partial q_k}{\partial \tau} \right)^2 + \omega_k^2(\tau) q_k^2 \right)$$
(1)

$$\overline{T}_{ii} = \frac{a_i^2}{32\pi^3 g} \sum_k \left( \left( \frac{\partial q_k}{\partial \tau} \right)^2 + \left( \frac{2k_i^2 g}{a_i^2} - \omega_k^2(\tau) \right) q_k^2 \right)$$
(2)

The scalar field can be quantized mode by mode by defining

$$p_k = \frac{dq_k}{d\tau}$$

and imposing the usual commutation relations. The number operator is defined as

$$N_k = a_k^{\dagger} a_k$$

Defining

$$a_{k} = -i \frac{d\gamma}{d\tau} q_{k} + i\gamma p_{k}$$
$$a_{k}^{\dagger} = i \frac{d\gamma^{*}}{d\tau} q_{k} - i\gamma^{*} p_{k}$$

where  $\gamma(\tau)$  is a solution to the equation

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$$\frac{d^2q_k}{d\tau^2}+\,\omega_k^2(\tau)q_k^2=\,0$$

such that

$$\gamma^* \frac{d\gamma}{d\tau} - \gamma \frac{d\gamma^*}{d\tau} = i$$

From the above equations we get

$$q_k = \gamma^* a_k + \gamma a_k^\dagger \tag{3}$$

$$p_k = \frac{dq_k}{d\tau} = \frac{d\gamma^*}{d\tau} a_k + \frac{d\gamma}{d\tau} a_k^{\dagger}$$
(4)

## 3. SQUEEZED COHERENT STATE REPRESENTATION

Formally, squeezed states are generated from coherent states by appropriate squeezing operators. Fan and Xiao [7] introduced an entirely new approach to calculate the normally ordered form of single- and two-mode squeezed operators by a method called integration within ordered product (IWOP). This newly defined squeezed coherent state is adopted here and the expectation values of the stress-energy tensor of a scalar field are calculated. These expectation values are shown to be split into a classical term and an (infinite) vacuum fluctuation term.

The squeezed coherent state is defined as

$$|Z\rangle_{g} = \exp\left(\frac{-1}{2} |Z|^{2} + (fZ + gZ^{*}) \hat{a}^{\dagger} - fg\hat{a}^{\dagger^{2}}\right)|0\rangle$$
(5)

In this squeezed coherent state Z is the displacement parameter. The eigenvalue equation is

$$(\hat{a} + 2fg\hat{a}^{\dagger})|Z\rangle_g = (fZ + gZ^*)|Z\rangle_g \tag{6}$$

a and  $a^{\dagger}$  are the usual annihilation and creation operators; f and g are complex numbers satisfying the condition

$$|f|^2 + |g|^2 = 1$$

Define

$$\hat{b} = \frac{\hat{a} + 2fg}{\sqrt{1 - 4|fg|^2}} = \hat{a}\cosh\sigma + \hat{a}^{\dagger}e^{i\theta}\sinh\sigma$$
(7)

$$\hat{b} = \mu \hat{a} + \nu \hat{a}^{\dagger}; \qquad \hat{b}^{\dagger} = \mu \hat{a}^{\dagger} + \nu^* \hat{a}$$
 (8)

where

$$\mu = \cosh \sigma = \frac{1}{\sqrt{1 - 4|fg|^2}};$$

$$\nu = (\sinh \sigma) e^{i\theta} = \frac{2fg}{\sqrt{1 - 4|fg|^2}};$$

$$e^{i\theta} = \frac{fg}{|fg|}$$
(9)

The operators  $\hat{a}$  and  $\hat{b}$  are connected by a unitary transformation,

$$\hat{b} = UaU^{-1};$$
  $\hat{b}^{\dagger} = Ua^{\dagger}U^{-1}$ 

Coherent states are defined to be eigenstates of the formal annihilation operator,

$$\hat{a}|Z,g\rangle = \kappa(g|Z,g\rangle$$

Here

$$\kappa(g) = \frac{fZ + gZ^*}{\sqrt{1 - 4|fg|^2}}$$

Now, to calculate the expectation values we can work with

$${}_{g}\langle Z|f(a, a^{\dagger})|Z\rangle_{g} = \langle Z, g|U^{-1}f(a, a^{\dagger})U|Z, g\rangle$$
(10)

