S.m.9. Ramachandran Nair, V.K.–Queueing Models with rest to the server after serving a random number of units–1988–Dr. A. Krishnamurthy

Queues with rest to the server were analyzed by several authors. The doctoral thesis of Miller (1964) is concerned with an M/G/1 queue with rest to the server. The doctoral thesis of Jacob K. Daniel (1985) also treats some queueing models with vacation to the server; instead of exhaustive service, he assumes that the server takes rest either after serving k consecutive units or whenever the system becomes empty, whichever occurs first.

In this thesis Miller's and Daniel's results are generalised by assuming that the server goes on vacation after serving a random number of units. In all the models discussed in this thesis the rest times of servers are assumed to be i.i.d exponential variates which are futher independent of the arrival and service processes. The number of units served between consecutive rests takes values from 1 to m (finite and fixed). The probability of taking vacation after serving the k^{th} unit is p_k, $k = 1,2,\ldots\ldots\ldots$ m with $p_0 = 0$ and, of course $p_m = 1$. In most of the cases the condition for existence of steady state is explicitly obtained. Wherever it is impossible to find, sufficient conditions are given for the existence of steady state. For the first time in the literature, queue with vacation is introduced for a multiserver queue.

Chapter II concentrates on the simplest type of queueing model - the arrival is according to a homogeneous Poisson Process, service times are i.i.d exponential variates. In addition to the usual assumption it is assumed that each unit instantaneously feedsback with probability θ ($0 < \theta < 1$) after being served. The stability condition for the system is obtained, steady state probability distribution of the system is computed, and the waiting time distribution of a unit in the system (here defined as the total time spent in the system until served satisfactorily) by considering a bivariate Markov process, is derived. The situation where the server goes for rest either after serving k consecutive units or whenever the system becomes empty, whichever occurs first, is also discussed.

In Chapter III, a queueing problem in which inter arrival times of customers are i.i.d random variables following some arbitrary distribution is considered. Service times of units are i.i.d exponential variates and are independent of the arrival process. Using the embedding techniques, a bivariate Markov process is constructed. The associated transition probability matrix is obtained. Using this the steady state behaviour of the system and the necessary and sufficient condition for its existence are obtained. The waiting time distribution of a unit in queue is given. An optimization problem associated with the model is discussed.

In the IV chapter the assumption of exponentially distributed service times is dropped. Instead, service times are taken as i.i.d gamma variates of order k. The arrival process forms a homogeneous Poisson Process. Here again a bivariate Markov process is constructed and its steady state probability distribution studied. The condition for the stability of the process is derived. The waiting time distribution of a unit in the queue in the steady state is also given.

In chapter V a bulk service queueing model is considered, in which the arrival process is a homogeneous Poisson process. The size of each batch taken for service lies between c and d. The server goes for rest after serving a random number of batches (atleast one and atmost m(fixed)). A bivariate Markov process to analyse the problem is studied. A sufficient condition for the stability of the process is given. Also the waiting time distribution of a unit in the queue in the steady state is derived. Finally some results are given for the above model with interarrival times being i.i.d. random variables having arbitrary distribution.

The VI chapter is about a queueing problem with S servers. The arrival to the system is according to a homogeneous Poisson process. Service times are i.i.d. exponential variates. A unit is served by a single server. Initially all servers are available. After serving a unit if there is no unit waiting in the queue the server goes for rest of random duration. No server goes for rest without serving atleast one unit. The rest times of servers are i.i.d. exponential variates. If on return after rest, the server finds no unit waiting in the quaue he remains idle. Again this system is studied by considering a bivariate Markov process. The steady state distribution of the system is indicated and waiting time of a unit in the queue is derived.

In the VII chapter a buffer model is considered. The system consists of two servers in series and a finite intermediate waiting room between them. The arrival is according to a Poisson process and the first server remains idle either when the first waiting room is empty or when the second waiting room is full. The second server goes for rest of exponentially distributed duration after serving a random number of units (atleast one and atmost m, where without loss of generality we may also assume m to be the capacity of the second waiting room). The steady state behaviour of this model is investigated and waiting time distribution is found out.