Let us define

$$\hat{b} = \mu \hat{a} + \nu \hat{a}^{\dagger};$$
  $\hat{b}^{\dagger} = \mu \hat{a}^{\dagger} + \nu \hat{a}^{\dagger}$   
 $\hat{b} = U a U^{-1};$   $\hat{b}^{\dagger} = U a^{\dagger} U^{-1}$ 

and

$$U^{-1}aU = \mu \hat{a} - \nu \hat{a}^{\dagger}$$

Using the above equations, we can obtain the following relations:

$$_{g}\langle Z|\hat{a}|Z\rangle_{g} = \mu\kappa - \nu\kappa^{*} \tag{11}$$

$$_{g}\langle Z|\hat{a}^{\dagger}|Z\rangle_{g} = \mu\kappa - \nu^{*}\kappa \tag{12}$$

$${}_{g}\langle Z|\hat{a}^{2}|Z\rangle_{g} = \mu^{2}\kappa^{2} - \mu\nu(1+2|k|^{2}) + \nu^{2}\kappa^{*2}$$
(13)

$${}_{g}\langle Z|\hat{a}^{\dagger 2}|Z\rangle_{g} = \mu^{2}\kappa^{*2} - \mu\nu^{*}(1+2|k|^{2}) + \nu^{*2}\kappa^{2}$$
(14)

$${}_{g}\langle Z|\hat{a}\hat{a}^{\dagger}|Z\rangle_{g} = \mu^{2}(1+|\kappa|^{2}) - \mu\nu^{*}\kappa^{2} - \mu\nu\kappa^{*2} + |\nu|^{2}|\kappa|^{2}$$
(15)

$${}_{g}\langle Z|N|Z\rangle_{g} = {}_{g}\langle Z|\hat{a}^{\dagger}\hat{a}|Z\rangle_{g}$$

$$(1.5)$$

$$= \mu^{2} |\kappa|^{2} - \mu \nu^{*} \kappa^{2} - \mu \nu \kappa^{*2} + |\nu|^{2} (1 + |\kappa|^{2})$$
(16)

Expectation values of q,  $q^2$ , p, and  $p^2$  in the squeezed coherent state are

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$${}_{g}\langle Z|q|Z\rangle_{g} = \mu \cdot 2 \operatorname{Re}(\gamma \kappa^{*}) - 2 \operatorname{Re}(\gamma \nu^{*} \kappa)$$
(17)

$$_{g}\langle Z|p|Z\rangle_{g} = \mu \cdot 2 \operatorname{Re}\left(\frac{d\gamma}{d\tau}\kappa^{*}\right) - 2 \operatorname{Re}\left(\frac{d\gamma}{d\tau}\nu^{*}\kappa\right)$$
 (18)

$${}_{g}\langle Z|q^{2}|Z\rangle_{g} = \mu^{2} \cdot 2 \operatorname{Re}(\gamma^{2}\kappa^{*2}) - 2 \operatorname{Re}(\gamma^{2}\nu^{*2}\mu)(1+2|\kappa|^{2}) + 2 \operatorname{Re}(\gamma^{2}\nu^{2}\kappa^{2}) + |\gamma|^{2}(\mu^{2}+|\nu|^{2}-4\mu \operatorname{Re}(\nu^{*}\kappa^{2}) + 2\mu^{2}|\kappa|^{2} + 2 |\nu|^{2}|\kappa|^{2})$$
(19)

$${}_{g}\langle Z|p^{2}|Z\rangle_{g} = \mu^{2} 2 \operatorname{Re}\left(\left(\frac{d\gamma}{d\tau}\right)^{2} \kappa^{*2}\right) - 2 \operatorname{Re}\left(\left(\frac{d\gamma}{d\tau}\right)^{2} \nu^{*2} \mu\right)(1+2|\kappa|^{2}) + 2 \operatorname{Re}\left(\left(\frac{d\gamma}{d\tau}\right)^{2} \nu^{2} \kappa^{2}\right) + \left|\frac{d\gamma}{d\tau}\right|^{2} (\mu^{2} + |\nu|^{2} - 4\mu \operatorname{Re}(\nu^{*} \kappa^{2}) + 2\mu^{2}|\kappa|^{2} + 2|\nu|^{2}|\kappa|^{2})$$
(20)

$${}_{g}\langle Z|(\Delta q)^{2}|Z\rangle_{g} = {}_{g}\langle Z|q^{2}|Z\rangle_{g} - {}_{g}\langle Z|q|Z\rangle_{g}^{2}$$
$$= |\gamma|^{2}[|\nu|^{2} + \mu^{2}] - \mu[\gamma^{*2}\nu + \gamma^{2}\nu^{*}]$$
(21)

$${}_{g}\langle Z|(\Delta p)^{2}|Z\rangle_{g} = {}_{g}\langle Z|p^{2}|Z\rangle_{g} - {}_{g}\langle Z|p|Z\rangle_{g}^{2}$$
$$= \left|\frac{d\gamma}{d\tau}\right|^{2}[|\nu|^{2} + \mu^{2}] - \mu \left[\left(\frac{d\gamma^{*}}{d\tau}\right)^{2}\nu + \left(\frac{d\gamma}{d\tau}\right)^{2}\nu^{*}\right]$$
(22)

# 4. STRESS-ENERGY TENSOR EXPECTATION VALUES IN SQUEEZED COHERENT STATE

The stress-energy tensor expectation values in the squeezed coherent state can be calculated using Eqs. (1), (2), (19), and (20); we obtain

$$32\pi^{3} g_{g} \langle Z | \overline{T}_{00} | Z \rangle_{g}$$

$$= \sum_{k} \left\{ 2 \operatorname{Re} \left[ (\mu^{2} \kappa^{*2} - \mu \nu^{*} (1 + 2 |k|^{2}) + \nu^{*2} \kappa^{2}) \left( \left( \frac{d\gamma}{d\tau} \right)^{2} + \omega_{k}^{2} \gamma^{2} \right) \right] + \left( \left| \frac{d\gamma}{d\tau} \right|^{2} + \omega_{k}^{2} |\gamma|^{2} \right) [1 + 2(|\nu|^{2} + \mu^{2} (1 + |\kappa|^{2}) - \mu \nu^{*} \kappa^{2} - \mu \nu \kappa^{*2} + |\nu|^{2} |\kappa|^{2})] \right\}$$

$$(23)$$

The expectation value of  $\overline{T}_{00}$  is seen to split into two terms:

$$32\pi^3 g_g \langle Z | \overline{T}_{00} | Z \rangle_g = \rho = \rho_0 + \rho_{sq}$$
(24)

where  $\rho_0$  is the classical term:

$$\rho_0 = \frac{1}{32\pi^3 g} \sum_k \left( \left| \frac{d\gamma}{d\tau} \right|^2 + \omega_k^2 |\gamma|^2 \right)$$
(25)

and  $\rho_{sq}$  is due to squeezing:

$$\rho_{sq} = \frac{1}{16\pi^{3}g} \sum_{k} \left\{ \operatorname{Re} \left[ (\mu^{2}\kappa^{*2} - \mu\nu^{*}(1+2|k|^{2}) + \nu^{*2}\kappa^{2}) \left( \left( \frac{d\gamma}{d\tau} \right)^{2} + \omega_{k}^{2}\gamma^{2} \right) \right] + \left( \left| \frac{d\gamma}{d\tau} \right|^{2} + \omega_{k}^{2} |\gamma|^{2} \right) [2(|\nu|^{2} + \mu^{2}(1+|\kappa|^{2}) - \mu\nu^{*}\kappa^{2} - \mu\nu\kappa^{*2} + |\nu|^{2} |\kappa|^{2})] \right\}$$

$$(26)$$

 $\rho_0$  is the classical energy density and  $\rho_{\it sq}$  is the contribution to energy density due to squeezing. Similarly, we find

$$32\pi^{3}g_{g}\langle Z|\overline{T}_{ii}|Z\rangle_{g}$$

$$=\sum_{k}\left\{2\operatorname{Re}\left[(\mu^{2}\kappa^{*2} - \mu\nu^{*}(1+2|k|^{2}) + \nu^{*2}\kappa^{2})\left(\left(\frac{d\gamma}{d\tau}\right)^{2} + \left(\frac{2k_{i}^{2}g}{a_{i}^{2}} - \omega_{k}^{2}\right)\gamma^{2}\right)\right] + \left(\left|\frac{d\gamma}{d\tau}\right|^{2} + \omega_{k}^{2}|\gamma|^{2}\right)[1+2(|\nu|^{2} + \mu^{2}(1+|\kappa|^{2}) - \mu\nu^{*}\kappa^{2} - \mu\nu\kappa^{*2} + |\nu|^{2}|\kappa|^{2})]\right\}$$

$$(27)$$

$${}_{g}\langle Z|\overline{T}_{ii}|Z\rangle_{g} = P = P_{i0} + P_{sq}$$
<sup>(28)</sup>

where  $P_{i0}$  is the anisotropic pressure

$$P_{i0} = \frac{1}{32\pi^3 g} \sum_{k} \left( \left| \frac{d\gamma}{d\tau} \right|^2 + \left( \frac{2k_i^2 g}{a_i^2} - \omega_k^2 \right) |\gamma|^2 \right)$$
(29)

and  $P_{sq}$  is the contribution due to squeezing

$$P_{sq} = \frac{1}{16\pi^3 g} \sum_{k} \left\{ \operatorname{Re} \left[ (\mu^2 \kappa^{*2} - \mu \nu^* (1+2|k|^2) + \nu^{*2} \kappa^2) \left( \left( \frac{d\gamma}{d\tau} \right)^2 + \left( \frac{2k_i^2 g}{a_i^2} - \omega_k^2 \right) \gamma^2 \right) \right] + \left( \left| \frac{d\gamma}{d\tau} \right|^2 + \left( \frac{2k_i^2 g}{a_i^2} - \omega_k^2 \right) |\gamma|^2 \right)$$

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$$\times \left[2(|\nu|^2 + \mu^2(1 + |\kappa|^2) - \mu\nu^*\kappa^2 - \mu\nu\kappa^{*2} + |\nu|^2|\kappa|^2)\right]\right\}$$
(30)

## 5. SQUEEZED VACUUM STATE EXPECTATION VALUES

We are now interested in squeezed vacuum state expectation values. Using Eqs. (17)–(20) and setting Z = 0, we get

$$_{g}\langle 0|q|0\rangle_{g} = 0 \tag{31}$$

$${}_{g}\langle 0|p|0\rangle_{g} = 0 \tag{32}$$

$${}_{g}\langle 0|q^{2}|0\rangle_{g} = |\gamma|^{2}(\mu^{2}|\nu|^{2}) - \gamma^{*2}\mu\nu - \gamma^{2}\mu\nu^{*2}$$
(33)

$${}_{g}\langle 0|p^{2}|0\rangle_{g} = \left|\frac{d\gamma}{d\tau}\right|^{2}(\mu^{2}|\nu|^{2}) - \left(\frac{d\gamma^{*}}{d\tau}\right)^{2}\mu\nu - \left(\frac{d\gamma}{d\tau}\right)^{2}\mu\nu^{*2}$$
(34)

$${}_{g}\langle 0|N|0\rangle_{g} = |\nu|^{2} \tag{35}$$

Equation (35) shows that squeezing results in  $|\nu|^2$  particles on the average. Expectation values of the stress-energy tensor can be found in the squeezed vacuum. Using Eqs. (23), (27), (33), and (34) we find

$$32\pi^{3}g_{g}\langle 0|\overline{T}_{00}|0\rangle_{g}$$

$$=\sum_{k}\left\{(\mu^{2}+|\nu|^{2})\left(\left|\frac{d\gamma}{d\tau}\right|^{2}+\omega_{k}^{2}|\gamma|^{2}\right)\right.$$

$$\left.-\mu\nu\left(\left(\frac{d\gamma^{*}}{d\tau}\right)^{2}+\omega_{k}^{2}\gamma^{2}\right)\right)-\mu\nu^{*}\left(\left(\frac{d\gamma}{d\tau}\right)^{2}+\omega_{k}^{2}\gamma^{2}\right)\right)\right\}$$

$$32\pi^{3}g_{g}\langle 0|\overline{T}_{ii}|0\rangle_{g}$$

$$=\sum_{k}\left\{(\mu^{2}+|\nu|^{2})\left(\left|\frac{d\gamma}{d\tau}\right|^{2}+\left(\frac{2k_{i}^{2}g}{a^{2}}-\omega_{k}^{2}\right)|\gamma|^{2}\right)\right\}$$
(36)

$$-\mu\nu\left(\left(\frac{d\gamma^{*}}{d\tau}\right)^{2} + \left(\frac{2k_{i}^{2}g}{a_{i}^{2}} - \omega_{k}^{2}\right)\gamma^{*2}\right) + \mu\nu^{*}\left(\left(\frac{d\gamma}{d\tau}\right)^{2} + \left(\frac{2k_{i}^{2}g}{a_{i}^{2}} - \omega_{k}^{2}\right)\gamma^{2}\right)\right)$$
(37)

## 6. PARTICLE PRODUCTION IN THE SQUEEZED VACUUM

In cosmological space-times the inequivalance of vacua appears at different times of evolution. If we consider initially a vacuum state  $|0\rangle$  at  $t_0$ , such

that  $a_k|0\rangle = 0$ , at a later time  $t_1$  a new vacuum state  $|0\rangle\rangle$  can be defined such that  $b_j|0\rangle\rangle = 0$ . Here the annihilation operators  $a_k$  (at  $t_0$ ) and  $b_j$  (at  $t_1$ ) are not equal, but they are related by a set of Bogoliubov transformations

$$b_j(t_1) = \sum_k (\alpha_{jk}(t)a_k + \beta_{jk}^*(t)a_k^{\dagger})$$

If the Bogoliubov coefficients  $\alpha_{jk}$  and  $\beta_{jk}$  are nonzero, there is a nonzero probability of particle creation as the field evolves from the in-region to the out-region.

Parker [2] has shown that the probability of observing particles at time *t* is

$$P_r = \frac{|\beta|^2}{1+|\beta|^2} = \frac{|\beta|^2}{|\alpha|^2}$$
(38)

where  $\alpha$  and  $\beta$  are Bogoliubov transformation coefficients. In our calculation these coefficients are identified as

$$|\alpha|^2 = |\mu|^2$$

and

$$|\beta|^2 = |\nu|^2$$

and they satisfy the condition

$$|\alpha|^2 - |\beta|^2 = 1$$

Therefore the probability of observing particles in the squeezed vacuum is given by

$$P_r = \frac{|\nu|^2}{|\mu|^2}$$
(39)

Using Eq. (9), we can write

$$P_r = 4|f|^2|g|^2 \tag{40}$$

A plot of  $P_r$  versus squeezing parameters (f, g) is shown in Fig. 1. From this we can infer that the particle production probability becomes unity when  $|f| = |g| = 1/\sqrt{2}$ .

The probability of observing  $n_k$  pairs at time *t* in the state  $|0\rangle$  where one particle is in the mode *k* and other in the mode -k for some set of occupied modes  $\{k\}$  is

$$|_{t}\{2_{n_{k}}\}|0\rangle|^{2} = \prod_{k} \left[\left\{\frac{|\beta|}{\alpha}\right\}^{2}\right]^{n_{k}} \frac{1}{|\alpha|^{2}}$$

$$(41)$$

Therefore the probability of observing the  $n_k$  pairs in the squeezed vacuum is

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$$|_{t}\{2_{n_{k}}\}|0\rangle_{sq}|^{2} = \prod_{k} \left[ \left( \left| \frac{\nu}{\mu} \right|^{2} \right)^{n_{k}} \frac{1}{|\mu|^{2}} \right] = \prod_{k} \left[ \left( \left| \frac{\sinh\sigma}{\cosh\sigma} \right|^{2} \right)^{n_{k}} \frac{1}{|\cosh\sigma|^{2}} \right]$$
(42)

## 7. CONCLUSIONS

We have constructed a single-mode squeezed coherent state representation of the scalar field which is valid near a singularity. The newly introduced squeezed coherent state is much simpler than the squeezed states defined by standard means. The squeezing parameters are not independent, so different degrees of squeezing are possible by adjusting the complex parameters (g, f).

The problem of particle creation near a cosmological initial singularity was reexamined by means of the squeezed coherent state. The particle produc-

tion probability was found to be fully dependent upon the squeezing parameters. A plot of  $P_r$  versus the squeezing parameters was drawn, and we found that the particle creation probability becomes unity when  $|f| = |g| = 1/\sqrt{2}$ .

The squeezed vacuum gives rise to fluctuations of the energy density and the anisotropic pressure. The particles are produced from the excitation of vacuum fluctuations (parametric amplification) by the changing background gravitational field. The squeeze parameter  $|\nu|^2$  measures the number of particles created.

Thus we can conclude that cosmological particle creation amounts to squeezing the vacuum by the background gravitational field.

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